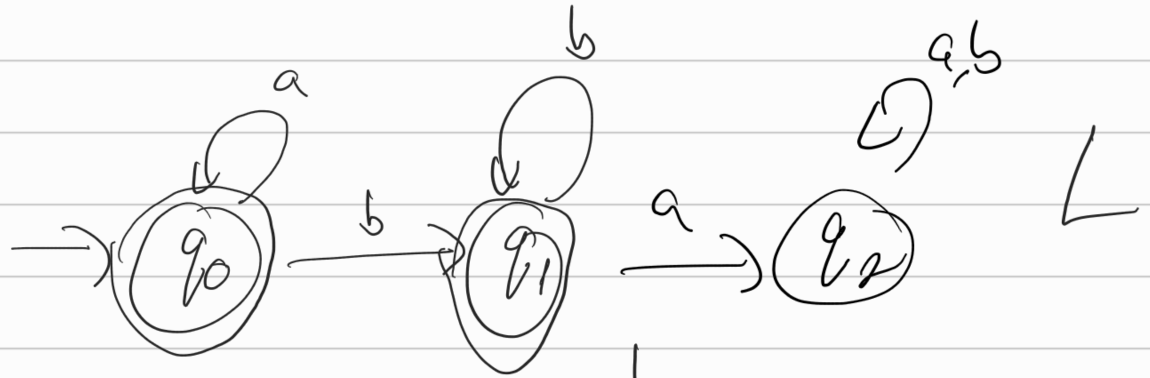
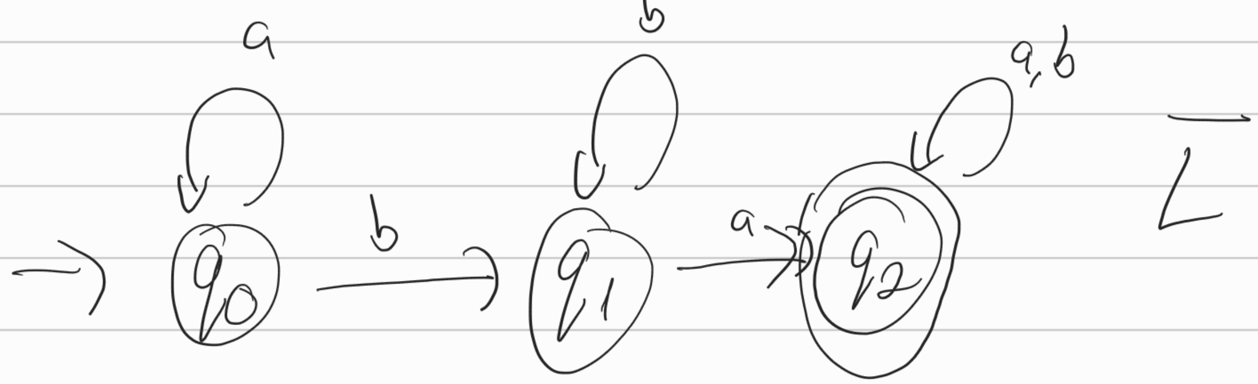
Seminar 3

L acceptat de un AFD complet definit $M=(Q, \Sigma, \delta, q_0, F)$

$\Rightarrow \bar{L} = \Sigma^* \setminus L$ acceptat de AFD $M'=(Q, \Sigma, \delta, q_0, Q\setminus F)$

$L = a^* b^*$





L_1 acceptat de un AFD complet definit

$M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$

L_2 acceptat de un AFD complet definit

$M_2 = (R_2, \Sigma, \delta_2, r_{02}, F_2)$

$\Rightarrow$ dacă $L = L_1 \cap L_2 = \{w \in \Sigma^* \mid w \in L_1 \text{ și } w \in L_2\}$

acceptat de $\quad M = (Q_1 \times R_2, \Sigma, \delta, (q_{01}, r_{02}), F)$

$$\delta((q_i, r_j), a) = (\delta_1(q_i, a), \delta_2(r_j, a))$$

$$F = F_1 \times F_2$$

dacă $L = L_1 \setminus L_2 = \{w \in \Sigma^* \mid w \in L_1 \text{ și } w \notin L_2\}$

$$= (L_1 \cap \overline{L_2})$$

$$F = F_1 \times (R_2 \setminus F_2)$$

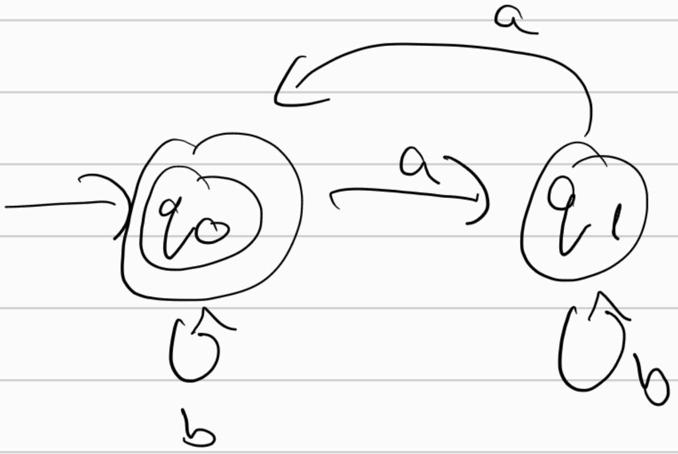
dacă $L = L_2 \setminus L_1 \Rightarrow F = (Q_1 \setminus F_1) \times F_2$

