Contact box close/open

0.1 Objective

These two problem exercise the contact algorithm on a simple split block that either opens or closes a gap driven by thermal expansion or contraction. The block is an orthogonal mesh, but it is rotated in space so that the contact surfaces are not aligned with any global axes. These problems test the rotation of stress and strain components and the contact algorithm.

0.2 Definition

E	Young's modulus
G	second Lame$'$ constant
T	temperature
T_{ref}	stress reference temperature
α	linear coefficient of thermal expansion
δ_{ij}	Kronecker delta
λ	first Lame$'$ constant
ν	Poisson's ratio
$\boldsymbol{\sigma}$, σ_{ij}	Cauchy stress tensor, stress components
$\boldsymbol{\epsilon}$,ϵ_{ij}	total strain tensor, strain components
$\boldsymbol{\sigma'}$, σ'_{ij}	rotated Cauchy stress tensor, components
$\boldsymbol{\epsilon'}$,ϵ'_{ij}	rotated total strain tensor, components

The block of solid material has a gap interface cutting it in half and is constrained on 5 of the six sides. The initial temperature is set to produce a closed or open gap and the block is cooled or heated from the two external surfaces parallel to the gap interface. The problem is run until the temperature is essentially uniform so that the resulting stress and strain state is also uniform. The unconstrained direction is one of the directions in the plane of the gap interface.

The analytic solution is calculated for the closed gap condition, which is the final state for the closing problem or the initial state for the opening problem. The unrotated solution is calculated as follows:

Components prescribed by boundary conditions.

$$\epsilon_{xx} = 0 \tag{1}$$

$$\epsilon_{zz} = 0 \tag{2}$$
$$\sigma_{yy} = 0 \tag{3}$$
$$\sigma_{xx} = \sigma_{zz} \tag{4}$$
$$\epsilon_{xy} = \epsilon_{xz} = \epsilon_{yz} = 0 \tag{5}$$
$$\sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0 \tag{6}$$

We can use the ϵ_{xx} component of isotropic generalized Hooke's law to get σ_{xx}.

$$\epsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) + \alpha(T - T_{ref}) \tag{7}$$

Since $\sigma_{xx} = \sigma_{zz}$, this leads to

$$\sigma_{xx} = \sigma_{zz} = -\frac{E\alpha(T - T_{ref})}{1-\nu} \tag{8}$$

The ϵ_{yy} component can be found from the σ_{yy} component of isotropic generalized Hooke's law:

$$\sigma_{yy} = \lambda\epsilon_{yy} + 2G\epsilon_{yy} - (3\lambda + 2G)\alpha(T - T_{ref}) \tag{9}$$

or

$$\epsilon_{yy} = \frac{(3\lambda + 2G)\alpha(T - T_{ref})}{\lambda + 2G} \tag{10}$$

The box is rotated by angle ϕ around the y axis and then angle θ about the x axis. The stress and strain components must be rotated to the new coordinate system. The rotation matrices $[R_x]$ and $[R_y]$ are multiplied in reverse order to get the transformation matrix.

$$[R_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{11}$$

$$[R_y] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \tag{12}$$

The transformation matrix $[A]$ is

$$[A] = [R_x][R_y] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ -\sin\theta\sin\phi & \cos\theta & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \sin\theta & \cos\theta\cos\phi \end{bmatrix} \tag{13}$$

The transformed stress and strain components are found by:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{14}$$

$$[\boldsymbol{\epsilon}] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \tag{15}$$

The transformed stresses and strains are

$$[\boldsymbol{\sigma'}] = [A][\boldsymbol{\sigma}][A]^T \tag{16}$$

$$[\boldsymbol{\epsilon'}] = [A][\boldsymbol{\epsilon}][A]^T \tag{17}$$

0.3 Metrics

The stress and strain components for the uniform temperature closed gap cases are compared with the analytical solution in the table below. The gap normal traction should also equal σ_{xx} for the unrotated frame of reference. The gap displacement should be small.

The verification of the contact algorithm is more difficult. The transient process as the gap closes is somewhat unstable, and the results will vary with architecture and parallelism. For now we will choose time steps just before and just after the surfaces come into contact and compare the results to a previous solution.

0.4 Truchas Model

The problem uses an orthogonal mesh with one gap surface. The boundary conditions are straightforward for solid mechanics. It is necessary to carefully choose the temperature BCs and time step so that the contact happens fairly gradually. Convection BCs on the two external surfaces parallel to the gap interface are used and the heat transfer solution is symmetric with the gap surface as the symmetry plane. The gap surfaces do not stay planar or parallel to the mesh boundary due to the temperature gradients, so the surfaces do not contact at the same time at all nodes.

The time stepping is somewhat awkward for the closing problem in that the contact occurs in the first 5 seconds, but it takes ≈ 500 seconds for the temperature to be sufficiently uniform to get a nearly uniform stress–strain solution. Results will be used at 0, 5 and 600 seconds.

0.5 Results

Analytical results for the rotated stresses and strains resulting from the provided input file.

Variable	Close	Open
σ'_{xx}	-2.288e8	-2.288e7
σ'_{yy}	5.72e7	5.72e6
σ'_{zz}	-1.716e8	-1.716e7
σ'_{xy}	0	0
σ'_{xz}	0	0
σ'_{yz}	9.90733062e7	9.90733062e6
ϵ'_{xx}	0	0
ϵ'_{yy}	3.3e-3	3.3e-4
ϵ'_{zz}	1.1e-4	1.1e-4
ϵ'_{xy}	0	0
ϵ'_{xz}	0	0
ϵ'_{yz}	1.90525589e-3	1.90525589e-4
gap normal traction	-2.288e8	-2.288e7

0.6 Critique

The gap closing problem is a very good candidate for a restart problem. We can use a small time step through the contact phase, stop and restart with a much larger time step and write interval for the uniform temperature solution. This would also test restarting with contact active.