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Master's thesis - Critical Slowing Down

M1 Econometrics, Statistics

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Abstract

Cryptocurrencies are an electronic money issued peer-to-peer, without the need for a bank or central bank or human intermediary, used through a decentralized computer network based on a blockchain integrating cryptographic technologies for the processes of issuing and settling transactions. Therefore, they have emerged as highly volatile financial assets.

Today, the two cryptocurrencies with the highest market capitalisation in the world are Bitcoin and Ethereum. Bitcoin, particularly, is the first and most well-known cryptocurrency, introduced in 2009 by an anonymous entity under the pseudonym of Satoshi Nakamoto. Ethereum is therefore, the second most important cryptocurrency in the world, it was created in 2015 by Vitalik Buterin.

Because of the decentralization and the peer-to-peer system, cryptocurrencies are facing extreme price fluctuations, compared to traditional currencies. That's why we chose to work with a database of daily bitcoin and ethereum prices since 2014. We aim to determine whether or not early warning signals can be identified in the bitcoin market, in order to explain critical transitions such as sharp price drops or sudden increases.

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1 Introduction

Over the year, Bitcoin, and even cryptocurrencies in general have become synonymous with extreme price volatility. They were marked by sudden spikes and dramatic crashes, so this raises questions about the underlying mechanisms driving these critical transitions, that's why we will focus on investigating early warning signals that may precede critical transitions in both Bitcoin market and Ethereum market. According to Dakos et al, critical transitions are defined as a bifurcation, which is mathematically "an abrupt qualitative change in the behavior of a dynamical system when one or more control parameters change". This control parameter is affecting the dynamics of the state variable. This parameter is either constant or changes gradually in tiny steps, so that the state variable may be regarded as rapid in relation to the control variable. In our application, the state variable is either the closing price of bitcoin or that of ethereum, and the control parameter could be the interest rate. Indeed, Dakos[4] explained that the interest rate of the central bank for instance can be considered as a control parameter since it influences the dynamics of the financial markets and, consequently, the long-term behavior of the markets by modifying the quantity and/or the stability of the equilibria. More concretely, a critical transition represents sharp and deep shifts in a state of a system, set off when the system comes near to a tipping point of instability. It is therefore the process causing crucial transitions in complex systems. In their 1994 study, Thompson et al.[14] examine several kinds of bifurcations in dissipative dynamical systems. A so-called hazardous bifurcation, in which a stable equilibrium becomes unstable when a control parameter surpasses a critical value, is linked to critical transitions [12], and even those seen in complex systems. An analytical base for these transitions is provided by the idea of Critical Slowing Down (CSD). Indeed, a system's recovery from minor disturbances slows down as it gets closer to a tipping point, which makes irregularities more enduring. Variance (standard deviation squared) and autocorrelation (AR(1)), which are quantifiable indicators of system instability, rise as a result. These principles have been widely studied in ecology, climatology and epidemiology, where researchers have observed that ecosystems, weather patterns and even disease dynamics present warning signals prior to major changes.

One of the most influential studies on this subject is that of Dakos et al. (2008)[4] whose work (which also helped us a lot) showed that early warning signals (EWS) can successfully anticipate regime shifts in complex systems. Their study focused on paleoclimate records and ecological transitions, to demonstrate that natural and biological systems show predictable signs of loss of resilience prior to abrupt changes in state. They used historical data on lakes subject to eutrophication and climate transitions leading to ice ages, and they found that increases in variance and autocorrelation reliably preceded catastrophic changes. Their methodology has since been applied to a variety of fields, such as financial markets, where researchers wanted to know whether similar warning signals could be detected before major economic crashes or not. Our work aims then to detect early warning signals, quantifiable trends in market data that point to the imminence of these big shifts. A system's ability to recover from minor disturbances slows down as it gets closer to a critical threshold, which shows up as an increase in autocorrelation and variance. These indicators can therefore serve as warning signals for upcoming critical transitions. To achieve this, we will follow the methodology used by Dakos et al.

Firstly, we will detrend the time series by using the Gaussian smoothing method. This method uses a Gaussian filter to smooth the time series p, yielding a trend component and residuals (denoise = p-ma). Then, indicators like standard deviation and AR(1) are calculated using the residuals, which reflect local oscillations. As we said, our analysis will be based on the bitcoin and ethereum market, but we could have applied this analysis in other areas, such as meteorological or epileptic crises. Our goal is to shed light on the behavior of these extremely volatile systems and how they handle significant changes by focusing on cryptocurrency markets.

2 Literature Review

2.1 Critical transition

Before talking about critical slowing down as an early warning signal, we must look at what is a critical transition. In mathematics, a critical transition is related to what's called a bifurcation point. In many complex, dynamical systems, we observe such bifurcations that are also called a ”tipping point” at which a system will shift abruptly from one state to another [11]. There exist three different types of bifurcations (Thompson et al.,1994)[14]:

-**Safe Bifurcations**: ”... *A continuous attractor path with no fast dynamic jump or instantaneous enlargement of the attractor*”

-**Explosive Bifurcations**: ”*Catastrophic bifurcation that violates the continuity of the attractor path by causing the attractor to suddenly enlarge.*”

-**Dangerous Bifurcations**: ”*Catastrophic bifurcation characterized by the disappearance of the current attractor, giving rise to a jump to a remote attractor of any type*”

An attractor can be seen as a stable equilibrium that tends to become unstable when the control parameter affecting the dynamics of the state variable in the model approaches the tipping point.
For example, if we look at Figure 1 below, we have an example of a kind of catastrophic bifurcation which is a saddle-noddle bifurcation. It corresponds to a model with two stable equilibria represented by the two solid lines and an unstable equilibrium represented by the dotted line. We find this kind of bifurcation between two equilibria that will merge together, here for example the upper stable equilibrium and the unstable one (dotted line) which will lead them to disappear when the control parameter reach the critical value inducing the transition to the second stable equilibrium which is the lower one. To observe such scenarios when a solution is only **locally stable** and not **globally stable** which is the case of the majority of systems in physics or engineering [8]

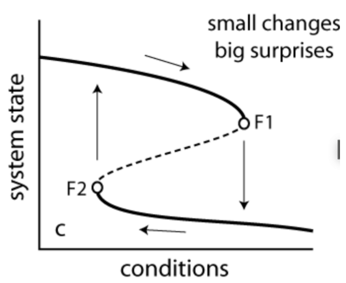


Figure 1: Critical transition induced by a Saddle-noddle bifurcation (source:Early warning signal toolbox)

Most of the works on critical transition try to predict and also understand this phenomenon which could seem complicated due to the different fields on which it could be used, but we observed that a critical transition often highlights the same characteristics [10]:

-"*Abrupt qualitative change in the dynamical system*"

-"*The change occurs rapidly compared to other dynamics*"

-"*Existence of a threshold near the transition*" (Similar to figure 1 at point F1 where the control parameter(not exogenous) or a variable slowly reach the tipping point)

-"*The new state of the system is far away from the previous state*"

Still, it seems like even though we are getting better at predicting critical transitions through tipping point, we still need to improve. We are not totally able to model such dynamical systems in order to predict with accuracy the threshold at which the transition will occur as it was mentioned by (Dakos et al., 2008)[4], in the case of climate change.

2.2 Critical Slowing Down

What is interesting about those critical transitions is that we can try to foreshadow them. When the control parameter increase toward the threshold, we can observe what is called a critical slowing down(CSD). A critical slowdown results in the fact that the system becomes slower in recovering from small shocks and to go back at the equilibrium state as the control parameter becomes close with the tipping point.

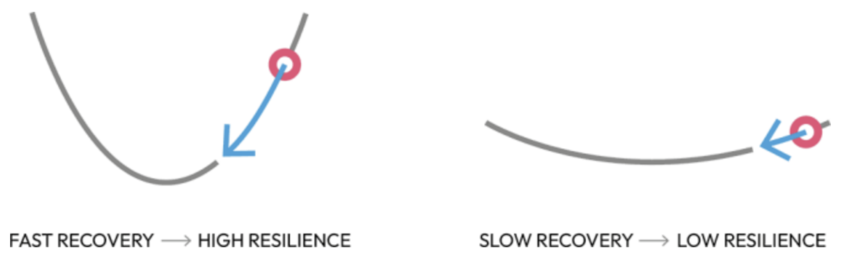


Figure 2: The left ball represents a system that is far from the tipping point as suggested by a steep incline. On the right, we see a shallower curve corresponding to a system close to the tipping point that will take a longer time to recover from a same perturbation.[3]

As we can see on the Figure 2, when we are far from the critical threshold, the ball goes back to the local minimum quickly meaning. The closer we get to the critical threshold, the shallower the curve becomes due to a shallower local minimum meaning that the system takes more time to recover from a small shock. Lastly, when we exceed the tipping point, if we take the image of a ball such as in Figure 2, the ball will go to a lower minimum representing the other stable equilibrium of the model after the critical transition which is represented in Figure 1 as the lower solid line. CSD is often used as an early warning signal in order to predict a critical transition...

