Lezione 7

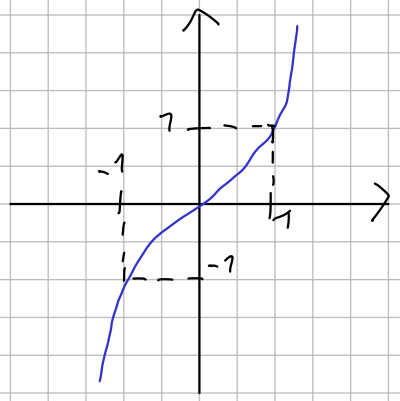
Grafici di funzioni potenza

Potenza a esponente naturale positivo

$f(x) = x^n \quad n \in \mathbb{N}$

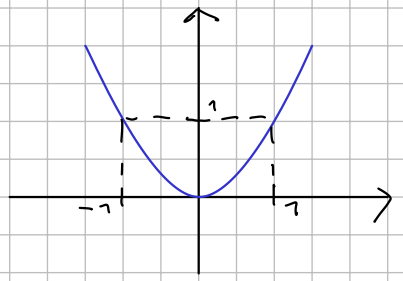
2 casi:

i) n dispari, $n \geq 3$, x^3



dispari, iniettiva e monotona st. crescente

ii) n pari, x^2



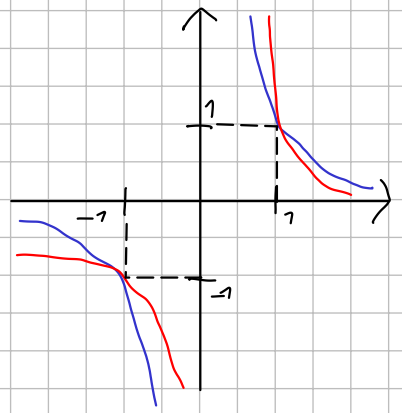
$A = \mathbb{R}$, pari e non iniettiva
monotona st. crescente su $[0, +\infty]$
decrescente su $[-\infty, 0]$

Potenza a esponente naturale negativo

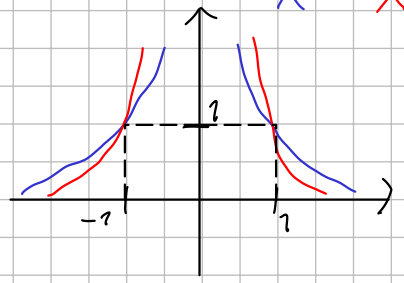
$$F(x) = x^{-n} = \frac{1}{x^n}, \quad n \in \mathbb{N}$$

2 casi:

i) n dispari $\frac{1}{x}$, $\frac{1}{x^3}$



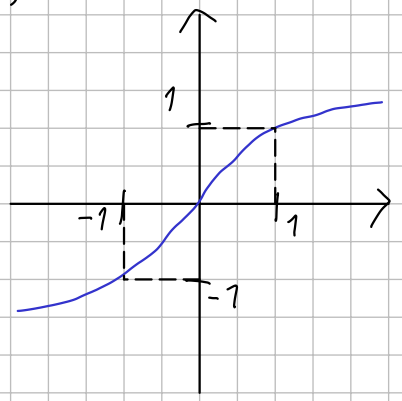
ii) n pari $\frac{1}{x^2}$, $\frac{1}{x^4}$



Potenza a esponente $\frac{1}{n}$ positivo

$$F(x) = x^{\frac{1}{n}} \quad n \in \mathbb{N}$$

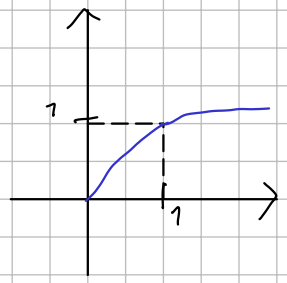
i) n dispari



funzione inversa di x^n, n dispari su $\mathbb{R}$

$$x^{\frac{1}{n}} = \begin{cases} \sqrt[n]{x} & x \geq 0 \\ -\sqrt[n]{-x} & x < 0 \end{cases}$$

ii) n pari

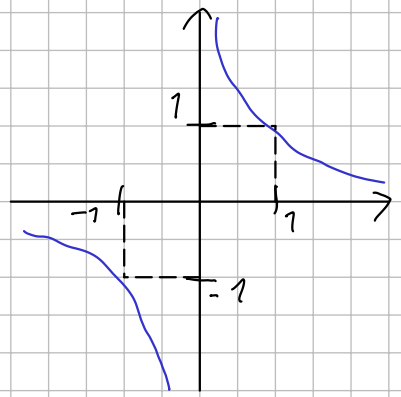


funzione inversa di x^n, n pari su $[0, +\infty)$

potenza a esponente $\frac{1}{n}$ negativa

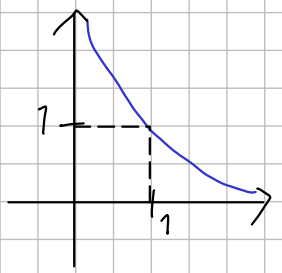
$F(x) = x^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{x}}$ $\quad n \in \mathbb{N}$

i) n dispari



funzione inversa di x^{-n} su $\mathbb{R} \setminus \{0\}$

ii) n pari



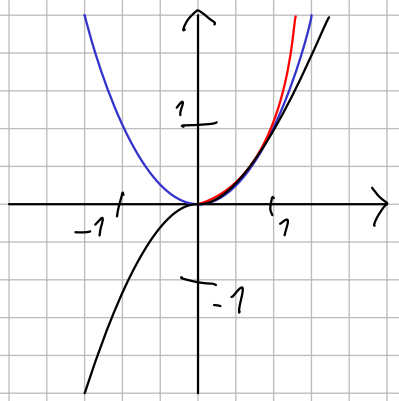
funzione inversa di x^{-n} su $(0, +\infty)$

