
Multiparty Computation based on Ring-LWE

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Abstract

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Chapter 1

Introduction

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Chapter 2

Review of literature

2.1 Ring LWE - A somewhat homomorphic encryption scheme

In the paper “Fully Homomorphic Encryption from Ring-LWE and Security for Key Dependent Messages” written by Zvika Brakerski and Vinod Vaikuntanathan [1] they describe a method for converting the Ring Learning with errors (RingLWE) problem into an encryption scheme which reduces to the worst-case hardness of problems on ideal lattices. We’ll shortly describe the encryption scheme here but will omit proofs and detailed discussions. Both system will be over the message space of $R_t = \mathbb{Z}_t[x]/\langle f(x) \rangle$.

2.1.1 Polynomial learning with errors

The polynomial learning with errors problem is made as a decision problem and is defined by in the Hermite normal form as the following.

The PLWE Assumption - Hermite Normal Form. for all $\kappa \in \mathbb{N}$, let $f(x) = f_\kappa(x) \in \mathbb{Z}[x]$ be a polynomial of degree $n = n(\kappa)$, let $q = q(\kappa) \in \mathbb{Z}$ be a prime integer, let the ring $R = \mathbb{Z}[x]/\langle f(x) \rangle$ and $R_q = R/qR$ with χ denoting a distribution over the ring R . Then the Polynomial learning with error ($PLWE_{f,q,\chi}$) assumption can be defined as

$$\{(a_i, a_i \cdot s + e_i)\}_{i \in [l]} \approx^c \{(a_i, u_i)\}_{i \in [l]}\}$$

where s is sampled from χ , a_i is uniform in R_q , the error polynomials e_i are sampled from χ and u_i are random ring elements from R_q .

2.1.2 Symmetric version

Let κ be the security parameter and let further q and t be prime numbers where $t \in \mathbb{Z}_n^*$. We also need a polynomial of degree n $f(x) \in \mathbb{Z}[x]$ and an error distribution χ over the ring $R_q = \mathbb{Z}_q[x]/\langle f(x) \rangle$, then we can define the following operations.

Key-gen

Let our secret key be a randomly sampled element from the error distribution $s \leftarrow^{\$} \chi$. Then given the security parameter κ sample a ring element s uniformly at random from κ and define the secret key vector by $(s^0, s^1, s^2, \dots, s^D) \in R_q^{D+1}$.

Encryption

Remember that all messages are encodeable in our message space R_t , thus we will encode our message m as a n degree polynomial with coefficient mod t . To encrypt we sample $(a, b = a \cdot s + t \cdot e)$ where $a \xleftarrow{\$} R_q$ and $e \xleftarrow{\$} \chi$, then compute

$$c_0 := b + m \quad c_1 := -a$$

and from this output the ciphertext $\mathbf{c} := (c_0, c_1) \in R_q^2$.

Decryption

Note that a ciphertext is on the form $(c_0, c_1, \dots, c_D) \in R_q^{D+1}$. Define the inner product over R_q as

$$\langle c, s \rangle = \sum_{i=0}^D c_i \cdot s^i$$

Then to decrypt, simply set m as the inner product of c and s and take modulo t .

$$m = \langle c, s \rangle \bmod t$$

m will then be the decrypted message.

Eval

To obtain the homomorphic abilities of the encryption scheme, Zvika Brakerski and Vinod Vaikuntanathan show how to obtain homomorphic addition and multiplication of ciphertexts.

Addition: Assume we have 2 ciphertexts $c \in R_q^{D+1}$ and $c' \in R_q^{D+1}$, then an encryption of the sum of the 2 underlying messages will be

$$c_{Add} = c + c' = (c_0 + c'_0, c_1 + c'_1, \dots, c_d + c'_d) \quad c_{Add} \in R_q^{D+1}$$

The decryption of c_{Add} will then be the sum of the unencrypted messages from c and c' .

