$f: A \to B$, $f = \{f(x) : x \in A\} = f(A)$

$A_1 \subseteq A \Rightarrow f(A_1) = \{f(x) : x \in A_1\}$

$B_1 \subseteq B \Rightarrow f^{-1}(B_1) = \{x \in A : f(x) \in B_1\}$

Preimaginea lui B_1 prin f $\exists$ chiar dacă $f \neq$ bij

Aplicații:

1) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x+2, & x < 2 \\ x^2 - 5x + 6, & x \geq 2 \end{cases}$

a) f - bij

b) $f([1,2])$

$f([3,4])$

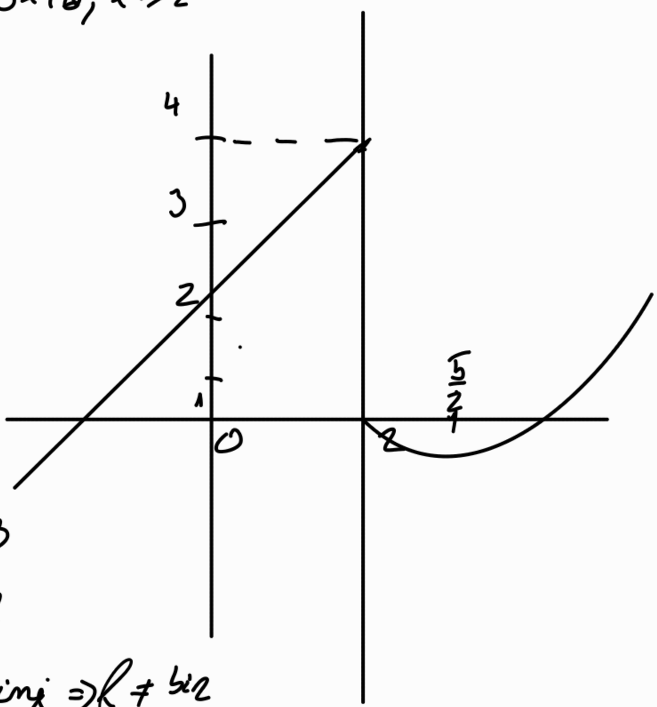
$f^{-1}((-\frac{1}{3}, 0))$

$f^{-1}([-2, 0])$

$x = 0 \Rightarrow f(0) = 2$

$x = 2 \to 4$

$\Delta = 1 \Rightarrow x_{1,2} = \frac{5 \pm 1}{2}$, $x_1 = 3$, $x_2 = 2$



Criteriul grafic $\Rightarrow f \neq$ inj $\Rightarrow f \neq$ bij

f - surj

b) $f([1,2]) = [3,4] \cup \{0\}$

$f([2,4]) = [0,2] \cup [-\frac{1}{4}, 0) = [-\frac{1}{4}, 2]$

$f^{-1}(-\frac{1}{4}, 0) = (2, \frac{5}{2}) \cup (\frac{5}{2}, 3) \cup (-\frac{9}{4}, -2)$

$f^{-1}([-2, 0]) = [-4, -2] \cup [2, 3]$

$f([0,5)) = [-2, 0] \cup (0, 2) \cup [3, \frac{5+\sqrt{2}}{2}) \cup \{2\}$

2) Dem că dacă $f: A \to B$ - inj $\Rightarrow \exists r: B \to A$ cu $r \circ f = 1_A$ (Dacă f - inj $\Rightarrow \exists r: B \to A$ cu $r \circ f = 1_A$ (retracta lui f) $\to$ e unică doar dacă f - bij)

Definim r astfel.

Alegem $a_0 \in A$

Dacă $a \in \mathrm{Im} f \Rightarrow r(b) = a_b$

$\hookrightarrow \exists!\, a_b$ cu $f(a_b) = b$

Dacă $b \notin \mathrm{Im} f \Rightarrow r(b) = a_0$

$(r \circ \ell)(ab) = r(\ell(ab)) = r(b) = ab$

PIE: $|A_1 \cup \ldots \cup A_m| = \sum_{i=1}^{m} |A_i| - \sum_{1\le i<j\le m}^{m} |A_i \cap A_j| + \sum_{1\le i<j<k\le m}^{m} |A_i \cap A_j \cap A_m| - \ldots + (-1)^{m+1} |A_1 \cap \ldots \cap A_m|$

