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# Multiparty Computation based on Ring-LWE

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# Abstract

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# **Chapter 1**

## **Introduction**

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## Chapter 2

# Review of literature

### 2.1 Encoding messages as polynomials

Encoding messages as polynomials follows simply by the fact that a letter can be represented by a number. These numbers can then be combined into a vector which can be encoded as a polynomial. As an example take the message: “CRYPTO”, using the ASCII table, this message can be represented by the vector  $[67, 82, 89, 80, 84, 79]$ . This vector can be represented as a polynomial by  $67 + 82x + 89x^2 + 80x^3 + 84x^4 + 79x^5$ . Trivially we can also decode any polynomial into a message, by reverting these steps one by one.

### 2.2 Ring LWE - A somewhat homomorphic encryption scheme

In the paper “Fully Homomorphic Encryption from Ring-LWE and Security for Key Dependent Messages” written by Zvika Brakerski and Vinod Vaikuntanathan [1] they describe a method for converting the Ring Learning with errors (RingLWE) problem into an encryption scheme which reduces to the worst-case hardness of problems on ideal lattices. We’ll shortly describe the encryption scheme here but will omit proofs and detailed discussions. Both system will be over the message space of  $R_t = \mathbb{Z}_t[x]/\langle f(x) \rangle$ .

#### 2.2.1 Symmetric version

Let  $\kappa$  be the security parameter and let further  $q$  and  $t$  be prime numbers where  $t \in \mathbb{Z}_n^*$ . We also need a polynomial of degree  $n$   $f(x) \in \mathbb{Z}[x]$  and an error distribution  $\chi$  over the ring  $R_q = \mathbb{Z}_q[x]/\langle f(x) \rangle$ .

#### Key-gen

Let our secret key be a randomly sampled element from the error distribution  $s \xleftarrow{\$} \chi$ . Then given the security parameter  $\kappa$  sample a ring element  $s$  uniformly at random from  $R_q$  and define the secret key vector by  $(s^0, s^1, s^2, \dots, s^D) \in R_q^{D+1}$ .

## Encryption

Remember that all messages are encodeable in our message space  $R_t$ , thus we will encode our message  $m$  as a  $n$  degree polynomial with coefficient mod  $t$ . To encrypt we sample  $(a, b = a \cdot s + t \cdot e)$  where  $a \leftarrow R_q$  and  $e \leftarrow \chi$ , then compute

$$c_0 := b + m \quad c_1 := -a$$

and from this output the ciphertext  $\mathbf{c} := (c_0, c_1) \in R_q^2$ .

## Decryption

Note that a ciphertext is on the form  $(c_0, c_1, \dots, c_D) \in R_q^{D+1}$ . Define the inner product over  $R_q$  as

$$\langle c, s \rangle = \sum_{i=0}^D c_i \cdot s^i$$

Then to decrypt, simply set  $m$  as the inner product of  $\mathbf{c}$  and  $s$  and take modulo  $t$ .

$$m = \langle c, s \rangle \bmod t$$

$m$  will then be the decrypted message.

## Eval

To obtain the homomorphic abilities of the encryption scheme, Zvika Brakerski and Vinod Vaikuntanathan show how to obtain homomorphic addition and multiplication of ciphertexts.

**Addition:** Assume we have 2 ciphertexts  $c \in R_q^{D+1}$  and  $c' \in R_q^{D+1}$ , then an encryption of the sum of the 2 underlying messages will be

$$c_{Add} = c + c' = (c_0 + c'_0, c_1 + c'_1, \dots, c_d + c'_d) \quad c_{Add} \in R_q^{D+1}$$

The decryption of  $c_{Add}$  will then be the sum of the unencrypted messages from  $c$  and  $c'$ .

**Multiplication:** Assume we have 2 ciphertexts  $c \in R_q^{D+1}$  and  $c' \in R_q^{D'+1}$  and let  $v$  be a symbolig value then calculate the updated ciphertext  $(\hat{c}_0, \hat{c}_1, \dots, \hat{c}_d + d') \in R_q^{D+D'+1}$  by

$$c_{mul} = \left( \sum_{i=0}^D c_i \cdot v^i \right) \cdot \left( \sum_{j=0}^{D'} c'_j \cdot v^j \right) = \sum_{i=0}^{D+D'} \hat{c}_i \cdot v^i \quad c_{mul} \in R_q^{D+D'+1}$$

The output of the multiplication operation will then be  $c_{mul} = (\hat{c}_0, \hat{c}_1, \dots, \hat{c}_{D+D'})$

### 2.2.2 Public key version

## 2.3 Circuit privacy

Each time we add or multiply ciphertexts the error term grows. As a result of this the error term will be larger for a ciphertext output by a call to **eval**, than for a ciphertext output by **encrypt**. This poses a problem, as an adversary might be able to derive information about the function computed by looking at the ciphertexts produced. To deal with this problem we would like the output distributions of the ciphertexts output by **eval** and **encrypt** to be identical, which is known as *circuit privacy*. This property along with how to achieve it has been described in [3].

To achieve *circuit privacy* we can make an encryption of 0 with a very large error term, and then add this ciphertext to the original ciphertext. By doing this we essentially drown out information about the error vector of the original ciphertext. This will not modify the encrypted data, as the new error term will be removed by the (mod  $t$ ) computation done in **decrypt** anyways.

## 2.4 Multiparty Computation

Multiparty Computation concerns itself with ...

## 2.5 Security of MPC protocols

## 2.6 Bootstrapping

## 2.7 Protocol for MPC based on SHE

Bla bla bla based on [2].