Multiplication: Assume we have 2 ciphertexts $c \in R_q^{D+1}$ and $c' \in R_q^{D'+1}$ and let v be a symbolig value then calculate the updated ciphertext $(\hat{c}_0, \hat{c}_1, \dots, \hat{c}_d + d') \in R_q^{D+D'+1}$ by

$$c_{mul} = \left(\sum_{i=0}^D c_i \cdot v^i \right) \cdot \left(\sum_{j=0}^{D'} c'_j \cdot v^j \right) = \sum_{i=0}^{D+D'} \hat{c}_i \cdot v^i \quad c_{mul} \in R_q^{D+D'+1}$$

The output of the multiplication operation will then be $c_{mul} = (\hat{c}_0, \hat{c}_1, \dots, \hat{c}_{D+D'})$

2.1.3 Public key version

To achieve a public scheme instead, we can make the following changes

- In the key generation we generate in addition to the secret key $sk = s \xleftarrow{\$} \chi$, we output a public key $pk = (a_0, b_0 = a_0 \cdot s + t \cdot e_0)$, where $a_0 \xleftarrow{\$} R_q, e_0 \xleftarrow{\$}$

- In the encryption algorithm, instead we use $(a_0 \cdot v + t \cdot e', b_0 \cdot v + t \cdot e'')$ where $v, e' \xleftarrow{\$} \chi$ and $e'' \xleftarrow{\$} \chi'$.

where we have then obtained a public key $pk = (a_0, a_0 \cdot s + t \cdot e_0)$ corresponding to the secret key $sk =$.

2.2 Circuit privacy

Each time we add or multiply ciphertexts the error term grows. As a result of this the error term will be larger for a ciphertext output by a call to **eval**, than for a ciphertext output by **encrypt**. This poses a problem, as an adversary might be able to derive information about the function computed by looking at the ciphertexts produced. To deal with this problem we would like the output distributions of the ciphertexts output by **eval** and **encrypt** to be identical, which is known as *circuit privacy*. This property along with how to achieve it has been described in [4].

To achieve *circuit privacy* we can make an encryption of 0 with a very large error term, and then add this ciphertext to the original ciphertext. By doing this we essentially drown out information about the error vector of the original ciphertext. This will not modify the encrypted data, as the new error term will be removed by the (mod t) computation done in **decrypt** anyways.

2.3 Protocol for Multiparty Computation based on SHE

The Multiparty Computation (MPC) problem is the problem where n players each with some private input x_i , want to compute some function f on the input, without revealing anything but the result.

In [3] the authors describe a protocol for MPC based on a SHE scheme. The protocol is able compute arithmetic formulas consisting of up to a single multiplication, along with a relatively large number of additions, while being statistically UC-secure against an active adversary and $n - 1$ corruptions.

The protocol proceeds in two phases. In the first phase, preprocessing, a global key $[\![\alpha]\!]$, random values in two representations $[\![r]\!]$, $\langle r \rangle$, and a number of multiplicative triples $\langle a \rangle, \langle b \rangle, \langle c \rangle$ satisfying $c = ab$ are generated.

In the second phase, the online phase, the players use the global key and secret-shared data generated in the preprocessing phase to do the actual computations.

The online phase therefore only makes indirect use of the SHE scheme, as it is only used in the preprocessing phase to generate input for the online phase.

These two phases are described in further detail in section 2.3.2 & 2.3.3.

Representations of shared values The protocol makes use of two different representations of shared values $[\![\cdot]\!]$, $\langle \cdot \rangle$. For a shared value $a \in F_{p^k}$ the $\langle \cdot \rangle$ representation is defined as follows:

$$\langle a \rangle := (\delta, (a_1, \dots, a_n), (\gamma(a)_1, \dots, \gamma(a)_n))$$

where $a = \sum_i a_i$ and $\alpha(\delta + a) = \sum_i \gamma(a)_i$. The $\gamma(a)_i$ values are thus MAC values used to authenticate a . Such a value is shared s.t. each party P_i has access to the global value δ

along with shares $(a_i, \gamma(a)_i)$. Multiplication by a constant and regular addition and are then defined entry-wise on the representation, while addition by a constant is defined as

$$c + \langle a \rangle := (\delta - c, (a_1 + c, \dots, a_n), (\gamma(a)_1, \dots, \gamma(a)_n))$$

When a value $\langle a \rangle$ is partially opened, it means that the value a is revealed without revealing a 's MAC values.

