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Channel Flow

Objective

The channel flow test problem is has an analytic solution. It can be used to test the pressure gradient and viscosity calculations, as well as the implementation of the pressure boundary conditions.

Definition

Channel flows are generated by pressure gradients, with applications primarily to ducts. In this case, 2-D steady laminar flow between two parallel plates separated by a width of 2h is considered. With both plates fixed and with uniform end pressures, the flow is one-dimensional assuming that the flow is fully developed and gravity forces are neglected. The fixed plates are subject to the no-slip/no-penetration conditions, while entrance and exit regions are defined by Dirichlet pressure boundary conditions. Using the Navier-Stokes equations, an analytical expression for the steady-state velocity profile is given by

$$u(y) = \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\left[1-\left(\frac{y}{h}\right)^2\right]$$

where $u(y)$ is the velocity in the x-direction and is a function of the y-position. $2h$ is the gap width between the plates, $\left({dp}/{dx}\right)$ is the pressure gradient, and μ is the dynamic viscosity.

For a unit channel depth (into the plane), the Reynolds number is calculated using the average velocity

$$u_{avg} = \frac{h^3}{3\mu}\left(-\frac{dp}{dx}\right)$$

and the Reynolds number is

$$\mathrm{Re} = \frac{4\rho h^4}{3\mu^2\left(2+2h\right)}\left(-\frac{dp}{dx}\right)$$

where the hydraulic diameter has been used as the characteristic length.

The time-dependent solution for the velocity profile is

$$u(y,t) = \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\left[1-\left(\frac{y}{h}\right)^2 - \frac{32}{\pi^2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\left(2k-1\right)^3}\cos\left\{\gamma y\right\}\exp\left\{\gamma^2\frac{\pi}{\rho}t\right\}\right]$$

where

$$\gamma = \frac{(2k-1)\pi}{2h}$$

Metrics

The quantification of the discrete error between results obtained using the simulation and the analytical solution for velocity at a particular time, t , will be defined by

$$E_t = \sqrt{\left(\left| u_{exact} - u_{discrete} \right|^2 \right)}$$

which is an approximation of the L2 norm. The sum is over all grid points, $u_{discrete}$ is the numerical solution given by a discrete approximation of the PDEs and a set of initial and boundary conditions.

Truchas Model

To simulate laminar pressure driven flow between two fixed plates, the flow physics model will be selected exclusively. The appropriate boundary conditions were imposed at each of the six planes flanking the flow region. For both bounding planes in the z-direction, the selection of no boundary conditions resulted in the default or 'freeslip' condition. At inflow and outflow locations 'dirichlet' pressure boundary conditions will be imposed. For the stationary plates, the 'noslip' condition was applied on both of the fringing y-planes. Because of the symmetry of the problem, the simulation can also be performed over half of the channel width.

Possible variations of this problem are:
1. The computational setup of the halfwidth channel is identical to the full channel except for the centerline condition. The symmetry boundary condition at the centerline must be imposed by selecting freeslip for the x=0 plane.
2. The mesh that is being used can be either an ortho or and unstructured mesh. In the unstructured mesh case we can test the quality of the support operator discretization.

Results

The results should be close to the exact.

Critique

To be added later.

References

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