



**POLITECNICO**  
MILANO 1863

# **ONLINE LEARNING APPLICATIONS PROJECT**

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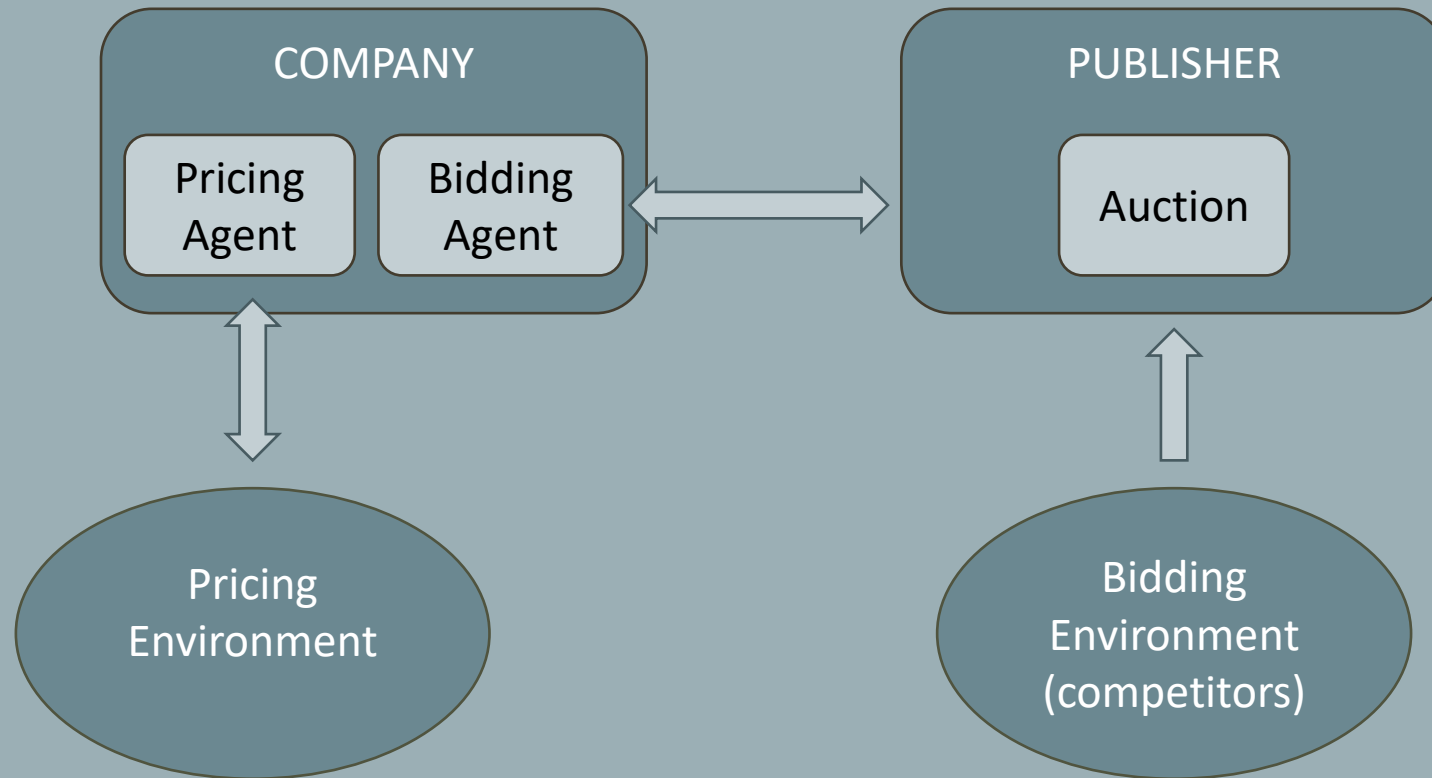
Davide Remondina

Sofia Yang

# REQUIREMENT 1

<b>BIDDING ENVIRONMENT (COMPETITORS)</b>	Stochastic with uniformly distributed bids
<b>PRICING ENVIRONMENT</b>	Stochastic with: <ul style="list-style-type: none"><li>• demands sampled from a binomial distribution with success probability given by the conversion rate <math>(1 - p)</math></li><li>• number of trials = number of visits obtained in the bidding interaction</li></ul>
<b>AUCTION TYPE</b>	Second Price (Truthful)
<b>BIDDING AGENTS</b>	<ul style="list-style-type: none"><li>• UCB-like</li><li>• Primal-Dual (Multiplicative Pacing)</li></ul>
<b>PRICING AGENT</b>	Gaussian Process (continuous action set)

# INTERACTION



# PRICING ONLY

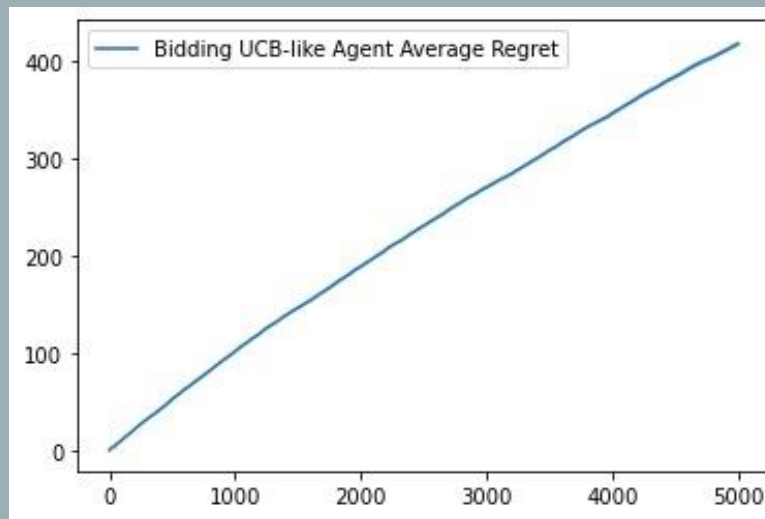
NUMBER OF INTERACTIONS	500
NUMBER OF CUSTOMERS	50
PRODUCT COST	0.1
NUMBER OF TRIALS	10

# BIDDING ONLY

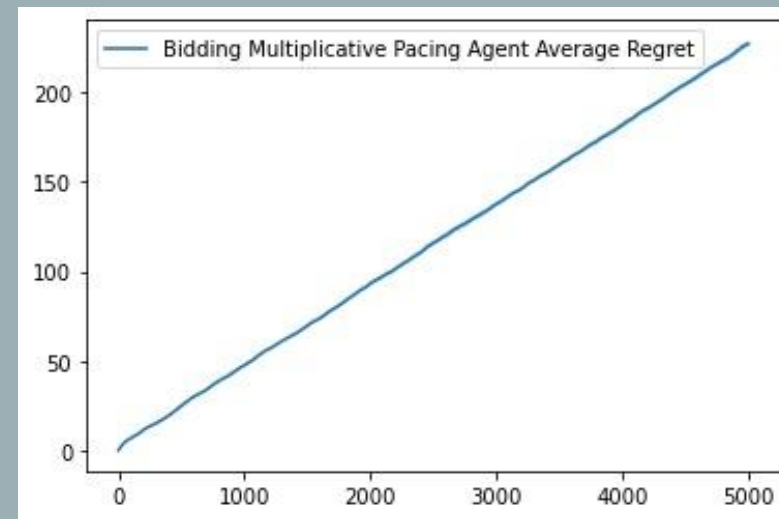
NUMBER OF INTERACTIONS	5000
BUDGET	1000
VALUATION	1
CTRs	[0.8 0.5 0.9 1]
NUMBER OF TRIALS	10



GPUCB



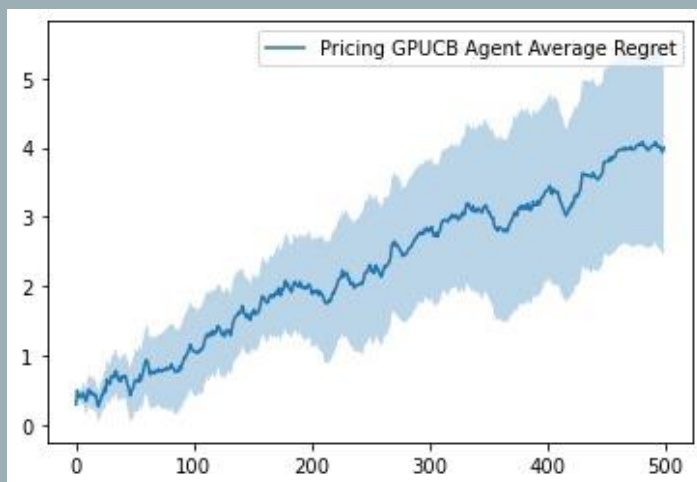
UCB-like



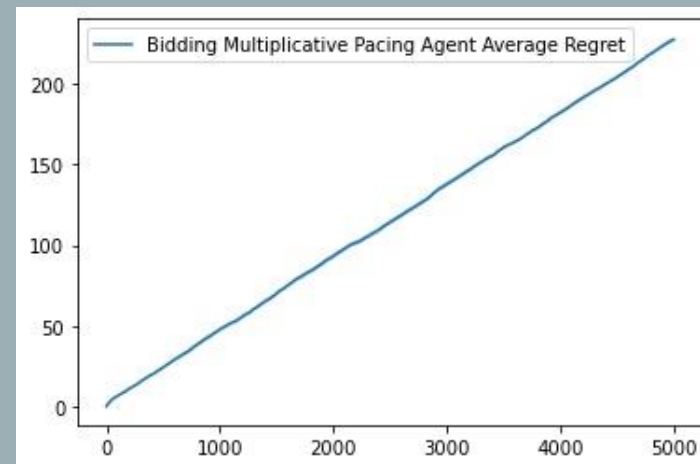
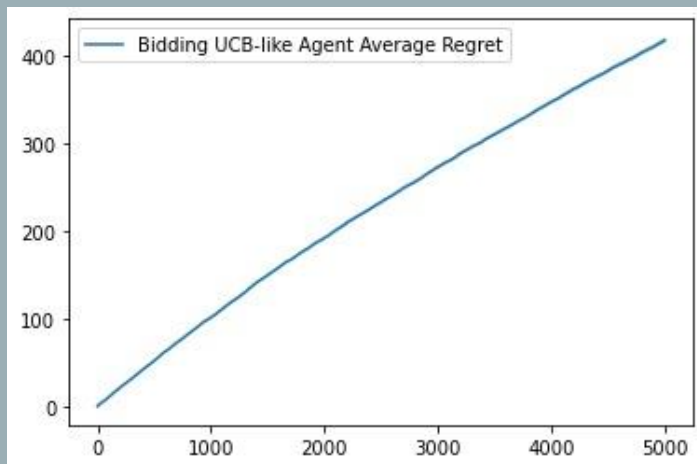
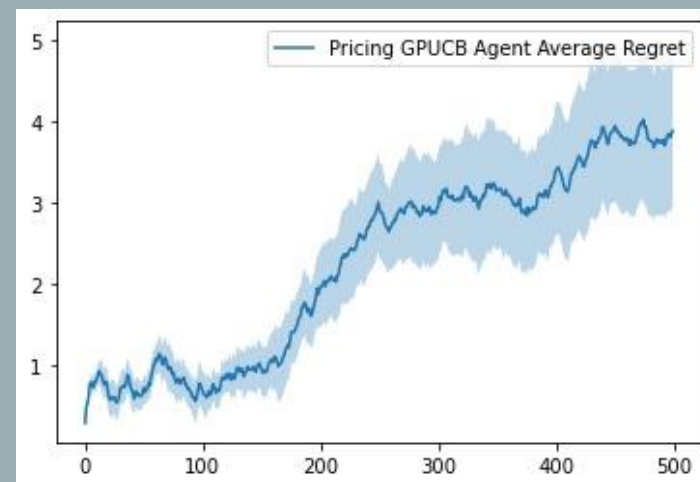
Multiplicative Pacing