We could then ask ourselves: How can we observe these kinds of slowing down? There exist different types of indicators named early warning signals that could give us information about this phenomenon such as the standard error, the AR(1) coefficient [4], Skewness, Kurtosis... In our case and for most of the studies, the variance and autocorrelation are the most common Early Warning Signals based on the critical slowing down. Both of them are giving information in order to observe the slow such as an increase of the variance or of the autocorrelation (AR(1) coefficient) near the tipping point.

2.3 Critical Slowing Down in Finance

In 2016, a study called *"Are critical slowing down indicators useful to detect financial crises?"* was conducted by Hayette Gatfaoui, Isabelle Nagot and Philippe de Peretti [7], in order to analyze the usefulness of critical slowing down indicators to detect financial crises.

In this study, financial markets are considered as complex dynamic systems where phase transitions can occur. It examines whether certain statistical indicators as autocorrelation, variance and asymmetry can predict financial crises as the 2007-2008 subprime financial crisis. The analyses are based on stock market data from 10 European countries and from the USA.

To carry out their analysis, the authors studied the main stock indices in logarithmic format and cumulative returns and used rolling windows to observe the evolution of indicators before the crisis. They used three main indicators in this study. First, the AR(1) coefficient which measures the persistence of shocks, the variance which signals the instability of the system and the asymmetry through Skewness which indicates structural changes in the market.

Regarding the main results of this study, the authors noticed first that autocorrelation does not predict the global financial crisis and can even generate false alarms. Secondly we can notice that for almost every country, the variance increases and the Skewness becomes negative before the crisis, signaling a change in the distribution of returns. This study shows that the critical slowing down indicators are not all effective in anticipating financial crises. Indeed, variance and skewness can be warning signals, whereas autocorrelation does not provide useful information.

Therefore, the study shows that the use of these indicators must be prudent and cannot be generalized to all crises, and that a combination of several indicators can improve the detection of financial crises.

3 Data and Methodology

3.1 Data

Table 1: Critical points and time series

Label	**Critical point**	**Time series**
(1)	10 Apr. 2021	Bitcoin Market Price
(2)	16 Dec. 2017	Bitcoin Market Price
(3)	02 Apr. 2022	Ethereum Market Price

Here, we'll dive into further detail about the data we worked with.
The Bitcoin and Ethereum markets[9] were at the core of our analysis, and we picked times of high fluctuation to help us understand the collapses. Our analysis will focus on the tipping point of April 10, 2021, taking into account 500 days before, when there were significant increases in prices and subsequent declines for Bitcoin. This dataset provided a thorough foundation for analyzing market trends, particularly the massive crash on April 10, 2021. To find early warning signs, we computed our indicators with a 250-day rolling window and looked at the 500 days before the tipping point. Using this method, we were able to do our Gaussian smoothing and see how important measures, such as standard deviation and AR(1), changed locally over time. We then applied the same methodology to other case studies, namely another period during which bitcoin increased significantly, then dramatically fell (another tipping point on December 16, 2017, also taking into account 500 days before). We did the same for ethereum, with which we chose the tipping point of May 8, 2021 (starts on December 25, 2019, i.e. 500 days before the tipping point).
The time periods we chose are good for identifying early warning signals since they offer distinct illustrations of crucial developments. These shifts, which are marked by quick price gains and sudden falls, present a special chance to examine how resilient bitcoin and ethereum markets have become. These durations are consistent with the methodology proposed by [4], which highlights the significance of choosing timeframes in which systems show distinct tipping points so that we can see how indicators like standard deviation and AR(1) change as the system gets closer to instability.
In order to catch significant trends without being overtaken by irrelevant market noise, a time frame of 500 days prior to the tipping point is used as we said. In addition to enabling localized study, the 250-day rolling window offers a dynamic perspective of the system's stability fluctuations before to the key shift. We focused on daily closing prices, or the "close" variable, in order to be more accurate and consistent. We choose this variable because it gives us a consistent point of reference for every day, which enables us to capture the general sentiment of the market. This approach is interesting because the closing price, which is unaffected by transient swings, provides a summary of the trading day in contrast to intraday prices, which can occasionally be loud and erratic. This variable is frequently utilized in financial analysis for the same reason.

Different external and market variables had an impact on the crashes observed at the times we chose. Due to speculation and institutional interest, Bitcoin reached an all-time high in April 2021, surpassing 64,000 dollars, supported by massive investor enthusiasm and increased interest from financial institutions. We had Tesla, MicroStrategy and other major companies that had invested in the asset, bolstering its credibility. The accommodative monetary policy of central banks (notably the Fed), had flooded the markets with liquidity, encouraging investors to turn to riskier assets such as cryptos. However, many things led to a huge correction in April 2021. A coal mine explosion in China led to a power outage, and that was one of the causes of this tipping point. Indeed, many Bitcoin mining infrastructures are located in the Xinjiang area, which was negatively impacted by this occurrence. Mining is essential to the smooth operation of the Bitcoin network, since it secures transactions and guarantees the system's stability. This power cut immediately raised concerns about the security and reliability of transactions. There was a sudden drop in computing power, which slowed down transactions and amplified market volatility, triggering a chain reaction among investors. Moreover, the US Federal Reserve's monetary policy worries were another element that led to this downturn. At the time, investors' desire for speculative assets like cryptocurrencies was diminished as markets started to brace for a potential tightening of monetary policy. Early discussions about a possible normalization of monetary policy sparked concerns about a decrease in the amount of liquidity available on financial markets, making riskier assets more susceptible to corrections. This was in contrast to the low-rate environment and massive injection of liquidity that had fueled Bitcoin's spectacular rise at the beginning of 2021.[1]

Additionally, in 2017, Bitcoin experienced a spectacular surge : it rose from 1,000 dollars in January to nearly 20,000 dollars in December. This sharp increase was caused by a large flurry of retail investors who were lured in by the promise of quick profits, as well as a spike in Initial Coin Offerings (ICOs), which mostly used Bitcoin and Ethereum as funding sources. Institutional interest was also increased and the market was given some credibility when the CME and CBOE announced in December 2017 that the first Bitcoin futures contracts will be launched. However, this growth proved unsustainable and in early 2018 several events caused the bubble to burst. China has banned ICOs and restricted cryptocurrency trading, while other countries such as South Korea have begun tightening exchange regulation, creating then a climate of uncertainty. Many startups that raised funds through ICOs started selling their tokens, which put pressure on the market. The price decline triggered a panic among retail investors, who liquidated their positions massively. In less than a year, Bitcoin lost nearly 80 percent of its value, falling below 3,200 dollars in December 2018.

Finally, we looked at the Ethereum market. In the autumn of 2021, ETH was still supported by the rise of NFT and decentralized finance (DeFi), which attracted new investors and that kept strong demand on the Ethereum network. However, the fact that the US Federal Reserve (Fed) could potentially tighten its monetary policy marked a turning point. This led to a drop in the liquidity available on the markets and prompting investors to move away from risky assets such as cryptos (government bonds for example). Additionally, in May 2021, We had chinese authorities who have essentially prohibited financial institutions from providing services linked to cryptocurrency, probably in order to curb the flow of funds and restrict access to exchange platforms. Therefore, it is clear that China tightened its cryptocurrency laws, which may have increased market pessimism and lowered Ethereum's value. Beijing has intensified its anti-mining campaign, which has inevitably raised environmental issues and financial stability threats. The forced closure of several mining farms, primarily located in areas like Sichuan and Xinjiang, reduced the Ethereum network's processing capacity and sparked questions about its decentralization and security. Investors became alarmed as a result, fearing a miners' flight and a drop in liquidity. Massive sales that followed accelerated the downward pressure.

