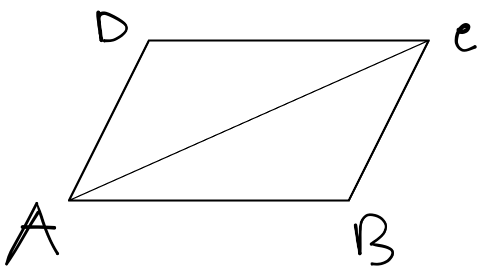
Problemi di trigonometria - Ferrante

1) 

$2p = ?$

$A = ?$

$\overline{AC} = 6\sqrt{2} + 6\sqrt{6}$

$C\hat{A}B = \pi/6$

$A\hat{C}B = \pi/4$

$$A\hat{B}C = \pi - (C\hat{A}B + A\hat{C}B) = \pi - \left(\frac{3\pi + 2\pi}{12}\right)$$

$$= \pi - \frac{5\pi}{12} = \frac{7}{12}\pi$$

$$\frac{\overline{AB}}{\sin A\hat{C}B} = \frac{\overline{AC}}{\sin A\hat{B}C}$$ Teorema dei seni

$$\frac{\overline{AB}}{\frac{\sqrt{2}}{2}} = \frac{6\sqrt{2} + 6\sqrt{6}}{\sin\left(\frac{7}{12}\pi\right)}$$

$$\frac{2\,\overline{AB}}{\sqrt{2}} = \frac{6\sqrt{2} + 6\sqrt{6}}{\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)}$$

$$\sqrt{2}\,\overline{AB} = \frac{6\sqrt{2} + 6\sqrt{6}}{\sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}}$$

$$\sqrt{2}\,\overline{AB} = \frac{6\sqrt{2} + 6\sqrt{6}}{\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}}$$

$$\sqrt{2}\,\overline{AB} = \frac{6(\sqrt{2} + \sqrt{6})}{\frac{\sqrt{2} + \sqrt{6}}{4}} \cdot \frac{4}{4}$$

$$\sqrt{2}\,\overline{AB} = \frac{6\cancel{(\sqrt{2} + \sqrt{6})}\cdot 4}{\cancel{\sqrt{2} + \sqrt{6}}}$$

$$\sqrt{2}\,\overline{AB} = 24$$

$$\overline{AB} = \frac{24}{\sqrt{2}} \Rightarrow \overline{AB} = \frac{\cancel{24}\sqrt{2}}{\cancel{2}} = 12\sqrt{2}$$

$$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB}\,\overline{AC}\cos C\hat{A}B$$ Teorema dei seni

$$\overline{BC}^2 = (12\sqrt{2})^2 + (6\sqrt{2} + 6\sqrt{6})^2 - \cancel{2}(12\sqrt{2})(6\sqrt{2} + 6\sqrt{6})\frac{\sqrt{3}}{\cancel{2}}$$

$$\overline{BC}^2 = 288 + 72 + 72\sqrt{12} + 216 - (12\sqrt{2})(6\sqrt{2} + 6\sqrt{6})(\sqrt{3})$$

$$\overline{BC}^2 = 288 + 72 + 144\sqrt{3} + 216 - 12\sqrt{6}(6\sqrt{2} + 6\sqrt{6})$$

$$\overline{BC}^2 = 288 + 72 + 144\sqrt{3} + 216 - 72\sqrt{12} - 432$$

$$\overline{BC}^2 = 288 + 72 + \cancel{144\sqrt{3}} + 216 - \cancel{144\sqrt{3}} - 432$$

$$\overline{BC}^2 = 576 - 432 = 144$$

$$\overline{BC} = \sqrt{144} = 12$$

$$2p = 2\overline{AB} + 2\overline{CB}$$

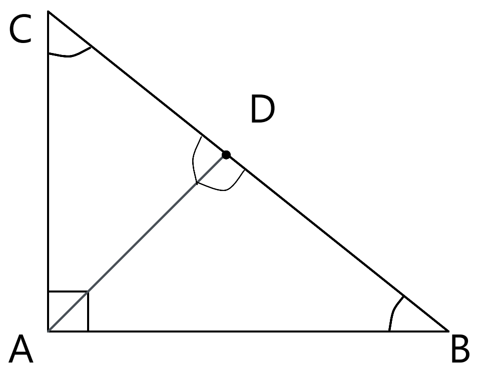
$$2p = 2\cdot 12\sqrt{2} + 2\cdot 12$$

$$2p = 24\sqrt{2} + 24$$

$$A = \frac{\overline{AB}\cdot\overline{BC}}{2}$$

$$A = \frac{12\sqrt{2}\cdot 12}{2} = \frac{144}{2}\sqrt{2} = 72\sqrt{2}$$

2)



AD = ?
CD = ?
BD = ?

