Determining the wavelength of light emitted by an LED via interference fringes' spacial frequency using Lloyd's mirror setup

Gabriel Bristot, January 2025

Abstract

We first verified the qualitative characteristics of the Lloyd's mirror setup. Then we measured the wavelength (λ) of the light emitted from an LED. Our chosen method was to use a moiré pattern created by the interference between a single slit reflecting off a mirror and a raster. This technique does not require an initial slit to achieve coherent light in contrast to the classic Young's arrangement, therefore, its setup is simpler. The final value calculated for the wavelength was $(530 \pm 9)nm$. This matches the measured value through a spectrometer of $(528.5 \pm 0.1)nm$ to one standard error.

1 Introduction

1.1 Intention

Interference phenomena play a critical role in many optical systems. Theoretical work in this area has many applications in different fields including: interferometry, metrology, and Interference-based microscopy. So it is clear that this experiment intends to replicate results that are in accordance with the standard theoretical model and explore ways in which the experiment diverged from the expected results. This report is laid out as follows: subsection 1.2 explores the historical development of light theories, section 2 lays the theory needed to grasp the experiment, section 3 describes the setup and method used, section 4 covers the results and compares them to other measurements, and section 5 outlines the most important results of this report and how this experiment could be improved.

1.2 Historic background

Interest in the nature of light and its properties has been manifested by many scholars throughout history. Eventually, culminating in the development and maturity of the corpuscular theory of light that was expanded and popularized by Sir Isaac Newton [1]. Although the wave model of light was developed by contemporaries of Newton, such as Christiaan Huygens [2], it was not until the early 19th century that the concept gained widespread recognition, largely due to Thomas Young's groundbreaking work on interference phenomena [3].

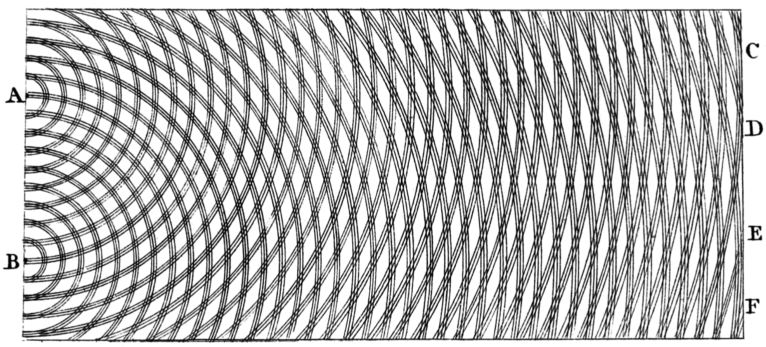


Figure 1: Young's sketch of two-source interference of water waves, 1845. Figure from [4].

One of the most decisive predictions of the wave theory of light was the fact that all waves, including light, can create interference patterns in certain special conditions (Figure 1). Young's famous double-slit experiment demonstrated that light has these key properties, such as the ability to diffract and interfere with other light. And it is amidst these new discoveries that Humphrey Lloyd performed his experiment [5] which provided further confirmation of the phenomena of interference of light.

2 Theory

The object of this section is to find an formula for the separation of successive fringes in the plane of observation.

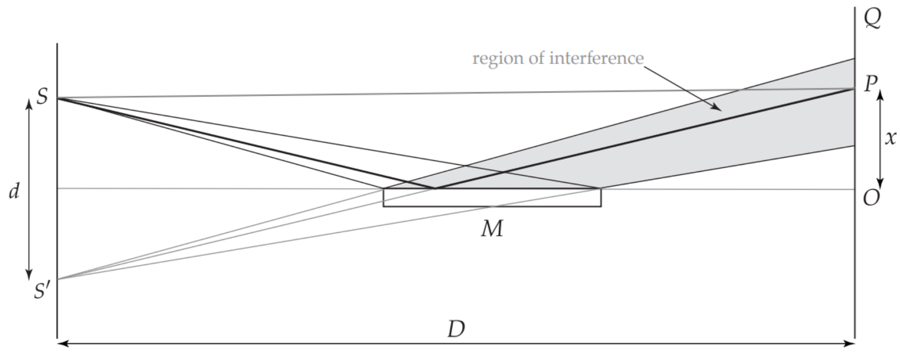


Figure 2: Sketch of the light rays in the experiment. Figure from [6].

