$\varphi = v$ ; $\psi = v_2$

$\neg v \to v_2$

$\neg(\neg v \to v_2) = \lambda = var(\mathcal{X}) = \{v, v_2\}$

$e : V \to \{0, 1\}$

T1.10 : $e^+ Form \to \{0, 1\}$

1) $v \in V$, $e^+(v) = e(v)$

formulă variabilă propozițională

2) $\varphi \in Form$, $e^+(\neg\varphi) = \neg \underbrace{e^+(\varphi)}_{\in \{0,1\}}$

3) $\varphi, \psi \in Form$, $e^+(\varphi \to \psi) = e^+(\varphi) \to e^+(\psi)$

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S1.1. Să se demonstreze că pt (b)

$x_0, x_1, x_3, x_4 \in \{0, 1\}$ avem:

i) $((x_0 \to x_1) \to x_0) \to x_0 = 1$

ii) $(x_3 \to x_4) \to ((x_4 \to x_1) \to (x_3 \to x_1)) = 1$

| $x_0$ | $x_1$ | $x_0 \to x_1$ | $(x_0 \to x_1) \to x_0$ | Tot |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |

S1 2.2 (=) P. 110

$e : V \to \{0,1\}$, $\varphi, \psi \in$ Form

$e^+(\varphi \vee \psi) = e^+(\neg p \to q) =$

3) $\varphi, \psi \in$ Form, $e^+(\varphi \to \psi) = e^+(\varphi) \to e^+(\psi)$

$\Rightarrow = e^+(\neg p) \to e^+(q) =$

2) $\varphi \in$ Form, $e^+(\neg\varphi) = \neg \underbrace{e^+(\varphi)}$

$\Rightarrow = \neg e^+(p) \to e^+(q) \overset{(*)}{=} e^+(p) \vee e^+(q)$

$V = \{v_1, v_2, v_3, \dots, v_n\}$

$e_1 : V \to \{0, 1\}$

$e_1(v_1) = 0$

$e_1(v_i) = 1 \ (\forall) \ i \in \overline{2, n}$

$e_2 : V \to \{0, 1\}$

$e_2(v_1) = 0$

$e_2(v_2) = 1$

$e_2(v_i) = 0 \ (\forall) \ i \in \overline{3, n}$

$\varphi : v_1 \to v_2$

$e_1^+(\varphi) = e_1^+(v_1 \to v_2) = e_1^+(v_1) \to e_1^+(v_2)$

$= e_1(v_1) \to e_1(v_2) = 0 \to 1 = 1$

$e_2^+(\varphi) = \dots = e_2(v_1) \to e_2(v_2) = 0 \to 1 = 1$

P.1.12

S.1.3 Să se găsească câte un model pt.

i) $v_0 \to v_2$

ii) $v_0 \land v_3 \land \neg v_4 := \varphi$

i) Căutăm $e_1 : V \to \{0, 1\}$ a. î. $e_1 \models \varphi$

$$e_1(v_i) = \begin{cases} 0, & i = 0 \\ 1, & \text{altfel} \end{cases} : v_i \in V \setminus \{v_0\}$$

$e_1^+(v_0 \to v_2) = e_1^+(v_0) \to e_1^+(v_2) = e_1(v_0) \to e_1(v_2)$

$= 0 \to 1 = 1 \Rightarrow e_1 \models v_0 \to v_2$

$e_2 : V \to \{0, 1\}$

$e_2(v) = 0 \quad (\forall) v \in V$

$e_2^+(v_0 \to v_2) = \ldots = e_2(v_0) \to e_2(v_2) =$

$0 \to 0 = 1 \Rightarrow e_2 \models v_0 \to v_2$

ii) $e_3 : V \to \{0, 1\}$

$$e_3(v_i) = \begin{cases} 1, & i \in \{0,3\} \\ 0, & i = 4 \\ 0, & \text{altfel} \end{cases}$$