The second representation used for the protocol, $[[\cdot]]$, is defined in the following way:

$$[[a]] = ((a_1, \dots, a_n), (\beta_i, \gamma(a)_1^i, \dots, \gamma(a)_n^i)_{i=1, \dots, n})$$

where $a = \sum_i a_i$ and $a\beta_i = \sum_j \gamma(a)_i^j$. Thus the $\gamma(a)_i^j$ values are used to authenticate a under P_i 's personal key β_i . Each player P_i then has the shares $(a_i, \beta_i, \gamma(a)_1^i, \dots, \gamma(a)_n^i)$. To open a $[[\cdot]]$ value each player P_j sends $a_j, \gamma(a)_i^j$ to P_i , who checks that $a\beta_i = \sum_j \gamma(a)_i^j$. Afterwards P_i can compute $a = \sum_i a_i$.

2.3.1 Abstract SHE scheme and instantiation

The cryptosystem used as the SHE scheme in the protocol has to have certain properties to be admissible. The authors of the MPC protocol present a concrete instantiation of such an abstract SHE scheme, using the Ring-LWE based public key encryption scheme by Zvika Brakerski and Vinod Vaikuntanathan [] described in section x.x. To be able to use this scheme we have to define the functions $encode : (\mathbb{F}_{p^k})^s \mapsto \mathbb{Z}^N$ and $decode : \mathbb{Z}^N \mapsto (\mathbb{F}_{p^k})^s$ where $M = (\mathbb{F}_{p^k})^s$ denotes the message space.

In addition to the aforementioned requirements, we also want the cryptosystem to implement a functionality $\mathcal{F}_{KeyGenDec}$. This functionality will on receiving "start" from all honest players generate a keypair (pk, sk) , and then distribute pk to the players and store sk . The players can then use the functionality to cooperate in decrypting a ciphertext encrypted under pk .

2.3.2 Preprocessing phase

The preprocessing phase is implemented by the Prep protocol, which consists of the steps **initialize**, **pair**, and **triple**. These steps use the additional protocols Reshare, PAngle, and PBracket as subroutines.

Protocol Reshare: The Reshare protocol takes a ciphertext e_m as input and a parameter enc , which can be set to either *NewCiphertext* or *NoNewCiphertext*. The protocol then outputs a share m_i of m to each player along with a new fresh ciphertext e'_m if $enc = \text{NewCiphertext}$, where e'_m contains $\sum_i m_i$. To do this the players first each sample $f_i \in \mathbb{F}_{p^k}$, and then broadcast

$$e_{f_i} \leftarrow Enc_{pk}(f_i)$$

Each P_i then runs the ZKPoPK protocol while acting as a prover on the previously generated ciphertext e_{f_i} , and if any of these proofs fail, then parties abort. Now, each player homomorphically adds each encrypted share

$$e_f \leftarrow e_{f_1} \boxplus \dots \boxplus e_{f_n}$$

and then homomorphically adds e_m and e_f to get e_{m+f} . The players now use $\mathcal{F}_{KeyGenDec}$ to decrypt e_{m+f} so that they get $m + f$. Now P_1 sets $m_1 \leftarrow m + f + f_1$, while the rest of the players P_i set $m_i \leftarrow -f_i$. If $enc = NewCiphertext$, then the players each compute

$$e'_m \leftarrow Enc_{pk}(m + f) \boxplus e_{f_1} \boxplus \dots \boxplus e_{f_n}$$

where default randomness is used for the encryption.