Taking into account the fact that reflection of the light in the mirror M introduces an additional phase change of π radians. The overall phase change difference the waves from S to S' (Figure 2) is:

$$\phi = \frac{2\pi}{\lambda}(S'P - SP) + \pi \tag{1}$$

Where ϕ is the phase difference and λ is the wavelength. And provided that $x \pm \frac{d}{2} \ll D$. Where x, d, and D are shown in Figure 2:

$$\phi = \frac{2\pi}{\lambda}\frac{xd}{D} + \pi \tag{2}$$

Therefore, the n^{th} bright fringe happens at:

$$\phi = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d} \tag{3}$$

Hence, the fringe separation Δx is:

$$\Delta x = \frac{\lambda D}{d} \tag{4}$$

And the spatial frequency f (the number of fringes per unit length) is found using:

$$f = \frac{1}{\Delta x} = \frac{d}{\lambda D} \tag{5}$$

And finally by noticing that $d = 2d_{measured}$ due to the mirror and adding some offset of d_0 to $d_{measured}$ due to zero error in readings, we obtain a formula for $d_{measured}$ in terms of f, D, and λ:

$$d_{measured} = \frac{f\lambda D}{2} + \frac{d_0}{2} \tag{6}$$

Further detail in the derivation of these results can be found at [6].

3 Setup and method

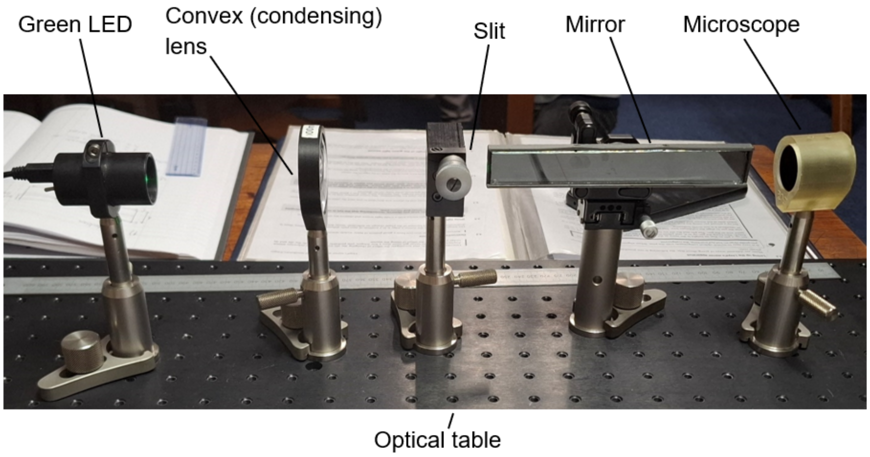
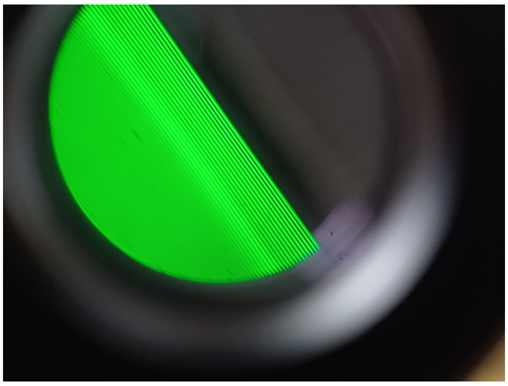
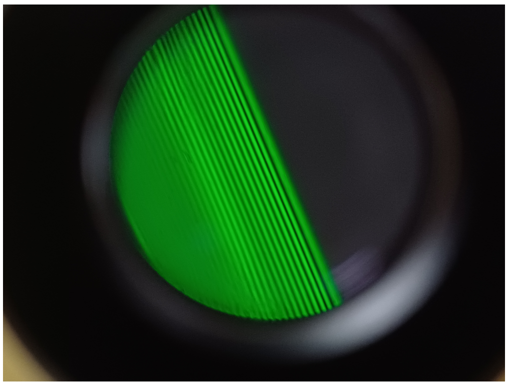


Figure 3: Photograph of the experimental setup used.

As seen in figure 3, the setup we used consists of the green LED shinning upon a convex lens that focuses the light into the slit, that reflects off the mirror and is detected using the microscope. A number of precautions were taken to ensure that the equipment used was aligned and properly focused. An optical pin was used to check for a consistent height of the components above the optical table. The LED, convex lens, and slit were initially aligned in a straight line along the holes in the optical table, serving as a visual guide, and the lens was moved along the optical axis until it formed a sharp image of the LED onto the slit. Then the mirror was placed a few millimetres to the right of the slit and it's position was changed using one of its knobs until 2 slits could be easily seen. Finally, the position and angle of the microscope were adjusted until the fringe pattern became straight and aligned with the centre of the eyepiece.

