LG-OAM Link Security using Antenna Arrays

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***Abstract*—The application of Orbital Angular Momentum (OAM) beams in wireless communication is a relatively new area of study. Unlike conventional transmissions with a plane wavefront, OAM beams feature a helical wavefront and require different receiving architectures to decode the transmitted signal. This research studies the capabilities of eavesdropping on wireless communication systems using OAM. Demonstrating that eavesdropping has a low impact on OAM link security by simulating a Laguerre-Gaussian beam capable of carrying OAM and using different eavesdropping locations to analyze the receiving capabilities of each configuration. The research also provides valuable insight into designing and manufacturing antenna array hardware using computer design software and 3D printing techniques.**

I. Introduction

The rotational motion of Electromagnetic waves is comprised of Spin Angular Momentum Orbital and Orbital Angular Momentum, while the former describes the degree of twist of a beam on its own axis, Orbital angular momentum describes the degree of twist of a beam around an axis, Fig. I In a beam carrying OAM the initial plane wave is multiplied by the azimuth phase factor $\mathrm{e}^{\mathrm{i}\phi l}$ where ϕ is the angle about the propagation axis [1].

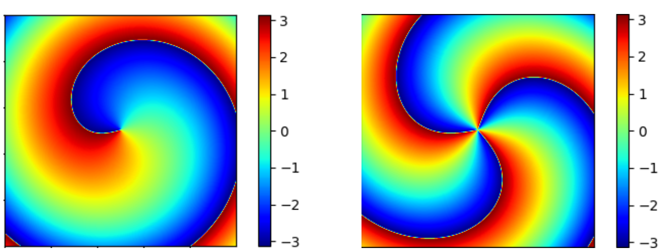

Fig. 1. The phase varies by the amount of $2\pi l$ as the angle goes from 0 to 2π. The figure on the left shows the phase as l=1, while the figure on the right shows the phase as l=3.

A number of different optical beam modes can carry OAM, including higher order Bessel or Hermite–Gauss beams [5], however, for the sake of simplicity this research centers on Laguerre-Gaussian beams as they are capable of carrying OAM and its electric field equation can be used to easily represent the characteristics of OAM in a simulated environment, the field of the LG-beam that carries OAM is represented as (1) where $E_0 = 1$, $w(z) = w_0\sqrt{1 + (z/z_R)}$ is the beam radius at a distance of z in meters, $k = (2\pi n)/\lambda$ where n is the refractive index of the environment being used, $Z_R = \frac{\pi w_0^2}{\lambda}$ is the Rayleigh range, $L_p^{|l|}$ refers to the generalized Laguerre polynomial from p to $|l|$, and the arctangent term represents the Gouy phase of the beam [6].

$$E_{lp}(r,z,\phi) = \frac{E_0}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} \mathrm{e}^{-r^2/w^2(r)} L_p^{|l|}\left(\frac{2r^2}{w^2(r)}\right) \times \mathrm{e}^{-\mathrm{i}kr^2z}/\left(2\left(z^2+z_R^2\right)\right)\mathrm{e}^{-\mathrm{i}\phi l+\mathrm{i}(2p+|l|+1)\arctan(z/z_R)} \tag{1}$$

Lastly, r represents the distance from the axis in the transverse plane, and the index p characterizes the radial structure of the l mode.

A. Uniform Circular Array Antenna

A UCA is defined as a set of multiple connected antennas that are equidistant to each other and work together as a single unit Fig.2, this type of antenna requires precise Tx - Rx alignment and is able to produce OAM by shifting the phase of each antenna.

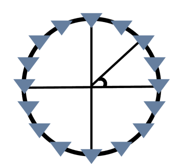

Fig. 2. 16 Element UCA.

UCA-A's are capable of generating multiple OAM modes of up to $l < N/2$ where N is the number of antennas and l the OAM mode, the phase difference of the UCAA is defined by $2\pi l/N$ The electric field of a UCAA is defined with the following Equation (2) where k is the angular wave number $2\pi L/\lambda$, r is the position vector of point P, R_n, and $\phi_n = 2\pi n/N$ is the position vector, and azimuth angle of the n^{th} element, respectively.

$$E(\boldsymbol{r}) = \sum_{n=0}^{N-1} \frac{A_n e^{-ik|\boldsymbol{r}-\boldsymbol{r}_n|}e^{il\phi_n}}{|\boldsymbol{r}-\boldsymbol{r}_n|} \approx A_n\frac{e^{-ikr}}{r}\sum_{n=0}^{N-1} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}_n+l\phi_n)} \approx A_n\frac{e^{-ikr}}{r}Ni^l e^{il\phi}J_l(ka\sin\theta) \tag{2}$$

Lastly the term $J_l(ka\sin\theta)$ signifies the lth order Bessel function of the first kind, and A_n represents the amplitude of each antenna [5].