3.2 Methodology

3.2.1 Detrending

The detrending consists in removing the trend pattern of our original time series in order to obtain a mean-stationarity which will make us able to observe other structures that don't rely on trend and will enable us to make the calculations for our indicators. We will use the technique of the moving average [4].Here, we use a weighted moving average in order to give different multiplying factors to the mean of our data. Here in our case, we use the Gaussian kernel function in order to apply Gaussian weights and to smooth the trend of our time series.

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

We will then be allowed to compute our weighted moving average on our data before the date we used as a critical point in order to filter the trend. Here, in our case, we chose T=500 days before the tipping point. In order to compute our leading indicators we still need to obtain the difference between our original time series and our moving average:

$$y_t = z_t - MA_t \quad t = 1, ..., T \tag{2}$$

Here, z_t are the values of the cryptocurrencies at the end of the day. MA_t is our moving average and y_t is the "residuals" which are our detrended time series. We still need to be careful about the size of our bandwidth σ which is very important if we want to observe meaningful results. If we take a large bandwidth, this could lead to an oversmoothed time series with a biased estimation of our local means leading to the loss of trend details (peak and troughs) [5].On the other hand, a small bandwidth could give us a large variance of the detrended signal. By carefully choosing the size of the bandwidth, we could avoid these issues and be able to obtain a detrended time series which underlines other effects rather than the trend. In our case we chose to take a $\sigma = 12$ by trying different values and comparing them through an arbitrage between a more detailed detrended time series (lower σ) and smoother one (higher σ).

3.3 indicators

Now that we have our detrended time series, we are able to calculate our indicators, our Early Warning Signals in order to observe a critical slowing down. To proceed, we will make our estimations based on a rolling window of size $\frac{T}{2}$ [4]. In our analysis, we used two indicators that are the Standard Deviation and the AR(1) coefficient in order to capture the slowing resilience of the model to back to the equilibrium.

3.3.1 Standard Deviation

Standard deviation (SD) observes the spread around the mean of the observations of our time series prior to the critical point. Various studies have shown that when a system is approaching the tipping point, an increase in variance and in the spread is observed because the system takes longer to recover from small shocks [11, 13]. Nevertheless, we should be careful about such a result with the standard deviation since the variance could increase due to a poor detrending or the presence of one-off external shock, that could distort interpretations and not denote the effect we want to observe (i.e. generate a rise in variance without necessarily signaling a critical slowing down). In order to compute it, we took, as previously said, a moving window of $n = \frac{T}{2}$ [4] which in our case corresponds to 100 observations since we have 500 days before the tipping point. Still, we need to be careful of the size of our window because if it's too small we could have less precise indicators which are represented by the size of the bandwidth (σ) another important parameter.

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{3}$$

3.3.2 AR(1)

An other leading indicator to capture the slowing down in a system recovery is the coefficient we obtain from a lag-1 auto-regressive model (AR(1)). When the model approaches the critical point, we should observe an increase in the AR (1) coefficient because the observations become more and more similar to each other (autocorrelation), representing the slowdown.

This method is usually employed in the literature to observe critical slowing down [11].The application of AR(1) in early warning signals has been largely documented in complex systems, such as financial markets, ecology, and climate research. The mechanism is that when a system becomes less resilient, it becomes more reliant on its previous states, which causes variations to exhibit more autocorrelation. The characteristics of financial markets, including cryptocurrency, are similar; before significant crashes, price swings are persistent due to liquidity shortages and changes in investor attitude. We then used a simple auto-regressive model. We calculated the AR(1) coefficient through our moving window ($n = \frac{T}{2}$) of 250 observations sliding from left to right.

$$y_t = \lambda y_{t-1} + \epsilon_t \tag{4}$$

Here, as we said before, y_t denotes our detrended time series, λ corresponds to the autocorrelation. It is equal to zero for white noise and when the noise is auto-correlated it's close to one [11]. As stated before, when we get close to the critical point, λ get close to one, which denotes an increase in the autocorrelation and then a lower resilience of the model. In simple terms, represents how much today's value depends on yesterday's value. If = 0, there is no correlation, meaning that past values have no influence on the present. If is close to 1, it means that price movements become highly dependent on their past values, which indicates that the system will be slow to recover from shocks.

3.4 Skewness

First of all, let's define Sknewness. Skewness is a measure of asymmetry. The asymmetry of a distribution reflects the regularity (or not) with which observations are distributed around the central value. This indicator can be used to detect whether the extreme values of a time series are more concentrated on one side or the other of the mean, making it a potentially relevant indicator in the study of critical transitions. Mathematically, skewness is defined by the following expression :

$$S = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma} \right)^3 \tag{5}$$

where :

- x_i represents an observation of the time series studied, corresponding here to the closing prices of Bitcoin and Ethereum after detrending
- $\bar{x}$ is the average of the observations calculated on a rolling window.
- σ is the standard deviation, representing the dispersion of Bitcoin and Ethereum closing prices around their mean.
- n represents the total number of observation used in a rolling window

As we said, skewness measures the asymmetry of a distribution and identifies whether extreme price variations are more frequent in one direction or the other. The exponent 3 takes into account the direction of this asymmetry. We have positive deviations (above the mean) that remain positive when cubed, and negative deviations (below the mean) that remain negative. This directional component is essential because it allows to show us whether a distribution leans to the left (negative asymmetry) or to the right (positive asymmetry). Sharp declines are more frequent when the skewness is negative, while extreme rises are more frequent when the skewness is positive. Here, we're interested in the closing prices on the Bitcoin and Ethereum markets, and in this context, this indicator can be used to detect progressive imbalances in price dynamics. According to Doane and Seward (2011)[6], skewness is often underestimated in statistical analysis, even though it provides essential information on the structure of fluctuations. By using skewness, we may determine whether these fluctuations become increasingly asymmetrical as the market gets closer to a tipping point. A balanced market has a relatively symmetric distribution of price increases and decreases around the mean. Its loss of stability, however, results in a distorted distribution of variations, which drastically changes the skewness. Thus, skewness is an interesting indicator but it needs to be nuanced. Indeed, cryptocurrency markets are by definition very volatile, so it is worth accompanying our analysis with other indicators.

3.5 Kendall's Tau

The Kendall's tau or Kendall correlation rank will enable us to establish trends in order to observe the similarities between two variables. We can use it on our indicators of critical slowing down in order to observe if we have an increase in the variance (standard deviation) or in the autocorrelation (AR(1)). It's a non parametric statistical measure that we will employ here in the case of our AR(1) indicator by looking at the correlation between our variable and lag(1) of this variable (here, the detrended closing price of Bitcoin)[2].It is denoted on Eq5. below:

$$\tau = \frac{C - D}{\frac{n(n-1)}{2}} \tag{6}$$

Here, C is the number of concordant pairs (if the ranking of values is the same for the two variables), D the number of discordant pairs and $\frac{n(n-1)}{2}$ is the number of different pair combination. The Kendall's tau is within the interval [-1;1]. If $\tau < 1$ then we observe a negative trend and inversely a $\tau > 1$ denotes a positive trend which is what we expect to observe when we are looking for a critical slowing down. The closer τ is getting to 1 (positively or negatively) means a stronger trend and for example , if it's positive, more similarities between the variables.

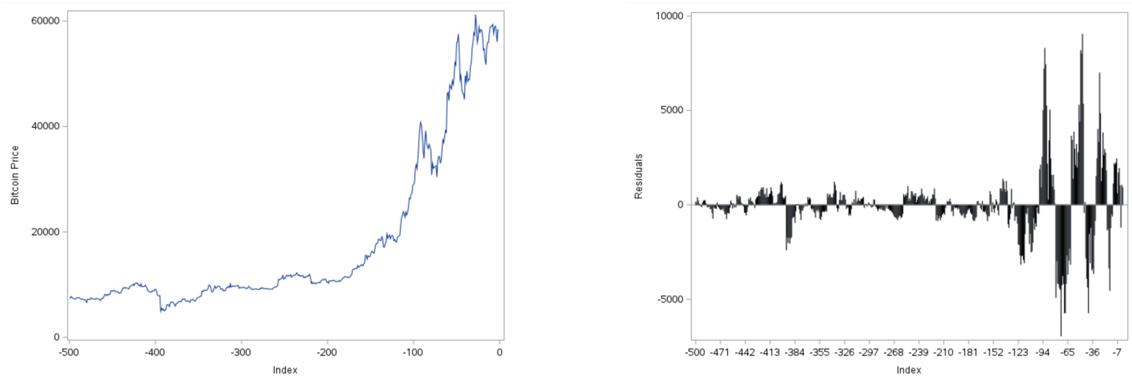
4 Results

We are going to look at the results we obtained by trying to find evidence of critical slowing down within three time series showing us a big crisis, big drops in the values of cryptocurrencies such as Bitcoin or Etherum. Here, in this section, we are going to focus on the analysis of our Early Warning Signals (SD and AR(1) coefficient) and see if we can observe a critical slowing down when the system gets closer to the critical point for our three time series. Then, to finish our analysis, we will do a robustness check of what we analyzed.

