__/__/__

Lista 2

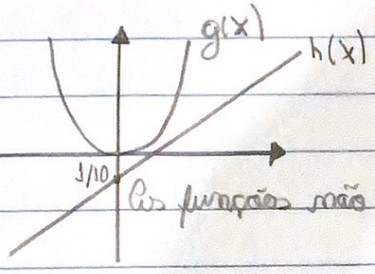
1) a) $[(x-0.3)^2+0.01]+[1/(x-0.9)^2+0.04]-6$

$x^2-0.6x+0.1=0$

$x^2=0.6x-0.1$

$g(x)=x^2$

$h(x)=0.6x-0.1$



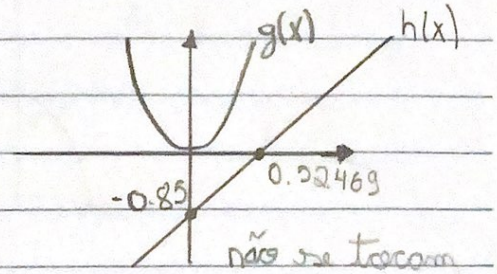
As funções não se tocam

$x^2-1.62x+0.85=0$

$x^2=+1.62x-0.85$

$g(x)=x^2$

$h(x)=1.62x-0.85$



não se tocam

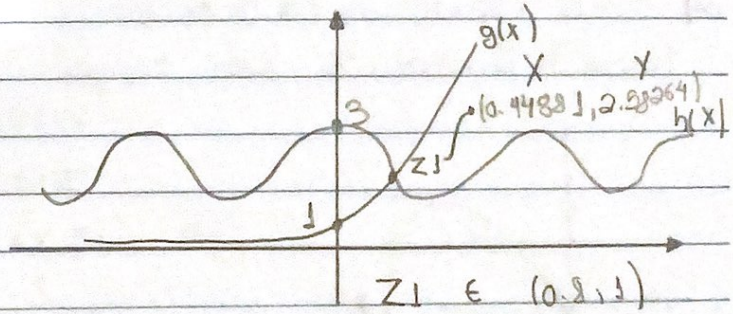
b) $f(x)=e^x-\cos(x)-2$

$e^x-\cos(x)-2=0$

$e^x=\cos(x)+2$

$g(x)=e^x$

$h(x)=\cos(x)+2$



$Z1 \in (0.8, 1)$

c) $f(x)=2x^3+\ln(x)-5$

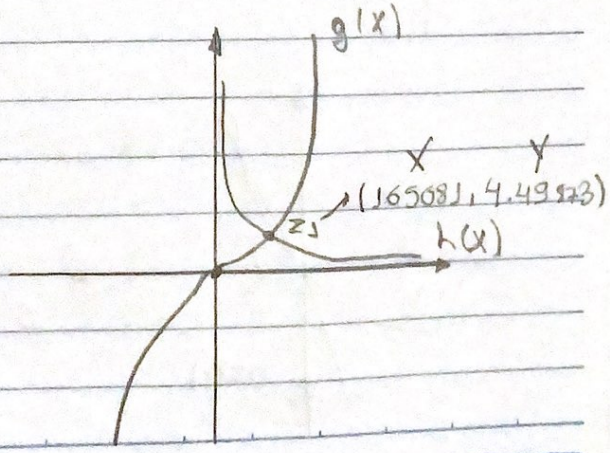
$2x^3+\ln(x)-5=0$

$2x^3=-\ln(x)+5$

$g(x)=2x^3$

$\ln(x)=-\ln(x)+5$

$Z1 \in (1,2)$



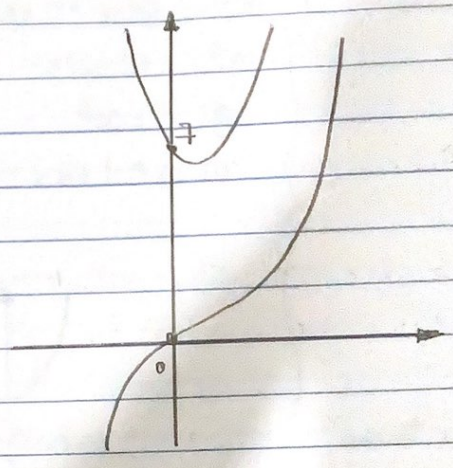
__/__/__

d) $f(x) = x^5 - 6.7x^4 + 8.4x^3 - 10.8x^2 + 8x - 6.8$

$x^5 - 6.7x^4 + 8.4x^3 - 10.8x^2 + 8x - 6.8 = 0$

$x^5 = 6.7x^4 - 8.4x^3 + 10.8x^2 - 8x + 6.8$

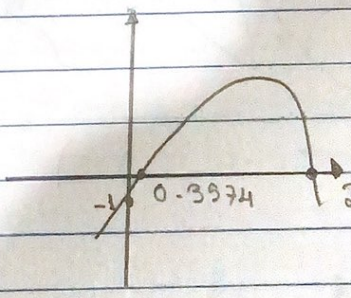
$g(x)$ $\qquad$ $h(x)$



não se tocam

2) $F(x) = 4x - e^x \quad \epsilon \leq 0.0060$

Passo 1: Isolamento das raízes



$I_1 = [0,1]$

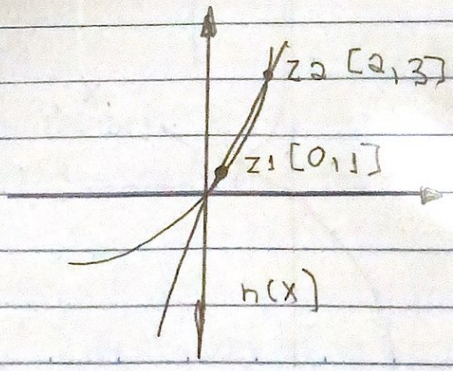
$I_2 = [2,3]$

$4x - e^x = 0$

$e^x = +4x$

$g(x) = e^x$

$h(x) = +4x$



Defi de intervalo inicial

$Z_1 = -0.2039$

$Z_1 \in [0,1]$

$Z \in [2,3]$

Passo 2: Número de iterações / particões (K)

$$K \geq \frac{\ln(b-a) - \ln(\varepsilon)}{\ln(2)}$$

