A Method for Estimation and Filtering of Gaussian Noise in Images

Fabrizio Russo, *Member, IEEE*

***Abstract*—A new approach to the restoration of images corrupted by Gaussian noise is presented. The proposed method combines a nonlinear algorithm for detail preserving smoothing of noisy data and a technique for automatic parameter tuning based on noise estimation. As a key feature, the method does not require any *a priori* knowledge about the amount of noise corruption. Experimental results show that the filtering performance of the proposed approach is very satisfactory.**

***Index Terms*—Gaussian noise, image enhancement, image processing, image restoration, nonlinear filters.**

I. Introduction

THE development of techniques for noise removal is of paramount importance for image-based measurement systems. Indeed, noise can significantly decrease the accuracy of very critical operations such as feature extraction and object recognition. In this respect, noise having Gaussian-like distribution is very often encountered in acquired data [1]. In order to smooth out this kind of noise, many filtering architectures have been proposed in the literature [2]. Since the goal of the filtering action is to cancel noise while preserving the integrity of edge and detail information, nonlinear approaches generally provide more satisfactory results than linear techniques [3]–[6]. However, a common drawback of the practical use of these methods is the fact that they usually require some "a priori" knowledge about the amount of noise corruption. This information is necessary to perform the optimal choice of parameter values and/or threshold selections. Unfortunately, such information is very often not available in real applications. Even if some methods for automatic threshold selection have been recently proposed in very specific areas (medical images showing one-peak or bi-peak histograms [7]), the problem of automatic parameter estimation still remains an open issue.

In this paper, a new approach to estimation and filtering of Gaussian noise is presented. The proposed technique is based on a simple nonlinear algorithm for detail-preserving smoothing, whose filtering behavior depends on one parameter only. Automatic parameter tuning is performed by a new step-by-step procedure that takes into consideration the mean square error (MSE) between subsequent pairs of processed images. As a result, an estimate of the optimal parameter value can be obtained without any "a priori" information about the variance of the Gaussian noise. This paper is organized as follows. Section II presents the new filtering architecture, Section III describes the method for automatic parameter tuning, Section IV discusses the experimental results, Section V analyzes the filtering performance, Section VI focuses on potential applications and, finally, Section VII reports conclusions.

$x_1(\mathbf{n})$	$x_2(\mathbf{n})$	$x_3(\mathbf{n})$
$x_8(\mathbf{n})$	$x(\mathbf{n})$	$x_4(\mathbf{n})$
$x_7(\mathbf{n})$	$x_6(\mathbf{n})$	$x_5(\mathbf{n})$

Fig. 1. 3×3 window.

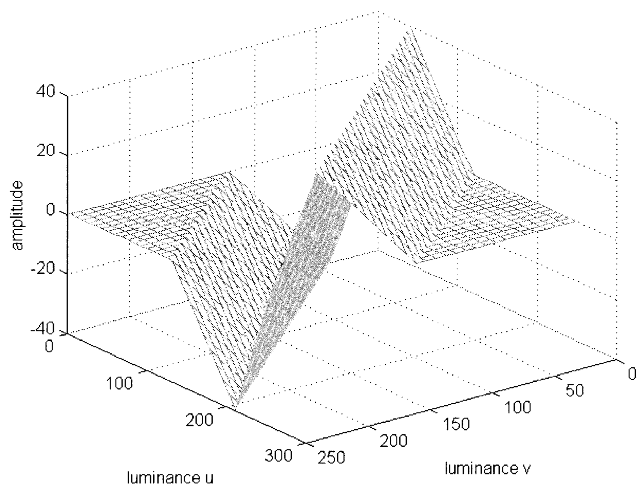


Fig. 2. Graphical representation of relation (2) ($p = 40$).

II. Proposed Filter

Let us suppose we deal with digitized images having L gray levels. Let $x(\mathbf{n})$ be the pixel luminance at location $\mathbf{n} = [n_1, n_2]$ in the noisy image. Let $x_1(\mathbf{n}), x_2(\mathbf{n}), \ldots, x_N(\mathbf{n})$ be the group of $N = 8$ neighboring pixels that belong to a 3×3 window around $x(\mathbf{n})$ (Fig. 1).

The output $y(\mathbf{n})$ of the filter (here called "zed filter") is defined by the following relationship:

$$y(\mathbf{n}) = x(\mathbf{n}) + \frac{1}{N} \sum_{i=1}^{N} \zeta\left(x_i(\mathbf{n}), x(\mathbf{n})\right) \tag{1}$$

Manuscript received June 15, 2002; revised January 17, 2003. This work was supported by the University of Trieste, Trieste, Italy.