### 2.7.1 Preprocessing phase

### 2.7.2 Online protocol

### 2.7.3 Parameter setting

## Chapter 3

# Implementation

We implemented a somewhat-homomorphic public-key encryption scheme in the Rust programming language, as well as functions for adding and multiplying ciphertexts for that encryption scheme.

### 3.1 Polynomials (`poly.rs`)

Since our public-key encryption scheme uses polynomials to represent all of the values we work on (messages, ciphertexts, secret keys, and public keys), we made an implementa

### 3.2 Quotient ring (`quotient_ring.rs`)

Encryption, decryption, and key generation involves adding, subtracting, multiplying, and negating elements in the quotient ring  $R_q = \mathbb{Z}_q[x]/\langle x^n + 1 \rangle$ . To be able to do these computations we made a quotient ring implementation, which can be found in `quotient_ring.rs`.

#### 3.2.1 `Rq`

The quotient ring module contains a struct definition `Rq`, which represents an instantiation of a quotient ring  $\mathbb{Z}_q[x]/\langle f(x) \rangle$ . It therefore has fields `q` and `modulo`, where `q` is a `BigInt`, and `modulo` is a `Polynomial` representing  $f(x)$ . The `new` function takes `q` and `modulo` as input, and is used to make a new instantiation of `Rq`.

#### 3.2.2 `reduce`

The `reduce` method found in the quotient ring module is called on an `Rq` struct, takes a polynomial `pol` as input, and returns the normal form of the element `pol` with respect to `modulo`.

To achieve this the method first does polynomial long division with `pol` as the dividend and the `modulo` from the `Rq` struct as the divisor. The remainder computed in this way is then stored in the variable `r`.

Dette skal nok opdateres når vi er færdige med implementationen. Skal inkludere det om MPC.

Hvor meget skal forklares her? At det er Brakersky-Vaikuntanathan vi implementerede? At det er baseret på Ring-LWE? Eller bliver det alt sammen beskrevet et andet sted?



Lastly, we reduce the coefficients of the resulting polynomial  $r$  modulo  $q$ , by using the remainder operation(`%`) defined in the `poly.rs` module, and then return the result.

### 3.2.3 `add, times, neg, mul`

The methods `add`, `times`, `neg`, `mul` are called on an `Rq` struct. These methods first use the addition, scalar multiplication, negation, and polynomial multiplication methods defined in the `poly.rs` module on the input. Then a `reduce` call is done with the result as input to get a new  $R_q$  element.

## 3.3 Public-key encryption scheme (`encryption.rs`)

The encryption scheme, as usual, has three major components:

- the `generate_key_pair` function
- the `encrypt` function
- and the `decrypt` function

### 3.3.1 `generate_key_pair`

The `generate_key_pair` function takes as input an instance of the `Parameters` struct. This struct essentially just defines the parameters that nearly all functions in the encryption scheme use in some form or another. This includes  $r$ ,  $n$ ,  $q$ ,  $t$ , and the quotient ring  $R_q$ , which are all relevant for the key generation function.

The function starts by sampling polynomials  $sk$  and  $e_0$  from a Gaussian distribution with standard deviation  $r$ , and the polynomial  $a_0$  uniformly from  $\mathbb{Z}_q$ .

It then calculates the public-key as  $pk = (a_0, a_0 \cdot sk + e_0 \cdot t)$ , and finally returns the key pair  $(pk, sk)$ .

### 3.3.2 `encrypt`

The `encrypt` function takes a `Parameters` instance, as described above, and additionally takes a polynomial  $m$  and a public key  $pk$ .

We extract the two polynomials of the public key,  $a_0$  and  $b_0$ .

First, we make sure that the message polynomial we are trying to encrypt is in  $R_t$ . Then, we sample polynomials  $v$  and  $e'$  from a Gaussian distribution with standard deviation  $r$ , and the polynomial  $e''$  from a Gaussian distribution with standard deviation  $r'$  (which is also defined in the `Parameters` struct).

We then calculate  $a = a_0 \cdot v + e' \cdot t$  and  $b = b_0 \cdot v + e'' \cdot t$ . Finally, we create the ciphertext as a `Vec` (a contiguous growable array type)  $c = [b + m, a]$  and return it.

### 3.3.3 `decrypt`

The `decrypt` function takes a `Parameters` instance, as well as a ciphertext  $c = [c_0, c_1, \dots]$  and a secret key  $sk$ .

We start by constructing the secret key vector  $\mathbf{s} = [1, s, s^2, \dots]$  from the secret key. We only create the first  $|c|$  entries of the secret key vector, as those are the only ones we'll need.

We then initialize a polynomial  $msg = 0$ , which will become the decrypted message. Then, iterating over each element  $c_i$  in the ciphertext, we add  $c_i \cdot s_i$  to  $msg$ , where  $s_i$  is the  $i$ 'th entry (zero-indexed) in the secret key vector.

Currently, the message has coefficients in  $\mathbb{Z}_q$ , but we need them to be in the range  $-\frac{q}{2}$  to  $\frac{q}{2}$ . Therefore, we iterate through all coefficients  $x$  and map them to the new range such that if  $x > \frac{q}{2}$ , then we let  $x' = x - q$ , and otherwise  $x' = x$ .

Finally, we reduce the message modulo  $t$  to remove the  $e \cdot t$  part of the encryption, and return the result.

Skal nok lige have nogle andre til at tilpasse denne - jeg har svært ved at finde ud af hvordan man forklarer det.

Er det inklusive eller eksklusivt i intervallet?

## **Chapter 4**

## **Conclusion**

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# Acknowledgments

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## **Appendix A**

### **First appendix**