B. Superposition of OAM modes

Since a characteristic of OAM beams is the orthogonality between different values of l, a UCA antenna is able to transmit ± 1, 2, 3, ... l modes in multiplexed data packets, as there is no interference/cross-talk between them as they travel through the same space.

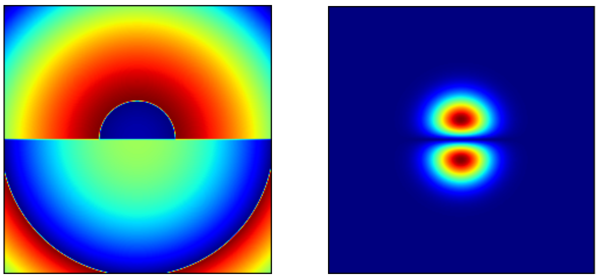

Fig. 3. Phase (Left) and Intensity (Right) of superposition of l modes ± 1.

Superposition of states allows for a higher spectral efficiency of a system, leading to better overall packet transmission as the information can be loaded onto different OAM modes and then multiplexed, in the receiving side, the packet can then be demultiplexed using many methods, however, an interesting one is the geometrical transformations of the helical wave into a tilted plane wave seen in Fig.4 initially proposed by [7].

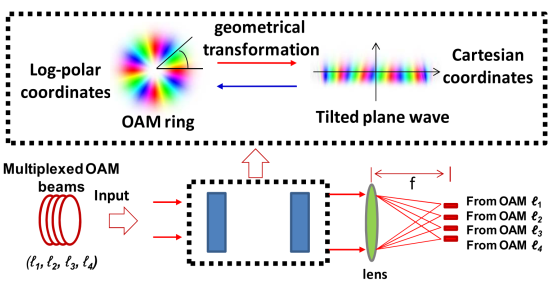

Fig. 4. Method for multiplexing - demultiplexing of OAM modes. Figure taken from [7]

II. UCA DESIGNING PROCESS

In order to achieve reliable OAM modes using a UCA-A, we require a precise level of angle and length adjustment for each antenna, which in the case of this project the UCA has been designed to house up to 32 antennas in various radii ranging from 15 cm to 50 cm.

Each circle of the array has a diameter of 9.25mm at an angle of 11.25 degrees from each other. In order to achieve this level of precision, a set of 32 3D - printed parts have been designed in CAD in order to form the UCA.

The design consists on 16 Female/Male parts that combine to form a circle of a 1-meter radius and as seen in Fig.5 the design features a hollow center in order to reduce printing time and use of material.

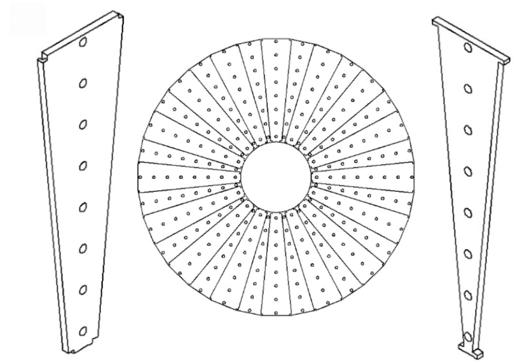

Fig. 5. Solid-works design of 3D printed UCA Male(Left)-Female(Right).

III. UCA CONSTRUCTION PROCESS

- The entire design was printed with 5 kilograms of Polylactic Acid Filament (PLA) on a Raise3D 2 Dual over a time period of 2 weeks for all 32 individual parts.

- The initial assembly had weak structural support so a wood frame was added for better handling.

- The final product is able to house all 32 antennas with little to no angle/length errors as seen in Fig.6 however, even though this final assembly meets the design criteria to a relatively close degree, there's still room for improvement as the plastic material is still fairly radio reflective, and as such a radio wave absorbing material could be used to coat the assembly, other improvements would be on the structural support of the assembly as the antenna requires a high level of alignment in order to prevent transmission errors.

