Lista 3

1)a)

x_i	0	1.5	3.0	4.5	6.0
$f(x_i)$	0.001	0.016	0.028	0.046	0.057

(column indices above: 0, 1, 2, 3, 4)

$$P_M(x) = \sum_{i=0}^{4} l_i \cdot f(x_i) = L_0 \cdot f(x_0) + L_1 \cdot f(x_1) + l_2 \cdot f(x_2) + l_3 f(x_3) + l_4 f(x_4)$$

$$L_0 = \frac{(x-x_1)\cdot(x-x_2)\cdot(x-x_3)\cdot(x-x_4)}{(x_0-x_1)\cdot(x_0-x_2)\cdot(x_0-x_3)\cdot(x_0-x_4)} = \frac{(x-1.5)\cdot(x-3)\cdot(x-4.5)\cdot(x-6)}{121.5}$$

$$L_1 = \frac{(x-x_0)\cdot(x-x_2)\cdot(x-x_3)\cdot(x-x_4)}{(x_1-x_0)\cdot(x_1-x_2)\cdot(x_1-x_3)\cdot(x_1-x_4)} = \frac{(x)\cdot(x-3)\cdot(x-4.5)\cdot(x-6)}{-30.375}$$

$$L_2 = \frac{(x-x_0)\cdot(x-x_1)\cdot(x-x_3)\cdot(x-x_4)}{(x_2-x_0)\cdot(x_2-x_1)\cdot(x_2-x_3)\cdot(x_2-x_4)} = \frac{(x)\cdot(x-1.5)\cdot(x-4.5)\cdot(x-6)}{20.25}$$

$$L_3 = \frac{(x-x_0)\cdot(x-x_1)\cdot(x-x_2)\cdot(x-x_4)}{(x_3-x_0)\cdot(x_3-x_1)\cdot(x_3-x_2)\cdot(x_3-x_4)} = \frac{(x)\cdot(x-1.5)\cdot(x-3)\cdot(x-6)}{-30.375}$$

$$L_4 = \frac{(x-x_0)\cdot(x-x_1)\cdot(x-x_2)\cdot(x-x_3)}{(x_4-x_0)\cdot(x_4-x_1)\cdot(x_4-x_2)\cdot(x_4-x_3)} = \frac{(x)\cdot(x-1.5)\cdot(x-3)\cdot(x-4.5)}{121.5}$$

$$P(x) = 0.001 \cdot \frac{(x-1.5)\cdot(x-3)\cdot(x-4.5)\cdot(x-6)}{121.5} + 0.016 \cdot$$

$$\frac{(x)\cdot(x-3)\cdot(x-4.5)\cdot(x-6)}{-30.375} + 0.028 \cdot \frac{(x)\cdot(x-1.5)\cdot(x-4.5)\cdot(x-6)}{20.25}$$

$$+ 0.046 \cdot \frac{(x)\cdot(x-1.5)\cdot(x-3)\cdot(x-6)}{-30.375} + 0.057 \cdot \frac{(x)\cdot(x-1.5)\cdot(x-3)(x-4.5)}{121.5}$$

b) $P(1.1) = \dfrac{0.001 \cdot (1.1-1.5) \cdot (1.1-3) \cdot (1.1-4.5) \cdot (1.1-6)}{121.5} +$

$\dfrac{0.016 \cdot (1.1-0) \cdot (1.1-3) \cdot (1.1-4.5) \cdot (1.1-6)}{-30.375} +$

$\dfrac{0.028 \cdot (1.1-0) \cdot (1.1-1.5) \cdot (1.1-4.5) \cdot (1.1-6)}{20.25} +$

$\dfrac{0.046 \cdot (1.1-0) \cdot (1.1-1.5) \cdot (1.1-3) \cdot (1.1-6)}{-30.375} +$

$\dfrac{0.057 \cdot (1.1-0) \cdot (1.1-1.5) \cdot (1.1-3) \cdot (1.1-4.5)}{121.5} =$

$0.0001 + 0.018 + (-0.01) + 0.006 + (-0.001) = \boxed{0.0131}$

$P(2.2) = \dfrac{0.001 \cdot (2.2-1.5) \cdot (2.2-3) \cdot (2.2-4.5) \cdot (2.2-6)}{121.5} +$

$\dfrac{0.016 \cdot (2.2-0) \cdot (2.2-3) \cdot (2.2-4.5) \cdot (2.2-6)}{-30.375} +$

$\dfrac{0.028 \cdot (2.2-0) \cdot (2.2-1.5) \cdot (2.2-4.5) \cdot (2.2-6)}{20.25} +$

$\dfrac{0.046 \cdot (2.2-0) \cdot (2.2-1.5) \cdot (2.2-3) \cdot (2.2-6)}{-30.375} +$

$\dfrac{0.057 \cdot (2.2-0) \cdot (2.2-1.5) \cdot (2.2-3) \cdot (2.2-4.5)}{121.5} =$

$-0.60004 + 0.008 + 0.019 + (-0.007) + 0.001 = \boxed{0.02}$

$P(3.4) = \dfrac{0.001 \cdot (3.4-1.5) \cdot (3.4-3) \cdot (3.4-4.5) \cdot (3.4-6)}{121.5} +$