Protocol PAngle: PAngle takes as input a ciphertext e_v along with privately held shares v_1, \dots, v_n . These are then used to generate a value in the angle representation $\langle v \rangle$. To achieve this all players first compute

$$e_{v \cdot \alpha} \leftarrow e_v \boxplus e_\alpha$$

Reshare is then used with $e_{v \cdot \alpha}$ as input, such that each player P_i receives a share γ_i of $v \cdot \alpha$. Finally, $\langle v \rangle = (0, (v_1, \dots, v_n), (\gamma_1, \dots, \gamma_n))$ is output.

Protocol PBracket: PAngle takes as input a ciphertext e_v along with privately held shares v_1, \dots, v_n . These are then used to generate a value in the angle representation $\llbracket v \rrbracket$. For $i = 1, \dots, n$ all players compute

$$e_{\gamma_i} \leftarrow e_{\beta_i} \boxtimes e_v$$

and then generate $(\gamma_i^1, \dots, \gamma_i^n)$ by calling Reshare with e_{γ_i} and $NoNewCiphertext$ as input. The representation $\llbracket v \rrbracket = ((v_1, \dots, v_n), (\beta_i, \gamma(v)_1^i, \dots, \gamma(v)_n^i)_{i=1, \dots, n})$ is then output.

Initialize: The **initialize** step generates the global and personal keys. This is achieved by the players first calling "start" on $\mathcal{F}_{KeyGenDec}$, so that every player obtains the public key pk . Then each player samples $\alpha_i, \beta_i \in \mathbb{F}_{p^k}$, and broadcasts

$$e_{\alpha_i} \leftarrow Enc_{pk}(Diag(\alpha_i)), \quad e_{\beta_i} \leftarrow Enc_{pk}(Diag(\beta_i))$$

where $Diag(a) = (a, a, \dots, a) \in (\mathbb{F}_{p^k})^s$. Now each P_i runs the ZKPoPK protocol twice with $diag = true$, while acting as a prover, where the inputs are the ciphertexts e_{α_i} and e_{β_i} repeated sec times. Finally, the players homomorphically add the encrypted shares e_{α_i} to get e_α , which they use along with their share $Diag(\alpha_i)$ to generate $\llbracket Diag(\alpha) \rrbracket$ using a call to PBracket. Then $\llbracket Diag(\alpha) \rrbracket$ is the global key, while β_i is P_i 's personal key.

Pair: In **pair** the players generate random values in the two representations $\llbracket r \rrbracket, \langle r \rangle$. This is done by each player first sampling a share $r_i \in (\mathbb{F}_{p^k})^s$, then broadcasting. Each player then encrypts their share to get

$$e_{r_i} \leftarrow Enc_{pk}(r_i)$$

which they then broadcast. Once again P_i will now run the ZKPoPK protocol acting as a prover with input e_{r_i} , and if the ZK proof fails, then the protocol is aborted. The players then homomorphically add the encrypted shares to get e_r , which they use along with their share r_i as input to PBracket and PAngle to generate $\llbracket r \rrbracket, \langle r \rangle$.

Triple: The **triple** step generates triples $(\langle a \rangle, \langle b \rangle, \langle c \rangle)$ satisfying $c = ab$. To do this the players start off by sampling shares $a_i, b_i \in (\mathbb{F}_{p^k})^s$. The players then encrypt their shares and broadcast the result

$$e_{a_i} \leftarrow \text{Enc}_{pk}(a_i), \quad e_{b_i} \leftarrow \text{Enc}_{pk}(b_i)$$

Now each P_i acts as a prover running the ZKPoPK protocol first with e_{a_i} and then with e_{b_i} as input, and if any proof fails, then the protocol is aborted. The players then homomorphically add the encrypted shares to get e_a and e_b , and use these along with their shares a_i, b_i to generate $\langle a \rangle, \langle b \rangle$ using calls to PAngle. Following this each player homomorphically multiplies e_a and e_b to get e_c , which the players use as input to Reshare to get shares of c along with a new ciphertext $(c_1, \dots, c_n, e_{c'})$. Then the players then use their shares of c along with $e_{c'}$ to generate $\langle c \rangle$ by calling PAngle. Finally, the triple $(\langle a \rangle, \langle b \rangle, \langle c \rangle)$ is output.