# PRICING AND BIDDING

## UCB-LIKE BIDDER



## PRIMAL-DUAL BIDDER

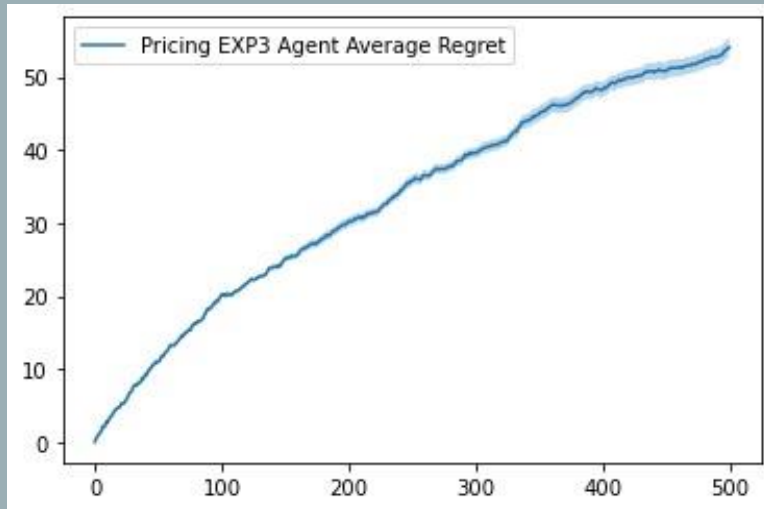


## REQUIREMENT 2

BIDDING ENVIRONMENT (COMPETITORS)	<ul style="list-style-type: none"><li>• Adversarial with uniformly distributed bids</li><li>• Extremes of the interval sampled from a beta distribution each round</li></ul>
PRICING ENVIRONMENT	Adversarial with: <ul style="list-style-type: none"><li>• demands sampled from a binomial distribution with success probability given by the conversion rate <math>(1 - \theta * p)</math></li><li>• <math>\theta</math> sampled from a beta distribution each round</li><li>• number of trials = number of visits obtained in the bidding interaction</li></ul>
AUCTION TYPE	Generalized First-Price (Non-truthful)
BIDDING AGENTS	Primal-Dual (Multiplicative Pacing)
PRICING AGENT	EXP3 (discretized prices)

# PRICING ONLY

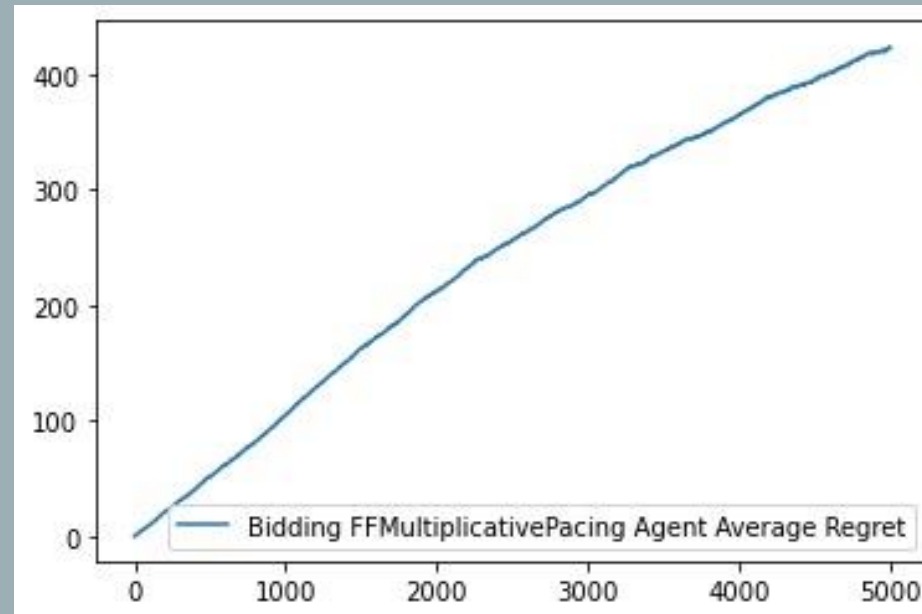
NUMBER OF INTERACTIONS	500
NUMBER OF CUSTOMERS	50
PRODUCT COST	0.1
NUMBER OF TRIALS	10



EXP3

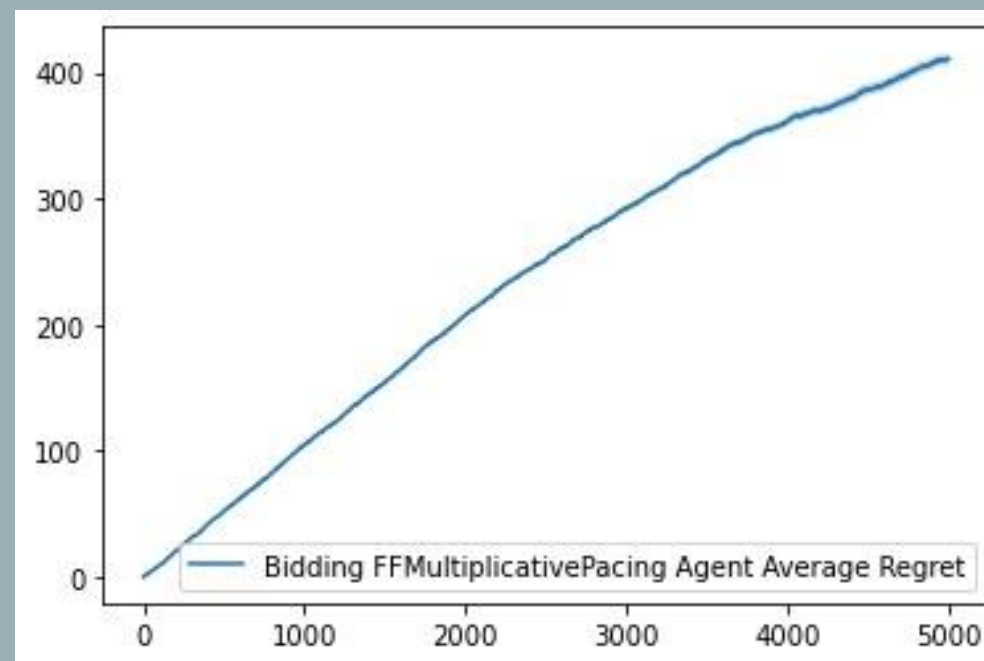
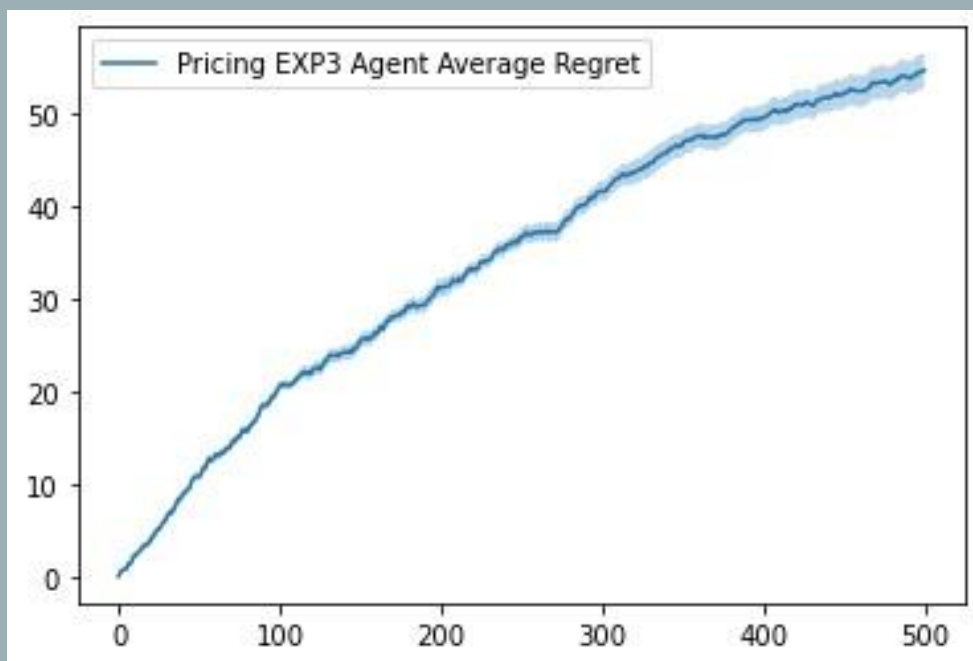
# BIDDING ONLY

NUMBER OF INTERACTIONS	5000
BUDGET	1000
VALUATION	1
CTRs	[0.8 0.5 0.9 1]
LAMBDA <sub>s</sub>	[1 0.9]
NUMBER OF TRIALS	10



FF  
Multiplicative  
Pacing

# PRICING AND BIDDING





# REQUIREMENT 3

- Focus on the pricing problem
- Non-stationary environment
  - Abrupt changes
  - Noisy demand curve changes each interval

