Diagonal Advection

Objective

This test problem is intended to test the advection algorithms within the fluid flow section of Truchas. It exercises the PLIC algorithm for tracking a fluid interface, and the flux limited second order advection algorithm for moving enthalpy with two materials.

This problem will coincidentally test mesh generation, specified velocity field, region initialization, and the relationship between enthalpy and temperature for a simple material.

Definition

Advect a circle of water whose temperature is 273 along the diagonal of a unit square in an atmosphere of air, whose temperature is 200. There is to be no thermal conduction in this problem.

Metrics

The centroids of the circle should translate according to

$$\boldsymbol{x}_{final}\left(t\right) = \boldsymbol{x}_{cent}\left(0\right) + \boldsymbol{v}\,\Delta t \tag{1.1}$$

The size and shape of the circle should remain constant. There should not be any mixture of the air and water at any point in the domain at the completion of the simulation. Relevant metrics for this problem include:

1. The centroids of the water region at the beginning and end of the simulation. Compare the result to the prescribed locations.
2. The volume fraction of water that would be in every cell for the circle in its initial and final locations. Sum over all cells the square of the difference between the computed volume fraction and this analytic value.
3. Calculate the temperature of every cell based on the analytic translation of the circle at the start and end of the simulation. Sum over all cells the square of the difference between this temperature and the computed result.
4. Compare the initial and final water and air mass.
5. Compare the initial and final total enthalpy.

We note in passing that Truchas advects the mixture enthalpy rather than independent material enthalpy. Currently, the regression test computes the total enthalpy for each material internally, but only the total enthalpy integrated over the domain is used as a metric.

The metrics implemented for this regression test are

- $\left|1-\frac{\boldsymbol{x}_{cent}}{\boldsymbol{x}_{final}}\right|$
- $\left|1-\frac{m_{H20}\big|_{final}}{m_{H20}\big|_{initial}}\right|$
- $\left|1-\frac{m_{air}\big|_{final}}{m_{air}\big|_{initial}}\right|$
- $\left|1-\frac{H\big|_{final}}{H\big|_{initial}}\right|$

where the final centroid location $\boldsymbol{x}_{final}$ is computed using Eq. (1.1). The centroid, mass and enthalpy are computed as

$$\boldsymbol{x}_{cent} = \frac{\sum_{\boldsymbol{Nel}} \boldsymbol{x}_{centroid} F_{H2O} V}{\sum_{\boldsymbol{Nel}} F_{H2O} V} \tag{1.2}$$

$$m_k = \sum_{Nel} \rho_k F_k V \tag{1.3}$$

$$H_k = \sum_{Nel} \rho_k F_k h_k V \tag{1.4}$$

where $k = air,\ H2O$. The total enthalpy for the domain is $H = H_{air} + H_{H2O}$.

Truchas Model

The problem domain is a unit square in the x-z plane divided into 10 mesh cells in each direction. (The y direction is a single cell of thickness 0.03125). A constant, uniform velocity field is imposed in the diagonal direction. The x and z components of this velocity are both unity. There is an initial circle of water embedded in a background of air. The radius of the circle is 0.2, and its center is initially at x=z=0.3. The initial temperatures of the two regions are specified as described in the problem definition. The simulation is run for 40 time steps, each of duration 0.008. The VOF algorithm is used to move the material interface, and flux limited second order advection is applied to the enthalpy.

Variations: Change the plane of the simulation and direction of the velocity field to investigate symmetry. Change the energy advection to donor cell. Switch the initial liquid and gas volumes. Change the air to a void.

Results

To be added.

Critique

To be added.