1) Fie $m \ge 1$

$\varphi(n)$ = fct lui Euler = nr întregi poz $\le m$ și primi cu m

exp: $\varphi(12) = 4$

$\underline{1}$ 2 3 4 $\underline{5}$ 6 $\underline{7}$ 8 9 10 $\underline{11}$ 12

$\varphi(m) = m\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_s}\right)$; unde $p_1, p_2, \ldots p_s$ - fact primi din desc lui m

$\varphi(12) = 12\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = \cancel{12}^{4} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} = 4$

a, b - prime între ele $\Leftrightarrow (a,b) = 1$

Det nr de nr $\le m$ și neprime cu m

$m = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_s^{e_s}$

$A_{p_1} = \{x : x = p_1 \cdot b, b \in \mathbb{N}, x \le m\}$

$\vdots$

$A_{p_m} = \{x : x = p_k \cdot g, g \in \mathbb{N}, x \le N\}$

$N = |A_{p_1} \cup \ldots A_{p_s}|$ - nr de nr $\le m$ și neprime cu m

$|A_{p_k}| = \frac{m}{p_k}$

$|A_{p_i} \cap \ldots \cap A_{p_{ig}}| = \frac{m}{p_{i_1} \cdot p_{i_2} \cdot \ldots \cdot p_{i_g}}$

$N = \sum_{i=1}^{s} \frac{m}{p_i} - \sum_{1\le i<j\le s} \frac{m}{p_i \cdot p_j} + \ldots + (-1)^{s+1} \cdot \frac{m}{p_i \cdot \ldots \cdot p_g} = m\left(\frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_s} - \frac{1}{p_1 p_2} - \ldots - \frac{1}{p_{s-1} p_s} + \ldots + (-1)^{s+1} \frac{1}{p_1 \cdots p_s}\right)$

$\varphi(m) = m - A = m\left(\frac{1}{p_1} + \frac{1}{p_2} + \ldots + \frac{1}{p_s} - \ldots (-1)^s \frac{1}{p_1 \cdots p_s}\right)$

I) $P(2)$: $m = p_1^{e_1} \cdot p_2^{e_2}$

$\varphi(m) = m\left(1 - \frac{1}{p_1} - \frac{1}{p_2} + \frac{1}{p_1 \cdot p_2}\right) = m\left(1 - \frac{1}{p_1} - \frac{1}{p_2}\left(1 - \frac{1}{p_1}\right)\right) = m\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)$ (A)

II Pp $P(k)$ adev $\Rightarrow$ dem $P(k+1)$

$\left(1 - \frac{1}{p_1} - \frac{1}{p_2} - \ldots - \frac{1}{p_k} + \frac{1}{p_1 p_2} + \ldots + \frac{1}{p_{k-1} \cdot p_k} + \ldots + (-1)^k \frac{1}{p_1 \cdots p_k}\right) = \left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_k}\right) \Big| \left(1-\frac{1}{p_{k+1}}\right)$

$P(k) \cdot \left(1 - \frac{1}{p_{k+1}}\right) = P - \frac{1}{p_{k+1}} + \frac{1}{p_1 p_{k+1}} + \ldots + (-1)^{k+1} \cdot \frac{1}{p_1 \cdot \ldots \cdot p_{k+1}} = \left(1-\frac{1}{p_1}\right) \ldots \ldots \left(1 - \frac{1}{p_{k+1}}\right) = P(k+1)$ (A)

2) Fie A o mulțime de n elem

Det nr de partiții ale lui A cu k elem

Partiția unei mulțimi M

$M_1, M_2 \ldots M_k$ - submulțimi ale lui M

1) $M_1 \cup M_2 \cup \ldots \cup M_k = M$

2) $M_i \cap M_j = \emptyset$, $\forall i, j = \overline{1, p}$

ex: $k = 3$

$A = \{1, 2, 3, 4\}$

$\{1\}, \{2\}, \{3, 4\} \to \{1, 2, 3\}$

$\{1\}, \{2, 3\}, \{4\} \to \{1, 2, 3\}$

$\{2\}, \{1, 3\}, \{4\} \to \{1, 2, 3\}$

$S(n, k)$ = nr de part cu k elem ale unei mulț cu n elem

Câte fct surj $f: A \to B$ există?

$$S = k^n - \sum_{i=1}^{k} C_k^i (k-i)^n \cdot (-1)^{i+1}$$

$$S = k! \cdot S(n, k) \Rightarrow S(n, k) = \frac{S}{k!}$$