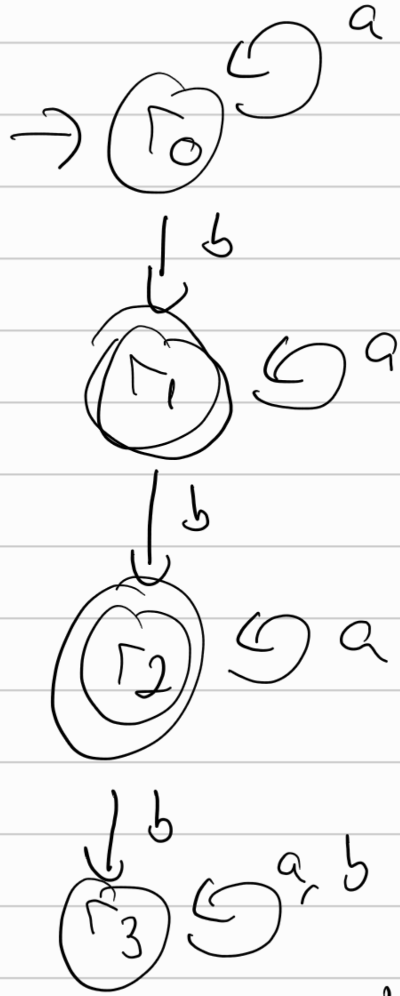
dacă $L = L_1 \cup L_2 \Rightarrow F = (F_1 \times (R_2 \setminus F_2)) \cup (Q_1 \setminus F_1) \times F_2) \cup F_1 \times F_2$

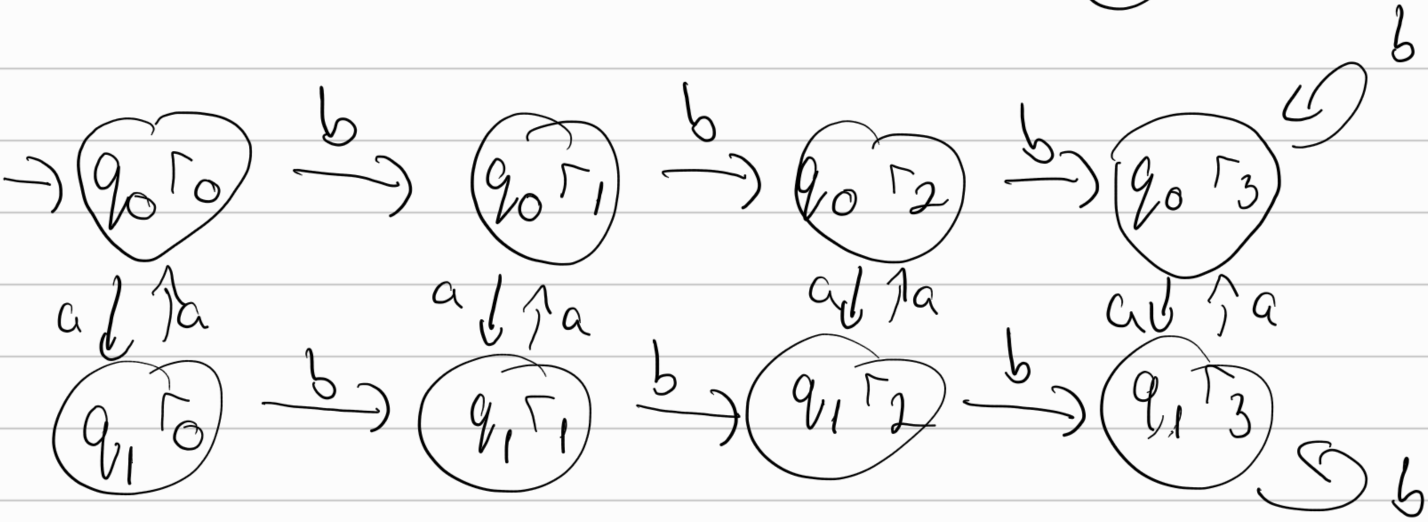
$$= F_1 \times R_2 \cup Q_1 \times F_2$$

