

## Question 1

### (a) Binary Integer Linear Program.

we define variables  $x_1, \dots, x_n$ . where  $x_i \in \{0,1\}$ .  $x_i = 1$  means Anu will visit city  $i$ ,  $x_i = 0$  means Anu will not visit city  $i$ . the binary Integer Linear Program is as follows:

$$\text{maximize : } \sum_{i=1}^6 x_i \cdot h_i$$

$$\text{subject to : } \sum_{i=1}^6 c_i \cdot x_i \leq B$$

$$x_3 + x_5 \leq 1$$

$$x_1 + x_5 + x_6 \leq 1$$

$$x_3 + x_4 - x_5 \geq 0$$

$$x_3 + x_4 - x_6 \geq 0$$

$$x_1 + x_2 + x_4 + x_5 + x_6 \geq 1$$

$$x_1 + x_2 + x_4 + x_5 + x_6 \leq 3$$

$$x_i \in \{0,1\} \quad \forall i = 1,2,3,4,5,6$$

### (b) Dual of relaxation of part(a).

we first write the symmetrical form of relaxation of part(a) as follows:

$$\text{maximize : } \sum_{i=1}^6 x_i \cdot h_i$$

$$\text{subject to : } \sum_{i=1}^6 c_i \cdot x_i \leq B$$

$$x_3 + x_5 \leq 1$$

$$x_1 + x_5 + x_6 \leq 1$$

$$x_5 - x_3 - x_4 \leq 0$$

$$x_6 - x_3 - x_4 \leq 0$$

$$-x_1 - x_2 - x_4 - x_5 - x_6 \leq -1$$

$$x_1 + x_2 + x_4 + x_5 + x_6 \leq 3$$

$$x_i \geq 0 \quad \forall i = 1,2,3,4,5,6$$

Then we write the dual of relaxation of part(a) as follows:

$$\text{minimize : } B \cdot y_1 + y_2 + y_3 - y_6 + 3 \cdot y_7$$

$$\text{subject to : } c_1 \cdot y_1 + y_3 - y_6 + y_7 \geq h_1$$

$$c_2 \cdot y_1 - y_6 + y_7 \geq h_2$$

$$c_3 \cdot y_1 + y_2 - y_4 - y_5 \geq h_3$$

$$c_4 \cdot y_1 - y_4 - y_5 - y_6 + y_7 \geq h_4$$

$$c_5 \cdot y_1 + y_2 + y_3 + y_4 - y_6 + y_7 \geq h_5$$

$$c_6 \cdot y_1 + y_3 + y_5 - y_6 + y_7 \geq h_6$$

$$y_i \geq 0 \quad \forall i = 1, 2, 3, 4, 5, 6$$