Activity Set 1.4
ARRAYS WITH BLACK AND RED TILES

PURPOSE
To learn how to model rectangular arrays with black and red tiles and determine the net value of the arrays. To explore the result of flipping over columns and rows on the net value of a rectangular array. To learn how to use edge pieces to keep track of flipped columns and rows in a rectangular array of black and red tiles. To learn about minimal arrays, minimal collections and the empty array.

MATERIALS
Black and red tiles with black and red edge pieces

✓ No calculators

INTRODUCTION

Rectangular Arrays
A Rectangular Array is a rectangular arrangement of numbers or objects.

Rectangular Array Examples

This is a rectangular array of numbers.
This array has *3 rows* and *4 columns;* it is a
3 × 4 Array
Row then column

1	4	5	2
5	5	4	7
2	0	10	1

This is a rectangular array of black tiles
This array has *2 rows* and *5 columns*; it is a
2 × 5 Array

Rectangular Array Terms

Edge Pieces and Edge Sets are defined in context, see activity 2

Minimal Arrays, Minimal Collections and Minimal Edge Sets are defined in context, see activity 8.

The Empty Array and the Non-Empty Array are defined in context, see activity 8.

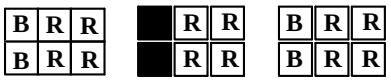
EQUIVALENT ARRAYS

Orientation on the page does not distinguish arrays, a 1 × 3 array and a 3 × 1 array with the same edge sets are equivalent arrays and only one should be given as a solution to a question.

Equivalent Arrays

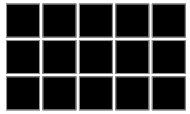
SKETCHING TIPS

Sketching Arrays
When sketching an array of black and red tiles, show the rectangular shape of the array and the square shape of the tiles. It may be easiest to sketch blank squares, lightly shade the black tiles and label the interiors of the red tiles R or to skip shading and just label both the black and the red squares with a B or an R, respectively.



Tiles touching or Tiles not touching
Sketching options

1. Use your black and red tiles and form the following 3 × 5 array of black tiles.



Array 1

a. (*) What is the net value of Array 1?

b. (*) Using your model of Array 1, pick one column and flip over all of the tiles in that column; what is the ending net value of the array? Return Array 1 to all black tiles and flip over all of the tiles in another column. Does the ending net value depend on which column you flip over? Sketch two such arrays and explain.

c. Return Array 1 to all black tiles; pick two columns and flip over all of the tiles in both columns; what is the ending net value of the array? Does the ending net value depend on which two columns you flip? Explain.

d. Return Array 1 to all black tiles and flip over all of the tiles in Column 1 (C1) and then flip over all of the tiles in Row 1 (R1). Note the tile that is in both C1 and R1 will be flipped twice. What is the ending net value of the array? Does the ending net value depend on which column and row you flip? Experiment with several combinations (C2 and R3, C3 and R1, etc.) and explain what happens if you flip one column and then flip one row. Sketch at least one such array.

e. Does the order of flipping matter? If you start with an all black Array 1, flip one row and then flip one column, what happens to the ending net value? Is the ending net value different than if you flip first the column and then the row?

f. Return Array 1 to all black tiles and flip over all of the tiles in Column 1 (C1), in Column 2 (C2) and then in Row 1 (R1). What is the ending net value of the array? Does the ending net value depend on which two columns and one row you flip? Experiment with several combinations of two columns and one row and explain what happens. Sketch and label at least one such ending array.

g. Summarize the ending net values you obtain by flipping various combinations of rows and columns in Array 1. Use black and red tiles to model each changing array as you fill out the table.