$$K \geq \frac{\ln(1-0) - \ln(0.0060)}{\ln(2)}$$

$$K \geq \frac{0 + 5.1160}{0.6931}$$

$$K \geq 7.9552 \Rightarrow \boxed{K \geq 8}$$

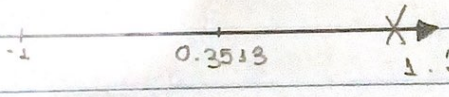
Passo 3: fazer o processo iterativo [até $K \geq 8$]

$K = 1 \quad c = (a+b)/2 = (0+1)/2 = \boxed{+0.5}$

$K = 1 \quad f(c) = f(0.5) = 4(0.5) - e^{0.5} = 0.3513$

$f(a) = f(0) = 4.0 - e^{0} = -1$

$f(b) = f(1) = 4.1 - e^{1} = 1.2817$

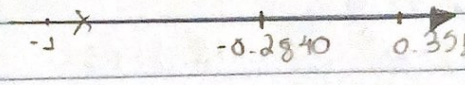


$K = 2 \quad c = (0 + 0.5)/2 = +0.25$

$f(c) = f(+0.25) = 4(+0.25) - e^{+0.25} = -0.2840$

$F(a) = 4(0) - e^{0} = -1$

$f(b) = F(0.5) = 4(0.5) - e^{0.5} = 0.3513$

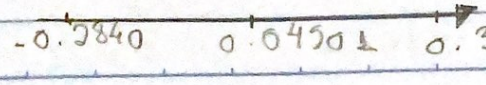


$K = 3 \quad c = (0.25 + 0.5)/2 = 0.375$

$f(c) = F(0.375) = 4(0.375) - e^{0.375} = 0.04501$

$f(a) = F(0.25) = 4(0,25) - e^{0.25} = -0.2840$

$f(b) = f(0.5) = 4(0.5) - e^{0.5} = 0.3513$



/ / /

k=4 $c = (0.25 + 0.375)/2 = 0.3125$

$F(c) = F(0.3125) = 4(0.3125) - e^{0.3125} = -0.1168$

$F(a) = F(0.25) = 4(0.25) - e^{0.25} = 0.2840$

$F(b) = F(0.375) = 4(0.375) - e^{0.375} = 0.0450$

-0.2840 -0.1168 0.0450 [c, b]

k=5 $c = (0.3125 + 0.375)/2 = 0.3438$

$F(c) = F(0.3438) = 4(0.3438) - e^{0.3438} = -0.0351$

$F(a) = F(0.3125) = 4(0.3125) - e^{0.3125} = -0.1168$

$F(b) = F(0.375) = 4(0.375) - e^{0.375} = 0.0450$

-0.1168 -0.0351 0.0450 [c, b]