2.3.3 Online phase

The Online protocol implements the online phase, and consists of the steps **initialize**, **input**, **add**, **multiply**, and **output**. These steps are executed as needed to evaluate the arithmetic circuit that we wish to evaluate.

Initialize: The **initialize** step simply consists using the Prep protocol to generate a global key, along with enough multiplicative triples and random values in the two representations showed earlier, for the circuit that we want to evaluate.

Input: The *input* step lets a player P_i share their private input x_i . The input is shared by taking a pair $[\![r]\!], \langle r \rangle$, and then opening $[\![r]\!]$ to P_i so that P_i gets r . Following this P_i computes and broadcasts $\varepsilon \leftarrow x_i - r$. All players finally set $\langle x_i \rangle \leftarrow \langle r \rangle + \varepsilon$.

Add: To add two values $\langle x \rangle, \langle y \rangle$, we simply perform the component-wise addition $\langle z \rangle = \langle x \rangle + \langle y \rangle$, meaning that each player adds their shares locally $z_i = x_i + y_i, \gamma(z)_i = \gamma(x)_i + \gamma(y)_i$.

Multiply: To multiply two values $\langle x \rangle, \langle y \rangle$, we use two multiplicative triples $(\langle a \rangle, \langle b \rangle, \langle c \rangle)$ and $(\langle f \rangle, \langle g \rangle, \langle h \rangle)$. We use the second triple to check that $ab = c$, but this could also be done in preprocessing instead. To do this check we first open $[\![t]\!]$ to get t , then partially open $t * \langle a \rangle - \langle f \rangle$ and $\langle b \rangle - \langle g \rangle$ to get ρ and σ respectively. Finally, we compute and partially open

$$t * \langle c \rangle - \langle h \rangle - \sigma \langle f \rangle - \rho \langle g \rangle - \sigma \rho$$

if the result is 0, then we conclude that $ab = c$ and move on, otherwise the players abort.

Now, partially open $\langle x \rangle - \langle a \rangle$ and $\langle y \rangle - \langle b \rangle$ to get ε and δ respectively. Finally, we output

$$\langle z \rangle \leftarrow \langle c \rangle + \varepsilon \langle b \rangle + \delta \langle a \rangle + \varepsilon \delta$$

Output: To output a value y given $\langle y \rangle$, the players do the following. First, a random value $\llbracket e \rrbracket$ is opened so each player gets e , which is used to compute

$$a = \sum_j e^j \cdot a_j$$

where a_j are all of the opened values of the form $\langle a_j \rangle$. Each player P_i then uses the commitment functionality \mathcal{F}_{Com} to commit to $\gamma_i \leftarrow \sum_j e^j \gamma(a_j)_i$ along with y_i and $\gamma(y)_i$. Then, the global key $\llbracket \alpha \rrbracket$ is opened. The players then use \mathcal{F}_{Com} to open γ_i and then check that indeed

$$\alpha(a + \sum_j e^j \delta_j) = \sum_j \gamma_i$$

If this is not the case, then the players abort. Finally, to make sure that all players end up with y the commitments to y_i and $\gamma(y)_i$ are opened. Now the players check that $\alpha(y + \delta) = \sum_i \gamma(y)_i$, and if this is the case, then $y = \sum_i y_i$ is output.

2.4 Reuse of unrevealed secret-shared data

In [2] a technique that allows for reuse of unrevealed secret-shared data is used. This technique revolves around not having to open $\llbracket \alpha \rrbracket$, and in fact we do not even need the $\llbracket \cdot \rrbracket$ representation when using this technique.