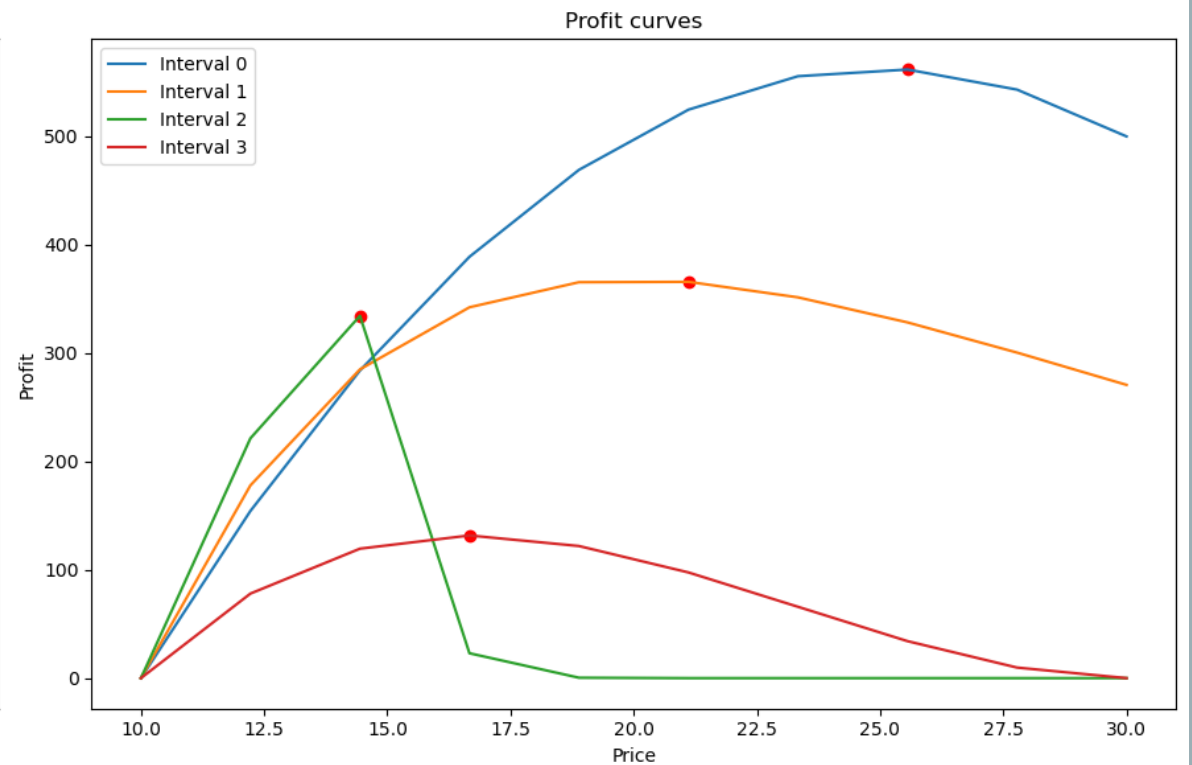
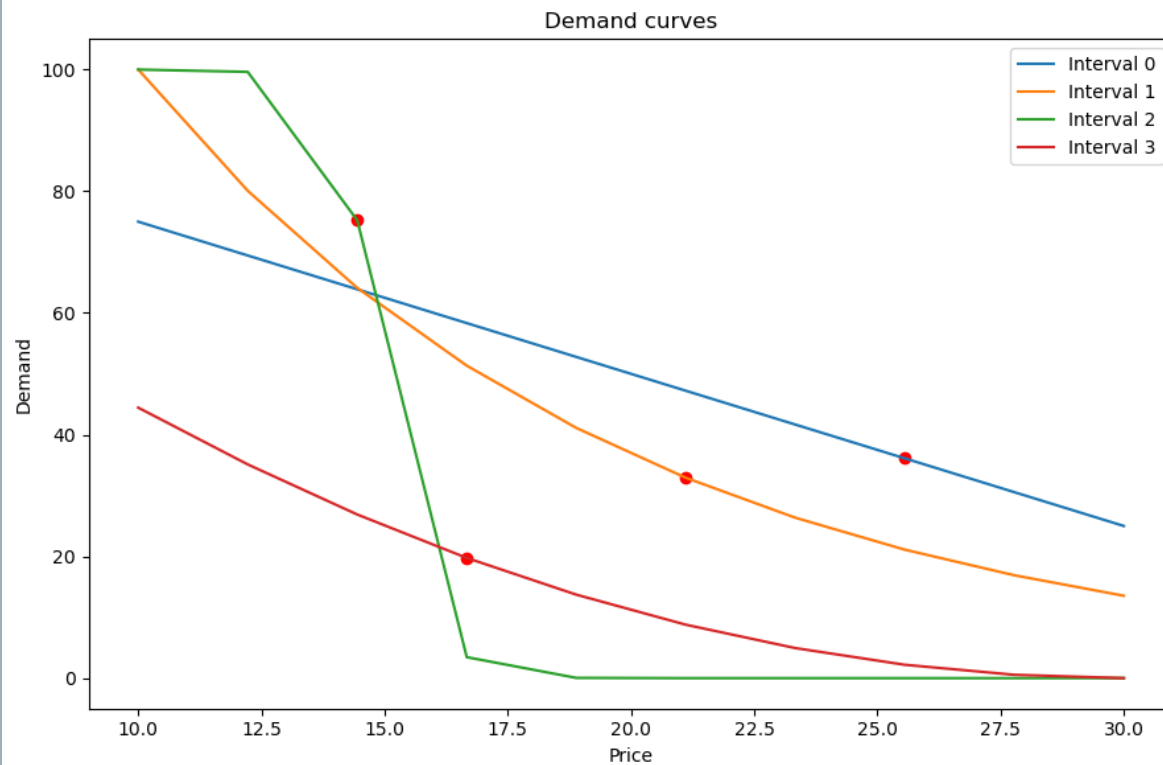
## GOAL

Pricing strategy for discretized set of prices  $p \in [0,1]$  using:

- Sliding Window
- CUSUM

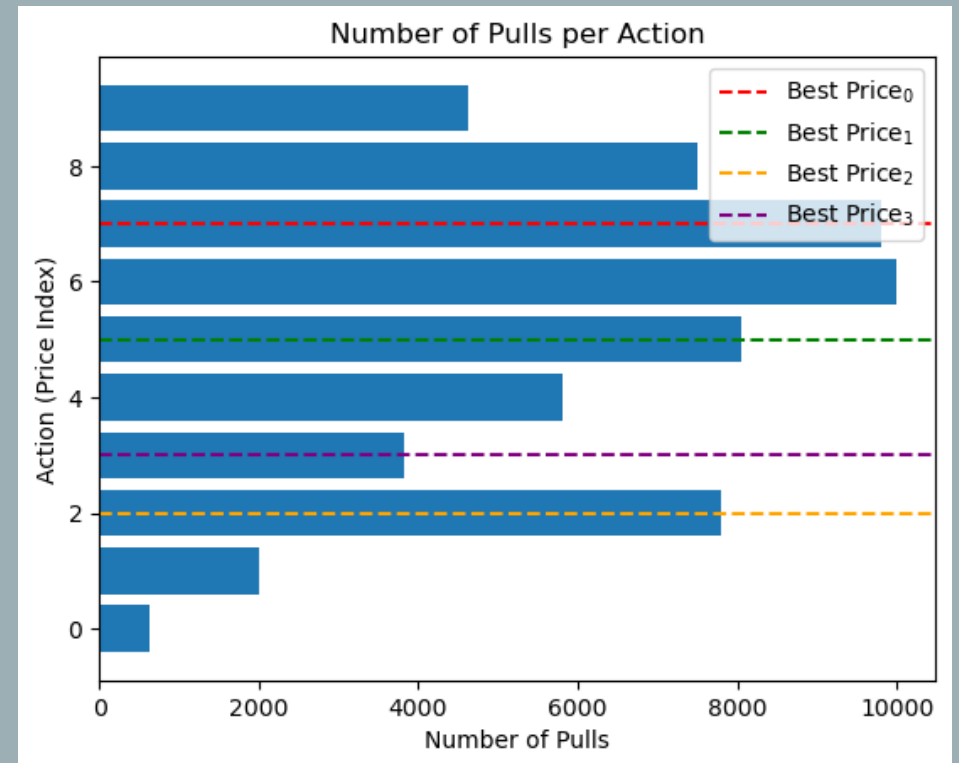
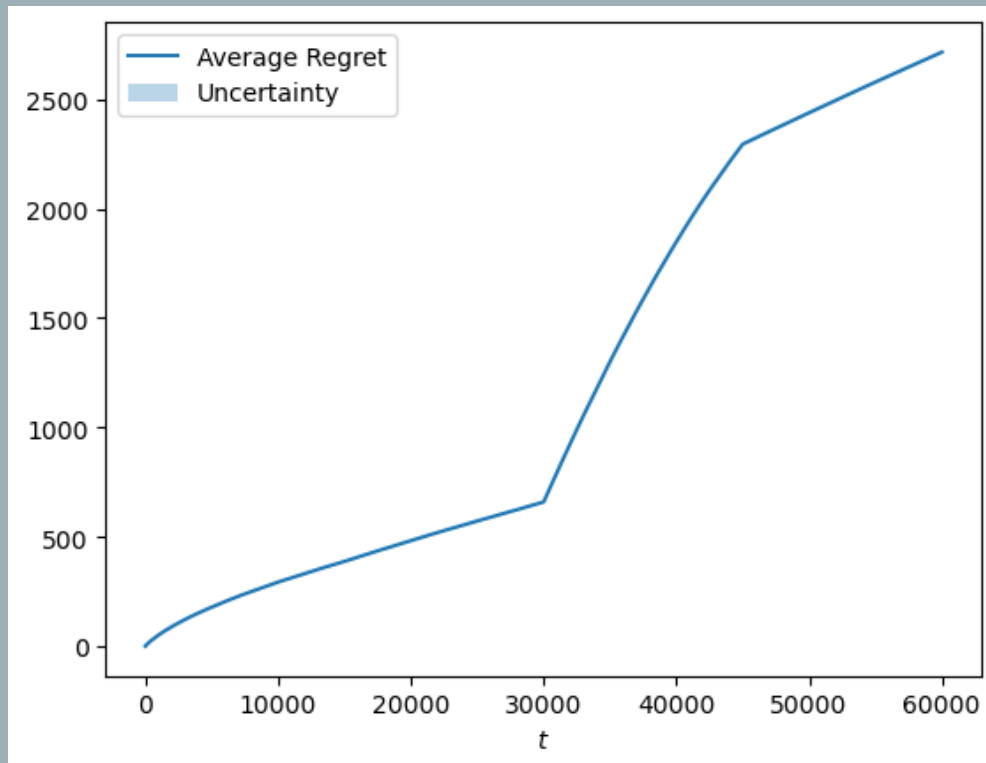
# SETUP

