MATERIAL SEMANA #3

# Reglas de derivación - Continuación

| **Regla** | **Función** | **Derivada** |
|---|---|---|
| Regla del Producto | $f(x)\cdot g(x)$ | $f'(x)g(x)+f(x)g'(x)$ |
| Regla del Cociente | $\dfrac{f(x)}{g(x)}$ | $\dfrac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$ |
| Regla de la Cadena | $f(g(x))$ | $f'(g(x))\cdot g'(x)$ |

| **Función** | **Derivada** |
|---|---|
| $e^x$ | $e^x$ |
| $\ln x$ | $\dfrac{1}{x}$ |

EJERCICIOS. Aplique las reglas de derivación y determine la derivada en cada caso

| | |
|---|---|
| a)<br>$f(x) = \underbrace{e^x}\,\underbrace{(2x^2+5)}$<br>$f'(x) = e^x\,(2x^2+5) + e^x(4x)$<br>$= e^x(2x^2+4x+5)$ | b)<br>$g(x) = \underbrace{\sqrt{x}}\,\underbrace{(2e^x - 2x^3)}$<br>$g'(x) = \dfrac{1}{2}x^{-\frac{1}{2}}\,(2e^x-2x^3) + \sqrt{x}\,(2e^x - 6x^2)$ |
| c)<br>$y = (4x^3-5x)\ln x$<br>$y' = (12x^2-5)\ln x + (4x^3-5x)\cdot\dfrac{1}{x}$<br>$y' = (12x^2-5)\ln x + 4x^2 - 5$ | d)<br>$y = (5e^x + x)(2\ln x - x^3)$<br>$y' = (5e^x+1)(2\ln x - x^3) + (5e^x+x)\left(\dfrac{2}{x} - 3x^2\right)$ |

e) $f(x) = \dfrac{4x^3+5}{1-3x}$ $\qquad \left(\frac{f}{g}\right)' = \frac{f'g - f\,g'}{g^2}$

$$f'(x) = \frac{\overbrace{(4x^3+5)'}\ (1-3x) - (4x^3+5)\overbrace{(1-3x)'}}{(1-3x)^2}$$

$$f'(x) = \frac{(12x^2)(1-3x) - (4x^3+5)(-3)}{(1-3x)^2}$$

$$f'(x) = \frac{12x^2 - 36x^3 + 12x^3 + 15}{(1-3x)^2}$$

$$f'(x) = \frac{12x^2 - 24x^3 + 15}{(1-3x)^2}$$

f) $g(x) = \dfrac{2e^x}{x^2-1}$

$$g'(x) = \frac{2e^x\ (x^2-1) - 2e^x\ (2x)}{(x^2-1)^2}$$

$$g'(x) = \frac{2e^x\ (x^2-1-2x)}{(x^2-1)^2}$$

g) $f(x) = \frac{x^3 \ln x}{(x+1)}$ $\quad \rightarrow combina\ regla\ del\ cociente\ y\ regla\ del\ producto$

$$f'(x) = \frac{\overbrace{(x^3 \ln x)'}^{producto}(x+1) - x^3 \ln x\ \ (x+1)'}{(x+1)^2}$$

$$f'(x) = \frac{\left(3x^2 \ln x + x^3 \cdot \dfrac{1}{x}\right)(x+1) - x^3 \ln x}{(x+1)^2}$$

$$f'(x) = \frac{(3x^2 \ln x + x^2)(x+1) - x^3 \ln x}{(x+1)^2}$$

h) $h(x) = \frac{e^x - 2x}{2x^3 - 3}$ $\qquad \left(\frac{f}{g}\right)' = \frac{f'g - f\,g'}{g^2}$

$$h'(x) = \frac{(e^x - 2)(2x^3-3) - (e^x - 2x)(6x^2)}{(2x^3-3)^2}$$

EJERCICIOS DE PRACTICA. Aplique las reglas de derivación y determine la derivada en cada caso

| | |
|---|---|
| a) $f(x) = \frac{4}{x^3} - 2\sqrt[3]{x} + \frac{4\sqrt{x}}{x}$ | b) $h(x) = \frac{4x^3 - 2x - 3\sqrt[5]{x}}{x^3}$ |
| c) $y = (4x^3 - 2e^x)(x^2 - 5\ln x)$<br>$y' = (12x^2 - 2e^x)(x^2 - 5\ln x) + (4x^3 - 2e^x)\left(2x - \frac{5}{x}\right)$ | d) $y = \frac{2x-1}{x+2}$<br>$y' = \frac{2\,(x+2) - (2x-1)\cdot 1}{(x+2)^2}$<br>$y' = \frac{2x + 4 - 2x + 1}{(x+2)^2}$<br>$y' = \frac{5}{(x+2)^2}$ |
| e) $f(x) = \frac{1-2x^2}{3x+2}$<br>$f'(x) = \frac{(-4x)(3x+2) - (1-2x^2)\cdot 3}{(3x+2)^2}$<br>$f'(x) = \frac{-12x^2 - 8x - 3 + 6x^2}{(3x+2)^2}$<br>$f'(x) = \frac{-6x^2 - 8x - 3}{(3x+2)^2}$ | |

