1D AXISYMMETRIC JOULE HEATING (EM1)

1. Test Objective

This test problem exercises the Joule heating model in the simplest possible setting. Several other associated features are exercised in addition to the core EM solver. The mapping of constant material parameters from the hex mesh to the tet mesh and the mapping of the Joule heat from the tet mesh to the hex mesh are exercised in the trivial case the tet mesh conforms to the hex mesh. Also exercised is the time-dependent source capability.

2. Problem Description

An infinite-length, electrically-conductive cylinder is positioned in a uniform alternating magnetic field directed along the axis of the cylinder. The electric conductivity of the cylinder is axisymmetric with one constant in the core and an larger constant in a surface layer. The induced EM fields, current, Joule heat, and temperature fields are all 1D, depending only on the radial distance from the axis.

This problem has an analytic solution for the time-dependent Joule heat and EM fields, both in the conductor and free space [1].

3. Truchas Model

The axisymmetry is exploited by solving on one quarter of a thin cross-sectional slab that includes some of the free space. Appropriate symmetry conditions are imposed on the symmetry planes. The domain is meshed with an unstructured hex mesh 1-cell thick, and the tet mesh is obtained from it by simple subdivision of each hex cell into 6 tets. The hex mesh includes the free space region (void).

Heat conduction is modeled using constant properties throughout the cylinder, with no flux conditions on the symmetry planes (and no flux from the cylinder into the meshed free space region).

The characteristics of the magnetic source field are changed at several times during the course of the simulation, requiring the Joule heat to be recalculated, either directly or by a shortcut in special cases:

time	magnitude	frequency	expected action
0.0	2000	1000	full computation
25	4000	1000	$\times 4$ scaling of the Joule heat
50	0	500	Joule heat set to 0
75	4000	500	full computation

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4. Test Metrics

Although there is an analytic EM solution for this problem (and even in principle for associated heat conduction) we cannot afford to use the sufficiently fine mesh required to usefully compare with the analytic solution. Instead we will compare against reference 'golden' results obtained from a prior run. This ensures only that the results do not drift too far from the reference results, but is sufficient if used in conjunction with other detailed verification tests.

Ideally we should test the time-dependent EM solution on the tet mesh, but this information is not currently available in the output. Thus we are limited to testing the time-averaged Joule heat *after* it has been mapped onto the heat conduction hex mesh. This is probably adequate to detect significant changes to the EM solution and field mappings, but will not pinpoint the source of an error should a change be observed.

A constant step size of 5 is used, and output is obtained at every step (20 steps total). The mesh is partitioned into 3 element blocks: IDs 1 and 2 form the cylinder, and ID 3 is the free space. Let Q_j $j = 0, 1, \ldots, 20$ denote the Joule heat after step j (0 is the initial Joule heat), and let $\bar{Q}_j$ denote the corresponding reference field. Then the Joule heat will be tested as follows:

1. $\|Q_0 - \bar{Q}_0\|_{1,2} < \epsilon$ and $Q_0 = 0$ in region 3.
2. $Q_j = Q_0$, $j = 1, \ldots, 5$.
3. $\|Q_6 - 4Q_0\|_{1,2} < \delta$
4. $Q_j = Q_6$, $j = 7, \ldots, 10$
5. $Q_j = 0$, $j = 11, \ldots, 15$
6. $\|Q_j = \bar{Q}_{16}\|_{1,2} < \epsilon$ and $Q_{16} = 0$ in region 3;
7. $Q_j = Q_{16}$, $j = 17, \ldots, 20$.

The norm $\|\cdot\|$ will be the maximum relative error.

Finally we want to verify that there are no changes in the way the Joule heat is used as a source in heat conduction. For this it will suffice to compare the temperature at the final output against reference temperature results, again using a maximum relative error criterion. In this case the results will be affected by any change to the heat conduction solver and associated input parameters.

References

[1] Neil Carlson, *Analytic solution of a 1-D axisymmetric eddy current problem*, 2004, unpublished.