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### Homework 1.1 (scores=7)

HW 1: Mean-square error (MSE) — Alternative calculation

Can you show that the mean squared error of an estimator  $\hat{\Theta}$  can also be calculated as:

$$\mathsf{MSE}_{\theta}(\hat{\Theta}) = \mathsf{E}_{\hat{\Theta}}[(\hat{\Theta} - \theta)^2] = \mathsf{Var}_{\hat{\Theta}}[\hat{\Theta}] + \mathsf{Bias}_{\theta}(\hat{\Theta})^2. \tag{19}$$

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# Homework 1.2 (scores=10)

#### HW 2: Example for sample mean and variance

Let  $X_1, X_2, ...$  be a random sample from a distribution with mean  $\mu$  and variance  $\sigma^2$ . Note that the empirical variance is given by the following equation and is not the same as the sample variance.

$$S_{\mu}^{2} = \frac{1}{n} \sum_{i} (X_{i} - \bar{X})^{2} \tag{20}$$

- Q1. Is the empirical variance, defined as Eq.(20), an unbiased estimator for variance  $\sigma^2$ ? [Check if  $\mathbf{E}[S_{\mu}^2] \sigma^2 = 0$ ?]
- Q2. Is sample variance  $S^2$  an unbiased estimator for variance  $\sigma^2$ ?
- Q3. At n=10,100,1000,10000, and simulate the bias of both estimators, i.e.,  $E[S_{\mu}^2]$  and  $E[S^2]$ . [Coding simulations are in Github]
  - $\bullet$  Write the description for each simulation at different n values.
  - 2 Provide **two sets of plots** for the estimators, *i.e.*,  $S^2$  and  $S^2_{\mu}$ , *i.e.*, the histogram of estimators, the mean of estimators, and the true variance.

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## Homework 1.3 (scores=3)

#### HW 3: Method of Moment for exponential distribution

Let  $X_1, X_2, ..., X_n$  be the random sample from an exponential distribution with parameter  $\lambda$ , *i.e.*,  $e^{rate=\lambda}$ .

- Q1. What is the estimator for  $\lambda$  using method of moments, *i.e.*,  $\lambda_{MME}$ ?
- Q2. Is the estimator biased or unbiased?
- Q3. What is the efficiency (MSE) of  $\lambda_{MME}$ ?
- Q4. Is  $\lambda_{MME}$  consistent for  $\lambda$  ?  $\Rightarrow$  HOMEWORK

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