ARRAY 1: 3 × 5 array of black tiles

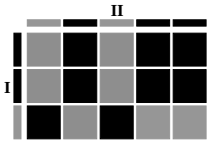
# of Columns To flip	# of Rows to flip	Ending Net Value
1	1	
1	2	
1	3	
2	1	
2	2	
2	3	
3	1	
3	2	
3	3	
4	1	
4	2	
4	3	
5	1	
5	2	
5	3	

Edge Sets

Note that it is difficult to look at an array of black and red tiles and determine which columns and rows have been flipped over. To keep track of the flipping information, we will use edge pieces and edge sets. Edge pieces indicate whether or not a row or column of an array has been flipped or turned over.

- ✓ An edge piece is a thin piece of black or red tile that is used to mark the edge dimensions of a black or red tiles (an edge piece has no height).
- ✓ A red edge piece at the end of a row or column indicates the row or column has been flipped.
- ✓ A black edge piece at the end of a row or column indicates the row or column has not been flipped.
- ✓ Edge pieces are designed to keep track of flipping and the dimensions of a rectangular array; the edge pieces themselves are not counted when determining the net value of an array.

Starting with an all black Array 1, these edge pieces indicate that Column 1, Column 3 and Row 3 have each been flipped over. The edge sets have been labeled I and II for reference.



Edge sets have net values just like tile collections as illustrated in this table:

Item	#Black	#Red	Net Value
Edge I	2B	1R	$^{+}1$
Edge II	3B	2R	$^{+}1$
Array	8B	7R	$^{+}1$

2. (*) Use black and red tiles, with edge pieces, to explore the connection between edge pieces and arrays. Use black and red tiles to model each array as you fill out the table.

Edge I		Edge II		Array		Net Values		
R	B	R	B	R	B	Edge I	Edge II	Array
3	0	0	2					
3	0	1	1					
3	0	2	0					
2	1	0	2					
2	1	1	1					
2	1	2	0					
1	2	0	2					
1	2	1	1					
1	2	2	0					
0	3	0	2					
0	3	1	1					
0	3	2	0					

3. In terms of edges: When is a tile in an array black? When is a tile in an array red?

4. For each row in the table, determine an array, with edge pieces, that has net value 0. Find five different collections of edge sets that work. Model with black and red tiles as needed.

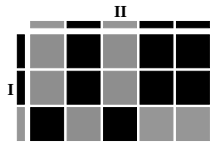
Edge I		Edge II		Array		Net Values		
R	B	R	B	R	B	Edge I	Edge II	Array
								0
								0
								0
								0
								0

5. (*) What do you notice about arrays of net value 0 and their corresponding edge sets? List your observations.

Inefficient Arrays

Many of the arrays and edge sets we have been working with contain net value zero pairs and do not seem to be the most efficient collections of black and red tiles or black and red edges.

6. Model the pictured *Array A-1* with black and red tiles and edge pieces.



Array A-1

a. (*) Remove two rows (and their edge pieces) without changing the net value of the array or the net value of either edge. Sketch the new array, Array A-2, and the new edges for Array A-2.

b. Now remove two columns (and their edge pieces) from Array A-2 without changing the net value of Array A-2 or the net value of either edge. Sketch this third array, Array A-3, include its edge pieces.

c. Can you remove any additional pairs of rows or columns without changing the net value of Array A-3 or the net value of either edge? Sketch any additional arrays, include their edge pieces.

d. Describe the pairs of rows and pairs of columns that you removed from the original array.

Minimal Arrays and Minimal Collections

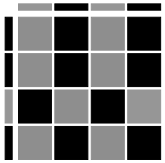
An array of black and red tiles in which all net value zero row pairs and all net value zero column pairs are removed is a Minimal Array.

In the same way, a collection of black and red tiles in which all net value zero pairs have been removed is a Minimal Collection. A Minimal Edge Set is a minimal collection of edge pieces.

7. Describe the characteristics of a minimal array:

8. Describe the characteristics of a minimal edge set:

9. Consider the following non-minimal array with net value 0.



Remove two columns or two rows (and their edge pieces) at a time without changing the net value of the array or the net value of either edge. Sketch each new array, include the edge pieces. Note which columns or rows you have removed.

