Shear-Constrain-Heat

0.1 Objective

This problem tests a simple uniform shear loading on a rectangular elastic block, and also solves a simple 1-D transient heat conduction problem. The steady state solution results in a uniform temperature and thermal strain field. Analytical solutions for the solid mechanics and heat transfer problem are available for verification of the results.

0.2 Definition

E	Young's modulus
e^{tot}	total strain tensor
G	second Lame$'$ constant
$\hat{n}$, $\hat{n}_i$	unit normal vector and components
T	temperature
T_{ref}	stress reference temperature
u_x,u_y,u_z	displacement components
x_i or $[x, y, z]$	global coordinates
α	linear coefficient of thermal expansion
δ_{ij}	Kronecker delta
λ	first Lame$'$ constant
ν	Poisson's ratio
σ	Cauchy stress tensor
τ_i	traction components

Two solid mechanics features and heat conduction are exercised in this problem. The relation ship between Cauchy stress and the combination of elastic and thermal strain is:

$$\sigma_{ij} = \lambda e^{tot}_{kk}\delta_{ij} + 2Ge^{tot}_{ij} - (3\lambda + 2G)\alpha(T - T_{ref})\delta_{ij} \tag{1}$$

The traction on a surface is related to the stress by:

$$\tau_j = \sigma_{ij} \cdot \hat{n}_i \tag{2}$$

If shear tractions are applied to the surfaces of a block aligned with the global Cartesian axes then a uniform shear strain component is the result.

If the block is constrained from expanding or contracting in one dimension and heated or cooled relative to the stress free reference temperature, then a uniform diagonal stress component in the constrained direction is the result.

0.3 Metrics

The the stresses, strains and displacements for the initial solution and final time step should be uniform and equal to the values in the table below. The maximum deviation from these values for any cell or node should be small.

The transient heat conduction problem also has an analytical solution, and in the future we may want to compare intermediate results with the exact solution. For now the intermediate temperature and solid mechanics results will be compared to a previous "golden" solution to see if it has changed.

0.4 Truchas Model

The problem domain is an orthogonal block of material, 2 cm x 3 cm x 4 cm. Tangential traction boundary conditions are applied to the x=0, x=1 cm, z=0 and z=4 cm surfaces, resulting in a uniform x-z shear stress and strain component over the entire domain. The y=0, z=0 and z=4 cm surfaces are specified to have zero normal displacement. The z=0 surface has a dirichlet temperature boundary condition with $(T - T_{ref})$ set 100 degrees above the initial temperature of the block, and all other surfaces are insulated. After 200 seconds, the temperature of the block is sufficiently uniform that the thermal stresses and strains are uniform to within 1 part in $10^1 0$ or better. For a uniform temperature, the thermal strain is $100 * \alpha$ where α is the coefficient of thermal expansion. The x-z shear strain and compressive z strain caused by thermal expansion do not interact. The final displacements, stresses and strains are given below. This problem can be run with either an orthogonal or non-ortho mesh.

0.5 Results

Result	Initial Solution	Final Solution
σ_{xx}	0	0
σ_{yy}	0	0
σ_{zz}	0	$-\alpha E(T - T_{ref})$
σ_{xy}	0	0
σ_{xz}	τ_{BC}	τ_{BC}
σ_{yz}	0	0
ϵ_{xx}	0	$100\alpha\nu$
ϵ_{yy}	0	$100\alpha\nu$
ϵ_{zz}	0	0
ϵ_{xy}	0	0
ϵ_{xz}	τ_{BC}/2G	τ_{BC}/2G
ϵ_{yz}	0	0
u_x	$z\epsilon_{xz}$	$x\epsilon_{xx} + z\epsilon_{xz}$
u_y	0	y ϵ_{yy}
u_z	0	0

0.6 Critique