The author is with the Dipartimento di Elettrotecnica Elettronica Informatica, University of Trieste, Trieste, Italy (e-mail: rusfab@univ.trieste.it).

Digital Object Identifier 10.1109/TIM.2003.815989

where

$$\zeta(\mathrm{u},\mathrm{v}) = \begin{cases} \mathrm{u}-\mathrm{v} & |\mathrm{u}-\mathrm{v}| \le \mathrm{p} \\ \left(\frac{3\mathrm{p}-|\mathrm{u}-\mathrm{v}|}{2}\right)\mathrm{sgm}(\mathrm{u}-\mathrm{v}) & \mathrm{p} < |\mathrm{u}-\mathrm{v}| \le 3\mathrm{p} \\ 0 & |\mathrm{u}-\mathrm{v}| > 3\mathrm{p} \end{cases} \tag{2}$$

and p is an integer ($0 < \mathrm{p} < \mathrm{L}$). A graphical representation of relation (2) is depicted in Fig. 2 ($\mathrm{p} = 40$). The filtering mechanism takes into account the pixel luminances in the neighborhood. According to (2), this mechanism aims at gradually excluding pixel values that are very different from the central element, in order to avoid blurring the image details during noise removal. In particular, two opposite effects can be examined.

1) Let the luminances of the neighbors be very close to the value of the central pixel

$$|\mathrm{x_i}(\mathbf{n}) - \mathrm{x}(\mathbf{n})| \le \mathrm{p} \quad \mathrm{i} = 1, \ldots, \mathrm{N}. \tag{3}$$

In this case

$$\zeta\left(\mathrm{x_i}(\mathbf{n}), \mathrm{x}(\mathbf{n})\right) = \mathrm{x_i}(\mathbf{n}) - \mathrm{x}(\mathbf{n}) \tag{4}$$

and (1) becomes

$$\mathrm{y}(\mathbf{n}) = \frac{1}{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{x_i}(\mathbf{n}). \tag{5}$$

As a result, the filter performs the arithmetic mean of the pixel luminances in the neighborhood, thus realizing the maximum smoothing action.

2) On the contrary, let the neighboring values be very different from the central element, according to the following relationship:

$$|\mathrm{x_i}(\mathbf{n}) - \mathrm{x}(\mathbf{n})| > 3\mathrm{p} \quad \mathrm{i} = 1, \ldots, \mathrm{N}. \tag{6}$$

In this case, (1) becomes

$$\mathrm{y}(\mathbf{n}) = \mathrm{x}(\mathbf{n}) \tag{7}$$

and the filter behaves as the identity filter, thus performing the maximum detail preservation (i.e., no filtering).

Intermediate situations are processed as a compromise between these opposite effects. In many cases the luminance of a neighboring pixel is not very close to the value of the central pixel, and not very far from this value, according to the following expression:

$$\mathrm{p} < |\mathrm{x_i}(\mathbf{n}) - \mathrm{x}(\mathbf{n})| \le 3\mathrm{p}. \tag{8}$$

Consequently, (2) can gradually reduce the importance of such neighboring pixels, as the luminance difference $|\mathrm{x_i}(\mathbf{n}) - \mathrm{x}(\mathbf{n})|$ becomes large. It should be observed that, according to (2), the filter behavior depends on the value of parameter p only. Large values produce a strong smoothing action. Small values, on the contrary, better preserve fine details and textures. The optimal choice of p depends on the amount of noise corruption in the input image and represents the value that would yield the minimum MSE between the filtered image and the original noise-free data. We can observe that the MSE by itself does not suffice to assess the quality of a processed image, especially where the final user is human. However, since we aim at implementing a fully automatic procedure for parameter tuning, the adoption of an objective criterion is necessary. We chose the MSE because it is conceptually simple and widely adopted in the literature. In the next section, we shall present the new method for estimating the optimal parameter value when only noisy data are available and the amount of noise corruption is unknown.

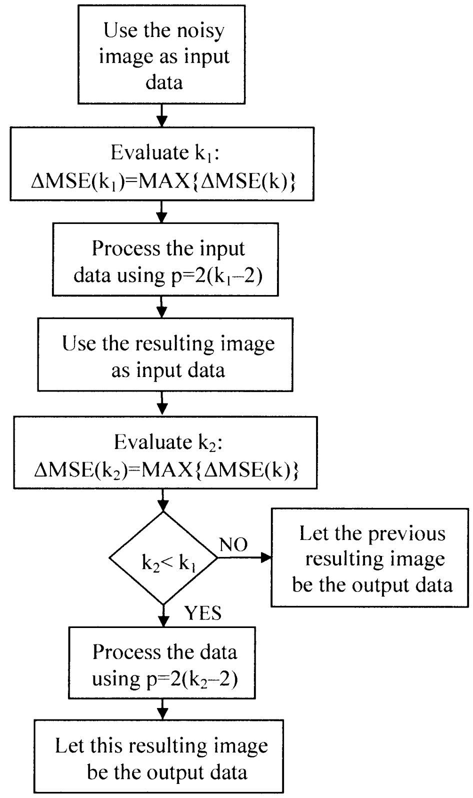


Fig. 3. Procedure for automatic parameter tuning.