The Empty Array

The array you end up with in the previous activity is a special type of minimal array, it is the Empty Array.

A Non-Empty Array is any array in which there are some black and/or red tiles.

10. Explain the following statement: The only minimal array with net value 0 is the Empty Array.

11. For each row in the table, determine an array, with edge pieces, that has net value $^{-}6$. Find four different collections of edge sets that work. Model with black and red tiles as needed.

Edge I		Edge II		Array		Net Values		
R	**B**	**R**	**B**	**R**	**B**	**Edge I**	**Edge II**	**Array**
								$^{-}6$
								$^{-}6$
								$^{-}6$
								$^{-}6$

12. (*) What do you notice about arrays of net value $^{-}6$ and their corresponding edge sets? List your observations; include notes on whether your arrays are or are not minimal arrays.

Homework Questions 1.4
ARRAYS WITH BLACK AND RED TILES

1. Sketch three different and non-equivalent non-empty arrays (Arrays A—C) with net value 0. Include edge pieces and use this information to fill out a table like this:

	Edge I		Edge II		Array		Net Values		
	R	**B**	**R**	**B**	**R**	**B**	**Edge I**	**Edge II**	**Array**
Array A									0
Array B									0
Array C									0

2. Sketch four different non-equivalent minimal arrays (Arrays A—D) with net value $^{+}4$. Include edge pieces and use this information to fill out a table like this:

	Edge I		Edge II		Array		Net Values		
	R	**B**	**R**	**B**	**R**	**B**	**Edge I**	**Edge II**	**Array**
Array A									$^{+}4$
Array B									$^{+}4$
Array C									$^{+}4$
Array D									$^{+}4$

3. For each of the following:
   i) Sketch a minimal array, with edge pieces, that satisfies the given conditions OR if no minimal array that meets the given conditions exists, explain why this is the case. Label the edge sets and arrays with their net value.

   ii) If more than one such minimal array exists, give two different examples of minimal arrays that work

   a. One edge has an odd number of black edge pieces and the net value of the array is $^{+}2$.

   b. One edge has an odd number of edge pieces and the net value of the array is $^{-}2$.

   c. Both edges have an odd number of edge pieces and the net value of the array is $^{+}8$.

   d. One edge is all red and one edge is all black; the net value of the array is $^{+}12$.

   e. One edge is all red and one edge is all black; the net value of the array is $^{-}10$.

Activity Set 1.5
MULTIPLYING INTEGERS WITH BLACK AND RED TILES

PURPOSE
To learn how to multiply integers using black and red tile arrays. To investigate the various rules of multiplying integers "Two positives is a positive, two negatives is a positive, etc." To explore the role of 0 in multiplication sentences.

MATERIALS
Black and red tiles with black and red edge pieces

✓ No calculators

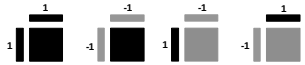
INTRODUCTION

Multiplication Terms

2 × 3 = 6: In this multiplication sentence, 2 and 3 are both Factors and 6 is the Product.

Using Black and Red Edge Pieces to Measure Side Dimensions (Lengths)

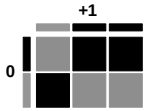
We saw in Activity Set 1.4 that we can use black and red edge pieces to indicate whether or not a row or column in an array of black and red tiles has been flipped. It also makes sense to think of a black and red edge pieces as measuring the side dimensions of arrays. For the rest of these materials, we will think of a black edge piece as measuring a side dimension of $^{+}1$ unit and a red edge piece as measuring a side dimension of $^{-}1$ unit. We will tend to call these "length 1" and "length $^{-}1$," although, of course, "length" is usually positive.



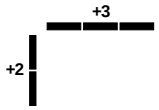
SKETCHING TIPS

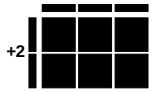
Labeling Edges

When sketching an array with edges, label the net value of each edge set as shown in this sketch.



