

Discontinuous Conductivity Orthogonal to the Heat Flux

Objective

This test problem tests the case of a discontinuous conductivity in the heat transfer model. All parameters are chosen such that we can test the results against an analytic solution.

Definition

We consider the 2D conductive heat transfer problem that is defined as follows.

$$-\nabla \cdot K \nabla T = 0.$$

The problem domain is the cube $[0, 1]^2$, which is discretized using an orthogonal mesh of size 10×10 . The conductivity is piecewise constant

$$K(T) = \begin{cases} 1, & x < 0.5 \\ 1000, & x > 0.5 \end{cases}$$

The boundary conditions are a constant Dirichlet boundary conditions on the boundaries $x = 0$ and $x = 1$

$$\begin{aligned} T(x, y, z) &= 0, & x = 0 \\ T(x, y, z) &= 100, & x = 1 \end{aligned}$$

All other boundaries have homogeneous Neumann boundary conditions (insulation). As a result, this problem has a one-dimensional solution

$$T(x) = \begin{cases} \frac{1000}{5005}x, & x < \frac{1}{2} \\ \frac{1000000}{5005}x + \frac{499500}{5005}, & x > \frac{1}{2} \end{cases}$$

Metrics

We compare the temperature at the final time step to the analytic solution. We use the ℓ^∞ -norm for this comparison.

Truchas Model

To run this example in truchas, we select the heat conduction model exclusively and a constant time step of 0.1. The boundary conditions are selected as described above. The interface between the two materials with different conductivities is modeled using the least squares model. The heat transfer problem is solved using Newton Krylov with flexible GMRES as a preconditioner, which is itself preconditioned by SSOR, and we run truchas for ten time steps. We use the ortho operator to discretize the gradient.

Results

The results of these runs are compared to the analytic solution.