III. New Procedure for Automatic Parameter Tuning

The method for automatic tuning of parameter p is completely based on experimentation. The proposed technique adopts a multipass procedure that takes into account the *progressive mean square error* between subsequent pairs of processed images. More formally, let $\mathrm{y}^{(\mathrm{k})}(\mathbf{n})$ represent the output of the filter when $\mathrm{p} = \mathrm{k}$. Thus, the progressive mean square error $\Delta\mathrm{MSE}(\mathrm{k})$ between the noisy image filtered with $\mathrm{p} = \mathrm{k}$ and the same image filtered with $\mathrm{p} = \mathrm{k} - 1$ is defined as follows:

$$\Delta\mathrm{MSE}(\mathrm{k}) = \frac{1}{\mathrm{M}} \sum_{\mathbf{n}\in\mathrm{F}} \left(\mathrm{y}^{(\mathrm{k})}(\mathbf{n}) - \mathrm{y}^{(\mathrm{k}-1)}(\mathbf{n})\right)^2 \tag{9}$$

where F denotes the set of M processed pixels. A brief flowchart of the procedure for automatic parameter tuning is shown in Fig. 3. This procedure operates as follows.

1) An image corrupted by Gaussian noise with unknown variance is assumed as input data.

TABLE I
FILTERING ERRORS ("BOATS")

σ	**MSE**	**MSE$_o$**
5	16.4	16.1
6	21.8	21.7
7	27.8	27.7
8	34.4	34.1
9	41.0	40.7
10	46.3	47.2
11	55.8	54.8
12	61.2	62.1

TABLE II
FILTERING ERRORS ("PENTAGON")

σ	**MSE**	**MSE$_o$**
5	18.6	18.0
6	23.8	23.8
7	30.2	29.9
8	35.9	36.5
9	43.5	43.5
10	49.5	50.9
11	60.8	58.8
12	66.0	67.1

TABLE III
FILTERING ERRORS ("CAMERAMAN")

σ	**MSE**	**MSE$_o$**
5	12.7	12.6
6	16.4	17.0
7	21.1	21.7
8	25.3	26.8
9	31.0	32.2
10	38.8	37.8
11	42.1	43.7
12	47.9	49.8

TABLE IV
FILTERING ERRORS ("BANK")

σ	**MSE**	**MSE$_o$**
5	12.9	12.8
6	16.7	17.2
7	22.0	21.9
8	26.1	27.2
9	32.5	32.8
10	37.2	38.7
11	44.6	45.1
12	51.1	51.9

2) By varying the value of parameter p from a minimum ($\mathrm{p}=2$) to a maximum ($\mathrm{p}=\mathrm{L}/4$), a collection of filtered images is obtained. At each step ($\mathrm{p}=\mathrm{k}$), the $\Delta\mathrm{MSE}(\mathrm{k})$ is evaluated. Let k_1 be the value that corresponds to the maximum: $\Delta\mathrm{MSE}(\mathrm{k}_1)=\mathrm{MAX}\{\Delta\mathrm{MSE}(\mathrm{k})\}$.
3) A heuristic estimate of the optimal value of parameter p is given by: $\mathrm{p}_1 = 2(\mathrm{k}_1 - 2)$. Thus, the input data are processed by setting $\mathrm{p}=\mathrm{p}_1$.
4) The resulting image is assumed as input data in order to (possibly) perform a second filtering action.
5) Again, by varying the value of parameter p, a collection of filtered images is obtained. Let k_2 be the value that corresponds to the maximum: $\Delta\mathrm{MSE}(\mathrm{k}_2)=\mathrm{MAX}\{\Delta\mathrm{MSE}(\mathrm{k})\}$.
6) If $\mathrm{k}_2 < \mathrm{k}_1$ proceed to the next step, otherwise stop the procedure and consider the previous resulting image as the output data.
7) A second-pass filtering is performed by choosing $\mathrm{p} = \mathrm{p}_2 = 2(\mathrm{k}_2 - 2)$.
8) The result represents the output image.

As above mentioned, the automatic tuning procedure is based on experimental results. Even if the estimate of p is currently based on a heuristic approach, the obtainable results are very promising, as it will be shown in the next section. In particular, the ability to perform a two-pass filtering action (if required), increases the effectiveness and the robustness of the overall method.