$\overline{AC} = 8\,cm \qquad \hat{C} = \frac{\pi}{3} \Rightarrow \hat{B} = \frac{\pi}{6}$

$C\hat{A}D = B\hat{A}D = \frac{\pi}{4}$

$A\hat{D}C = \pi - (\hat{C} + C\hat{A}D) =$

$= \pi - \left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \pi - \frac{7}{12}\pi = \frac{5}{12}\pi$

$\frac{\overline{AD}}{\sin\hat{C}} = \frac{\overline{AC}}{\sin A\hat{D}C}$ Teorema dei seni

$\frac{\overline{AD}}{\sin\frac{\pi}{3}} = \frac{8cm}{\sin\frac{5}{12}\pi}$

$\frac{\overline{AD}}{\frac{\sqrt{3}}{2}} = \frac{8cm}{\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)}$

$\frac{2\overline{AD}}{\sqrt{3}} = \frac{8cm}{\sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}}$

$\frac{2\sqrt{3}}{3}\overline{AD} = \frac{8cm}{\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}}$

$\frac{2\sqrt{3}}{3}\overline{AD} = \frac{8cm}{\frac{\sqrt{2}+\sqrt{6}}{4}}$

$\frac{2\sqrt{3}}{3}\overline{AD} = \frac{32cm}{\sqrt{2}+\sqrt{6}} \cdot \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}$

$\frac{2\sqrt{3}}{3}\overline{AD} = \frac{\cancel{32}(\sqrt{2}-\sqrt{6})}{\cancel{-4}}$

$\frac{2\sqrt{3}}{3}\overline{AD} = -8(\sqrt{2}-\sqrt{6})$

$\frac{3}{2\sqrt{3}} \cdot \frac{2\sqrt{3}}{3}\overline{AD} = -8\sqrt{2} + 8\sqrt{6} \cdot \frac{3}{2\sqrt{3}}$

$\overline{AD} = -\frac{\cancel{24}\sqrt{2}}{\cancel{2}\sqrt{3}} + \frac{\cancel{24}\sqrt{\cancel{6}}}{\cancel{2}\sqrt{\cancel{3}}}$

$\overline{AD} = -\frac{12\sqrt{2}}{\sqrt{3}} + 12\sqrt{2}$

$\overline{AD} = -4\sqrt{6} + 12\sqrt{2}$

$A\hat{D}B = \pi - (B\hat{A}D + \hat{B}) =$

$= \pi - \frac{\pi}{4} - \frac{\pi}{6} = \frac{12\pi - 3\pi - 2\pi}{12} = \frac{7}{12}\pi$

Secondo teorema dei triangoli rettangoli

$\overline{AB} = \overline{AC} \cdot \tan\hat{C} = 8\,cm \cdot \tan\frac{\pi}{3} = 8\sqrt{3}$

$\overline{BC} = \frac{\overline{AC}}{\sin\hat{B}} = \frac{8\,cm}{\sin\left(\frac{\pi}{6}\right)} = \frac{8cm}{\frac{1}{2}} = 8\,cm \cdot 2 = 16\,cm$ Primo teorema dei triangoli rettangoli

Circonferenza circoscritta al triangolo

$r_{ADC} = \frac{\overline{AD}}{2\sin(\hat{C})} = \frac{12\sqrt{2}-4\sqrt{6}}{2\sin\left(\frac{\pi}{4}\right)} = \frac{12\sqrt{2}-4\sqrt{6}}{\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}} = \frac{(12\sqrt{2}-4\sqrt{6})\cdot\sqrt{3}}{3} = \frac{12\sqrt{6}-4\sqrt{18}}{3} = \frac{12\sqrt{6}-12\sqrt{2}}{3} = \frac{\cancel{3}(4\sqrt{6}-4\sqrt{2})}{\cancel{3}} = 4\sqrt{6}-4\sqrt{2}\,cm$

Teorema della corda

$\overline{CD} = 2r_{ADC} \cdot \sin C\hat{A}D = 2\cdot(4\sqrt{6}-4\sqrt{2})\cdot\sin\frac{\pi}{4} = \cancel{2}\cdot(4\sqrt{6}-4\sqrt{2})\cdot\frac{\sqrt{2}}{\cancel{2}} = 4\sqrt{12} - 8 = 8\sqrt{3} - 8\,cm$

$\overline{BD} = \overline{BC} - \overline{CD} = 16\,cm - (8\sqrt{3}-8)\,cm = 16\,cm - 8\sqrt{3} + 8 = 24 - 8\sqrt{3}\,cm$

≈ 10,144
16 > 10,144

AD = $12\sqrt{2} - 4\sqrt{6}$ cm
CD = $8\sqrt{3} - 8$ cm
BD = $24 - 8\sqrt{3}$ cm