$$e^+(v_0 \wedge v_3 \vee \neg v_4) = e_3^+(v_0) \wedge e_3^+(v_3) \wedge e_3^+(\neg v_4) =$$

$$= e_3(v_0) \wedge e_3(v_3) \wedge \neg e_3(v_4) =$$

$$= 1 \wedge 1 \wedge (\neg 0) = 1 \wedge 1 = 1 \Rightarrow e_3 \vDash \varphi$$

$$e_1^+(\varphi) = 0 \wedge e_1^+(v_3 \wedge \neg v_4) = 0 \Rightarrow$$

$$\Rightarrow e_1 \nvDash \varphi$$

5.1.4 $(\forall)\ \varphi, \psi, \chi \in Form$

i) $\psi \vDash \varphi \to \psi$

ii) $\varphi \to (\psi \to \chi) \sim (\varphi \wedge \psi) \to \chi$

iii) $\varphi \vee (\varphi \wedge \psi) \sim \varphi$

iv) $\vDash \neg\psi \to (\neg\psi \leftrightarrow (\psi \to \varphi))$

<u>Obs:</u> $\forall a, b \in \{0, 1\}$

$a \to b = 1 \Leftrightarrow a \leq b$

$1 \to a = a \qquad 1 \wedge a = a$

$0 \to a = 1 \qquad 1 \vee a = 1$

$a \to 1 = 1 \qquad 0 \wedge a = 0$

$a \to 0 = \neg a \qquad 0 \vee a = a$

i) Fie $e: V \to \{0, 1\}$ a.î. $e \vDash \psi \Leftrightarrow e^+(\psi) = 1$

Vrem să dem. că și $e^+(\varphi \to \psi) = 1 \Leftrightarrow e \vDash \varphi \to \psi$

Adică dem. că $Mod(\psi) \subseteq Mod(\varphi \to \psi) \Leftrightarrow$

$\psi \overset{D.1.15}{\vDash} \varphi \to \psi$

$e^+(\varphi \to \psi) = e^+(\varphi) \to e^+(\psi) = e^+(\varphi) \to 1 = 1$

$\Rightarrow e \vDash \varphi \to \psi$

În concluzie, $\psi \vDash \varphi \to \psi$

ii) Fie $e \; V \to \{0, 1\}$

Trebuie să demonstrăm că $e^+(\varphi \to (\psi \to \chi)) = 1$

$\Leftrightarrow e^+((\varphi \wedge \psi) \to \chi) = 1$

Echivalent cu a dem. că $e^+(\varphi \to (\psi \to \chi))$

$\Leftrightarrow e^+((\varphi \wedge \psi) \to \chi)$

V1:

ECHIVALENTE

| $e^+(\varphi)$ | $e^+(\psi)$ | $e^+(\chi)$ | $e^+(\psi \to \chi)$ | $e^+(\varphi \to (\psi \to \chi))$ | $e^+(\varphi \wedge \psi)$ | $e^+((\varphi \wedge \psi) \to \chi)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

V2: $e^+(\varphi \to (\psi \to \chi)) = e^+(\varphi) \to e^+(\psi \to \chi)$

$e^+((\varphi \wedge \psi) \to \chi) = e^+(\varphi \wedge \psi) \to e^+(\chi)$

a) $e^+(\varphi) = 0 \Rightarrow e^+(\varphi \to (\psi \to \chi)) = 1$

$\Rightarrow e^+((\varphi \wedge \psi) \to \chi) =$

$= (0 \wedge e^+(\psi)) \to e^+(\chi) = 0 \to e^+(\chi) = 1$

b) $e^+(\varphi) = 1 \Rightarrow e^+(\varphi \to (\psi \to \chi)) =$

$= 1 \to e^+(\psi \to \chi) = \boxed{e^+(\psi) \to e^+(\chi)}$

$e^+((\varphi \wedge \psi) \to \chi) = (1 \wedge e^+(\psi)) \to e^+(\chi) =$

$= \boxed{e^+(\psi) \to e^+(\chi)}$