

Q2

(a)

$dp[1..2][0..n]$ :  $dp[i][j]$  denotes the max the sum of popularities of subtable(1..2, 1..j) when table(i,j) is put patio heater.

1. find the  $dp[x][y]$  with max value
2. table(x, y) is put patio heater
3. if  $dp[x][y] = p[x, y] + dp[x][y-2]$  then  $y = y-2$
4. if  $dp[x][y] = p[x, y] + dp[3-x][y-2]$  then  $x=3-x, y = y-2$
5. else if  $dp[x][y] = p[x, y] + dp[3-x][y-1]$  then  $x = 3-x, y=y-1$
6. repeat 2-5 until  $y \leq 0$

(b)

when  $i=1..2, j=0..n$

if  $j=0$ , then  $dp[i][j] = 0$

if  $j=1$ , then  $dp[i][j] = p[i,j]$

else  $dp[1][j] = p[1, j] + \max(dp[1][j-2], dp[2][j-2], dp[2][j-1])$

$dp[2][j] = p[2, j] + \max(dp[1][j-2], dp[2][j-2], dp[1][j-1])$

which can be presented in :

$dp[i][j] = p[i, j] + \max(dp[3-i][j-2], dp[i][j-2], dp[3-i][j-1])$

when  $j=0$ , it is obviously  $dp[i][j] = 0$

when  $j=1$ , it is obviously  $dp[i][j] = p[i,j]$

when  $j>1$ , if table(i,j) has been put patio heater then table(3-i, j) and table(i, j-1) must not been put patio heater, so the nearest tables are  $dp[3-i][j-2]$ ,  $dp[i][j-2]$  and  $dp[3-i][j-1]$ . Hence,  $dp[i][j] = p[i, j] + \max(dp[3-i][j-2], dp[i][j-2], dp[3-i][j-1])$ , which is correct.

(c)

algorithm( $p[1..2, 1..n]$ )

$dp[1..2][0..n]$  initial empty

$dp[1][0]=dp[2][0]=0$

$dp[1][1] = p[1,1]$

$dp[2][1] = p[2,1]$

for  $j=2$  to  $n$  do:

for  $j=1$  to 2 do:

$dp[i][j] = p[i, j] + \max(dp[i][j-2], dp[3-i][j-1])$

end for

end for

return  $dp$

We need to fill in table  $dp[1..2][0..n]$ , so:

running time:  $O(n)$

space time :  $O(n)$