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EE 231

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Convex Optimization Project

Optimal Downlink Beam Forming using Semidefinite Optimization

Introduction:

What is **Beamforming**, it is a signal processing technique used in **sensor arrays** for directional signal transmission. One type of sensor arrays is **antenna arrays**. When using antenna arrays at the base station of a cellular system, an optimal choice of beam formers for simultaneous transmission to several co channel users must be solved jointly for all users and the base stations in the area while taking into consideration the **SINR (Signal to interference plus noise ratio)** which is a quantity used to give theoretical upper bounds on the channel capacity or the rate of information transfer in wireless communication systems.

Problem formulation:

We consider the following problem, where we want to minimize the **total transmitted power** from the base stations to the mobile devices under the constraint that the received **SINR** at each mobile is above a threshold **γ**

Total Transmitted power **SINR**

$$P = \sum_{i=1}^{d} \|\mathbf{w}_i\|^2$$

$$\frac{\mathbf{w}_i^* \mathbf{R}_i \mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_n^* \mathbf{R}_{i,\kappa(n)} \mathbf{w}_n + \sigma_i^2}$$

Optmization Problem

$$\min \sum_{i=1}^{d} \|\mathbf{w}_i\|^2$$
$$\text{s.t. } \frac{\mathbf{w}_i^* \mathbf{R}_i \mathbf{w}_i}{\sum_{n \neq i} \mathbf{w}_n^* \mathbf{R}_{i,\kappa(n)} \mathbf{w}_n + \sigma_i^2} \geq \gamma_i,\ i = 1, \ldots, d\ ,$$

Notation:
w beam forming weight vector for transmission from base k to mobile i
$\mathbf{R}_{i,k}$ Correlation matrix of the channel from base station k to mobile i
σ^2 Variance of the noise in the transmitted signal

Convexity of the Problem

We can rewrite the problem as

$$\min \sum_{i=1}^{d} \mathbf{w}_i^* \mathbf{w}_i$$
$$\text{s.t. } \mathbf{w}_i^* \mathbf{R}_i \mathbf{w}_i - \gamma_i \sum_{n \neq i} \mathbf{w}_n^* \mathbf{R}_{i,\kappa(n)} \mathbf{w}_n \geq \gamma_i \sigma_i^2,\ i = 1, \ldots, d\ .$$

however this problem is not a convex optimization problem that can easily be solved because **the constraints are not convex**, if we **add a constraint for the special case of a rank 1 channels** and **rearrange the above constraint we get**

$$\min \sum_{i=1}^{d} \mathbf{w}_i^* \mathbf{w}_i$$
$$\text{s.t. } (\mathbf{w}_i^* \mathbf{h}_i)^2 \geq \gamma_i \left(\sum_{n \neq i} \mathbf{w}_n^* \mathbf{R}_{i,\kappa(n)} \mathbf{w}_n + \sigma_i^2 \right),$$
$$\mathbf{w}_i^* \mathbf{h}_i \geq 0,\ i = 1, \ldots, d\ .$$

Which is a convex optimization problem because we are minimizing a **linear function** over the **affine transformation of the convex second order cone.**

If we let **W = ww*** and use the fact that **w*Rw = Tr[Rww*] = Tr[Rww*]** and we **relax the rank of W** we can rewrite the problem as the **Semidefinite program**

$$\min \sum_{i=1}^{d} \mathrm{Tr}[\mathbf{W}_i]$$
$$\text{s.t. } \mathrm{Tr}[\mathbf{R}_i \mathbf{W}_i] - \gamma_i \sum_{n \neq i} \mathrm{Tr}[\mathbf{R}_{i,\kappa(n)} \mathbf{W}_n] = \gamma_i \sigma_i^2,$$
$$\mathbf{W}_i = \mathbf{W}_i^*,$$
$$\mathbf{W}_i \succeq 0,\ i = 1, \ldots, d\ .$$

Which is a **convex problem** because we are minimizing a **linear function of the variable** which is $\sum_{i=1}^{d} \mathrm{Tr}[\mathbf{W}_i]$ subject to a number of **linear equality constraints** $\mathrm{Tr}[\mathbf{R}_i \mathbf{W}_i] - \gamma_i \sum_{n \neq i} \mathrm{Tr}[\mathbf{R}_{i,\kappa(n)} \mathbf{W}_n] = \gamma_i \sigma_i^2$

And the variable is **symmetric positive semidefinite** $\mathbf{W}_i = \mathbf{W}_i^*,\ \mathbf{W}_i \succeq 0,\ i = 1, \ldots, d$. therfore it is the **standard form semidefinite program**

$$\begin{array}{ll} \text{minimize} & \mathbf{tr}(CX) \\ \text{subject to} & \mathbf{tr}(A_i X) = b_i, \quad i = 1, \ldots, p \\ & X \succeq 0, \end{array}$$

Numerical Result

1) The scenario is **3 mobile devices** served by **1 base station**,
2) one is located at **theta1 = 10 degrees**
3) The second and the third are located at **theta2 and theta3 = 10 +- delta** where **delta = 5 to 25 degrees**
4) Each user is surrounded by a large number of local scatterers corresponding to a **spread angle σ_{theta} = 2**
5) **The (k,l)th term of the channel covariance matrix** between the **base station** and the **mobile device** is $[\mathbf{R}(\theta, \sigma_\theta)]_{kl} = e^{j\pi(k-l)\sin\theta} e^{-\frac{(\pi(k-l)\sigma_\theta \cos\theta)^2}{2}}$
6) **SINR = 2** and **σ^2 = 1**

The Result