Potenza a esponente razionale

$F(x) = x^{\frac{m}{n}}$ $\quad m \in \mathbb{Z}, n \in \mathbb{N}, \frac{m}{n}$ ridotta ai min. termini

$F(x) = \left(x^{\frac{1}{n}}\right)^m$ F. composta

i) $\frac{m}{n} > 1$

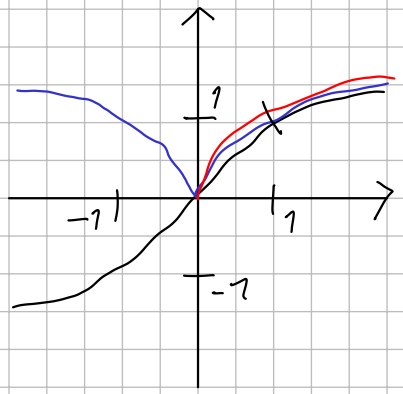


n dispari, m pari $\quad A = [0, +\infty)$

n pari, m dispari $\quad A = \mathbb{R}$

n dispari, m dispari $\quad A = \mathbb{R}$

ii) $0 < \frac{m}{n} < 1$

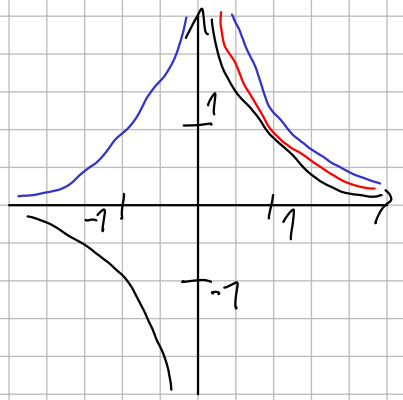


n dispari, m pari $\quad A = [0, +\infty)$

n pari, m dispari $\quad A = \mathbb{R}$

n dispari, m dispari $\quad A = \mathbb{R}$

iii) $\frac{m}{n} < 0$



n dispari, m pari $A = [0, +\infty)$

n pari, m dispari $A = \mathbb{R} \setminus \{0\}$

n dispari, m dispari $A = \mathbb{R} \setminus \{0\}$

iv) $m = n$ la funzione non è unica

es. $x^{\frac{2}{2}}$

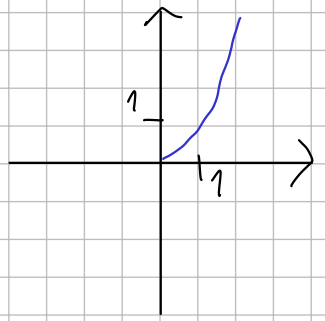
$(x^2)^{\frac{1}{2}} = \sqrt{x^2} = |x|$

$(x^{\frac{1}{2}})^2 = (\sqrt{x})^2 = x$

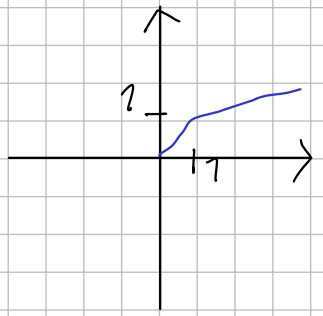
Potenza irrazionale

$F(x) = x^\alpha$ $\alpha \in \mathbb{R} \setminus \{0\}$ si approssima α con numeri razionali escludendo $x < 0$

i) $\alpha > 0$



ii) $0 < \alpha < 1$



iii) $\alpha < 0$

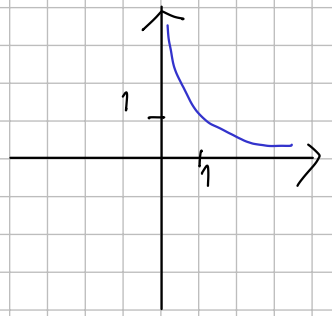
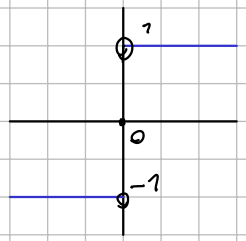


Grafico funzione signum x

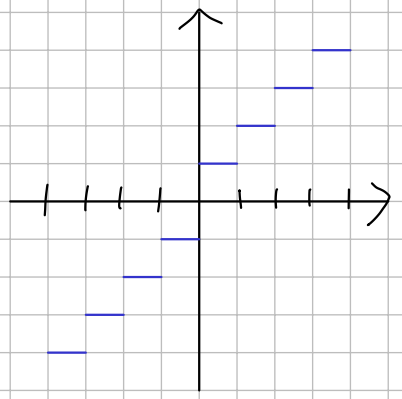
$$F(x) = \text{signum}\, x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$



Parte intera di x

La parte intera di x indicata con $[x]$ è il più grande numero intero minore o al più uguale a x

grafico



$y = [x] \quad A = \mathbb{R}$