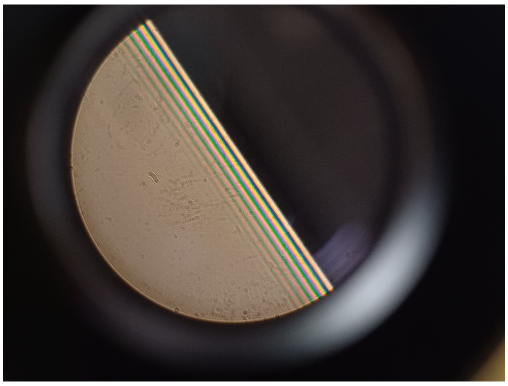


(a) Bigger d



(b) Smaller d

(c) Bigger slit width



(d) White light

Figure 4: Combination of all fringe photographs.

We then performed qualitative observations of the effects that changing d or the slit width had on the fringes. Firstly, we decreased the slit separation d from image (b) to image (c) and observed an decrease in the spacial frequencies f of the fringes, which is consistent with standard theory, since $f \propto d$ (5). Secondly, we increased the width of the slit in image (c) and observed an increase in the blur of the fringes, this was also expected as the slit stopped acting like a point source. Finally, a white light was shown on the optical system and separation of the wavelengths was observed in image (d), this is also expected as $\Delta x \propto \lambda$ (4).

The second part of the experiment consisted of using the moiré fringe technique to determine the wavelength λ. it consists of using a raster with n lines per millimetre, changing the distance of the mirror from the optical axis $d_{measured}$ and observing that when the spatial frequencies of the fringes and slits are nearly equal, a 'beat' pattern appears, and horizontal bands become clearly visible using the microscope. With this information we can modify equation (6) to obtain (variables being previously defined):

$$d_{measured} = \frac{n\lambda D}{2} + \frac{d_0}{2} \tag{7}$$

So by using multiple rasters with different number of lines per millimetre, the values of n can be changed and $d_{measured}$ can be recorded using the micrometer attached to the mirror. With a ruler, it is possible to measure D. Finally, finding the gradient m of the graph of $d_{measured}$ against n gives us the wavelength λ:

$$\lambda = \frac{2m}{D} \tag{8}$$

4 Results and error analysis

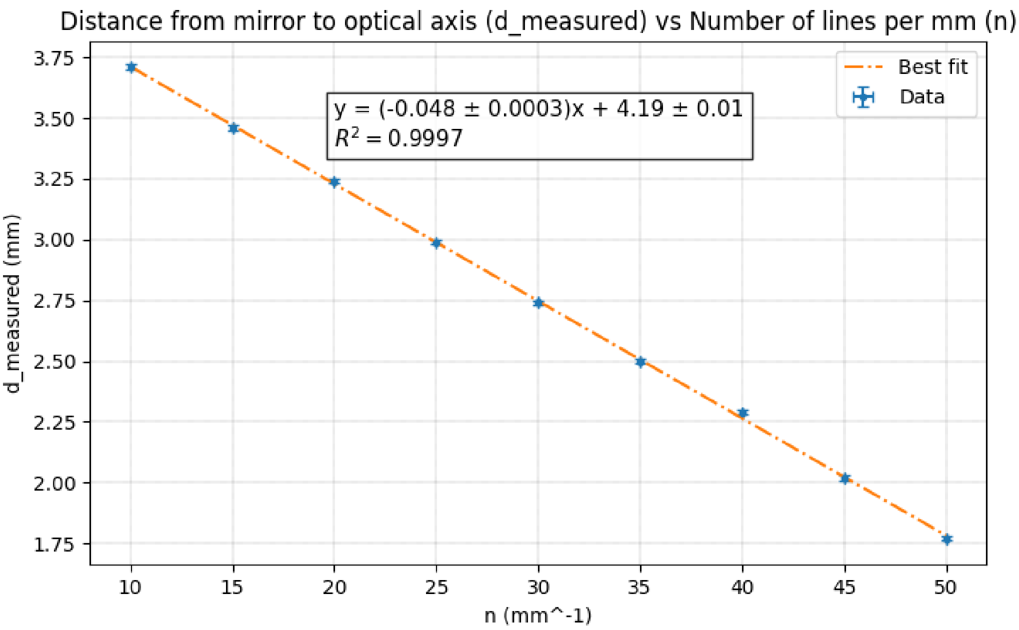


Figure 5: Graph of $d_{measured}$ against n. Clearly showing a linear relationship.