4.1 Analysis of our Early Warning Signals

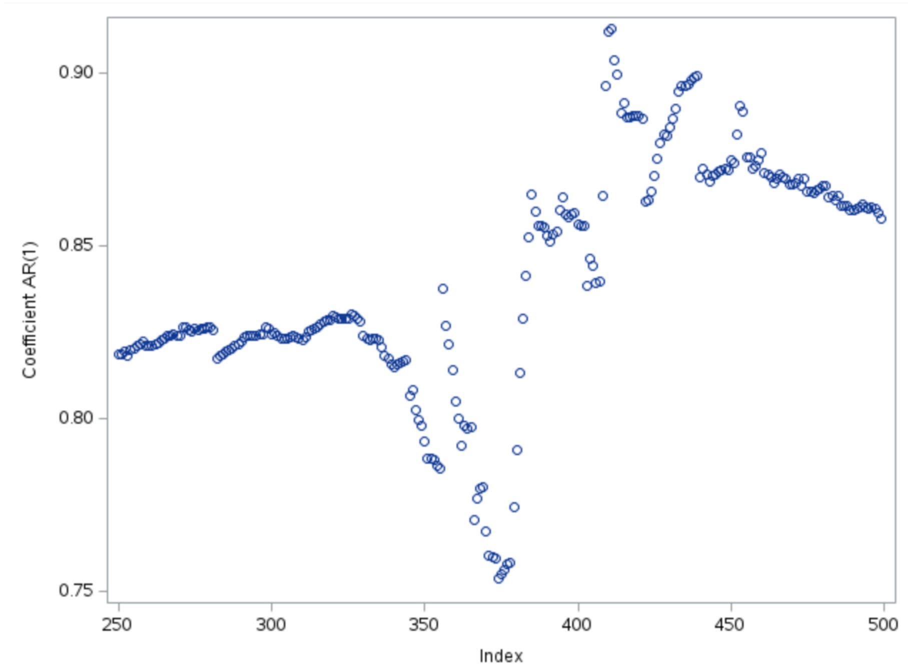
So, as we said before, we took three different time series in order to compute our leading indicators. Just the observations prior to the drop in the closing values per day have been analyzed in a range of T=500 observations.

In the first place, we are going to talk about the fall of the Bitcoin closing value that happened between **April and May 2021**. Precisely, we chose as our tipping point the 10th April 2021 before the huge drop that followed this date. The Figure 3 corresponds to the calculation of the Early Warning Signals for this period. The first graph shows us the closing price index of Bitcoin with T= 500 observations before the critical point. We used an index such as the point 0 on the x-axis corresponds to our critical point. Then Figure3.Graph(b) denotes the residuals of our detrended time series. We used a Gaussian smoother with a bandwidth $\sigma = 13$ in order to smooth our original time series and to obtain a moving average that we subtracted from the time series to obtain the residuals that we can observe that allowed us to obtain our leading indicators. The two lower graphs (Figure4.Graph(a)(b)) are plots of our leading indicators (AR(1) coefficient, Standard Deviation) and Figure5 of the Skewness. They are calculated using a moving window of $n = 250$ observations sliding from left to right.

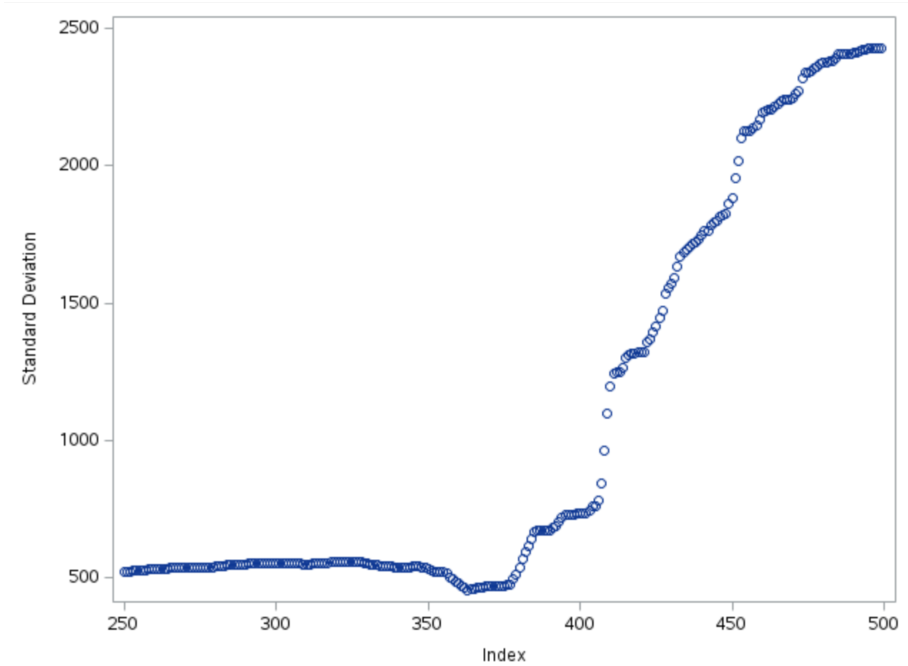


(a) (b)

Figure 3: (a) The closing price of Bitcoin over the period. (b) The residual time series after detrending using the Gaussian kernel smoothing method.



(a)



(b)

Figure 4: (a) AR(1) coefficient indicator. (b) Standard deviation indicator.

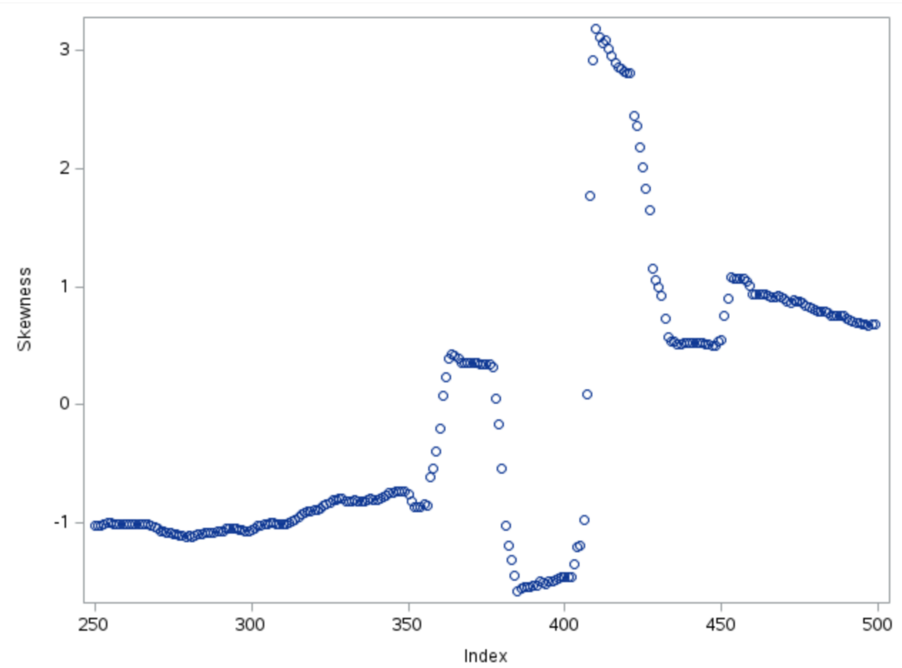


Figure 5: Skewness indicator

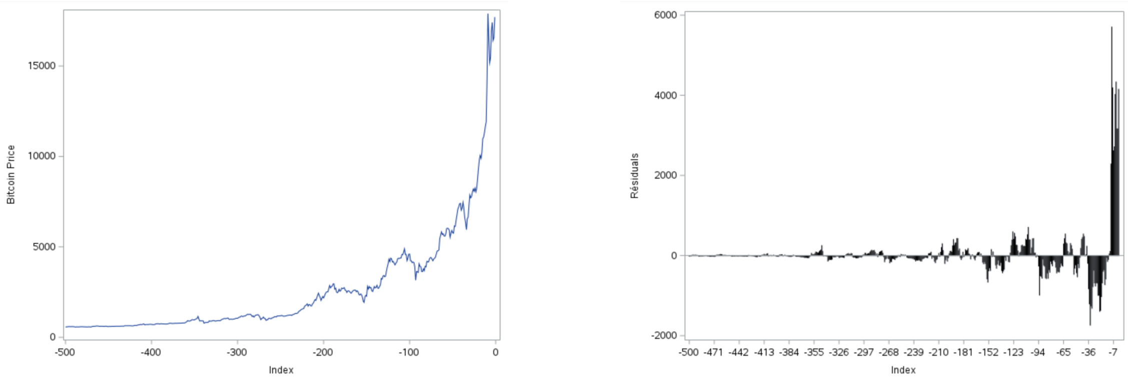
As we can see in this case of an abrupt shift in the Bitcoin closing price in April 2021, we observe in the Figure 4.Graph(a) and (b) and in the Figure5., an increase in the AR(1) coefficient, in the standard deviation and of the skewness during the period before the transition, suggesting a slow down of the system just before the tipping point. If we look at the standard deviation we observe an important rise around 180 days before our tipping point, which skyrockets as we get closer, aligning with the concept of critical slowing down.