$L_1 = \{ w \mid |w|_a \text{ este par} \}$

$L_2 = \{ w \mid |w|_b \in \{1, 2\} \}$







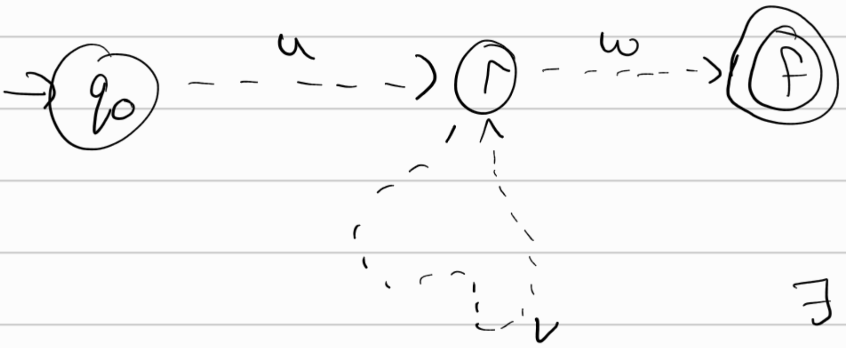
pt $L = L_1 \cap L_2 \Rightarrow F = \{q_0r_1, q_0r_2\}$

$L = L_1 \setminus L_2 \Rightarrow F = \{q_0r_0, q_0r_3\}$

$L = L_2 \setminus L_1 \Rightarrow F = \{q_1r_1, q_1r_2\}$

$L = L_1 \cup L_2 \Rightarrow F = \{q_0r_0, q_0r_1, q_0r_2, q_0r_3,$
$q_1r_1, q_1r_2\}$

Lema de pompare pentru limbaje regulate



Fie L un limbaj regulat. Atunci $\exists\, p \in \mathbb{N}^*$, $\forall\, \alpha \in L$ cuvânt, cu $|\alpha| \geq p$, $\exists$ o descompunere $\alpha = u \cdot v \cdot w$ cu proprietățile:

1) $|uv| \leq p$
2) $|v| \geq 1$
3) $u \cdot v^i \cdot w \in L, \ \forall\, i \geq 0$

$$(P \to Q) \equiv (\neg Q \to \neg P)$$

$$L_1 = \{ a^m b^{3n} c^{n+3} \mid m \geq 5,\ n \geq 1 \} \notin REG$$

Pp. prin reducere la absurd că $L_1 \in REG \Rightarrow \exists\, p \in \mathbb{N}^*$ din Lemă.

Alegem $\alpha = a^5 b^{3p} c^{p+3} \in L_1 \Rightarrow |\alpha| = 4p+8 \geq p$, $\forall\, p \in \mathbb{N}^*$

Fie $\alpha = uvw$ a.î. $|uv| \leq p$, $|v| \geq 1 \Rightarrow 1 \leq |v| \leq p \ (*)$

Caz I Fie $\left.\begin{array}{l} v = a^r b^t \\ 1 \leq r \leq 5 \\ 0 \leq t \leq p-5 \end{array}\right| \Rightarrow |v| = r+t \overset{*}{\Longrightarrow}$

$\Rightarrow 1 \leq r + t \leq p$

Alegem $i = 0 \Rightarrow \beta = uv^0w = uw = a^{5-r} b^{3p-t} c^{p+3} \notin L$

$\Leftrightarrow |\beta|_a \geq 5 \Leftrightarrow 5 - r \geq 5 \Leftrightarrow r \leq 0$ dar $1 \leq r$ $\Big|$ $\Rightarrow$ ⨯

Caz II

Fie $v = b^r, r > 0 \Rightarrow |v| = r \overset{(*)}{\Rightarrow} 1 \leq r \leq p$

Alegem $i = 2 \Rightarrow \beta = uv^2w = a^5 b^{3p+r} c^{p+3} \in L_1$

$\Leftrightarrow |\beta|_b = 3(|\beta|_c - 3) \Leftrightarrow 3p + r = 3(p + \cancel{3} - \cancel{3})$

$\Leftrightarrow r = 0$

dar $r > 0$ $\Big|$ $\Rightarrow$ ⨯ (2)

Din (1), (2) $\Rightarrow L_1 \in REG$

$L_2 = \{(a^n)^2 \mid n \geq 1\} = \{a, a^4, a^9, a^{16}, \dots\}$

$\notin REG$

Pp. prin reducere la absurd că $L_2 \in REG \Rightarrow \exists p \in \mathbb{N}^*$ din Lemă.

Alegem $\alpha = a^{p^2} \in L \Rightarrow |\alpha| = p^2 \geq p, \forall p \in \mathbb{N}^*$

Fie $\alpha = uvw$ a.î. $|uv| \leq p$ şi $|v| \geq 1 \Rightarrow$

$1 \leq |v| \leq p \quad (*)$

Fie $v = a^k \Rightarrow |v| = k \overset{(*)}{\Rightarrow} 1 \leq k \leq p$

Alegem $i = 2 \Rightarrow \beta = u v^2 w = a^{p^2+k} \in L_2$

$\Leftrightarrow |\beta| =$ pătrat perfect $\Leftrightarrow p^2 + k =$ p.p.

Știm $1 \leq k \leq p \Rightarrow p^2 < p^2 + 1 \leq \underbrace{p^2 + k}_{=|\beta|} \leq p^2 + p$

$< (p+1)^2 \Leftrightarrow p^2 < |\beta| < (p+1)^2 \Leftrightarrow |\beta|$ nu e p.p.

$\Rightarrow$ $\lightning$ $\Rightarrow L_2 \notin REG$