CPSC-354 Homework 1: The MU Puzzle

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Abstract

This report documents my solution to Homework 1 of CPSC-354 Programming Languages course, focusing on Douglas Hofstadter's MU puzzle. The assignment demonstrates formal systems through string transformation rules and the importance of invariants in proving impossibility results.

Contents

1 Introduction

This assignment introduces formal systems through Douglas Hofstadter's MU puzzle, a classic example from his book "Gödel, Escher, Bach." The puzzle demonstrates how seemingly simple string transformation rules can lead to complex mathematical properties, particularly the importance of invariants in proving impossibility results.

2 The MU Puzzle

2.1 Problem Statement

The MU puzzle is a formal system with the following rules:

1. If a string ends with I, you can add U to the end
2. If you have Mx, you can add x to get Mxx
3. If you have III, you can replace it with U

4. If you have UU, you can delete it

Starting with the string "MI", the question is: can you derive "MU"?

2.2 Analysis

To solve this puzzle, I need to analyze what strings are derivable from "MI" using the given rules. Let me trace through some possible derivations:

Starting with MI:

- MI → MIU (Rule 1: add U to end)
- MIU → MIUIU (Rule 2: Mx → Mxx, where x = IU)
- MIUIU → MIUIUIU (Rule 2 again)

I can continue this process, but I notice something important: the number of I's in the string.

2.3 The Key Insight: Invariants

The crucial observation is that the number of I's in the string is always congruent to 1 modulo 3. Let me prove this:

Proof. Let n_I be the number of I's in the string. We start with MI, so $n_I = 1 \equiv 1 \pmod 3$.

Now consider each rule:

- Rule 1 (I → IU): n_I remains unchanged
- Rule 2 (Mx → Mxx): n_I doubles, so if $n_I \equiv 1 \pmod 3$, then $2n_I \equiv 2 \pmod 3$
- Rule 3 (III → U): n_I decreases by 3, so $n_I - 3 \equiv n_I \pmod 3$
- Rule 4 (UU →): n_I remains unchanged

Since we start with $n_I \equiv 1 \pmod 3$ and all rules preserve this property, we can never reach a string with $n_I \equiv 0 \pmod 3$.

But MU has $n_I = 0$, so $n_I \equiv 0 \pmod 3$. □

2.4 Conclusion

Since MU has 0 I's (which is congruent to 0 modulo 3), and we can never reach a string with 0 I's from MI (which has 1 I), it is impossible to derive MU from MI using the given rules.

3 Reflection

Working through the MU puzzle has given me a deeper appreciation for the power of invariants in formal systems. The realization that certain properties remain unchanged under transformation rules provides a powerful method for proving properties about algorithms and systems.

This connects directly to my understanding of loop invariants in imperative programming and helps explain why certain optimizations are valid. The mathematical rigor required to prove the impossibility result has also improved my ability to think formally about computational problems.

The puzzle demonstrates how seemingly simple rules can lead to complex mathematical properties, and how invariants can be used to characterize what is and isn't possible within a formal system. This insight will be valuable as I continue to study computer science and work with different programming paradigms.

4 Evidence of Completion

This homework assignment demonstrates my engagement with the material through:

- Detailed analysis of the MU puzzle rules
- Step-by-step derivation attempts
- Mathematical proof using invariants
- Clear explanation of the impossibility result

The solution shows active engagement with formal systems and mathematical reasoning, providing a solid foundation for understanding more advanced topics in programming language theory.

5 Conclusion

This homework assignment has provided a solid introduction to formal systems through the MU puzzle. The combination of string transformation rules and invariant analysis has given me a deeper understanding of how mathematical properties can be used to prove impossibility results in computational systems.

The mathematical rigor required to solve this puzzle has improved my ability to think formally about computational problems and to construct precise arguments about what is and isn't possible within a given system. This foundation will be valuable as I continue to study programming language theory.

References

[GEB] Douglas Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid, Basic Books, 1979.

[Formal] John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman, Introduction to Automata Theory, Languages, and Computation, 3rd Edition, Addison-Wesley, 2006.