Mate 1: Curs #1

Profesor: Iulian Duca

30 Septembrie 2019

1 Matrici

Fie $A = \{1, 2, 3, ..., n\}$ si $B = \{1, 2, 3, ..., m\}, m, n \in \mathbb{N}^*$.
Se numeste matrice cu n linii si m coloane, orice aplicatie $f : A \times B \to I$, unde $(I, +, \cdot)$ inel.
Vom nota $f(i, j) = C_{ij},\ C = (C_{ij}), 1 \leqslant i \leqslant n,\ 1 \leqslant j \leqslant m$.
Consideram matricea extinsa: $\left(\begin{array}{cccc|c} x_{11} & x_{12} & \dots & x_{1n} & b_1 \\ x_{21} & x_{22} & \dots & x_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} & b_m \end{array}\right)$
a sistemului $A \cdot X = B,\ X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix},\ B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix},\ A \in M_n(\mathbb{R})$
Se observa ca daca se inverseaza doua linii pe matricea extinsa, se obtine o matrice extinsa a unui sistem echivalent cu cel initial.
Daca se inmulteste o linie cu un numar nenul si se adauga la alta linie, rezulta un sistem echivalent cu cel original.
Daca folosim cele 2 operatii enuntate anterior $\implies (A|B) \sim \dots \sim (I_n|B^*)$

Observatie:
Daca avem un sistem $(A|C1)$ si un alt sistem $(A|C2)$, se pot rezolva simultan cele 2 sisteme cu metoda Gauss-Jordan:
$(A|C1|C2) \sim \dots \sim (I_n|C_1^*|C_2^*)$
$(A|C1) \sim \dots \sim (I_n|C_1^*) \iff C_1^* = A^{-1} \cdot C_1$ Daca rezolva simultan n sisteme de ecuatii de forma $\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \dots \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$

$$\left(A \;\middle|\; \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix} \;\middle|\; \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix} \;\middle|\; \dots \;\middle|\; \begin{pmatrix}0\\0\\\vdots\\1\end{pmatrix} \right) \iff (A|I_n) \sim \dots \sim (I_n|A^{-1})$$