- Defined demand curves for each interval
- Derived respective profit curves



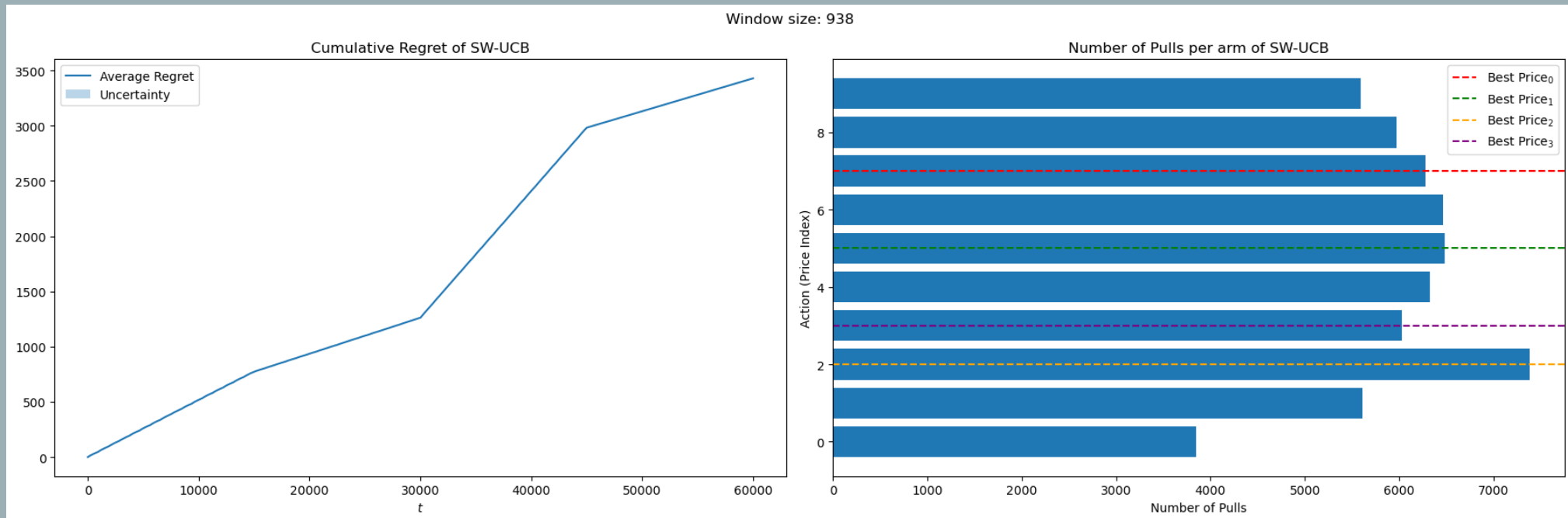
# NON-STATIONARITY CHECK

- Tested UCB1 on the environment
- Results indicate presence of non-stationarity

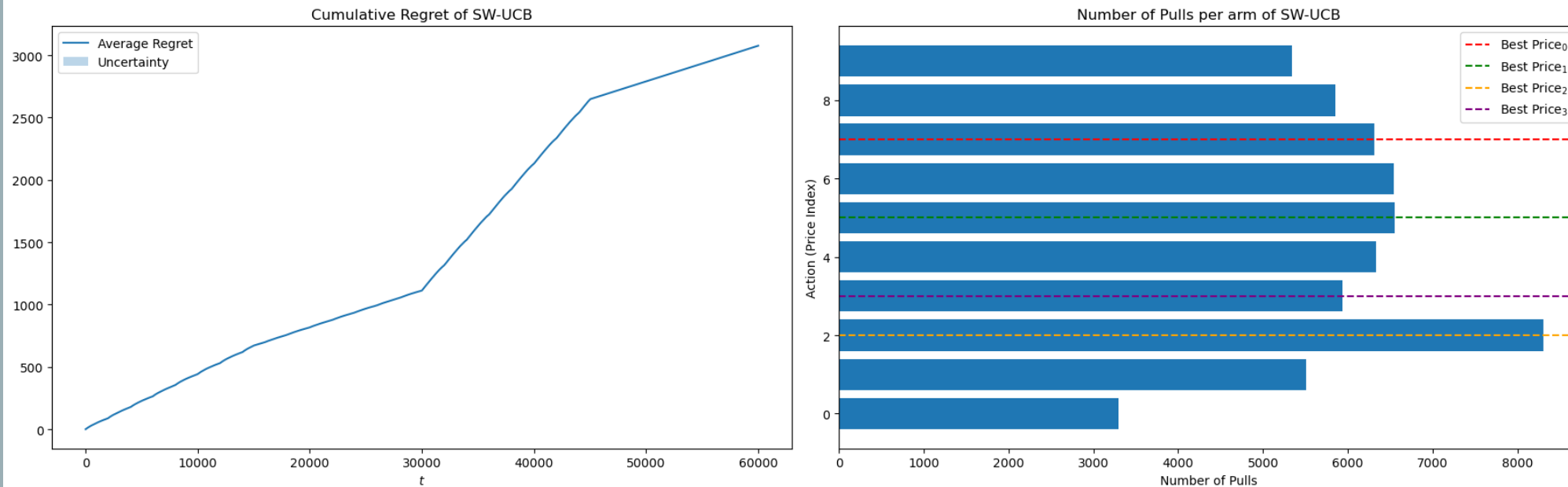


# SLIDING WINDOW

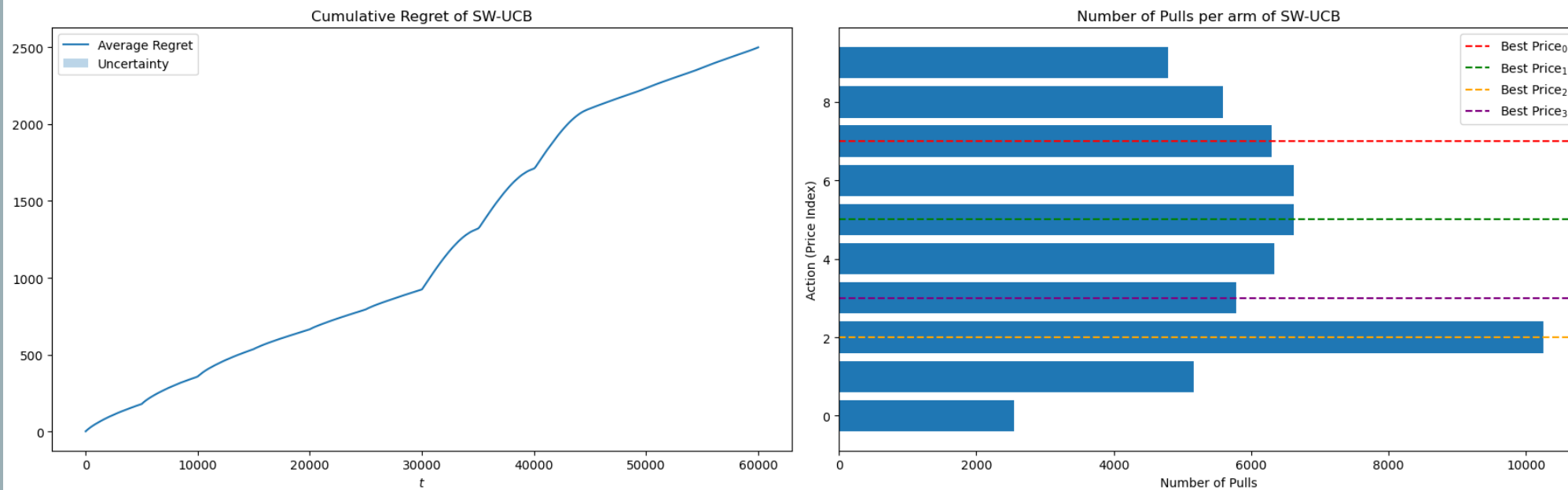
- Sliding Window UCB
- Initial trial failed
  - Decided to test different window sizes