IV. VALIDATION OF THE METHOD

Many computer simulations have been performed in order to validate the method. Some results are reported in Tables I–IV. In these experiments we have chosen a collection of four well-known 256×256 test images having $\mathrm{L}=256$ gray levels: "Boats," "Pentagon," "Cameraman," and "Bank". For each picture, we have generated a set of noisy images by adding zero-mean Gaussian noise with variance ranging from 25 to about 150. The filtering performance has been evaluated by considering the MSE of the processed data with respect to the original uncorrupted image. In the second column of each table are reported the filtering errors corresponding to the results obtained by using the automatic tuning procedure described in the previous section. For a comparison, the errors corresponding to the optimal choices of p (one-pass processing) are reported in the third column ($\mathrm{MSE_O}$). In this case, the optimal values of p have been found by a simple search routine that took into account the original noise-free data as a reference. On the contrary, the images generated by our automatic tuning procedure have been obtained without taking into account the original uncorrupted data, in order to simulate a real application where only noisy data are available. This is the key advantage of the proposed method.

A plot of the MSE values is also reported in Figs. 4–7. As an example, let us consider the MSE values reported in Fig. 4 ("Boats"). It can be seen that the automatic procedure for parameter estimation can yield very satisfactory results. Indeed, the filtering performance is quite near the optimal one. The effect of the two-pass algorithm is more apparent if we consider the plot of MSE values reported in Figs. 6 and 7. We can observe that, in many cases, the MSE values given by the proposed method are lower than those yielded by the optimal one-pass filtering. The occurrence of a second processing step improves the results because the filter is designed to preserve the image structure during noise removal. The automatic tuning procedure aims at performing an appropriate choice of parameter p in order to avoid blurring the image details during a repeated application of the filter.

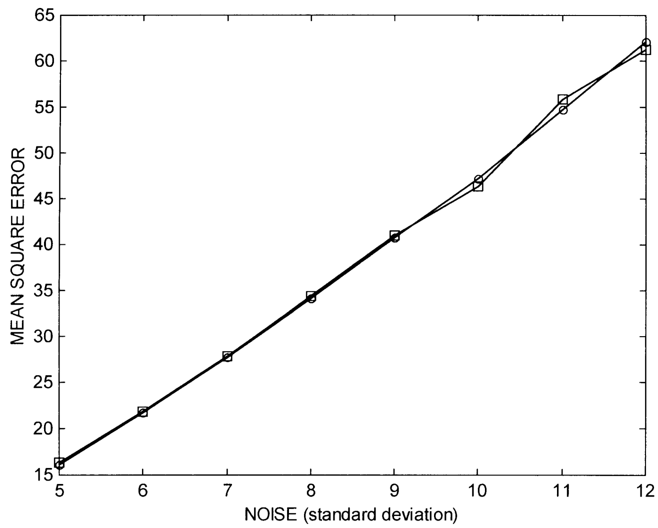


Fig. 4. Plot of the MSE values for the "Boats" image (squares denote the results yielded by the automatic tuning procedure; circles denote the optimal values corresponding to one-pass filtering).

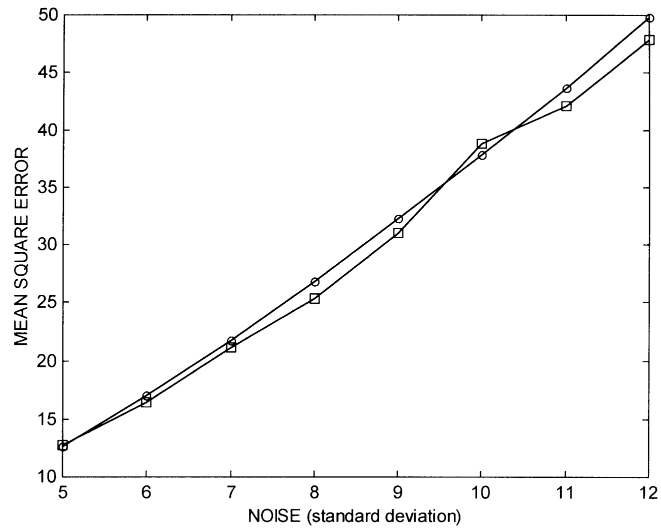


Fig. 6. Plot of the MSE values for the "Cameraman" image (squares denote the results yielded by the automatic tuning procedure; circles denote the optimal values corresponding to one-pass filtering).