K=6 $c = (0.3438 + 0.375)/2 = 0.3594$

$F(c) = F(0.3594) = 4(0.3594) - e^{0.3594} = 0.0051$

$F(a) = F(0.3438) = 4(0.3438) - e^{0.3438} = -0.0351$

$F(b) = F(0.375) = 4(0.375) - e^{0.375} = 0.0450$

-0.0351 0.0051 0.0450 [a, c]

k=7 $c = (0.3438 + 0.3594)/2 = 0.3516$

$F(c) = F(0.3516) = 4(0.3516) - e^{0.3516} = -0.0149$

$F(a) = F(0.3438) = -0.0351$

$F(b) = F(0.3594) = 0.0051$

-0.0351 -0.0149 0.0051 [c, b]

K=8 $c = (0.3516 + 0.3594)/2 = 0.3555$

$f(c) = f(0.3555) = 4(0.3555) - e^{0.3555} = -0.0049$

$F(a) = F(0.3516) = -0.0149$

$f(b) = P(0.3594) = 0.0051$

i) $(f|xK|) < 3$

$(4|0.3555) - e^{0.3555}) = 0.0049$ $\quad 0.0049 < 0.00060$

ii) $|bK - aK|/2 < \varepsilon$

$(0.3555 - 0.3516)/2 = 0.00195$ $\quad 0.00195 < 0.0060$

no intervalo $[0,1) \geq \hat{a}$ 0.3555

agora ≥ 2 E $[2,3]$

passo <

$$K \geq \frac{\ln(b-a) - \ln(\varepsilon)}{\ln(2)}$$

$$K \geq \frac{\ln(1) - \ln(\varepsilon)}{\ln(2)}$$

$\boxed{K \geq 8}$

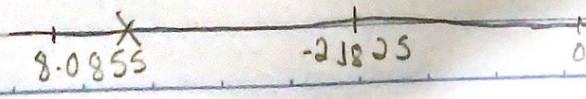
K=1 $c = (2+3)/2 = 2.5$

$f(c) = f(2.5) = 4(2.25) - e^{2.5} = -2.1825$

$f(a) = f(2) = 4(2) - e^{2} = 0.6109$

$f(b) = f(3) = 4(3) - e^{3} = -8.0855$

$[c,b]$



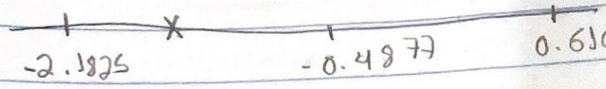
K=2 $c=(2.5+2)/2 = 2.25$

$f(c) = f(2.25) = 4(2.25) - e^{2.25} = -0.4877$

$f(a) = f(2.5) = -2.1825$

$f(b) = f(2) = 0.6109$

$[c,b)$



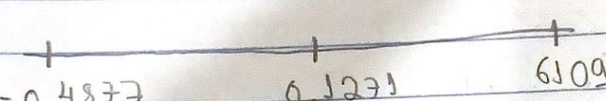
K=3 $c=(2.25+2)/2 = 2.125$

$f(c) = f(2.125) = 4(2.125) - e^{2.125} = 0.1271$

$f(a) = f(2.25) = -0.4877$

$f(b) = f(2) = 0.6109$

$[c,a)$



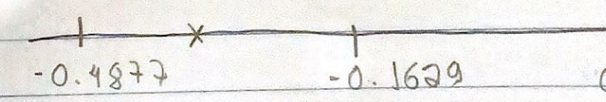
K=4 $c=(2.1254\ 2.25)/2 = 2.1875$

$f(c) = f(2.1875) = 4(2.1875) - e^{2.1875} = -0.1629$

$f(a) = f(2.125) = 0.1271$

$f(b) = f(2.25) = 0.4877$

$[a,c]$



K=5 $c=(2.125+2.1875)/2 = 2.1563$

$f(c) = f(2.1563) = 4(2.1563) - e^{2.1563} = -0.0139$

$f(a) = f(2.125) = 0.1271$

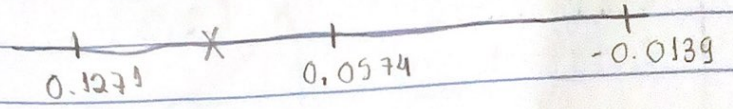
$f(b) = f(2.1875) = -0.1629$

K=6 $c = (2.125 + 2.1563)/2 = 2.1407$

$f(c) = f(2.1407) = 4(2.1407) - e^{2.1407} = 0.0574$

$f(a) = f(2.125) = 0.1271$

$f(b) = f(2.1563) = -0.0139$

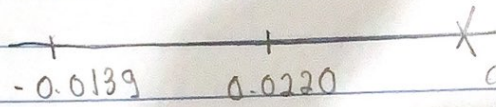


K=7 $c = (2.1407 + 2.1563)/2 = 2.1485$

$f(c) = f(2.1485) = 4.(2.1485) - e^{2.1485} = 0.0220$