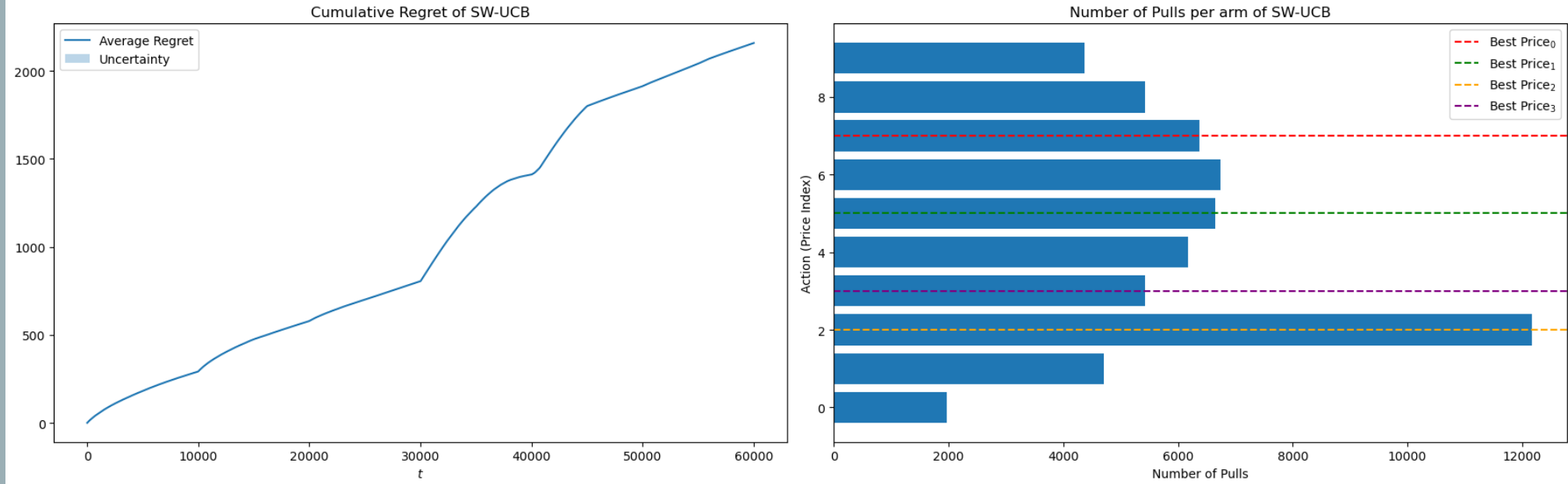
Window size: 2000



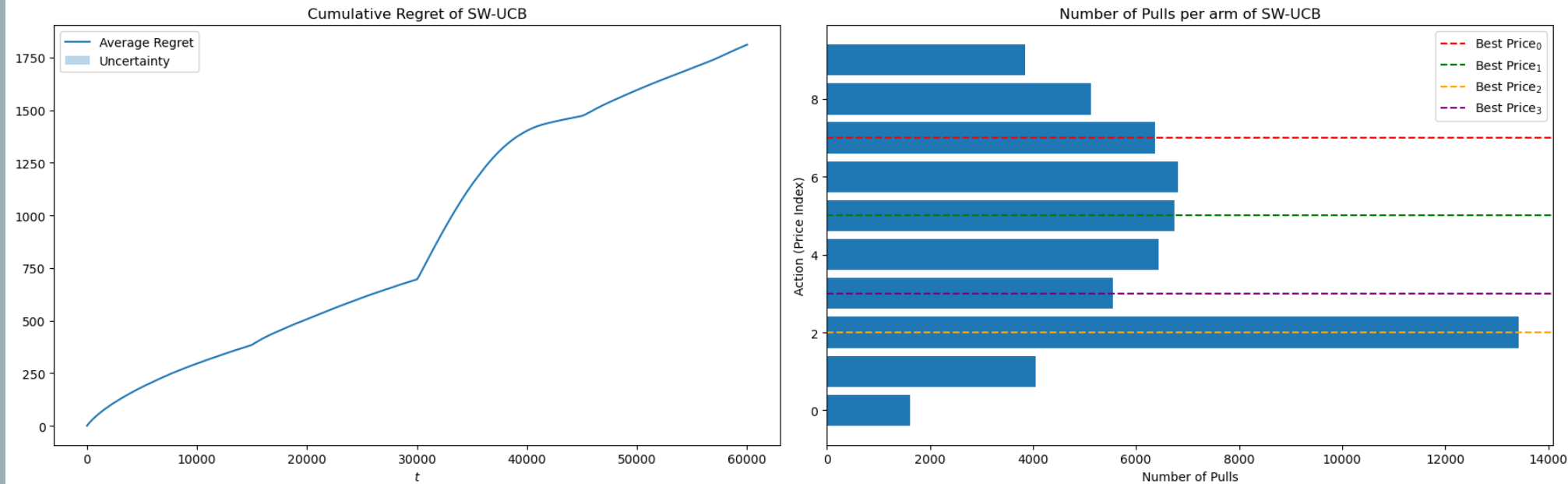
Window size: 5000



Window size: 10000



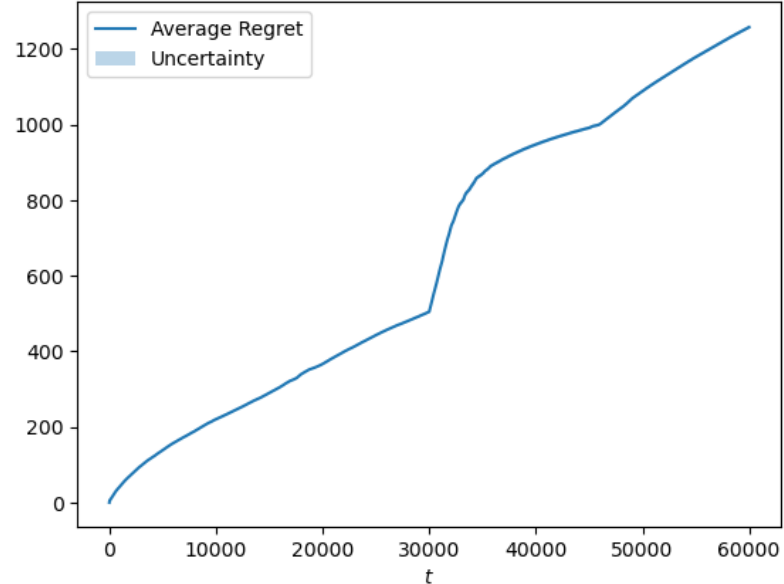
Window size: 15000



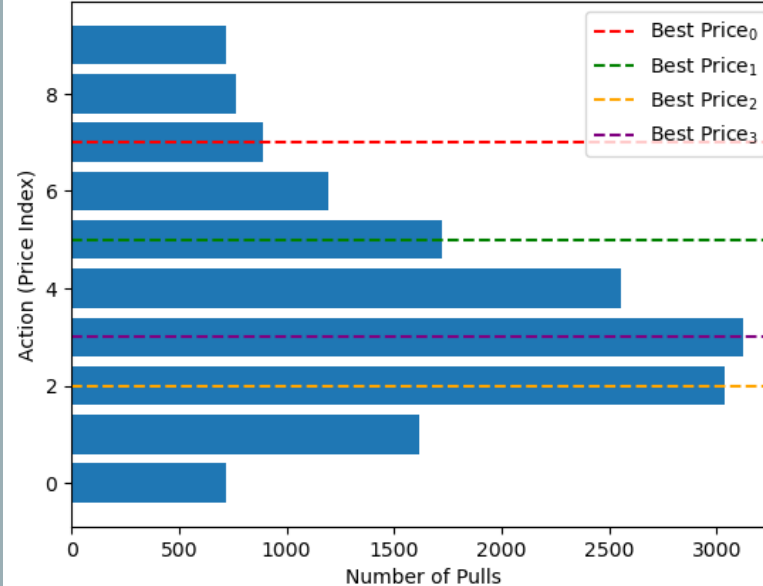
# CHANGE DETECTION

- CUSUM UCB
- Best performing method overall

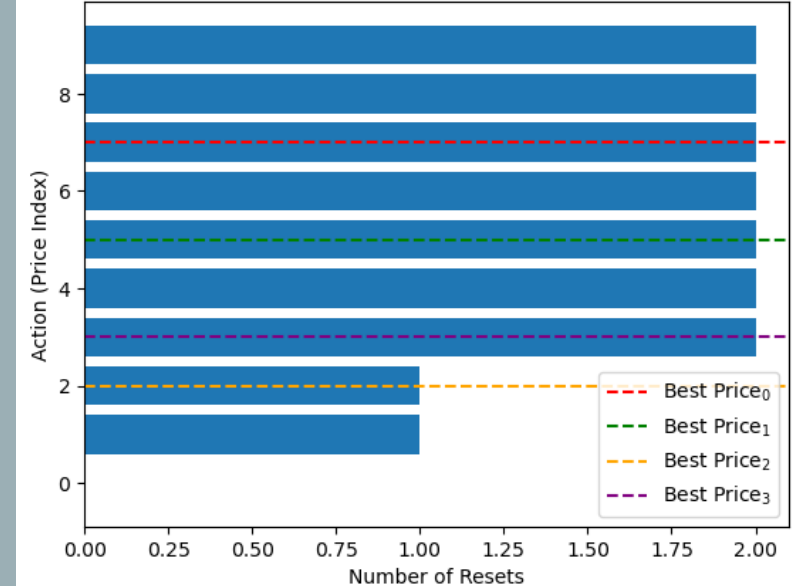
Cumulative Regret of CUSUM-UCB



Number of Pulls per arm of CUSUM-UCB



Number of Resets per arm of CUSUM-UCB

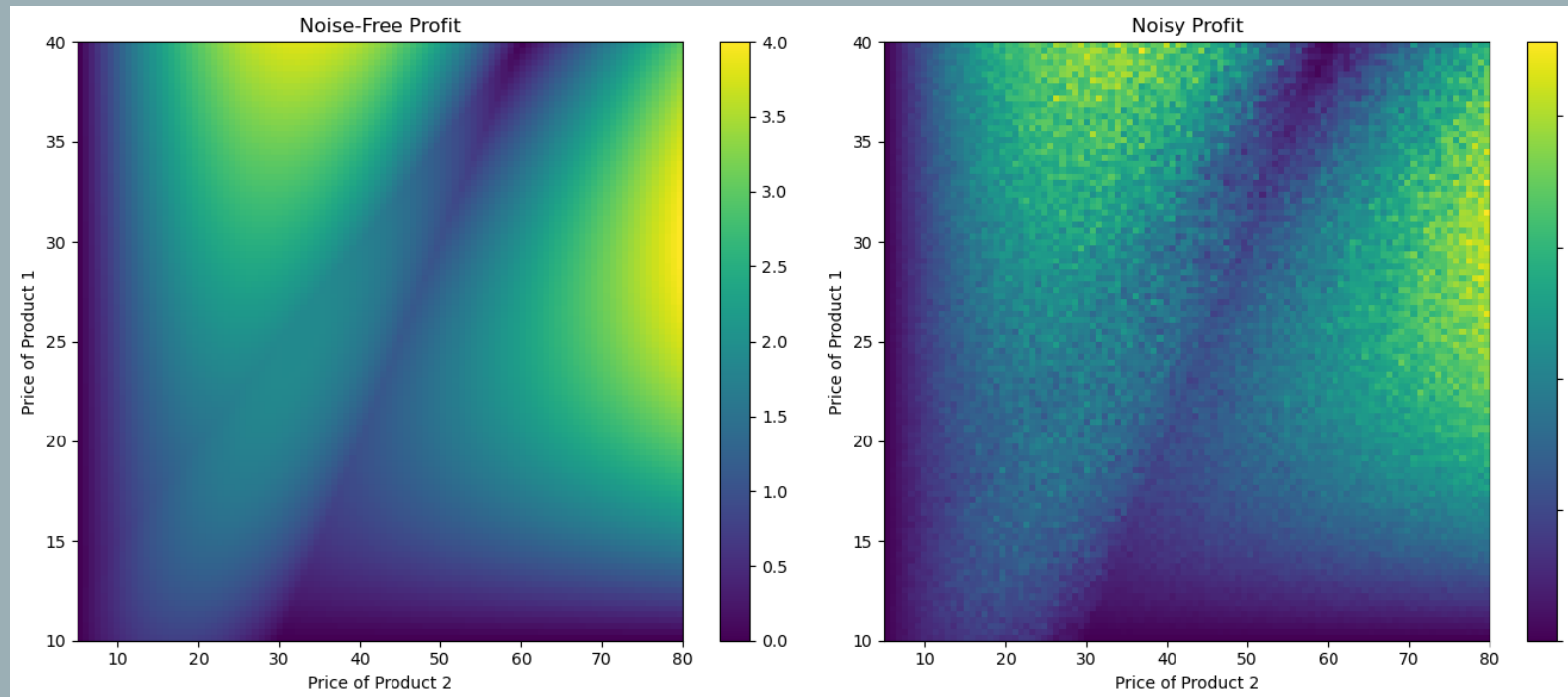


# BONUS POINT

- Two-item stochastic pricing environment
- Noisy demand curve  $D(p_1, p_2) + \eta$

