Prove TwoTrips is NP-complete:

To prove TwoTrips is NP complete, we first prove it is NP, then prove PARTITION problem can be reduced to it in polynomial time.

1. TwoTrips problem is NP.

Given a set C={c_1,…, c_{2n}}, and a sequence R∈{1 ,…, 2n}. in order to satisfy $\sum_{i\in R} c_i = \sum_{i\in\{1,\dots,2n\}\backslash R} c_i$, we first check $\sum_{i\in R} c_i = 0.5 \cdot \sum_{i\in\{1,\dots,2n\}} c_i$. in order to satisfy $R \cap \{2i-1, 2i\} = 1$, we check $\sum_{i\in R} ceil\left(\frac{i}{2}\right) = 1 + \cdots + n$. if both checks satisfied, the sequence R is a certificate. Since we totally cost O(n) time to check, so TwoTrips problem is NP.

2. PARTITION problem $\leq_P$ TWOTRIPS problem

Given a set T={a_1,…, a_n} of positive integers. we reconstruct a set C={c_1,…, c_{2n}} by split a_i to c_{2i} and c_{2i-1}, where $c_{2i} = a_i+1$, $c_{2i-1}=1$. So both c_{2i} and c_{2i-1} are positive integers. This convertion can be done in O(n) time, so this reduction is in polynomial time.

now we prove PARTITION problem has yes solution if and only if TWOTRIPS problem has yes solution.

- if PARTITION problem is yes, then T has a set S such that $\sum_{i\in S} a_i = \sum_{i\in\{1,\cdots,n\}\backslash S} a_i$,

  so for TWOTRIPS problem, if $i \in S$, we contain c_{2i} to R, if $i \in \{1,\cdots,n\}\backslash S$, we contain c_{2i-1} to R, so $\sum_{i\in R} c_i = 0.5 \cdot \sum_{i\in\{1,\dots,2n\}} c_i$, and $R \cap \{2i-1, 2i\} = 1$, which satisfy TWOTRIPS.

  So if PARTITION problem output yes, corresponding TWOTRIPS also output yes.

- if TWOTRIPS problem is yes, then set C has a set R∈{1 ,…, 2n} where $\sum_{i\in R} c_i = \sum_{i\in\{1,\dots,2n\}\backslash R} c_i$, and $R \cap \{2i-1, 2i\} = 1$.

  so for PARTITION problem, if $j \in R$ and $c_j \neq 1$, we contain a_i =floor(c_j/2) to S, so $\sum_{i\in S} a_i = \sum_{i\in\{1,\cdots,n\}\backslash S} a_i$, which satisfy PARTITION problem.

  So if TWOTRIPS problem output yes, corresponding PARTITION also output yes.

- So PARTITION problem is true if and only if it's corresponding TWOTRIPS is true.

- Since PARTITION is NP complete, and convertion can be done in polynomial time, so TWOTRIPS problem is NP complete.