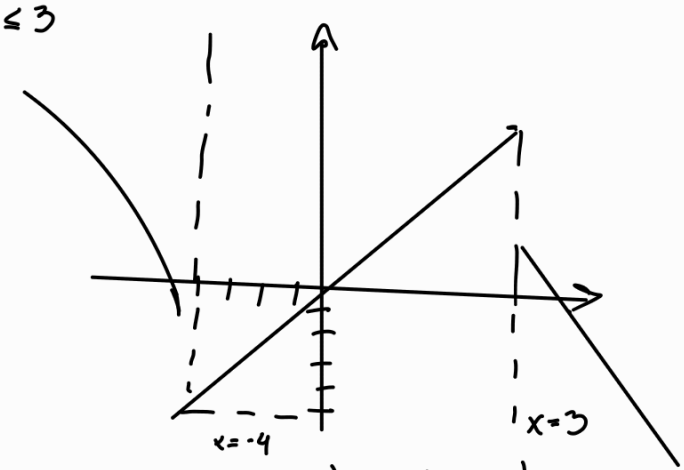
1) $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -5x^2 - 50x - 121, & x \le -4 \\ x, & -4 < x \le 3 \\ -x + 4, & x > 3 \end{cases}$$



a) $\operatorname{Im} f$

b) $f((-6,0))$

$f^{-1}((-2,2))$

$x=3 \Rightarrow 1$

$x=4 \Rightarrow 0$

$V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$

$\Delta = (-50)^2 - 4 \cdot (-5)(121) = 80 \Rightarrow V\left(-\frac{-50}{-10}, -\frac{80}{-20}\right) \Rightarrow V(-5,4)$

$Gf \cap Ox: -5x^2 - 50x - 121 = 0$

$\Delta = 80 \Rightarrow x_{1,2} = \frac{25 \pm 2\sqrt{5}}{-5} = -5 \pm \frac{2\sqrt{5}}{5}$

$2 < \sqrt{5} < 3 \mid \cdot 2 \Rightarrow 4 < 2\sqrt{5} < 6 \mid : 5 \Rightarrow \frac{4}{5} < \frac{2\sqrt{5}}{5} < \frac{6}{5}$

$x_1 \simeq -6$

$x_2 \simeq -4$

$f(-4) = -1$

a) $\operatorname{Im} f = (-\infty, 4]$

b) $f(-8,0) = (-4,4]$

$f^{-1}((-2,2)) = \left(-5-\frac{\sqrt{30}}{5}, -5-\frac{\sqrt{10}}{5}\right) \cup \left(-4, -5+\frac{\sqrt{10}}{5}\right) \cup (-2,2) \cup (3,6)$

Obs!

∀ fct strict monotonă $\Rightarrow$ f-inj

∀ fct inj nu e strict monotonă

1) $f: \mathbb{N} \to \mathbb{R}, f(x) = \begin{cases} -2x, & x\text{-par} \\ 2x+1, & x\text{-impar} \end{cases}$

2) $f(x) = \begin{cases} -x, & x > 0 \\ x, & x \le 0 \end{cases}$

3) $f: (-2, \infty) \to \mathbb{R}$

$f(x) = \begin{cases} x+2, & x \in (-2,1) \\ -x+1, & x \ge 1 \end{cases}$

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3) Funcția mersej

a) $\mathbb{Z} \to \mathbb{Z}$

b) $\mathbb{R} \to \mathbb{R}$

$f(x) = 2x$

4) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x-2, & x \le 2 \\ x^2-2x, & x > 2 \end{cases}$

a) f - inj, surj, bij = ?

b) f^{-1} = ?

f - bij $\Rightarrow f$ - inv $\Rightarrow \exists f^{-1}: \mathbb{R} \to \mathbb{R}$

$f(x) = y$

1) $x \le 2$: $x - 2 = y \Rightarrow x = y + 2$

$y + 2 \le 2 \Rightarrow y \le 0$

$f^{-1}(x) = \begin{cases} x+2, & x \le 0 \\ 1+\sqrt{1+x}, & x > 0 \end{cases}$

2) $x > 2$, $x^2 - 2x \Rightarrow x^2 - 2x - y = 0 \Rightarrow \Delta = 4 + 4y$

$x_{1,2} = \frac{2 \pm \sqrt{4+4y}}{2} \to 1 \pm \sqrt{1+y}$

$\sqrt{1+y} > 0 \Rightarrow 1 \pm \sqrt{1+y} < 1 \Rightarrow x = 1 + \sqrt{1+y} > 2 \Rightarrow \sqrt{1+y} > 1 \Rightarrow y > 0$

Relații

A, B - multimi nevide

ρ - rel pe $A \times B$ dacă $\rho \subseteq A \times B$

Dacă $|A| = n \Rightarrow \exists 2^n$ rel ρ pe A

Prop:

$A \neq \emptyset$, ρ rel pe A

1) Reflexivă: ρ reflexivă dacă pe $x \rho y \Rightarrow x = y$

2) Simetrică: ρ = simetrică dacă $\forall x, y \in A$ cu $x \rho y \Rightarrow y \rho x$

3) Antisimetrică ρ = anti... dacă $\forall x, y$ cu $x \rho y$, $y \rho x \Rightarrow x = y$

4) Tranzitivă dacă $\forall x, y, z \in A$ cu $x \rho y$, $y \rho z \Rightarrow x \rho z$

Exerciții:

1) $R \subseteq \mathbb{Z}$

a) $x \sim y \Leftrightarrow x - y \in \mathbb{Z}$

R: $\forall x \in \mathbb{R}$, $x - x = 0 \in \mathbb{Z} \Rightarrow x \sim x$

S: $\forall x, y \in \mathbb{R}$ cu $x \sim y \overset{?}{\Rightarrow} y \sim x$

$x \sim y \Rightarrow x - y \in \mathbb{Z} \Rightarrow -(x-y) \in \mathbb{Z} \Rightarrow y - x \in \mathbb{Z} \Rightarrow y \sim x$

T: $\forall x, y, z$ cu $x \sim y$, $y \sim z \Rightarrow x - y \in \mathbb{Z}$, $y - z \in \mathbb{Z}$ (+)

$x - z \in \mathbb{Z} \Rightarrow x \sim z$

b) $x \sim y \Rightarrow x \leq y$

R: $\forall x \in \mathbb{R} \Rightarrow x \sim x \Rightarrow x \leq x$ (A)

S: $\forall x, y \in \mathbb{R}$ cu $x \sim y \overset{?}{\Rightarrow} y \sim x$

$x \sim y \Rightarrow x \leq y$

C. Exemplu $1 \sim 2 \Rightarrow 1 \leq 2$ dar $2 \leq 1 \Rightarrow$ nu e simetric

AS: $\forall x, y \in \mathbb{R} \Rightarrow x \leq y$

$x \sim y \Rightarrow x \leq y$
$y \sim x \Rightarrow y \leq x$ $\Big/ \Rightarrow x = y$

$\forall x, y, z \in \mathbb{R}$ cu $x \sim y$ și $y \sim z \overset{?}{\Rightarrow} x \sim z$

$x \sim y \Rightarrow x \leq y$
$y \sim z \Rightarrow y \leq z$ $\Big/ \Rightarrow x \leq z \Rightarrow x \sim z$

$\sim \neq$ simetrică $\Rightarrow \sim$ - nesimetrică

Dacă nesimetric = antisimetric $\Rightarrow \sim$ - antisimetric

b) simetric $\neq$ antisimetric

$\hookrightarrow A = \{1, 2, 3\}$ $\quad \rho_{\sim} = \{(1,2), (1,3), (2,3)\}$