## GOAL

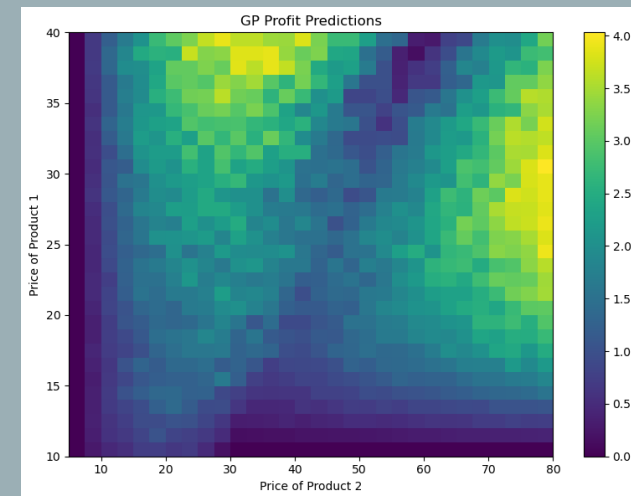
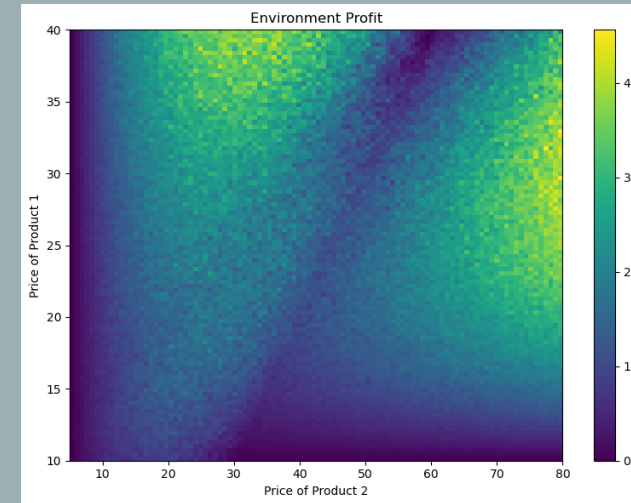
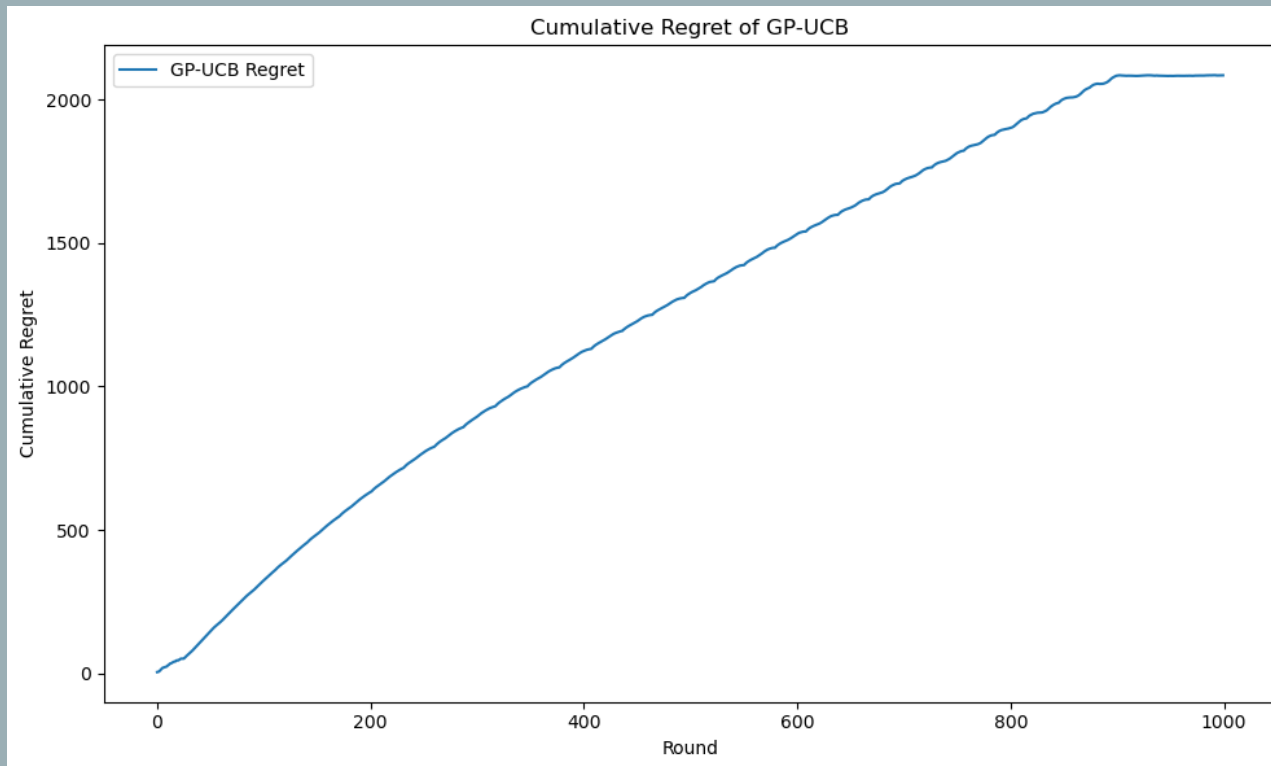
Build a regret minimizer for the continuous action set  $[0,1]^2$  using 2D Gaussian Processes





# RESULTS

- GP UCB
- Great results obtained



## REQUIREMENT 4

BIDDING ENVIRONMENT	Adversarial with full feedback
AUCTION TYPE	Generalized First-Price (Non-truthful)
BIDDING AGENTS	<ul style="list-style-type: none"><li>• Primal-Dual for Truthful (Multiplicative Pacing)</li><li>• Primal-Dual for Non-truthful (Multiplicative Pacing with Hedge)</li><li>• UCB-like (UCB1)</li><li>• UCB-like Updating <math>\rho</math> (our version of the algorithm)</li></ul>
GOAL	Compare the performances of different algorithms under various setups