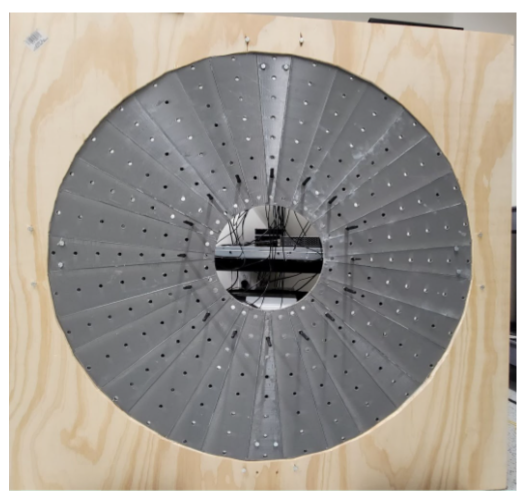

Fig. 6. Final assembled UCA.

IV. Simulating offset of an LG-OAM beam

The simulated Offset of EVE is defined by X, and Y coordinates that are later converted into polar and fed to the electric field of the LG-OAM equation.

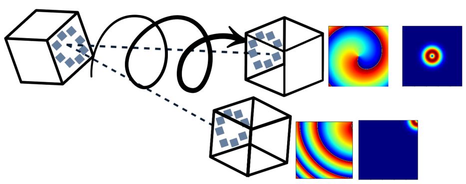

Fig. 7. Alice - Bob - Eve environment.

Using the simulated offset in Fig.7 we can see that TX - EVE Link power is dramatically decreased as the beam changes position. However, In addition to alignment, EVE is required to apply the conjugate phase in order to receive the OAM mode, thus requiring an exceptionally strong EVE in order to accurately intercept the beam.

V. Purity of an offset LG-OAM beam

A method to calculate the quality of an OAM beam is by comparing the phase difference between two phase points with the phase difference between two offset phase points [8], this is done in order to understand the decrease in power link between Tx-Rx as it changes positions to a defined offset.

To calculate the purity of the mode, we implement the phase gradient method [8] which uses two beam phases, and the angle between the two measurements given by $\beta_{1,2}$ the l number of the OAM mode can then be calculated using the following expression (3).

$$l_{\mathrm{R}} = \frac{\phi_1 - \phi_2}{\beta_{1,2}} \tag{3}$$

However, if instead of calculating the purity between Alice-Bob we want to calculate the quality of Eve's reception we need to choose two phase points that correlate to the phase received by the eavesdropper and assign a radius that corresponds with the position of Eve. Plugging these two points into (3) and then dividing by the angle $\beta_{1,2}$ from Eve's relative origin as seen in Fig. 8.

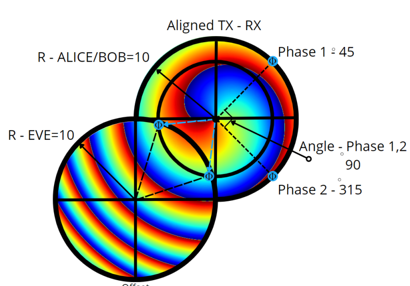

Fig. 8. Calculating purity of TX-RX (Alice-Bob) and Offset (Eve).

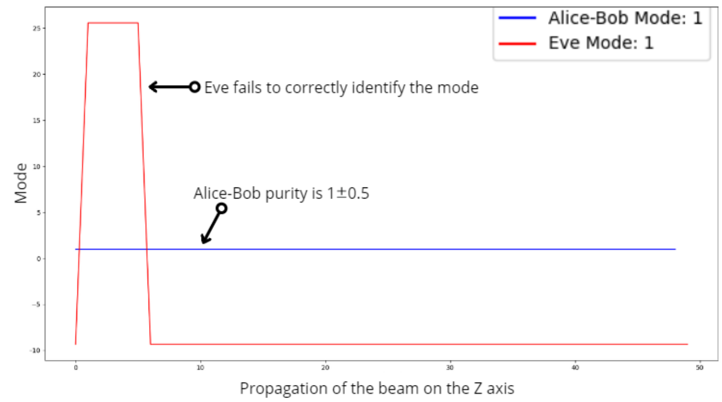

Fig. 9. Plot of estimated OAM purity, Alice-Bob vs Eve, mode l=1.

While this method is relatively exact for big angular separations, it results in poor approximations when used in small angular separations leading to mode estimation errors as seen in Fig.9, and while it can still be used to approximate the quality of the receiver it is not as reliable as the full phase gradient method seen in [8].

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