$f(a) = f(2.1407) = 0.0574$

$f(b) = f(2.1563) = -0.0139$

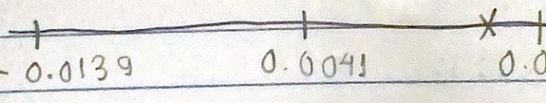


K=8 $c = (2.1485 + 2.1563)/2 = 2.1524$

$f(c) = f(2.1564) = 4(2.1564) - e^{2.1564} = 0.0041$

$f(a) = f(2.1485) = 0.0220$

$f(b) = f(2.1563) = -0.0139$



i) $|f(xk)| < \varepsilon$ $\quad 0.0041 < 0.0060$

ii) $|bK - aK| < 3$ $\quad |2.1524 - 2.1485|/2 = 0.0020$

$0.0020 < 0.0060$

3) $f(x) = x^2 - x - 2.5$ $\quad \varepsilon \leq 0.00050$

P1

↳ $x^2 - x - 2.5 = 0$

$x = \sqrt{x + 2.5}$

$g(x) \quad h(x)$

$[2,3]$

P2

↳ $x = \sqrt{x+2.5}$

$L(x) = \sqrt{x+2.5}$

$L'(x) = \frac{1}{\sqrt{x \cdot 2.5}}$

$|L'(x)| < 1$

$[2,3]$ ✓

P3

↳ $x_0 = (a+b)/2 = 2+3/2 = 2.5$

$x_1 = L(x_0) = L(2.5) = \sqrt{2.5+2.5} = 2.23607$

$x_2 = \sqrt{2.23607+2.5} = 2.17625$

$x_3 = \sqrt{2.17625+2.5} = 2.16246$

$x_4 = \sqrt{2.16240+2.5} = 2.15927$

$x_5 = \sqrt{2.15927++2.5} = 2.15853$

$x_6 = \sqrt{2.15853+2.5} = 2.15836$

$x_7 = \sqrt{2.15836+2.5} = 2.15832$

$x_8 = \sqrt{2.15832+2.5} = 2.15831$

P4

↳ $(x_k - x_{k-1})/2 < \varepsilon$

$(x_8 - x_{71})/2 < \varepsilon$

$(12.15831 - 2.158321)/2 = 0.000009\,9/2 =$

$0.000004 < 0.00050$

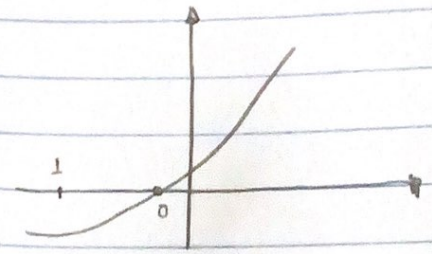
i) $|(x_k - x_{k-1})|/2 < \varepsilon$ OK

ii) $|f(x_k)| < 3 \quad f(2.15831) \leq 0.000007 < 3$

$x \approx 2.15831$ OK

4) $f(x) = e^x + 0.5x - 0.5 \quad \varepsilon \leq 0.000055$

Passo 1: isolamento da raiz $[-1, 0]$



Passo 2: derivador

$f'(x) = e^x + 0.5$

Passo 3: Processo iterativo

$X_0 = (a + b)/2 = (-1 + 0)/2 = -0.5$

$X_{K+1} = X_K - \frac{f(x)}{f'(x)}$

$K = 1 \quad X_0 = -0.5$

$X_1 = -0.5 - \left(\frac{(e^{-0.5} + 0.5(-0.5) - 0.5)}{e^{-0.5} + 0.5)} \right)$

$= -0.5 - \left(\frac{0.143469}{1.106531} \right) = -0.370343$

$X_2 = X_1 - \left(\frac{e^{X_n} + 0.5(X_1) - 0.5)}{e^{X_1} + 0.5)} \right)$

$= X_1 - \left(\frac{0.005326}{1.19497} \right) = -0.374217$

$$X_3 = X_2 - \left(\frac{(e^{x_2} + 0.5(x_2) - 0.5)}{e^{x_2} + 0.5} \right)$$

$$= X_2 - \left(\frac{0.000007}{1.187415} \right) = -0.374823$$

i) $|(x_K - x_{K-1})| / 2 < 3$

$|(-0.374823 + 0.374817)| / 2 = 0.000003$

$0.000003 < 0.000055$ ✓

ii) $|f(x_k)| < 3 \rightarrow 0.000007 < 0.000055$

✓