Sommes

#algebre #suites

Règles de calcul

- $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$
- $\sum_{k=1}^{n} \lambda a_k = \lambda \sum_{k=1}^{n} a_k$
- $\sum_{k=1}^{n} a_k \times b_k \neq \sum_{k=1}^{n} a_k \times \sum_{k=1}^{n} b_k$
- $\frac{1}{\sum_{k=1}^{n} a_k} \neq \sum_{k=1}^{n} \frac{1}{a_k}$

Changement d'indice

- Ecrire la relation entre les deux indices
- Changer les bornes de sommations
- Changer l'expression contenue dans la somme

Somme d'une suite arithmétique ou géométrique

- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1}^{n} k^3 = (\frac{n(n+1)}{2})^2$
- $\forall x \neq 1, \sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1 - x}$ si $x = 1$ alors $\sum_{k=0}^{n} x^k = n + 1$

Suite arithmétique

$$\begin{cases} u_{n+1} = u_n + r \\ u_n = u_0 + nr \end{cases}$$

- $\displaystyle\sum_{k=0}^{n} u_k = (n+1)u_0 + \frac{n(n+1)}{2} r = \frac{n+1}{2}(u_0 + u_n)$

Suite géométrique

$$\begin{cases} u_{n+1} = u_n q \\ u_n = u_0 q^n \end{cases}$$

- $\displaystyle\sum_{k=p}^{n} u_k = \frac{u_p - u_{n+1}}{1-q}$

Somme télescopique

$$\sum_{k=0}^{n} f(k+1) - f(k) = f(n+1) - f(0)$$

Somme double

Somme rectangulaire

$$S = \sum_{i=1}^{n} \sum_{j=1}^{p} a_{i,j} = \sum_{j=1}^{p} \sum_{i=1}^{n} a_{i,j} = \sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq p}} a_{i,j}$$

Somme triangulaire

$$S = \sum_{i=1}^{n} \sum_{j=1}^{i} a_{i,j} = \sum_{1 \leq j \leq i \leq n} a_{i,j} = \sum_{j=1}^{n} \sum_{i=j}^{n} a_{i,j}$$

Produit de deux sommes finies

$$\sum_{k=1}^{n} a_k \times \sum_{j=1}^{p} b_j = \sum_{k=1}^{n} \sum_{j=1}^{p} a_k b_j = \sum_{j=1}^{p} \sum_{k=1}^{n} a_k b_j$$

Quelques formules

Combinaisons

- $\displaystyle\binom{n}{p} = \frac{n!}{p!(n-p)!}$
- $\displaystyle\binom{n}{0} = 1, \binom{n}{1} = n, \binom{n}{2} = \frac{n(n-1)}{2}$
- $\displaystyle\binom{n}{p} = \binom{n}{n-p}$
- $\displaystyle\binom{n+1}{p} = \binom{n}{p} + \binom{n}{p+1}$

Triangle de Pascal

$n \backslash k$	0	1	2	3	4	5	$\cdots$
0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$$

Formules du binôme de Newton

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $\displaystyle (a+b)^n = \sum_{k=0}^{n} \binom{k}{n} a^{n-k} b^k = \sum_{k=0}^{n} \binom{k}{n} a^k b^{n-k} \quad (a,b) \in \mathbb{R}$

Identité remarquable $a^n - b^n$

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^n - b^n = (a-b)\sum_{k=0}^{n-1} a^k b^{n-1-k}$

Signe produit Π

- $n! = \prod_{k=1}^{n} k$
- $p^n = \prod_{k=1}^{n} p$
- $\ln(\prod_{k=1}^{n} a_k) = \sum_{k=1}^{n} \ln(a_k)$
- $\exp(\sum_{k=1}^{n} a_k) = \prod_{k=1}^{n} \exp(a_k)$