The technique works as follows. First, when we generate the global key we now just need each player P_i to have a share α_i of α . In the output step the player will then instead of the current MAC check instead invoke the new MACCheck protocol on all of the values in the $\langle \cdot \rangle$ representation that have been opened, and if this succeeds the player invokes MACCheck on the output value $\langle y \rangle$. The MACCheck protocol works as follows:

Protocol MACCheck First each P_i samples a seed s_i and \mathcal{F}_{Commit} to broadcast $\tau_i^s \leftarrow Commit(s_i)$. Following this each player opens all commitments using \mathcal{F}_{Commit} to get all n seeds s_j . Now, all players set

$$s \leftarrow s_1 \oplus \dots \oplus s_n$$

Players then use s as seed to sample a random vector of length t with entries in the interval $[0, p)$. All players then first compute

$$a \leftarrow \sum_{j=1}^t r_j \cdot \gamma(a_j)_i$$

where the a_j 's are the opened values. Now P_i computes

$$\gamma_i \leftarrow \sum_{j=1}^t r_j \cdot \gamma(a_j)_i \text{ and } \sigma_i \leftarrow \gamma_i - \alpha_i \cdot a$$

Player i then uses \mathcal{F}_{Commit} to broadcast $\tau_i^\sigma \leftarrow Commit(s_i)$. All players invoke \mathcal{F}_{Commit} to open the commitments received to get the σ_j 's. Finally, the players check that $\sigma_1 + \dots + \sigma_n = 0$, and if this is not the case, then they abort.

Chapter 3

Implementation

For the implementation part of the project we first implemented the Ring-LWE cryptosystem described in section x.x. We then used this for implementing the MPC protocol described in section x.x, while making slight deviations to allow for reuse of secret-shared values, as described in section x.x. The code for the protocol was written in the programming language Rust. In this chapter we describe the implementation details of these systems.

3.1 Ring-LWE cryptosystem

3.1.1 Polynomials (`poly.rs`)

Since our public-key encryption scheme uses polynomials to represent all of the values we work on (messages, ciphertexts, secret keys, and public keys), we made an implementa

3.1.2 Quotient ring (`quotient_ring.rs`)

Encryption, decryption, and key generation involves adding, subtracting, multiplying, and negating elements in the quotient ring $R_q = \mathbb{Z}_q[x]/\langle x^n + 1 \rangle$. To be able to do these computations we made a quotient ring implementation, which can be found in `quotient_ring.rs`.

Rq

The quotient ring module contains a struct definition `Rq`, which represents an instantiation of a quotient ring $\mathbb{Z}_q[x]/\langle f(x) \rangle$. It therefore has fields `q` and `modulo`, where `q` is a `BigInt`, and `modulo` is a `Polynomial` representing $f(x)$. The `new` function takes `q` and `modulo` as input, and is used to make a new instantiation of `Rq`.

reduce

The `reduce` method found in the quotient ring module is called on an `Rq` struct, takes a polynomial `pol` as input, and returns the normal form of the element `pol` with respect

Dette skal nok opdateres når vi er færdige med implementationen. Skal inkludere det om MPC.

Hvor meget skal forklares her? At det er Brakersky-Vaikuntanathan vi implementerede? At det er baseret på Ring-LWE? Eller bliver det alt sammen beskrevet et andet sted?

to modulo.

To achieve this the method first does polynomial long division with `pol` as the dividend and the `modulo` from the `Rq` struct as the divisor. The remainder computed in this way is then stored in the variable `r`.

Lastly, we reduce the coefficients of the resulting polynomial `r` modulo `q`, by using the remainder operation(`%`) defined in the `poly.rs` module, and then return the result.

add, times, neg, mul

The methods `add`, `times`, `neg`, `mul` are called on an `Rq` struct. These methods first use the addition, scalar multiplication, negation, and polynomial multiplication methods defined in the `poly.rs` module on the input. Then a `reduce` call is done with the result as input to get a new R_q element.