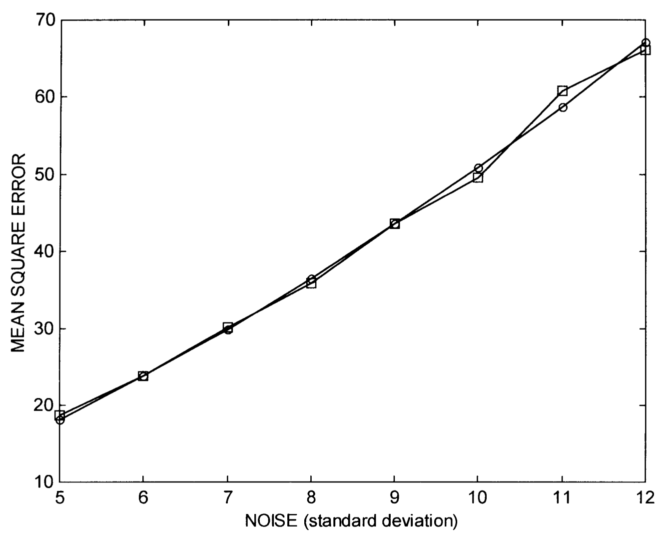


Fig. 5. Plot of the MSE values for the "Pentagon" image (squares denote the results yielded by the automatic tuning procedure; circles denote the optimal values corresponding to one-pass filtering).

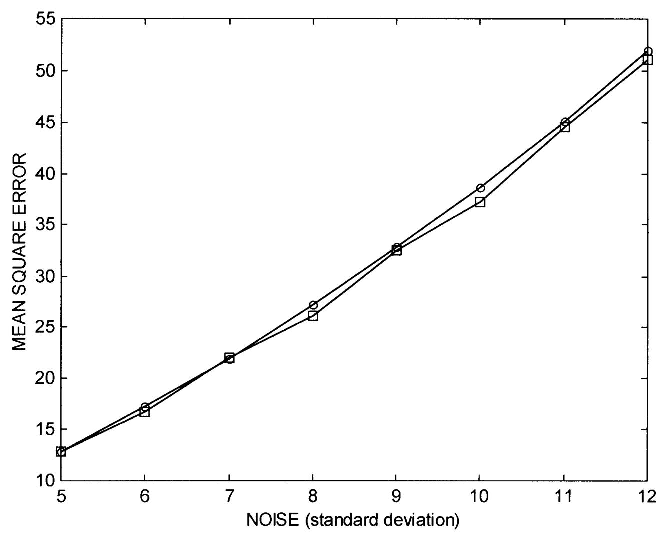


Fig. 7. Plot of the MSE values for the "Bank" image (squares denote the results yielded by the automatic tuning procedure; circles denote the optimal values corresponding to one-pass filtering).

V. Analysis of the Filtering Performance

In order to appraise the filtering performance of the proposed method, we considered for a comparison the rational filter RF1 [5] and the anisotropic diffusion technique adopting the Tukey edge-stopping function [8]. Rational filters and anisotropic diffusion algorithms presently represent some of the most powerful techniques in the literature. These filters are very effective for detail-preserving noise smoothing of image data and perform very well for *short-tailed* noise distributions such as Gaussian noise. Some results of the application of the mentioned filters to "Boats" and "Pentagon" corrupted by Gaussian noise are reported in Tables V and VI, respectively. We adopted the same input data as in Tables I and II in order to directly compare these results to those given by our method (see Section IV). Since anisotropic diffusion is typically an iterative process, a repeated application of this kind of filter was considered (third and fourth columns in Tables V and VI.).

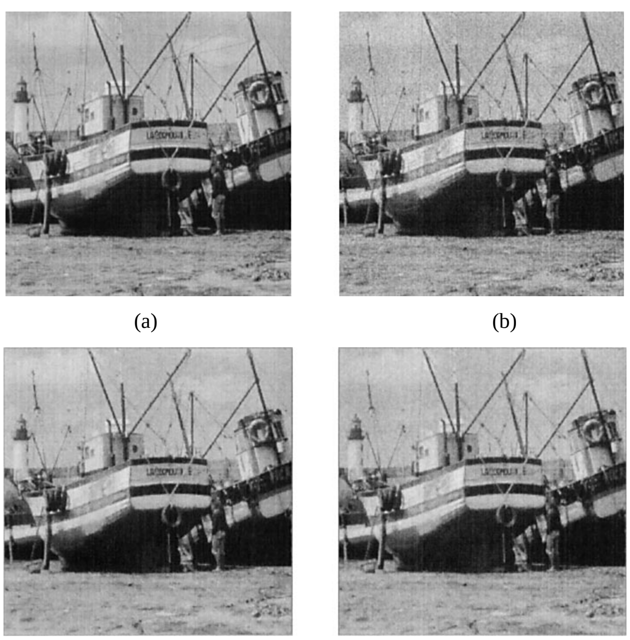
We chose the parameter settings for the RF1 filter by adopting $A = 6$ and by searching, for each noisy image, the optimal value of the parameter k [5]. Notice that no general method for automatic parameter tuning is currently available for this filter, so we chose the parameter values that minimized the MSE and the noise-free image was needed as a reference. We adopted a similar approach for the anisotropic diffusion technique. For a given number of iterations, the behavior of this filter basically depends on the parameter value that defines the threshold of diffusion. For each noisy image, we chose the values that minimized the MSE in three different cases (one-pass, five-passes, ten-passes). The noise-free image was needed as a reference for this filter too. Observing the results, we can conclude that the performance of our filter is