We measured the value of D as $(181 \pm 3)mm$, the error in D is an estimate derived from the fact that the ruler has an intrinsic uncertainty of $\pm 1mm$ and that it was difficult to precisely determine the exact starting position of the slit. The value of the gradient m being equal to $(0.048 \pm 0.0003)mm^2$ was taken from graph 5. Using equation 8, it is possible to calculate λ. And since the errors in D and m are independent of each other, we can find the total error as:

$$\frac{\delta\lambda}{\lambda} = \sqrt{\left(\frac{\delta D}{D}\right)^2 + \left(\frac{\delta m}{m}\right)^2} \tag{9}$$

$$\frac{\delta\lambda}{\lambda} = \sqrt{\left(\frac{3}{181}\right)^2 + \left(\frac{0.0003}{0.048}\right)^2} \tag{10}$$

$$\frac{\delta\lambda}{\lambda} = 1.77\% \tag{11}$$

One interesting observation is that the contribution to the total error of the distance measurement D is much bigger than the gradient's (m) contribution ($\delta D/D \approx 3\,\delta m/m$). This suggests that if higher precision for the value of λ is desired, then effords should be direct towards minimising δD first. Consolidating all information, we obtain a value of $\lambda = (530 \pm 9)nm$. Which agrees with the values directly measured using a spectrometer of $\lambda = (528.5 \pm 0.1)nm$.

5 Conclusion

In this experiment we correctly observed qualitative phenomena of the Lloyd's mirror setup such as the pattern of the diffraction fringes. And also measured the wavelength λ of light from a green LED achieving a value of $(530 \pm 9)nm$, this matches the value found using a spectroscope of $(530 \pm 0.1)nm$. However, one way this experiment could be improved is by decreasing the uncertainty of λ. This could be done by either recording multiple measurements for each raster, thus decreasing the uncertainty in m or, more importantly, using the interior part of a large vernier calliper to measure D. These modifications would allow for a narrower constraint of λ, thus making it possible to test the theoretical model with further precision.

References

[1] Isaac Newton. *Opticks: or, A treatise of the reflections, refractions, inflexions and colours of light.* Printed for Sam. Smith, and Benj. Walford, London, 1704. URL: https://doi.org/10.5479/sil.302475.39088000644674 (visited on 01/27/2025).

[2] Christiaan Huygens. *Opticks: or, A treatise of the reflections, refractions, inflexions and colours of light.* Chez Pierre vander Aa, marchand libraire, A Leide, 1690. URL: https://doi.org/10.5479/sil.294285.39088000545160 (visited on 01/27/2025).

[3] Thomas Young. "II. The Bakerian Lecture. On the theory of light and colours". In: *Philosophical Transactions of the Royal Society of London* 92 (1802), pp. 12–48. DOI: 10.1098/rstl.1802.0004. eprint: https://royalsocietypublishing.org/doi/pdf/10.1098/rstl.1802.0004. URL: https://royalsocietypublishing.org/doi/abs/10.1098/rstl.1802.0004.

[4] Thomas Young and Philip Kelland. *A Course of Lectures on Natural Philosophy and the Mechanical Arts.* Vol. 2. Printed for Taylor and Walton, 1845, plate XX, figure 267. URL: https://archive.org/details/courseoflectures01younrich/courseoflectures01younrich/page/n7/mode/2up.

[5] Humphrey Lloyd. "On a New Case of Interference of the Rays of Light". In: *The Transactions of the Royal Irish Academy* 17 (1831), pp. 171–177. ISSN: 07908113. URL: http://www.jstor.org/stable/30078788 (visited on 01/27/2025).

[6] University of Oxford. *Lloyd's mirror and double-slit interference fringes.* 2019. URL: https://www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Optics/OP11/OP11.pdf.

Appendix A Raw data

Table 1: Raw data for Figure 5.

n (mm^{-1})	$d_{measured}(mm)$
10	3.71 ± 0.01
15	3.46 ± 0.01
20	3.24 ± 0.01
25	2.99 ± 0.01
30	2.74 ± 0.01
35	2.50 ± 0.01
40	2.29 ± 0.01
45	2.02 ± 0.01
50	1.77 ± 0.01