3.1.3 Public-key encryption scheme (encryption.rs)

The encryption scheme, as usual, has three major components:

- the `generate_key_pair` function
- the `encrypt` function
- and the `decrypt` function

generate_key_pair

The `generate_key_pair` function takes as input an instance of the `Parameters` struct. This struct essentially just defines the parameters that nearly all functions in the encryption scheme use in some form or another. This includes r , n , q , t , and the quotient ring R_q , which are all relevant for the key generation function.

The function starts by sampling polynomials sk and e_0 from a Gaussian distribution with standard deviation r , and the polynomial a_0 uniformly from \mathbb{Z}_q .

It then calculates the public-key as $pk = (a_0, a_0 \cdot sk + e_0 \cdot t)$, and finally returns the key pair (pk, sk) .

encrypt

The `encrypt` function takes a `Parameters` instance, as described above, and additionally takes a polynomial m and a public key pk .

We extract the two polynomials of the public key, a_0 and b_0 .

First, we make sure that the message polynomial we are trying to encrypt is in R_t . Then, we sample polynomials v and e' from a Gaussian distribution with standard deviation r , and the polynomial e'' from a Gaussian distribution with standard deviation r' (which is also defined in the `Parameters` struct).

We then calculate $a = a_0 \cdot v + e' \cdot t$ and $b = b_0 \cdot v + e'' \cdot t$. Finally, we create the ciphertext as a `Vec` (a contiguous growable array type) $c = [b + m, a]$ and return it.

decrypt

The decrypt function takes a Parameters instance, as well as a ciphertext $c = [c_0, c_1, \dots]$ and a secret key sk .

We start by constructing the secret key vector $\mathbf{s} = [1, s, s^2, \dots]$ from the secret key. We only create the first $|c|$ entries of the secret key vector, as those are the only ones we'll need.

We then initialize a polynomial $msg = 0$, which will become the decrypted message. Then, iterating over each element c_i in the ciphertext, we add $c_i \cdot s_i$ to msg , where s_i is the i 'th entry (zero-indexed) in the secret key vector.

Currently, the message has coefficients in \mathbb{Z}_q , but we need them to be in the range $-\frac{q}{2}$ to $\frac{q}{2}$. Therefore, we iterate through all coefficients x and map them to the new range such that if $x > \frac{q}{2}$, then we let $x' = x - q$, and otherwise $x' = x$.

Finally, we reduce the message modulo t to remove the $e \cdot t$ part of the encryption, and return the result.

Skal nok lige have nogle andre til at tilpasse denne - jeg har svært ved at finde ud af hvordan man forklarer det.

Er det inklusive eller eksklusiv i intervallet?

3.2 MPC protocol

3.2.1 \mathcal{F}_{Com} functionality (commit.rs)

Our implementation of the commitment functionality that is used in the online phase can be found in commit.rs.

3.2.2 ZKPoPK (zk.rs)

3.2.3 Preprocessing phase (prep.rs)

The file prep.rs contains all of the methods that are called in the preprocessing phase to generate values for the online phase.

reshare

Reshare takes as input a Parameters struct params, a Ciphertext e_m , a PlayerState state, and an Enc enum enc, which can take on values NewCiphertext or NoNewCiphertext.

The first thing done in this method is to sample a value f_i uniformly from Z_t using sample_single from prob.rs. Then, f_i is encrypted to get e_{f_i} . The facilitator is then used to broadcast e_{f_i} , and subsequently receive the encrypted shares from the other parties, which are then homomorphically added to get e_f . Then, we compute $e_{m_plus_f} = \text{add}(\text{params}, e_m, \&e_f)$.

The ddec method from mod.rs is then called with $e_{m_plus_f}$ as input to get the plaintext m_plus_f .