On the other hand if we look at the AR(1) coefficient, we still observe an increase in the autocorrelation of our system and also of his Kendall's tau reaching 0.712 at the end of our rolling window. Even if this result underlines that the system slows down and is less resilient. In fact, the level of the AR(1) coefficient dropped around 150 days before the transition before a huge jump just 25 days after. Then we observe a gentle fall of the autocorrelation just near the critical point. The rise in the indicator is less important than the SD making it less relevant to conclude that there was a Critical Slow Down before the shift.

We observe the same kind of result when we are looking at the Skewness. Even if we observe a rise in the indicator due to the asymmetry of the system, the fact that the huge jump occurs before the date we selected and then we observe a slight downturn when approaching the critical value we selected makes it also more complicated to interpret.

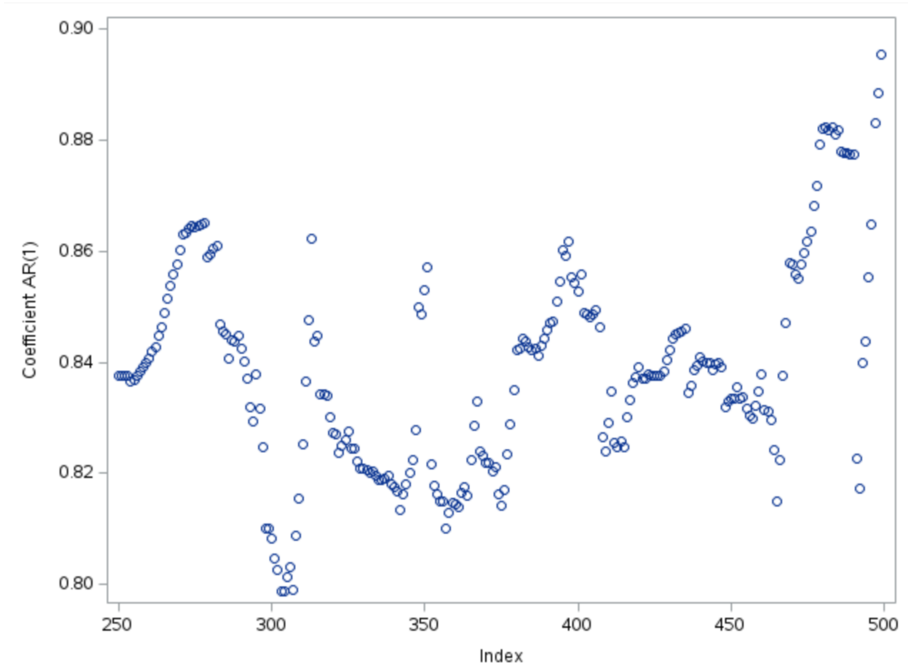
Overall, we saw with this case that it seems that there was indeed a slow down of the system before the fall in the Bitcoin closing price. All of our indicators tend to increase when we are getting closer to the bifurcation which means that the system tends to take more time to recover from a small shock and go back to the equilibrium. Still, only the standard deviation gave us really encouraging results, in fact even if we observe a global increase of the autocorrelation and the skewness it's more complicated to conclude with precision that there is indeed a clear slowing down of the system before the shift. This may be due to the data selection, as we are looking at our original time series, we observe a small drop in the bitcoin closing price a 100 days before the date we chose which may interfere with both of these indicators. It could also be the case of a False Positive, where we observe an early warning signal even without a bifurcation [11].

We are now going to analyze the case of the drop in Bitcoin closing price that occurred at the end of 2017, more precisely we chose the 16th December 2017 as our tipping point. The Figure 7 below represents the calculation of our leading indicators to find out evidence of a slow down during the period preceding this date. We took T=500 observations prior to the 16th. Our rolling window is still of n=250 observations ($\frac{T}{2}$) based on [4]. The two first graphs represents: Figure6.Graph(a) our original time series, Figure6.Graph(b) the residuals we obtain when we subtract the moving average that we obtained after detrending our time series with a Gaussian Kernel smoother. The two lower graphs still represent our leading indicators that are the standard deviation for Figure3.graph(c), and the AR(1) coefficient for Figure3.graph(d).

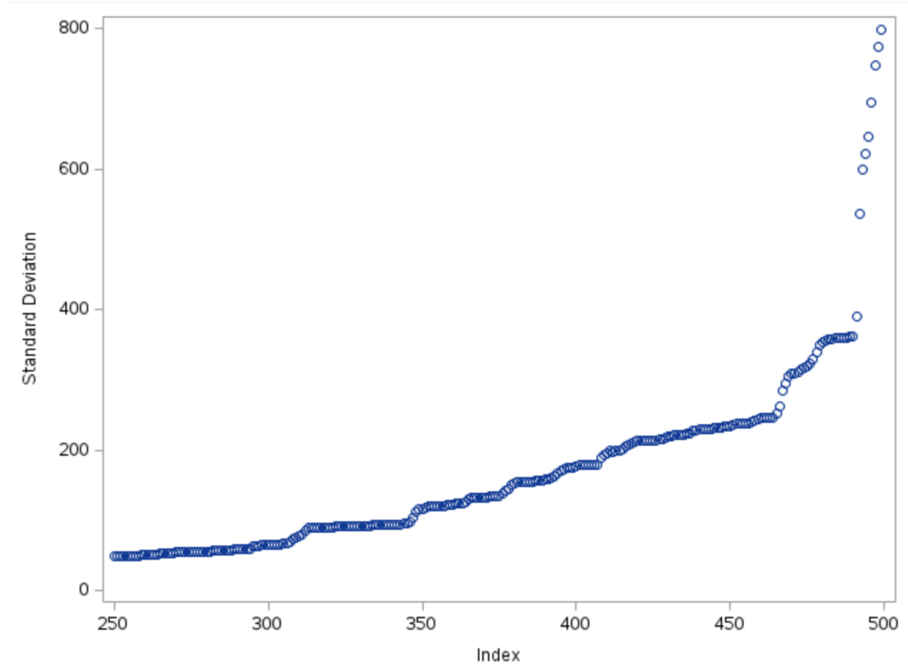


(a) (b)

Figure 6: (a) The closing price of Bitcoin over the period. (b) The residual time series after detrending using the Gaussian kernel smoothing method.



(a)



(b)

Figure 7: (a) AR(1) coefficient indicator. (b) Standard deviation indicator.