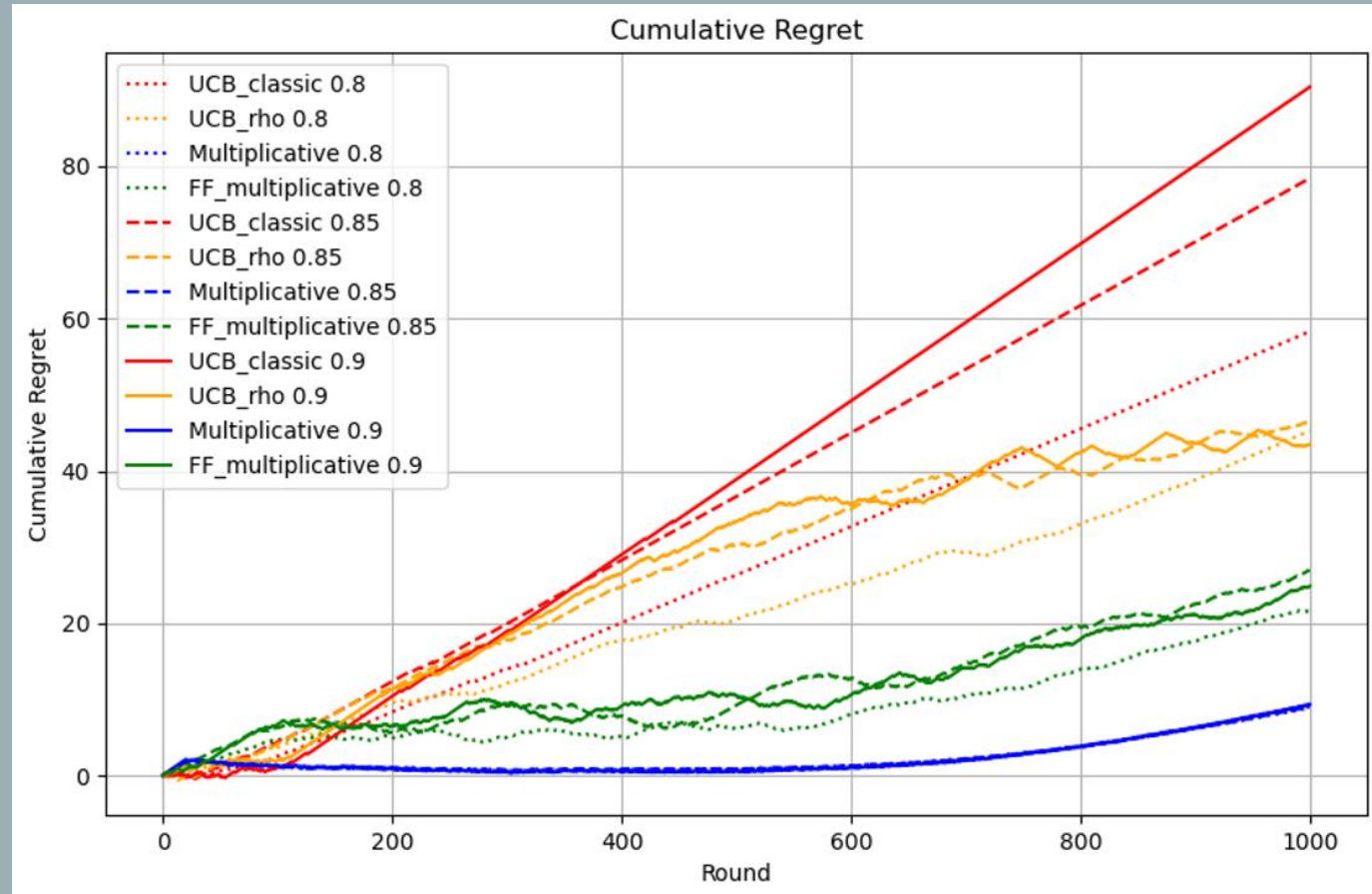
# STANDARD CASE

## SETUP

BIDDERS PER TYPE	3
VALUATIONS	[0.80 0.85 0.90]
NUMBER OF SLOTS	3
SLOT PROMINENCE	[0.8 0.9 1.0]
BUDGET	250

## OBSERVATION

- UCB\_classic has linear regret
- Multiplicative pacing has the smoothest curve



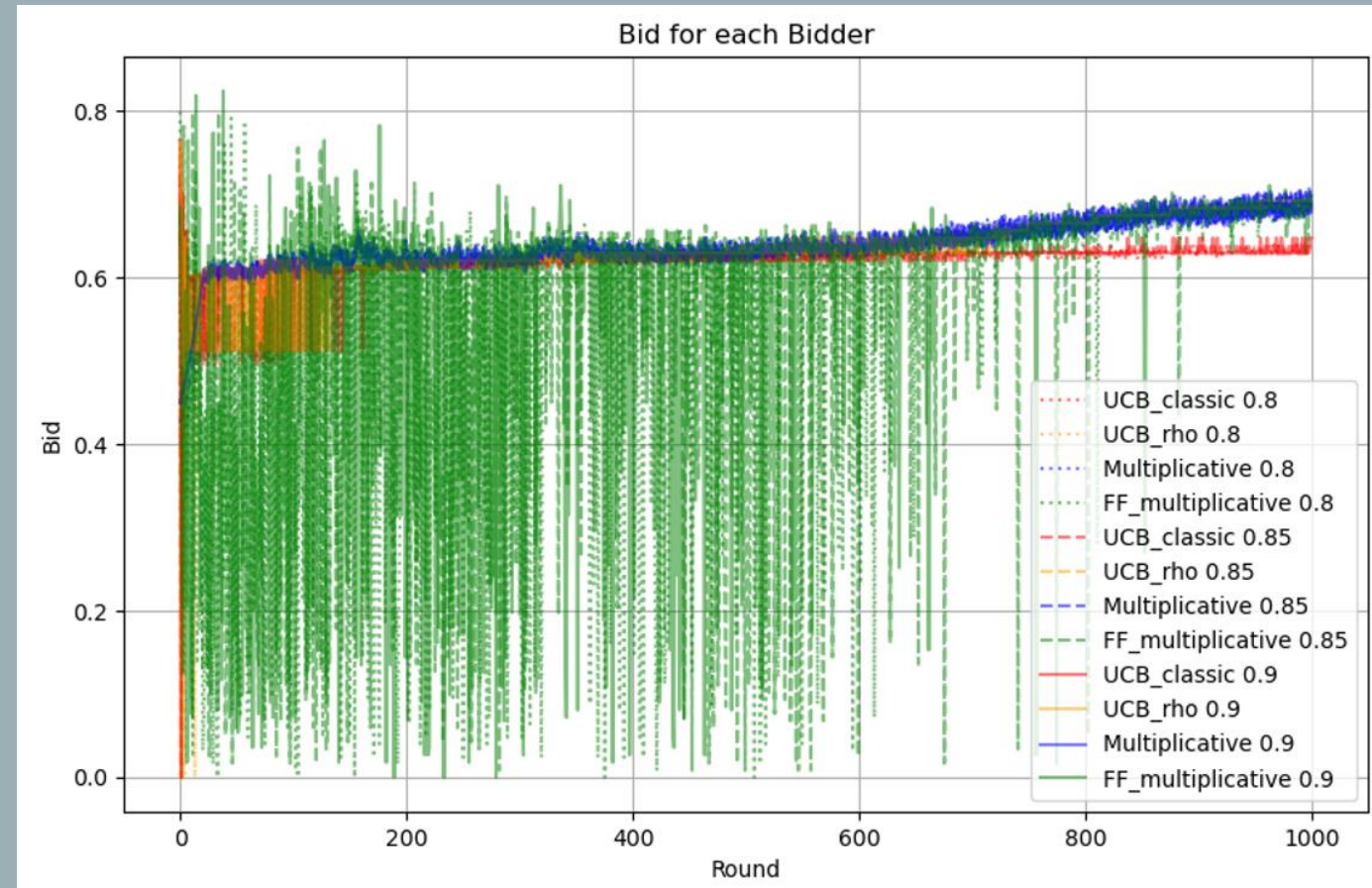
# STANDARD CASE

## SETUP

BIDDERS PER TYPE	3
VALUATIONS	[0.80 0.85 0.90]
NUMBER OF SLOTS	3
SLOT PROMINENCE	[0.8 0.9 1.0]
BUDGET	250

## OBSERVATION

- Bids increase over rounds
- UCB\_classic stops winning after round 500
- FF\_multiplicative needs time to be consistent with bids



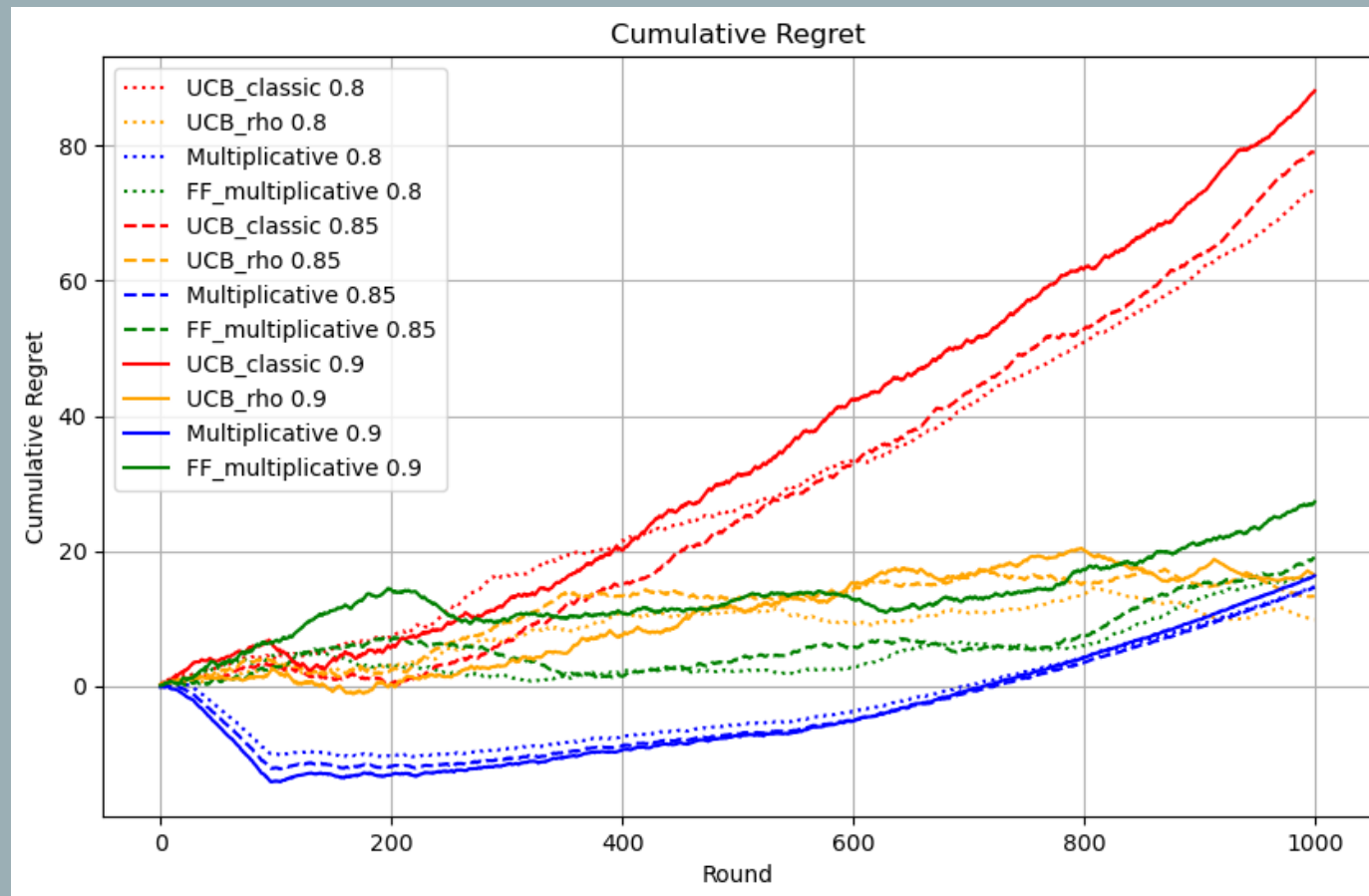
# MANY SLOTS CASE

## SETUP

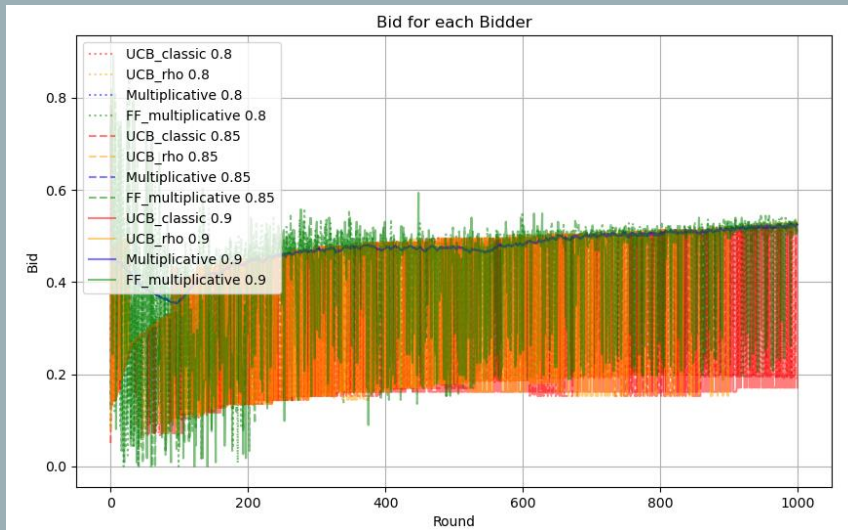
BIDDERS PER TYPE	3
VALUATIONS	[0.80 0.85 0.90]
NUMBER OF SLOTS	10
SLOT PROMINENCE	[0.1 ... 1.0]
BUDGET	250

## OBSERVATION

- All algorithms have more or less the same regret except for standard UCB\_like algorithm