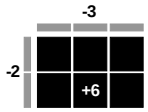
Sketching Multiplication
When sketching multiplication, show and briefly explain each step as shown in the following example. Label edges throughout.

<table>
<tr><td colspan="4">Find the solution to: $^{-}2 \times {}^{-}3 = ?$</td></tr>
<tr>
<td> 

Lay out all black edges; note black edge values</td>
<td> 

Fill in all black tiles</td>
<td> 

Flip row and corresponding edge pieces for $^{-}2$, note edge net value</td>
<td> 

Flip column and edge pieces for $^{-}3$, note edge net value</td>
</tr>
<tr><td colspan="4">Use the array to determine the final product
$^{-}2 \times {}^{-}3 = {}^{+}6$
 
</td></tr>
</table>

1. (*) Use your black and red tiles and form a minimal Array 1 with Edge Set I = $^{-}2$ and Edge Set II = $^{+}3$. Sketch and label your work

   a. What is the net value of Array 1?

   b. If you think of Array 1 as the multiplication of two factors; what are the two factors? What is the product of the two factors?

   c. What multiplication sentence do the edge sets and the array show? If you have not already done so; label the net values of the edge sets and the array on your sketch.

2. Use your black and red tiles to model each of the following multiplication questions. Sketch and label your work, including, for each step, labeling the net values of the edge sets and the array. Briefly explain each step. In each case, give the completed multiplication sentence the array and edge sets shows.

   a. $^{+}2 \times {}^{+}5 = ?$

   b. $^{-}2 \times {}^{+}5 = ?$

c. $^{+}3 \times {}^{-}4 = ?$

d. (*) $^{-}3 \times {}^{-}2 = ?$

3. Use your black and red tiles and form a non-empty Array 2 with Edge Set I = $^{+}3$ and Edge Set II = 0. Sketch and label your work

a. What is the net value of Array 2?

b. If you think of Array 2 and its edges as the multiplication of two factors; what are the two factors? What is the product of the two factors?

c. What multiplication sentence do the edge sets and the array show? If you have not already done so; label the net values of the edge sets and the array on your sketch.

d. Is this a minimal array? If you reduce Array 2 to a minimal array; what array will that be?

4. Use your black and red tiles and form a non-empty Array 3 with Edge Set I = 0 and Edge Set II = 0. Sketch and label your work

a. What is the net value of Array 3?

b. If you think of Array 3 and its edges as the multiplication of two factors; what are the two factors? What is the product of the two factors?

c. What multiplication sentence do the edge sets and the array show? If you have not already done so; label the net values of the edge sets and the array on your sketch.

d. Is this a minimal array? If you reduce Array 3 to a minimal array; what array will that be?

5. Complete each of the following sentences and explain why they are true. Use black and red tile array ideas to support your explanations.

a. The product of two positive factors is _____

b. The product of two negative factors is _____

c. The product of one positive factor and one negative factor is _____

d. The product of one positive or negative factor and the factor 0 is _____

e. The product of the factor 0 and the factor 0 is _____

Homework Questions 1.5
MULTIPLYING INTEGERS WITH BLACK AND RED TILES

1. For the following multiplication questions, use black and red tiles and edge pieces to model a sequence of minimal arrays showing the multiplication steps. Sketch and label your work; label the net values of each edge set for each array and also the net value of the product on the last array. Briefly explain each step. Identify factors and products. In each case, give the completed multiplication sentence the array and edge sets shows.

   a. $^{+}3 \times {}^{+}4 = ?$

   b. $^{+}3 \times {}^{-}4 = ?$

   c. $^{-}3 \times {}^{-}4 = ?$

2. Describe how you would explain to an elementary school student how the signs of factors relate to the sign of the product when multiplying integers. Be sure to explain the whole idea, not just how to short cut by changing signs. Use black and red tile arrays with edge pieces in your explanation.

3. Describe how you would explain to an elementary school student why multiplying an integer by a factor of 0 gives a product of 0. Use black and red tile arrays with edge pieces in your explanation.