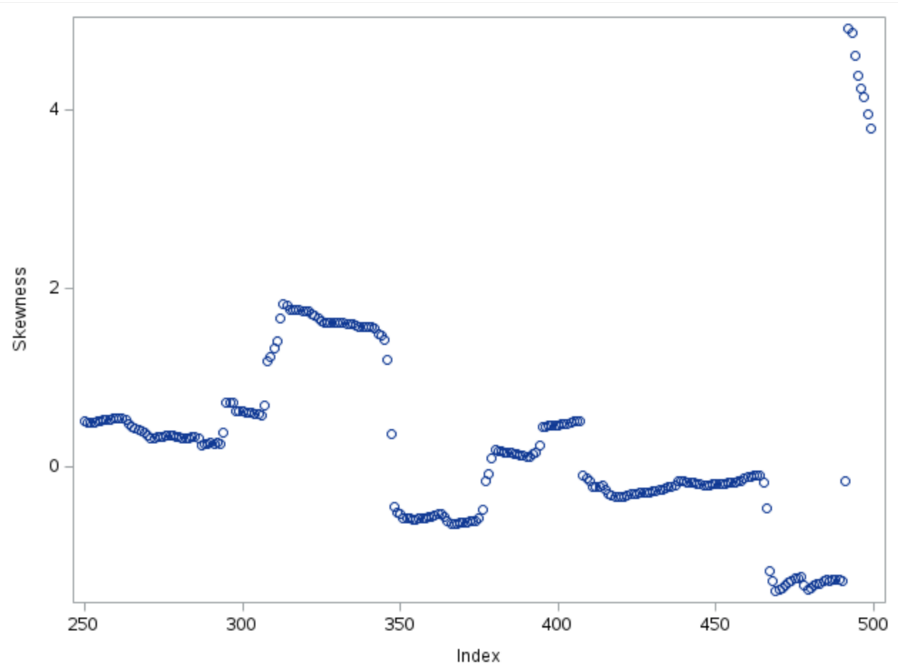


Figure 8: Skewness indicator

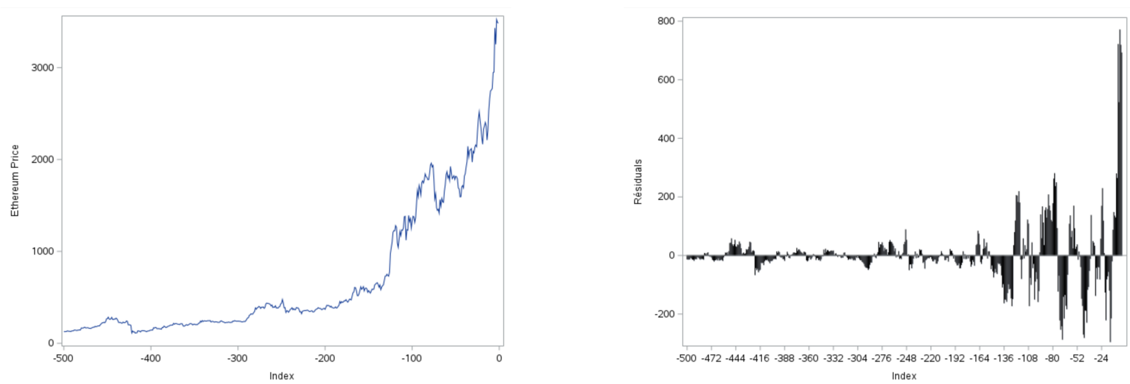
The figures just above show us the results of our early warning signal calculations for the December 2017 crisis. Just as before, we observe here an increase in the level of our three leading indicators suggesting that there was a slowdown of the system before the critical point.
Here, in this case, the results tends to give us a clearer evidence of the slow down before the shift. If we are looking at the Standard deviation for example, we observe a moderate increase all over the period and then we see that the spread skyrocket near the point .
Then, if we look at the autocorrelation and the skewness, which gave us mixed results when we looked at the drop of 2021. Here, we observe pretty good outcomes, if we look at the AR(1) coefficient, we observe a lot of fluctuations in the indicator throughout the period with drops and rises. Even though, we see an increase of the correlation as we are getting closer to the transition with a huge drop 10 days before our date followed by a huge increase. The variable is then getting more correlated with it's past which also underlines by a Kendall's tau of 0.7235 at the end of our window which is pretty high if we look at what we said earlier about the fact that a Kendall's tau close to 1 means a positive trend which is what we should observe in the case of a critical slowing down .
For the skewness, we observe a decrease of the indicator during the period and then, a sharp increase near the shift which also an interesting result that can makes us think that there really was a slowdown.
Overall, it seems that in this case, our leading indicators we're more relevant early warning signals than in the case we analysed before. All of them gave us positive results where we could capture what we should observe when we are looking at these kinds of indicators. It seems clearer here that in 2017 there was indeed a slowdown.

For our last analysis, we chose to take another cryptocurrency than the bitcoin in order to observe if the results will be approximately be the same.
We then took a time series about the evolution of Ethereum closing price. Such as the points before, we chose our transition point by eye and through a literature review in order to carefully select a relevant point for our calculations. Based on the informations we had, we selected May 2021 for our analysis, more precisely, the 8th of May 2021. We still chose a period of $T = 500$ observations for a rolling window of $n = 250$ observations ($n = \frac{T}{2}$) as we can observe on Figure9.graph(a). Figure9.graph(b) shows the residuals obtained after subtracting the moving average with our original time series obtained by using a Gaussian Smoother with a bandwidth of $\sigma = 13$.
Figure10.graph(a)(b) and figure11 denotes our leading indicators that are the same as before:

-AR(1) coefficient

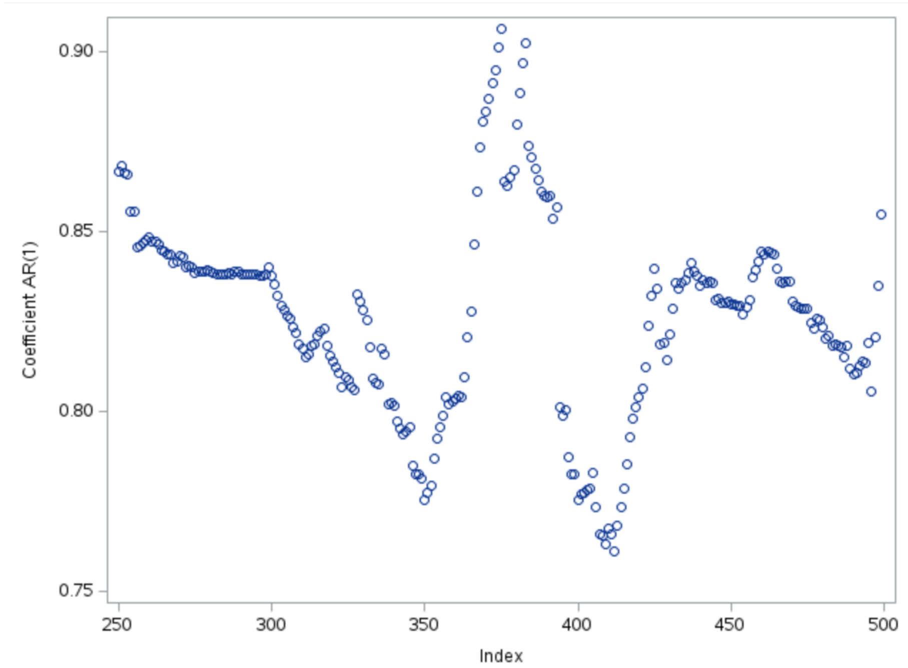
-Standard Deviation

-Skewness

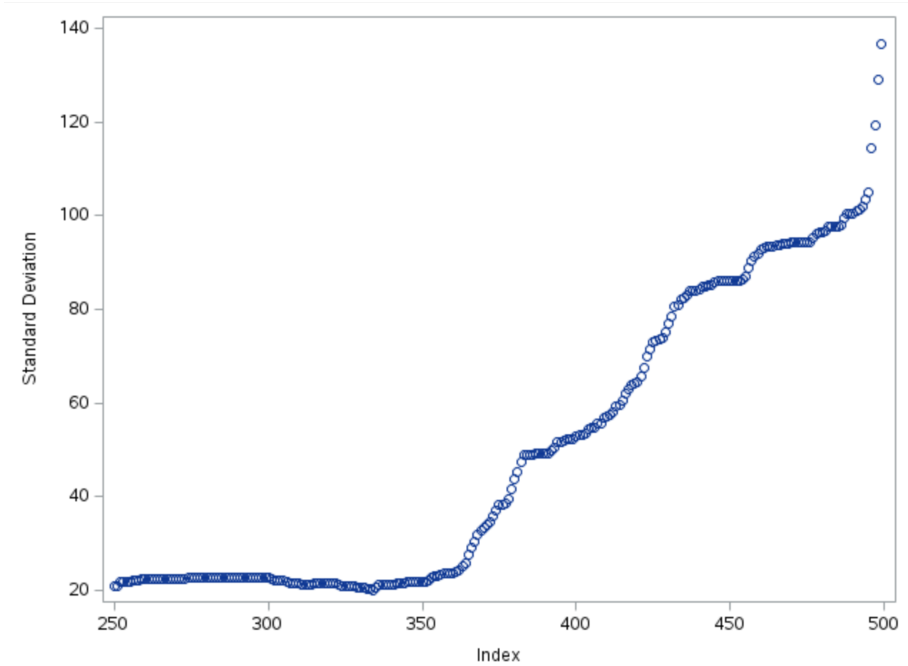


(a) (b)

Figure 9: (a) The closing price of Bitcoin over the period. (b) The residual time series after detrending using the Gaussian kernel smoothing method.