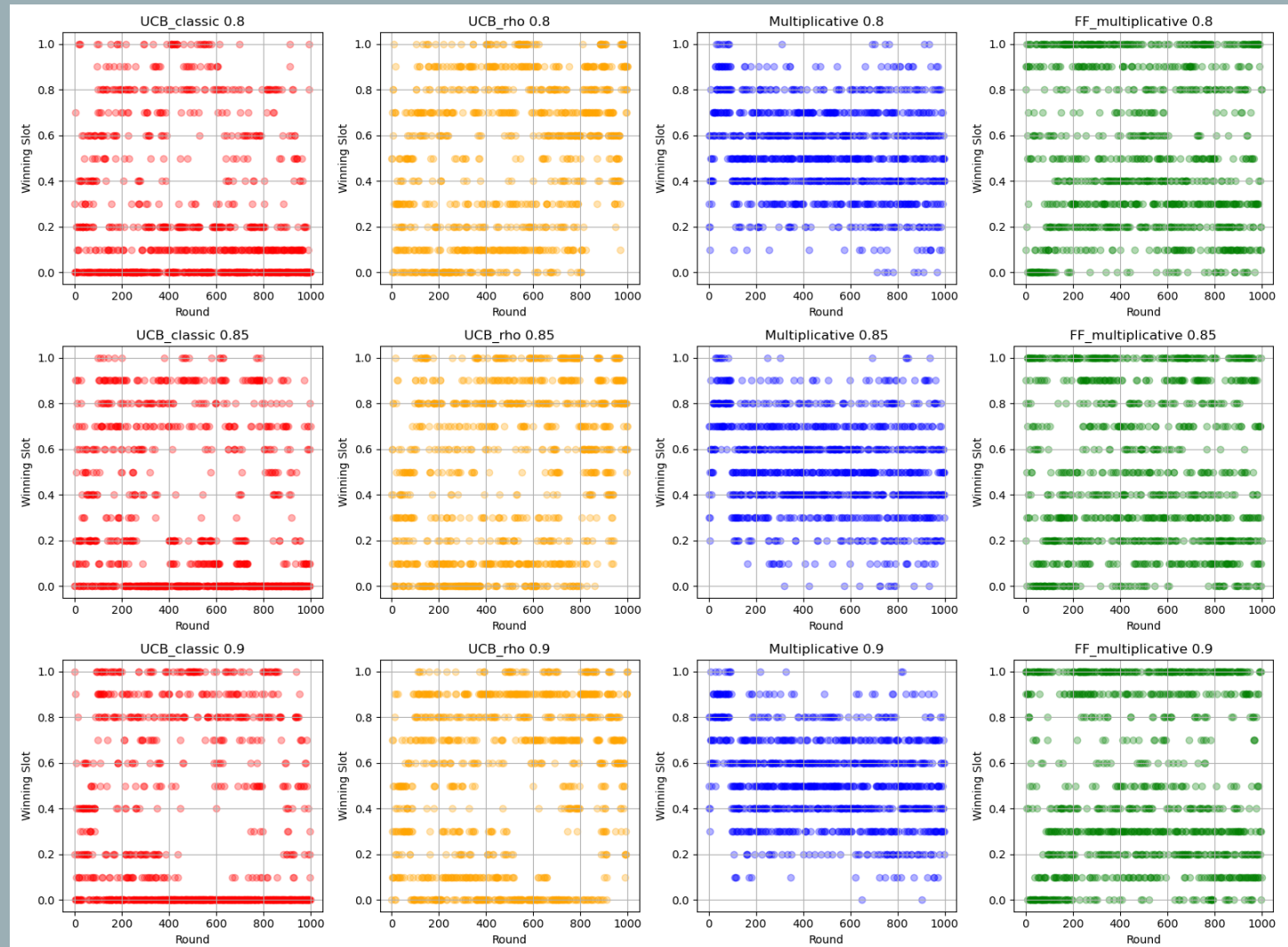


# MANY SLOTS CASE



## OBSERVATION

- Multiplicative algorithm wins middle slots
- FF\_multiplicative prefers going all or nothing, winning the best slot more often





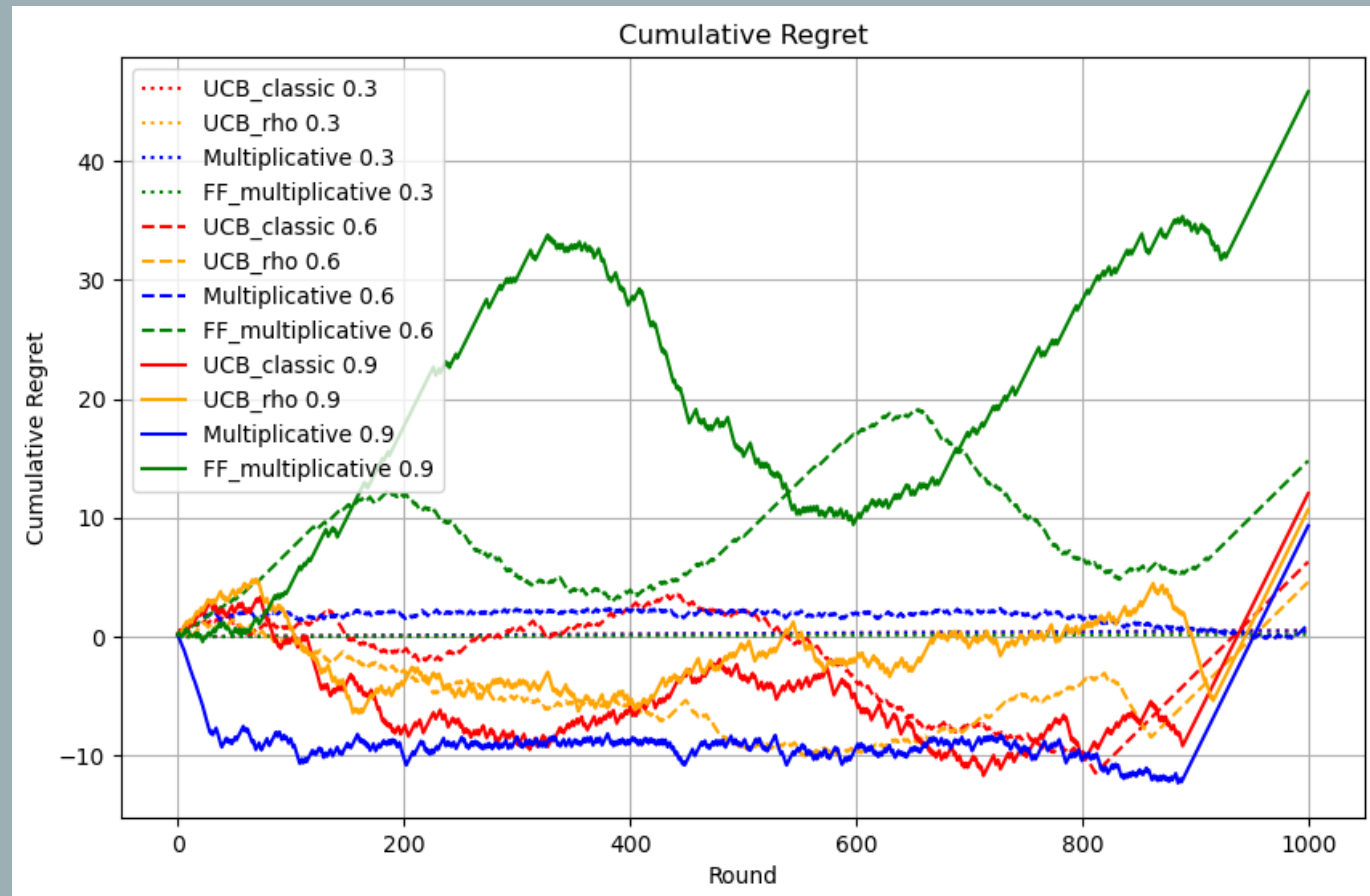
# LOW BUDGET - DIFFERENT VALUATION

## SETUP

BIDDERS PER TYPE	3
VALUATIONS	[0.3 0.6 0.9]
NUMBER OF SLOTS	3
SLOT PROMINENCE	[0.8 0.9 1.0]
BUDGET	100

## OBSERVATION

- UCB\_like performs better than FF\_multiplicative
- Our modification to the UCB\_like algorithm performs slightly better than the original
- Many agents deplete the budget about 100 rounds before the end



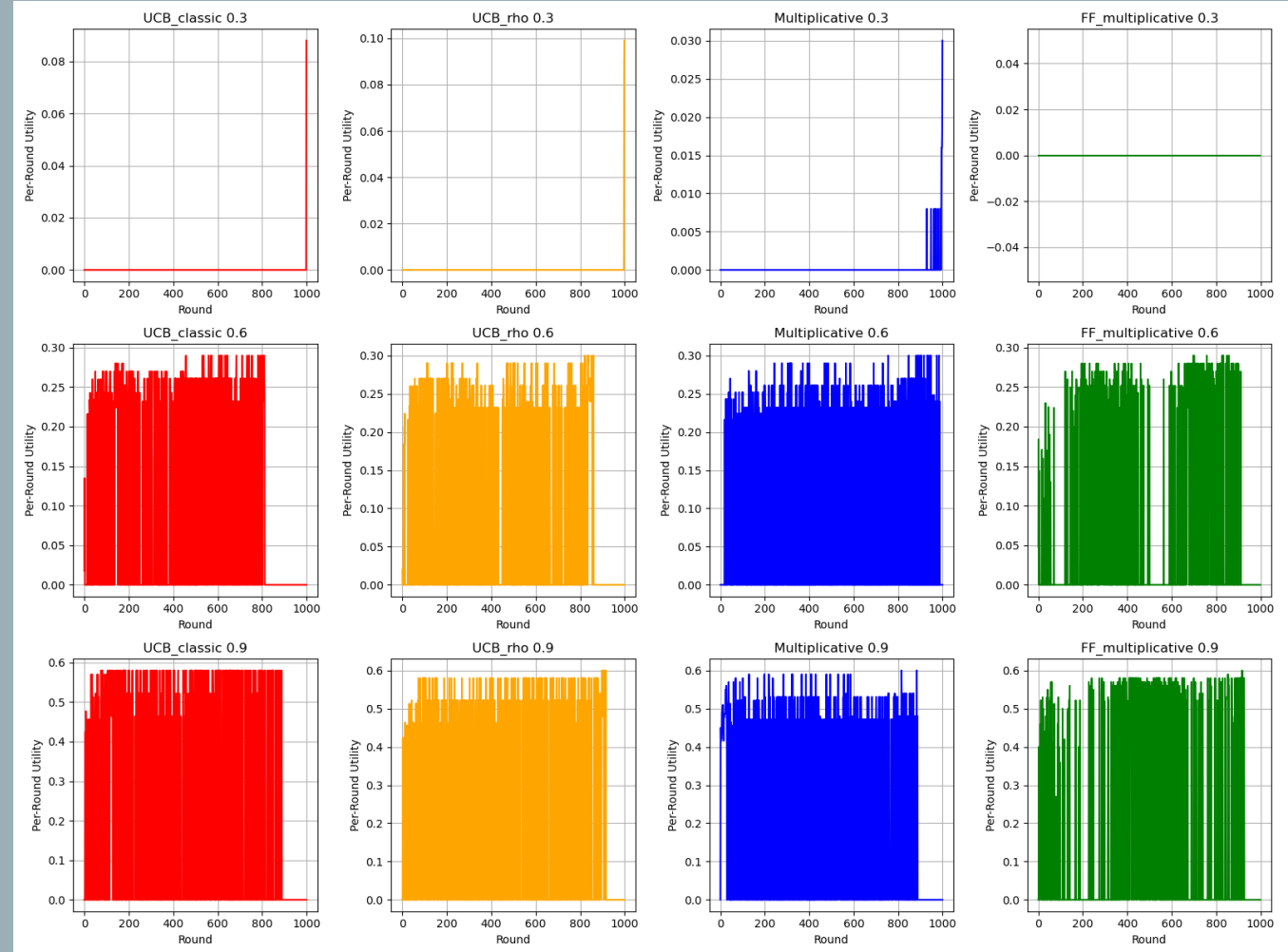
# LOW BUDGET - DIFFERENT VALUATION

## SETUP

BIDDERS PER TYPE	3
VALUATIONS	[0.3 0.6 0.9]
NUMBER OF SLOTS	3
SLOT PROMINENCE	[0.8 0.9 1.0]
BUDGET	100

## OBSERVATION

- Bidder with valuation of 0.3 have regret 0 since minimum bid for winning a slot is 0.4
- Unclear why FF\_multiplicative performs badly





# REQUIREMENT 4

## FINAL CONSIDERATIONS

- Best performing algorithm is:
  - Multiplicative, despite
  - FF\_multiplicative having better theoretical guarantees
- Our version of UCB-like algorithm for expert feedback worked well:
  - Almost always much better than UCB1
  - Slightly better in a single case

# THANK YOU

Source code available at this [GitHub repository](#)