Homework 1.1 (scores=7)

HW 1: Mean-square error (MSE) — Alternative calculation

Can you show that the mean squared error of an estimator $\hat{\Theta}$ can also be calculated as:

$$\mathrm{MSE}_\theta(\hat{\Theta}) = \mathrm{E}_{\hat{\Theta}}[(\hat{\Theta} - \theta)^2] = \mathrm{Var}_{\hat{\Theta}}[\hat{\Theta}] + \mathrm{Bias}_\theta(\hat{\Theta})^2. \tag{19}$$

Homework 1.2 (scores=10)

HW 2: Example for sample mean and variance

Let $X_1, X_2, \ldots$ be a random sample from a distribution with mean μ and variance σ^2. Note that the empirical variance is given by the following equation and is not the same as the sample variance.

$$S_\mu^2 = \frac{1}{n}\sum_i (X_i - \bar{X})^2 \tag{20}$$

- Q1. Is the empirical variance, defined as Eq.(20), an unbiased estimator for variance σ^2? [**Check if** $\mathbf{E}[S_\mu^2] - \sigma^2 = 0$**?**]
- Q2. Is sample variance S^2 an unbiased estimator for variance σ^2?
- Q3. At $n = 10, 100, 1000, 10000$, and simulate the bias of both estimators, *i.e.*, $\mathrm{E}[S_\mu^2]$ and $\mathrm{E}[S^2]$.
  1. Write the description for each simulation at different n values.
  2. Provide **two sets of plots** for the estimators, *i.e.*, S^2 and S_μ^2, *i.e.*, the histogram of estimators, the mean of estimators, and the true variance.

Homework 1.3 (scores=3)

HW 3: Method of Moment for exponential distribution

Let $X_1, X_2, ..., X_n$ be the random sample from an exponential distribution with parameter λ, *i.e.*, $e^{rate=\lambda}$.

- Q1. What is the estimator for λ using method of moments, *i.e.*, λ_{MME}?
- Q2. Is the estimator biased or unbiased?
- Q3. What is the efficiency (MSE) of λ_{MME}?

- Q4. Is λ_{MME} consistent for λ ? $\Rightarrow$ HOMEWORK