Benchmark 1: Analytical. Solitary Wave on a Simple Beach

The goal of this benchmark is to verify a model by comparing numerical and analytical solutions that describe propagation and runup of a 1-D solitary wave. The analytical solution to the solitary wave runup on a sloping beach was derived by Synolakis (1986, 1987). In this problem, the wave of height H is initially centered at distance L from the beach toe and is schematically shown in Figure 1. The beach bathymetry consists of an area of constant depth d, connected to a plane sloping beach of angle β=arccot(19.85). Note that the x coordinate increases monotonically seaward, $x = 0$ is the initial shore location, and the toe of the beach is located at x=X_0=$d \cot \beta$.

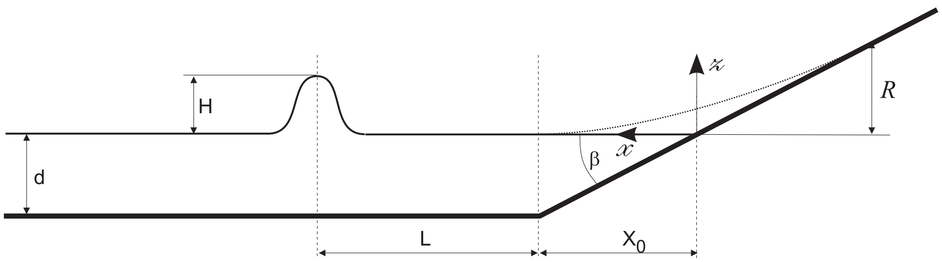

Figure 1: Non-scaled sketch of a canonical beach with a wave climbing up.

The value of L=arccosh $\left(\sqrt{20}\right) / \gamma$ is the half-length of the solitary wave, and the initial depth profile is given by

$$\eta(x, 0) = H \mathrm{sech}^2 \left(\gamma (x - X_1)/d\right), \tag{1}$$

where $X_1 = X_0 + L$, and $\gamma = \sqrt{3H/4d}$.

More information regarding this benchmark can be found in (Synolakis et al., 2007), or at the web-site http://nctr.pmel.noaa.gov/benchmark/Analytical/ of the NOAA Center for Tsunami Research.

This benchmark problem is focused on modeling runup of an incident non-breaking solitary wave such that its height H satisfies the condition: $H/d = 0.019$. In the computer experiment, this wave can simulated by specifying the initial wave profile according to formula (1) and by setting the initial wave-particle velocity, following Titov and Synolakis (1995) as:

$$u(x, 0) = -\sqrt{g/d}\; \eta(x, 0).$$

It is recommended to set the non-reflective boundary condition at the left side of the computational domain.

Figure 2 shows profiles and time series of the water level at the left and right plots, respectively. Extreme positions of the shoreline - the maximum runup and rundown occur $t{\approx}55(d/g)^{1/2}$ and $t{\approx}70(d/g)^{1/2}$, respectively. The water level dynamics at the locations $x/d = 0.25$, near the initial shoreline, predicts that the water retreats from $t{\approx}67(d/g)^{1/2}$ to $t{\approx}82(d/g)^{1/2}$. This point location temporally becomes dry, while the point $x/d = 9.95$ remains wet throughout the entire length of the computer experiment.

To accomplish this benchmark, it is suggested to:

1. Compare the numerically and analytically computed water level profiles at $t = 25(d/g)^{1/2}$, $t = 35(d/g)^{1/2}$, $t = 45(d/g)^{1/2}$, $t = 55(d/g)^{1/2}$, and $t = 65(d/g)^{1/2}$.
   Note that the numerical model must be run in appropriate (i.e. linear, non-dispersive, no friction) mode for the comparison and verification purposes.

2. Compare the numerically and analytically computed water level dynamics at locations $x/d = 0.25$ and $x/d = 9.95$ during propagation and reflection of the wave,

3. Compute the maximum runup,

4. (Optional) Show convergence of the numerical solution to the analytical one at $t = 55\sqrt{d/g}$ and $t = 70\sqrt{d/g}$.

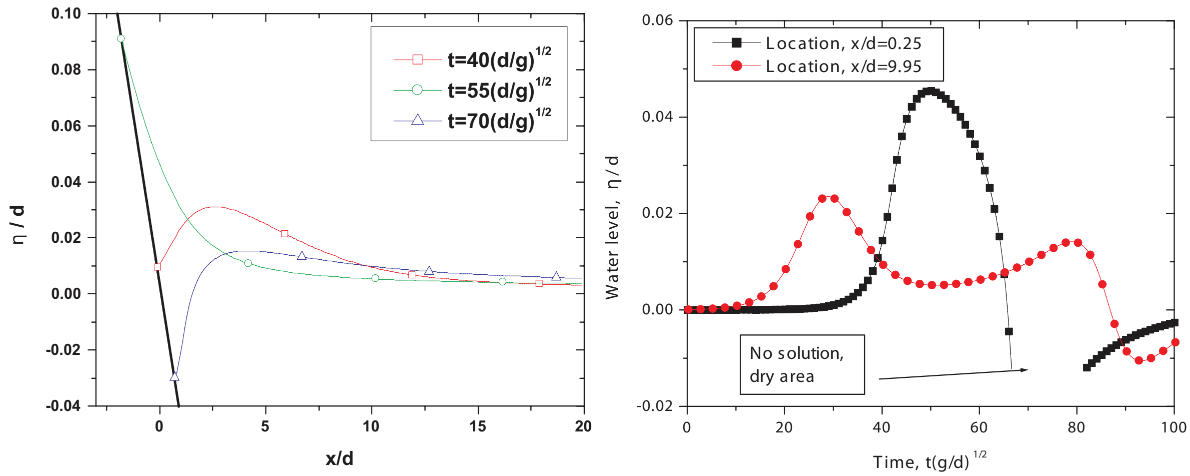

Figure 2: Left plot: the water level profiles during runup of the non-breaking wave in the case of $H/d = 0.019$ on the 1:19.85 beach.. Right plot: the water level dynamics at two locations $x/d = 0.25$ and $x/d = 9.95$. The analytical solution is according to Synolakis (1986).

Description of the data files

The analytically computed water level profiles at $t = 25(d/g)^{1/2}, ..., t = 70(d/g)^{1/2}$ are provided in file canonical_profile.txt. The analytically computed water level dynamics at two locations x/d=0.25 and x/d=9.95 are provided in file canonical_ts.txt. In both files the units are non-dimensional.

References

Synolakis, C., 1986. The Runup of Long Waves. Ph.D. thesis, California Institute of Technology, Pasadena, California, 228 pp..

Synolakis, C., 1987. The runup of solitary waves. Journal of Fluid Mechanics 185, 523–545.

Synolakis, C., Bernard, E., Titov, V., Kânoğlu, U., González, F., 2007. Standards, criteria, and procedures for NOAA evaluation of tsunami numerical models. OAR PMEL-135 Special Report, NOAA/OAR/PMEL, Seattle, Washington, 55 pp..

Titov, V., Synolakis, C., 1995. Evolution and runup of breaking and nonbreaking waves using VTSC2. Journal of Waterway, Port, Coastal and Ocean Engineering 121 (6), 308–316.