# Regla de la Cadena (para derivar composición de funciones: función dentro de otra)

| |
|---|
| Sea $y = f\big(g(x)\big)$, entonces $y' = f'\big(g(x)\big) \cdot g'(x)$<br>Sea $y = f\Big(g\big(h(x)\big)\Big)$, entonces $y' = f'(g\big(h(x)\big) \cdot g'\big(h(x)\big) \cdot h'(x)$ |

Ejemplos:

- $y = (x^2+1)^7 \quad \rightarrow \quad \underbrace{(\ )^7 \leftarrow (x^2+1)}$
- $y = \sqrt[5]{2x^3+x} \quad \rightarrow \quad \underbrace{\sqrt[5]{\quad} \leftarrow (2x^3+x)}$

| | |
|---|---|
| a) $f(x) = (3x^3-2x^2)^5$<br>$(\ )^5 \leftarrow (3x^3-2x^2)$<br>$f'(x) = 5(3x^3-2x^2)^4 \cdot (9x^2-4x)$ | b) $y = \sqrt{3x-2x^3} = (3x-2x^3)^{\frac{1}{2}}$<br>$\sqrt{\quad} \leftarrow (3x-2x^3)$<br>$y' = \frac{1}{2}(3x-2x^3)^{-\frac{1}{2}} \cdot (3-6x^2)$ |
| c) $g(x) = (1-3x+5x^7)^8$<br>$g'(x) = 8(1-3x+5x^7)^7 \cdot (-3+35x^6)$ | d) $y = \sqrt[3]{4x^2+3e^x} = (4x^2+3e^x)^{\frac{1}{3}}$<br>$y' = \frac{1}{3}(4x^2+3e^x)^{-\frac{2}{3}} \cdot (8x+3e^x)$ |
| e) $y = \ln\big(3x-\sqrt{x}\big)$<br>$\ln(\ ) \leftarrow \big(3x-\sqrt{x}\big)$<br>$y' = \dfrac{1}{3x-\sqrt{x}} \cdot \left(3-\frac{1}{2}x^{-\frac{1}{2}}\right)$<br>** $(\ln f(x))' = \frac{1}{f(x)} \cdot f'(x)$ | f) $y = e^{x^2-x}$<br>$e^{(\ )} \leftarrow (x^2-x)$<br>$y' = e^{x^2-x} \cdot (2x-1)$<br>** $\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)$ |
| g) $y = \frac{1}{x^2-2} = (x^2-2)^{-1}$<br>$y' = -(x^2-2)^{-2} \cdot (2x)$ | h) $y = \frac{4}{(x+2)^3} = 4(x+2)^{-3}$<br>$y' = -12(x+2)^{-4} \cdot 1$ |

i) $g(x) = \left(\frac{2x+5}{x^2+1}\right)^4 \leftarrow Cadena\ con\ regla\ del\ cociente\ para\ la\ función\ adentro$

$$g'(x) = 4\left(\frac{2x+5}{x^2+1}\right)^3 \cdot \underbrace{\left(\frac{2x+5}{x^2+1}\right)'}_{cociente}\ (aplicando\ cadena)$$

$$g'(x) = 4\left(\frac{2x+5}{x^2+1}\right)^3 \cdot \frac{2(x^2+1)-(2x+5)2x}{(x^2+1)^2}$$

$$g'(x) = 4\left(\frac{2x+5}{x^2+1}\right)^3 \cdot \frac{2x^2+2-4x^2-10x}{(x^2+1)^2}$$

$$g'(x) = 4\left(\frac{2x+5}{x^2+1}\right)^3 \cdot \frac{-2x^2-10x+2}{(x^2+1)^2}$$

j) $y = (x^2-4)^5\ (3x+5)^4 \quad \leftarrow Regla\ del\ producto\ con\ dos\ cadenas$

$$y' = \underbrace{[(x^2-4)^5]'}_{cadena}(3x+5)^4 + (x^2-4)^5\ \underbrace{[(3x+5)^4]'}_{cadena}$$

$$y' = [5(x^2-4)^4 \cdot 2x]\ (3x+5)^4 + (x^2-4)^5\ [4(3x+5)^3 \cdot 3]$$

k) $y = \frac{2e^x(3x-5)}{\sqrt{2x-1}} \quad \leftarrow (1)\ cociente; (2) producto\ arriba; (3) cadena\ abajo$