TABLE V
FILTERING ERRORS GIVEN BY OTHER TECHNIQUES ("BOATS" IMAGE)

σ	MSE rational filter	MSE anisotropic diffusion (1 pass)	MSE anisotropic diffusion (5 passes)	MSE anisotropic diffusion (10 passes)
5	16.2	16.1	17.1	18.6
6	21.7	21.8	22.7	25.2
7	27.7	28.1	28.6	32.0
8	34.3	35.0	34.6	39.2
9	40.7	42.3	40.9	46.3
10	46.9	50.0	46.8	53.3
11	54.9	58.4	53.6	61.3
12	63.4	67.1	60.1	68.9

TABLE VI
FILTERING ERRORS GIVEN BY OTHER TECHNIQUES ("PENTAGON" IMAGE)

σ	MSE rational filter (RF1)	MSE anisotropic diffusion (1 pass)	MSE anisotropic diffusion (5 passes)	MSE anisotropic diffusion (10 passes)
5	17.9	18.1	20.3	22.1
6	23.7	24.3	26.0	28.7
7	30.0	31.2	32.0	35.2
8	36.7	38.9	38.1	42.0
9	44.0	47.4	44.5	49.0
10	51.5	56.5	51.2	56.3
11	59.6	66.2	58.2	64.0
12	68.4	76.4	65.4	71.5



(a) (b) (c) (d)

Fig. 8. (a) Original image ("Boats"), (b) noisy image ($\sigma = 10$), (c) result yielded by the proposed method, and (d) result yielded by the RF1 technique.

comparable in terms of MSE values. However, as mentioned in Section II, the MSE cannot suffice to assess the quality of a processed image. For this reason, some results are depicted in Fig. 8, for visual inspection (for the sake of simplicity, only the result given by the rational filter has been reported for a comparison). In order to better appraise the noise cancellation behavior of the proposed method, the luminance values of a row are graphically depicted in Fig. 9. The original noise-free row number 20 (from top to bottom) is shown in Fig. 9(a). The presence of three masts is clearly perceivable as negative peaks along the waveform (dark pixels are represented by low luminance values). The data affected by Gaussian noise are shown in Fig. 9(b). The data processed by the proposed filter and the RF1 technique are depicted in Fig. 9(c) and (d), respectively. Finally, a detail of the processed images (first quadrant) is reported in Fig. 10 for visual inspection. Considering the data in Figs. 9 and 10, the good filtering behavior of the proposed method is apparent. Gaussian noise has been significantly reduced and the image details have been satisfactorily preserved. In particular, a (slightly) better performance of the proposed filter seems perceivable in the uniform regions of the image, where the effect of the noise can be more annoying from the point of view of the human perception.

VI. APPLICATIONS

Automatic restoration of image data is very likely to find potential applications in a number of different areas such as electromagnetic imaging of objects, medical diagnostics, non-destructive evaluation and testing, remote sensing, robotics, etc. Even if the proposed method has been described for gray level images, its extension to color pictures is rather straightforward. For example, if we deal with RGB images, we can apply the