(a)



(b)

Figure 10: (a) AR(1) coefficient indicator. (b) Standard deviation indicator.

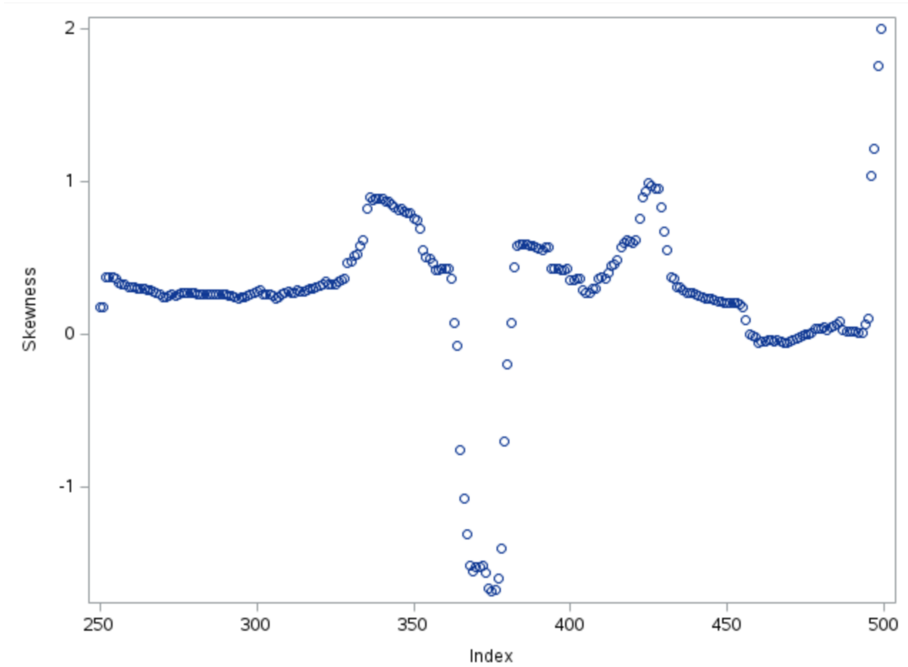


Figure 11: Skewness indicator

We will then analyse the results of our third and last case about the Ethereum. As we can see, we globally have an increase in the level of our indicators which can suggest that there is evidence of critical slowing down but before being able to conclude clearly that we observe it, we still need to look in detail at the evolution of each of them.
As we can observe, if we look at the standard deviation we see a sharp increase in the spread of our observations around the mean such as our two previous analysis. This result seems to be very relevant like we saw previously and tend to confirm us that there was a lost of resilience in the model.
We will then look at the Skewness. We see that around approximately 120 days before the date we chose, the indicator had a huge fall directly followed by a rapid growth. We can then question us on what could be the reasons of such a fall, maybe the fact that the Ethereum closing price saw a huge and instant increase that could have affected the symmetry around the mean of our distribution making it more symmetric. Still, if we look at the indicator near the shift we see that it skyrocketed which seems relevant .
Now for the AR(1) coefficient, the fluctuations seems to resemble to the ones we observed when we looked at the first Bitcoin crash that happened in April-May 2021 which is approximately the same period. What is interesting with the autocorrelation here apart from the huge spike (maybe due to our data selection or maybe a false positive where we observe an early warning signal without a bifurcation) we observe a 125 days before our crash, is that despite having a Kendall's tau of $\tau = 0.6642$ at the end of our rolling window which is quite high and can be a good explanation of the slowing down we see that globally, over our period the value decreased (from $\tau = 0.7078$). Still it is complicated to understand why especially if we look at the graph, near the shift we see that the level of the autocorrelation starts to increase a lot which is what we should observe as a relevant information for the slowing down.
Overall for this case, we still find evidence of a slowing down before the crisis for the stan-

dard deviation. The increase in the asymmetry seems to confirm this thought. However, the the autocorrelation give us mixed result and makes it more complicated to analyse.

So with our analysis, we find out that when we look at cryptocurrencies crises or drop in the closing value, it seems that the critical slowing down is a good Early Warning Signal in order to give information about the bifurcation of the system. This may be due to the fact that some complex systems are better understood than others [11].
If we look at our results from a more global point of view we see that all the indicators (autocorrelation, standard deviation, skewness) that we used are increasing which goes in par with what we said earlier(cf...) and what we should obtain when we want to shed light on this phenomenon.
Nevertheless, some of these indicators are more complicated to analyze. For example, in our study, we saw that globally the standard deviation or the variance is showed us some strong results giving us the opportunity to observe the decrease of resilience of the system with sharp increase of the indicator in every cases when approaching the tipping point that we set.
On the other hand, our others indicators have also shown globally good and relevant results of a critical slowing down among the three cases but it was more difficult to analyze and sometimes the increase in the autocorrelation and the skewness were not always as important as what we saw on the standard deviation. For example if we look at our results for the AR(1) coefficient, in two of our three cases (BTC:April 2021 ,ETH:May 2021), we observe that the autocorrelation has a big spike before our bifurcation point which can make us question ourselves. There may be different reasons, one of them is maybe due to the fact that both of these analysis took place during approximately the same period. Even if this is two different cryptocurrencies, they are pretty similar,and an external shock could have affected both of them resulting in an increase of the AR(1) coefficient before the tipping point, which could lead to a **false positive**(Could happen by chance or because of external perturbations). Another reason is that such signals could also be observed when the system approach a threshold that is not a catastrophic bifurcation which is something that already happened in a study of critical slowing down and also for the autocorrelation. "*Such non-catastrophic thresholds are related to catastrophic ones... Small forces can cause major changes in the state of a complex system*"[11]. We can observe such phenomenon on figure Figure12 below :

We could also ask ourselves if the AR(1) indicator is simply not a good indicator of the critical slowing down for finance [7, 5] and cryptocurrencies which are really close domains (but it seems complicated to come up with such results because the AR(1) still gave us good results). On the other hand, if we look at the skewness, what is interesting is that as we observed in our cases, it tends to increase when we are getting near the catastrophic bifurcation which correspond to a greater asymmetry of the fluctuations which will keep the system at the unstable point longer. It is not really a question of slowing down but more an effect of attraction that will keep the system in this state. Our three analysis gave us the opportunity to observe such a phenomenon.

To conclude the analysis of our leading indicators, based on our results, it seems that observing a critical slowing down of a system before a critical transition works for

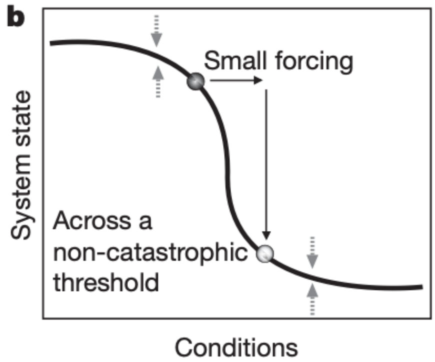


Figure 12: Early Warning Signal when the system approach a non-catastrophic bifurcation [11]

cryptocurrencies. Nevertheless, it is also possible to enhance our study by applying a bootstrap method in order to have better result [4, 5]. It could be also interesting using other techniques of filtering, or to test if our indicators are significant.

4.2 Robustness of parameters

After the calculation of our leading indicators, we wanted to observe the robustness of the parameters by varying two of the key parameters that we used.
The first parameter is the bandwidth that we used in order to compute our Gaussian kernel smoother to obtain our detrended time series. The residuals that we get from our detrending were used in order to compute the autocorrelation, the variance and also the skewness so this parameter has an important effect on our result. Note that we need to be careful with the size of the bandwidth because a filtering to important could remove all the trends we want to study. In the other hand if we not filter enough, it could leave some trends we don't want in our analysis and make our results less relevant.
The other key parameter is the size of our rolling window, since we compute our leading indicators throughout this window, results will also be impacted if we change its size. A too small window will not allow us to capture all the movement before the tipping point making our result less reliable.
In order to check the robustness of our parameters we calculated for our three different cases all the Kendall's tau obtained from the AR(1) coefficient computed for different rolling window sizes and different bandwidth. We then used a contour plot in order to observe our results. We chose a scale of 1 between 125 and 250 for the change in the size of the rolling window. For the filtering bandwidth, the scale is 0.01 between 5 and 20.