$$y' = \frac{\overbrace{[2e^x(3x-5)]'}^{producto}\sqrt{2x-1} - 2e^x(3x-5)\overbrace{\left[\sqrt{2x-1}\right]'}^{cadena}}{\left(\sqrt{2x-1}\right)^2}$$

$$y' = \frac{[2e^x(3x-5)+2e^x \cdot 3]\sqrt{2x-1} - 2e^x(3x-5)\left[\frac{1}{2}(2x-1)^{-\frac{1}{2}} \cdot 2\right]}{\left(\sqrt{2x-1}\right)^2}$$

Ejercicios de Práctica. Hallar la derivada de

1. $y = 3x^6 - \frac{7}{x^3} - 2\sqrt[3]{x}$

$$y' = 18x^5 + 21x^{-4} - \frac{2}{3}x^{-\frac{2}{3}}$$

2. $y = (2e^x - x^4)(\sqrt{x} - 2x)$

$$y' = (2e^x - 4x^3)\left(\sqrt{x} - 2x\right) + (2e^x - x^4)\left(\frac{1}{2}x^{-\frac{1}{2}} - 2\right)$$

3. $y = \frac{1-3x}{x-1}$

$$y' = \frac{-3(x-1) - (1-3x)}{(x-1)^2} = \frac{2}{(x-1)^2}$$

4. $y = \sqrt[3]{8x^2 - 1}$

$$y' = \frac{1}{3}(8x^2 - 1)^{-\frac{2}{3}} \cdot (16x)$$

5. $y = \frac{3}{(3x^2-x)^{\frac{2}{3}}} = 3(3x^2 - x)^{-\frac{2}{3}}$

$$y' = -2(3x^2 - x)^{-\frac{5}{3}} \cdot (6x - 1)$$

6. $y = x^2(x-4)^5$

$$y' = 2x(x-4)^5 + x^2 \cdot 5(x-4)^4$$

7. $y = \left(\frac{x-7}{x+4}\right)^{10}$

$$y' = 10\left(\frac{x-7}{x+4}\right)^9 \cdot \frac{(x+4) - (x-7)}{(x+4)^2} = 10\left(\frac{x-7}{x+4}\right)^9 \cdot \frac{11}{(x+4)^2}$$

8. $y = \frac{(2x+3)^3}{x^2+4}$

$$y' = \frac{[(2x+3)^3]'(x^2+4) - (2x+3)^3\,[x^2+4]'}{(x^2+4)^2}$$

$$y' = \frac{[3(2x+3)^2 \cdot 2](x^2+4) - (2x+3)^3\,(2x)}{(x^2+4)^2}$$

9. $y = 2x\sqrt{6x-1}$

$$y' = 2\sqrt{6x-1} + 2x\left[\frac{1}{2}(6x-1)^{-\frac{1}{2}} \cdot 6\right]$$

$$y' = 2\sqrt{6x-1} + 6x\,(6x-1)^{-\frac{1}{2}}$$

Ejemplos extra

- $(e^x)' = e^x;\quad \left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)$
- $(\ln x)' = \frac{1}{x};\quad (\ln f(x))' = \frac{1}{f(x)} \cdot f'(x)$

1. $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

$$y' = \frac{\sqrt{x^2+1} - x \cdot \left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x\right]}{\left(\sqrt{x^2+1}\right)^2} = \frac{\sqrt{x^2+1} - x^2(x^2+1)^{-\frac{1}{2}}}{x^2+1}$$

2. $f(x) = \ln\left(\dfrac{e^x+2}{x}\right)$

$$f'(x) = \frac{1}{\frac{e^x+2}{x}} \cdot \frac{(e^x)x - (e^x+2)}{x^2} = \frac{x}{e^x+2} \cdot \frac{xe^x - e^x - 2}{x^2}$$

$f(x) = \ln(e^x+2) - \ln(x)$

$$f'(x) = \frac{1}{e^x+2} \cdot e^x - \frac{1}{x}$$

3. $f(x) = \dfrac{e^{2x} + 1}{e^{x} - 2}$

$$y' = \frac{(e^{2x} \cdot 2)(e^{x} - 2) - (e^{2x} + 1)(e^{x})}{(e^{x} - 2)^2}$$

4. $y = \underbrace{\ln(2x^2 - 3x)}_{cadena} - \underbrace{2x^3 e^{2x+1}}_{producto}$

$$y' = \frac{1}{2x^2 - 3x} \cdot (4x - 3) - [6x^2\ e^{2x+1} + 2x^3 \cdot e^{2x+1} \cdot 2]$$