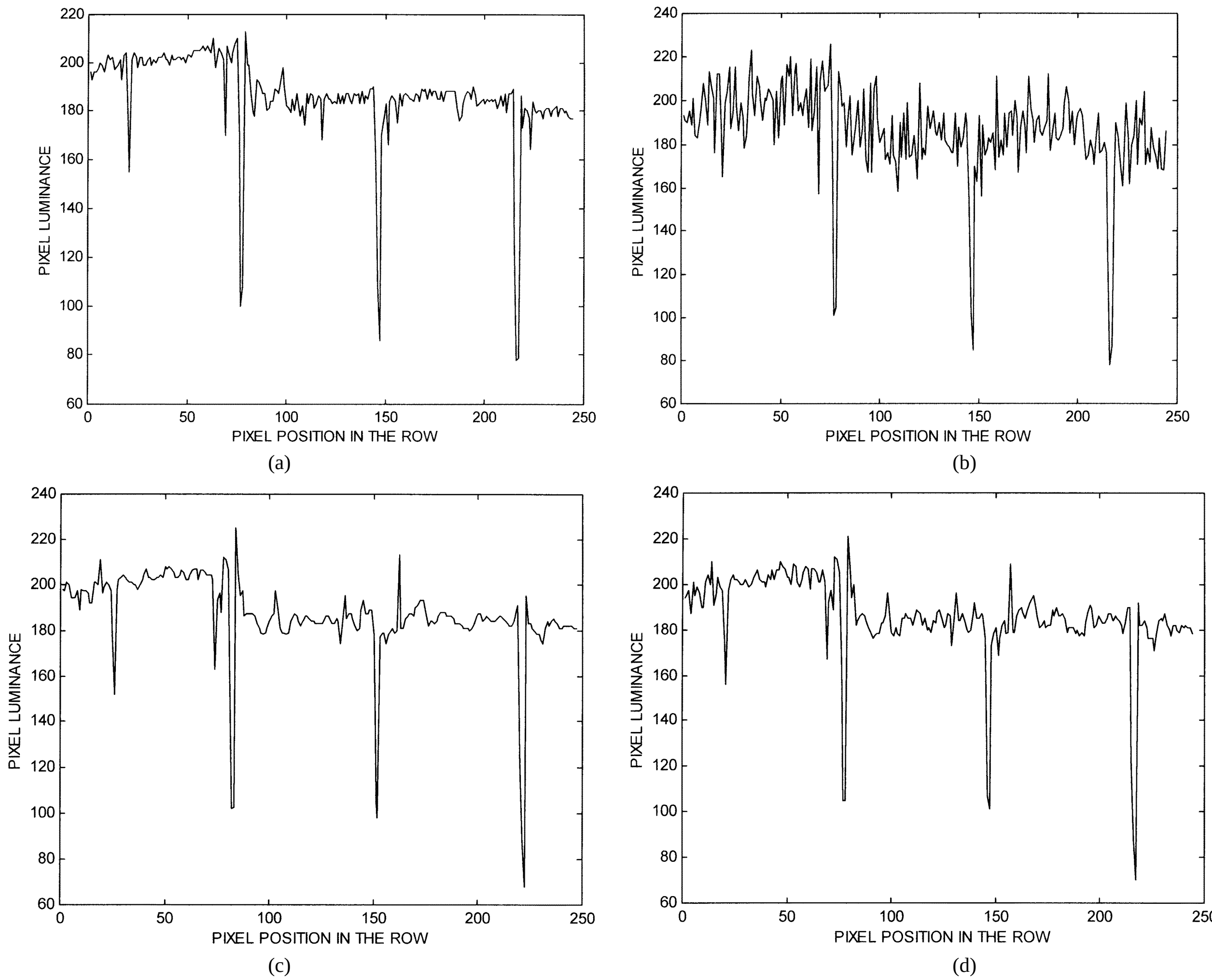


Fig. 9. Luminance values of the row 20. (a) Original image, (b) noisy image ($\sigma = 10$), (c) result yielded by the proposed method, and (d) result yielded by the RF1 technique.

method to each channel separately. The proposed technique can also be useful to reduce the data acquisition time, thus avoiding the occurrence of motion artifacts. An application example encompassing all the mentioned aspects is presented in Fig. 11. Fig. 11(a) shows a small detail of a 24-bit color picture taken by a 5-megapixel digital camera using a 2/3-in CCD sensor. The picture was obtained by disabling the built-in flash unit and by setting the sensitivity to ISO 800 equivalent. Notice that this choice is likely to produce noise in the resulting image. We chose the subject in the picture because it is rich of details and can generate motion artifacts. In order to reduce the noise, we applied our method to the R, G, and B components of the image. The result is shown in Fig. 11(b). We can observe that the noise has been reduced and the details have been satisfactorily preserved. In order to highlight the noise reduction effect, we considered a 50×50 detail of the background and we evaluated the standard deviation of the data in the original and in the processed picture, respectively. We obtained the following values: 5.5 (original image) and 3.7 (processed image). The routines for the proposed filter and the automatic tuning procedure have been written in C language. Since we used a look-up table for implementing relation (2), our filter requires only 33 ms to process a 256×256 image on a 1300-MHz Athlon-based PC.