$\dfrac{0.016 \cdot (3.4-0) \cdot (3.4-3) \cdot (3.4-4.5) \cdot (3.4-6)}{-30.375} +$

$\longrightarrow$

$$0.028 \cdot \frac{(3.4-0) \cdot (3.4-1.5) \cdot (3.4-4.5) \cdot (3.4-6)}{20.25} +$$

$$0.046 \cdot \frac{(3.4-0) \cdot (3.4-1.5) \cdot (3.4-3) \cdot (3.4-6)}{-30.375} +$$

$$0.057 \cdot \frac{(3.4-0) \cdot (3.4-1.5) \cdot (3.4-3) \cdot (3.4-6)}{121.5} =$$

$$0.000018 + (-0.002) + 0.026 + 0.01 + (-0.001) = \boxed{0.033} \text{ //}$$

$$p(4.9) = 0.001 \cdot \frac{(4.9-1.5) \cdot (4.9-3) \cdot (4.9-4.5) \cdot (4.9-6)}{121.5} +$$

$$0.016 \cdot \frac{(4.9-0) \cdot (4.9-3) \cdot (4.9-4.5) \cdot (4.9-6)}{-30.375} +$$

$$0.028 \cdot \frac{(4.9-0) \cdot (4.9-1.5) \cdot (4.9-4.5) \cdot (4.9-6)}{20.25} +$$

$$0.046 \cdot \frac{(4.9-0) \cdot (4.9-1.5) \cdot (4.9-3) \cdot (4.9-6)}{-30.375} +$$

$$0.057 \cdot \frac{(4.9-0) \cdot (4.9-1.5) \cdot (4.9-3) \cdot (4.9-4.5)}{121.5} =$$

$$-0.000023 + 0.002 + (-0.01) + 0.052 + 0.005 = \boxed{0.049} \text{ //}$$

2)a)

x_i	2	3	4
$f(x_i)$	0.6931	1.0986	1.3853

(columns labeled 0, 1, 2 above)

$$P_M(x) = \sum_{i=0}^{2} L_i \cdot f(x_i) = L_0 \cdot f(x_0) + L_1 \cdot f(x_1) + L_2 \cdot f(x_2)$$

$$L_0 = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-3)(x-4)}{(2-3)(2-4)} = \frac{(x-3)(x-4)}{2}$$

$$L_1 = \frac{(x-x_0)(x-x_2)}{(x_1-x_2)(x_1-x_2)} = \frac{(x-2)(x-4)}{(3-2)(3-4)} = \frac{(x-2)(x-4)}{-1}$$

$$L_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-2)(x-3)}{(4-2)(4-3)} = \frac{(x-2)(x-3)}{2}$$

$$P(x) = 0.6931 \cdot \frac{(x-3)(x-4)}{2} + 1.0986 \cdot \frac{(x-2)(x-4)}{-1} + 1.3863 \cdot \frac{(x-2)(x-3)}{2}$$

b) $P(2.3) = 0.6931 \cdot \frac{(2.3-3)(2.3-4)}{2} + 1.0986 \cdot \frac{(2.3-2)(2.3-4)}{-1} +$

$1.3863 \cdot \frac{(2.3-2)(2.3-3)}{2} = 0.4124 + 0.5603 + (-0.1456) =$

$$P(2.3) = \boxed{0.8271} \, \text{\textbackslash}$$

Passo 1: $f^{(m+1)}(x) = f^{(3)}(x) = \ln(x)$

$|\ln(x)| \le M$ para todo $x \in (2,4)$

$x = 4 \rightarrow |\ln(4)| = 1.3863$

$M \ge 1.3863$

Passo 2: Calculo do erro

$$|E| \le \prod_{k=0}^{m=2} |x-x_k| \frac{M}{(m+1)!}$$

$$|E| \le |(2.3-2)(2.3-3)(2.3-4)| \cdot \frac{1.3863}{(2+1)!} =$$

$$|(0.3)(-0.7)(-1.7)| \cdot \frac{1.3863}{3!} = 0.357 \cdot \frac{1.3865}{6} =$$

$$0.357 \cdot 0.2311 = \boxed{0.0825}$$

O valor $p(2.3) = 0.8271$ é uma aproximação para
$\ln(2.3)$ com erro ≤ 0.0825.

$P(3.3) = 0.6931 \cdot \dfrac{(3.3-3)(3.3-4)}{2} + 1.0986 \cdot \dfrac{(3.3-2)(3.3-4)}{-1} +$

$1.3863 \cdot \dfrac{(3.3-2)(3.3-3)}{2} = -0.0728 + 0.9997 + 0.2703 =$

$P(3.3) = \boxed{1.1972}$ //

passo 1: $f^{(3)}(x) = \ln(x)$ $M \geq 1.3863$

passo 2:

$|E| \leq |(3.3-2)(3.3-3)(3.5-4)| \cdot 0.2311 =$

$|(1.3)(0.3)(-0.7)| \cdot 0.2311 =$

$0.273 \cdot 0.2311 =$

$|E| \leq \boxed{0.063}$

O valor $P(3.3) = 1.1972$ é uma aproximação para $\ln(3.3)$ com erro ≤ 0.063.