Then we set m_i to be $e_{m+f} - f_i \bmod t$ if the player calling the method is the player with index 0, and $-f_i \bmod t$ otherwise. If $\text{enc} = \text{NewCiphertext}$, we now encrypt m_plus_f using encrypt_det where we use a triple of 1-polynomials instead of the randomness to make encryption deterministic. Now, we use add from encrypt.rs to

homomorphically subtract the encrypted shares e_{f_i} from the encryption of m_plus_f to get e_m_prime . Afterwards, we return $(Some(e_m_prime), m_i)$

If $enc = NoNewCiphertext$, we instead just return $(None, m_i)$.

p_angle

The `p_angle` method takes a `Parameters` struct `params`, a `BigInt` `v_i`, a `Ciphertext` `e_v`, and a `PlayerState` `state` as input.

The first thing done in `p_angle` is to homomorphically multiply `e_v` and `state.e_alpha` to get `e_v_mul_alpha`. Then, `reshare` is called to get `gamma_i`, which is a share of a `BigInt`, namely the plaintext in `e_v_mul_alpha`. Lastly, the method outputs (v_i, γ_i) , which is a share of $\langle v \rangle$.

As can be seen from the implementation we omit the public δ value as done in [2], such that the MAC's now instead satisfy $\alpha v = \sum_i \gamma(v)_i$.

initialize

The `initialize` method takes a `Parameters` struct `params`, and a mutable `PlayerState` `state` as input.

First, we sample a uniformly random `BigInt` from $[0, t)$ using the `sample_single` method with `params.t` as input, and set the `alpha_i` variable of the player state to this value. This represents the given players share of the global key. We then encrypt `state.alpha_i` to get an encrypted share `e_alpha_i`. Now, `e_alpha_i` is broadcast using `state.facilitator`, and each player then uses their facilitator to receive e_{α_i} from the other players. The e_{α_i} 's are then homomorphically added to get e_α , and the result is then assigned to the `e_alpha` variable of `state`. Finally, we run `zpopk` from `zk.rs` in a loop `sec` times.

Notice how we do not compute the personal keys β_i as done in [3]. As mentioned earlier these are not needed when we use the trick described in section 2.4.

pair

The `pair` methods takes a `Parameters` struct `params`, and a `PlayerState` `state` as input.

The method first samples a uniformly random `BigInt` `r_i` from Z_t . Now, `r_i` is encrypted to get `e_r_i`, which is the broadcast using the facilitator. All players then again use the facilitator to receive the encrypted shares from the other parties, which are then homomorphically added to compute `e_r`. Then, we call `zpopk` with `e_r_i` as input.

Following this we compute the given players share of $\langle r \rangle$ with a call to `p_angle` with `r_i` and `e_r` as arguments, which returns `r_angle`.

Again, we use the trick explained in section 2.4, so we don't need the values in the bracket representation, and therefore we just return (r_i, r_angle) .

triple

The `triple` method takes a `Parameters` struct `params`, along with a `PlayerState` `state` as arguments.

First, we sample a_i, b_i uniformly at random from Z_t , which are both then encrypted, and then encrypted shares are then broadcast.

Then, when the player receives the encrypted shares of a and b from the other players, then these are homomorphically added to get e_a and e_b .

Following this, we generate shares a_{angle} and b_{angle} with calls to `p_angle` using a_i, e_a and b_i, e_b as input respectively.

Now, the player calling the method has shares of $\langle a \rangle$ and $\langle b \rangle$, and we need to compute a share of $\langle c \rangle$.

To do this we compute e_c by homomorphically multiplying e_a and e_b , and then we call `reshare` with e_c and `NewCiphertext` as input to get a new ciphertext e_{c_prime} and c_i , which is a share of c .

This allows us to call `p_angle` using c_i and e_{c_prime} to get c_{angle} .

Finally, we return the shares $a_{\text{angle}}, b_{\text{angle}}$, and c_{angle} .

3.2.4 Online phase (prep.rs)

The code related to the online phase can be found in `prep.rs`.

give_input

receive_input

add

Add simply takes two `AngleShare` values x and y as input.

These are then used to compute $(x.0 + y.0, x.1 + y.1)$, and the resulting `AngleShare` is then returned.

partial_opening

triple_check

multiply

maccheck

output

Chapter 4

Conclusion

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Acknowledgments

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Appendix A

First appendix