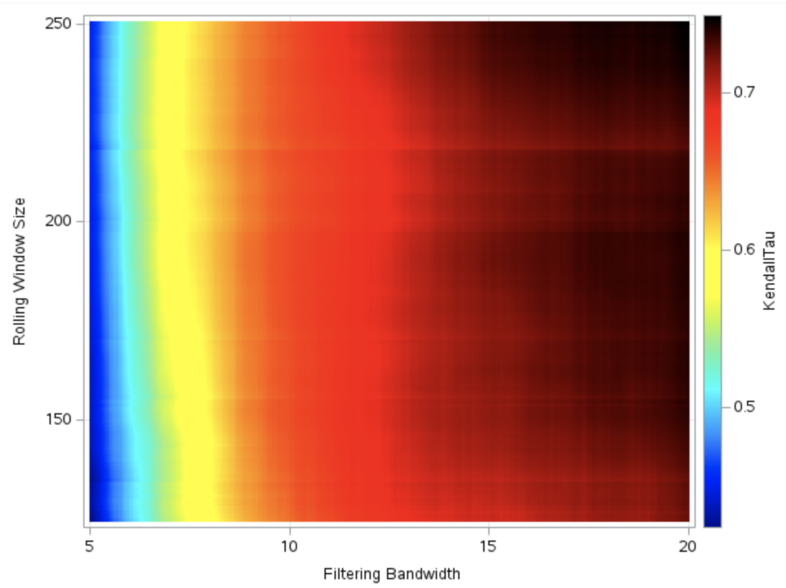


Figure 13: Analysis of the robustness of the parameters for the Bitcoin closing price in April 2021 (window size and bandwidth).

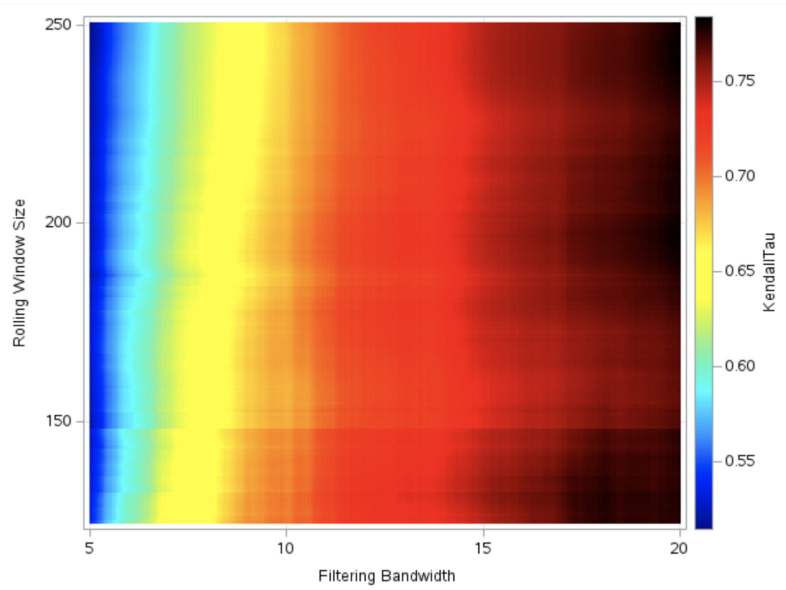


Figure 14: Analysis of the robustness of the parameters for the Bitcoin closing price in December 2017 (window size and bandwidth)

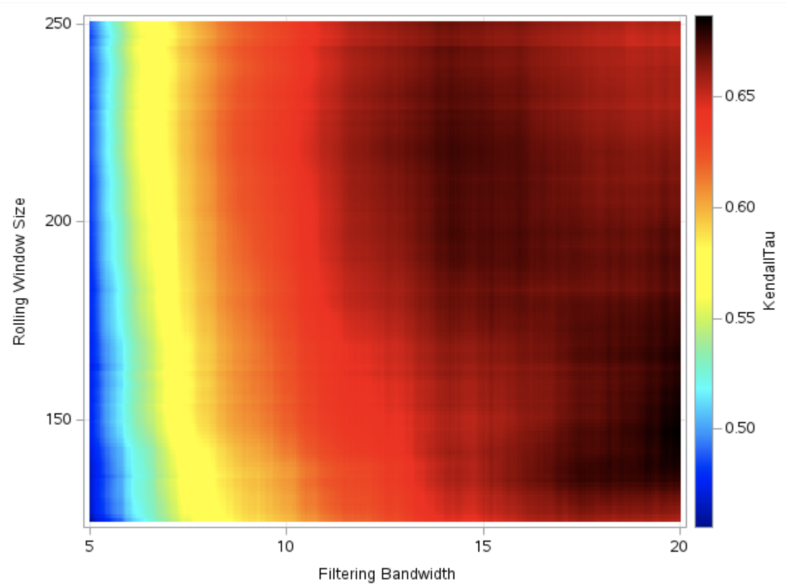


Figure 15: Analysis of the robustness of the parameters for the Ethereum closing price in May 2021 (window size and bandwidth)

It is important to denote that in our analysis we used a bandwidth of $\sigma = 13$ and a rolling window of the size $n = \frac{T}{2} = 250$ observations. The darker is the color on the Heat-maps just above, the higher is the Kendall's tau.
If we look globally at our results, we observe that the size of our rolling window is giving significant results for the three different cases. On the other hand, it seems that we chose a bandwidth maybe too small compared to what we can observe. The three analyses seem to tell us that a bandwidth close to 20 is in most cases better to capture a more significant positive trend of the Kendall's tau.
It is also interesting to observe that there is not a single answer to the question but if we look clearly, we observe that there is always a set of different parameters (different sizes of windows or of bandwidths) that give us significant result for the Kendall's tau we obtained from the AR(1).
These three figures underline what we said earlier about the fact that when we change their size, we could obtain very different and significant results.

5 Conclusion

In our study, we looked at critical slowing down as a way to forecast future critical transitions by using time series of the periods before. In order to do so, we detrended our original time series using a Gaussian Kernel Smoother and then we used a rolling window in order to compute our leading indicators that are the autocorrelation, the standard deviation (or variance) and also the skewness. We saw through our research that the increase in these indicators could be a signal of the slowdown of the system when approaching the bifurcation point.
This theory has been confirmed by different studies on different fields of research such as climate change with paleo climate[4],or also with finance[7].

We then decided to compute these indicators in the domain of cryptocurrencies in order to see if we could observe such slowdown. We used two different time series with different dates two for the **Bitcoin** and one for **Ethereum**. All of our calculation were done using a rolling window of 250 observations and a filtering bandwidth of 13. We then come up with the conclusion that yes, such slowdown of the system tends to be observable before a shift for cryptocurrencies. All of our three cases more precisely are the Bitcoin closing price in April 2021, in December 2017 and the Ethereum in May 2021. It was particularly the case for the standard deviation which seems to be really good to observe the slowdown. We also saw that it could be more complicated to interpret such results when we are looking at the autocorrelation for example, even if we had a positive and close to 1 Kendall's tau (0.7235 for December 2017)[5] which is favorable when we are talking about critical slowing down. This could be linked to the conclusion done for finance that the AR(1) coefficient is not a really a good indicator of the slowdown. Nevertheless, we conclude that overall, looking at slowing down is a good Early-warning-signal when we talk about cryptocurrencies.
In our final analysis, we looked at a robustness test of our parameters (rolling window size and bandwidth) and we saw that a large panel of parameters could give very positive and significant results which is encouraging.
Still, our analysis was really limited due to our knowledge and it is possible to improve in this field of research. On one hand, if we take our own experience, it could possible to enhance our analysis by implementing Bootstrap for example or by testing if our indicators are significant. On the other hand, if we look more generally, we see that these early warning signals are really good in order to predict critical transition. But, some very complex systems are way too complicated for us in order to have proper results. Our methods, even if they are improving are still limited and we do not really catch all the mechanisms. Sometimes, we still struggle a lot to find the moment of transition. One of the questions that is ask and is a subject of research and improvement for the future is about the real life application of such methods and if we are really able to detect a signal (such as the critical slowing down) in order to take action to prevent the transition? [11].

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