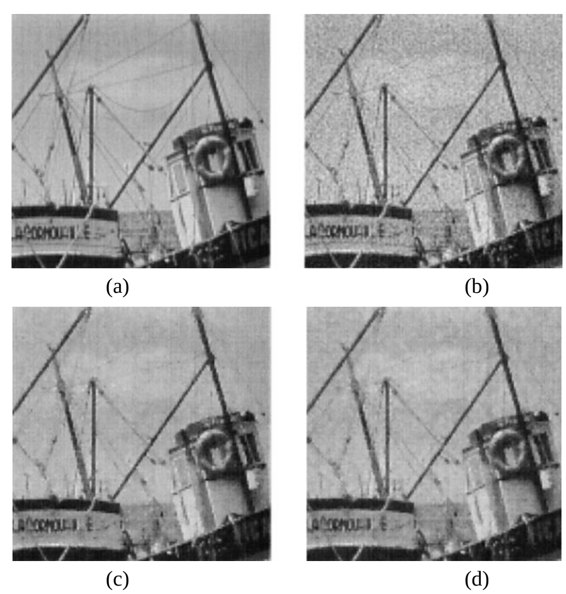
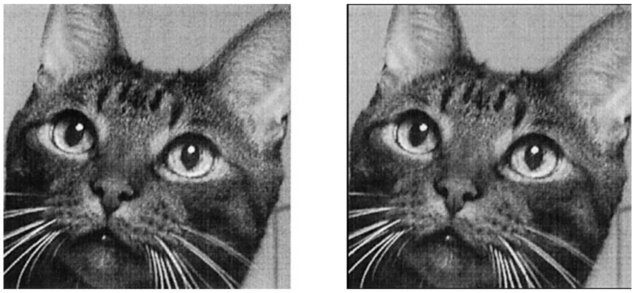


Fig. 10. (a) Detail of the original image ("Boats"), (b) detail of the noisy image ($\sigma = 10$), (c) result yielded by the proposed method, and (d) result yielded by the RF1 technique.

At present, we did not adopt any optimization for the routine implementing the automatic tuning procedure. This procedure typ-



(a) (b)

Fig. 11. (a) Noisy input data acquired by a digital camera (shutter $1/25\mathrm{s}$, aperture $\mathrm{f}/3.5$, and sensitivity ISO 800); (b) result of the application of the proposed method.

ically requires about 4 s to find the optimal parameter settings for a 256×256 image. Finally, we can observe that the memory requirements do not represent a very critical aspect of our approach. Basically, two memory buffers are currently required to store the intermediate images during the tuning procedure.

VII. Conclusion

A new technique for the restoration of images degraded by Gaussian noise has been presented. The innovative aspects of the proposed approach are a simple and effective algorithm for noise cancellation and a novel fully automatic procedure for parameter tuning that does not need any *a priori* information about the variance of the Gaussian noise.

Results of computer simulations have shown that the proposed method yields very satisfactory results. Experimental results dealing with color images produced by a digital camera have been reported. The application of the method to data preprocessing for color edge detection is a subject of present investigation.

References

[1] F. van der Heijden, *Image Based Measurement Systems*. New York: Wiley, 1994.

[2] A. K. Jain, *Fundamentals of Digital Image Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[3] I. Pitas and A. N. Venetsanopoulos, *Nonlinear Digital Filters: Principles and Applications*. Norwell, MA: Kluwer, 1990.

[4] S. K. Mitra and G. Sicuranza, Eds., *Nonlinear Image Processing*. New York: Academic, 2000.

[5] G. Ramponi, “Image processing with rational operators: Noise smoothing and anisotropic diffusion,” *Proc. IEEE-EURASIP 2nd Int. Symp. Image Signal Processing Anal.*, June 2001.

[6] F. Russo, “An image enhancement technique combining sharpening and noise reduction,” *IEEE Trans. Instrum. Meas.*, vol. 51, pp. 824–828, Aug. 2002.

[7] J. S. Jin, Y. Wang, and J. Hiller, “An adaptive nonlinear diffusion algorithm for filtering medical images,” *IEEE Trans. Inform. Technol. Biomed.*, vol. 4, pp. 298–305, Dec. 2000.

[8] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger, “Robust anisotropic diffusion,” *IEEE Trans. Image Processing*, vol. 7, pp. 421–432, Mar. 1998.

Fabrizio Russo (M’88) received the Dr. Ing. degree in electronic engineering (*summa cum laude*) from the University of Trieste, Trieste, Italy, in 1981.

In 1984, he joined the Dipartimento di Elettrotecnica Elettronica Informatica, University of Trieste, where he is currently an Associate Professor. His research interests are concerned with the application of computational intelligence to instrumentation, including fuzzy and neuro-fuzzy techniques for nonlinear signal processing, image filtering, and edge detection. His research results have been published in more than 70 papers in international journals, textbooks, and conference proceedings.

Dr. Russo is a Co-Chair of the IEEE Instrumentation and Measurement Technical Committee on Imaging Measurements.