POLITECNICO DI MILANO
DEPARTMENT OF AEROSPACE ENGINEERING
AEROSPACE SYSTEMS COURSE

PROVA FINALE: INTRODUZIONE ALL'ANALISI DI MISSIONI SPAZIALI

AA 2022-2023

Professor: Colombo Camilla

Juvara Matteo - 10726898 Mazzotti Luca - 10744992 Miccoli Francesco - 10782473

8 febbraio 2023

Indice

1 Introduction

One of the most important skills for a space engineer is knowing how to transfer in the most efficient way between two given orbits. The main goal of this project was to design and analyze the possible transfers from a lower MEO parking orbit to a higher MEO target orbit. The launch phase and the atmospheric stage are not taken into account, and all the maneuvers utilized will be assumed to be impulsive.

Every transfer strategy and its correspondent maneuvers were designed through scripts and functions purposely created within the MATLAB environment, as well as the plots and graphs provided.

We began by studying the standard strategy used for orbital transfers. We then tried varying some of the maneuvers within the standard strategy while still leaving the basic structure intact. By doing this, we were able to confront the results and gain a better understanding of how to make the transfer more time-efficient and less expensive. At this point, we attempted to develop various strategies that optimized the two primary constraints that are to be considered while designing an orbital transfer: the time it takes to complete the transfer and the amount of fuel required. We started by trying to optimize a bi-elliptic transfer, also known as the Sternfeld maneuver, and a three-impulse maneuver. We then tried to use direct transfers, which minimized the maneuver cost and the time required.

Once we knew the outcomes of each transfer, we were able to draw conclusions based on the data we found and the theory we studied during our course. We then wrote a report that contained our conclusions as well as the pertinent data, graphs, and plots to back them up. Finally, to make our results simpler to comprehend and communicate verbally, we summarized our findings into a presentation.

2 Initial and final orbit characterization

2.1 Initial orbit

2.1.1 Determine initial orbital parameters from given position and velocity

We were initially given the position and velocity vectors at the set starting point, both in geocentric-equatorial coordinates.

$$\vec{r}_1 = \begin{Bmatrix} -8048.2861 \\ -4171.3048 \\ 2895.9296 \end{Bmatrix} \mathrm{km} \qquad \vec{V}_1 = \begin{Bmatrix} 1.7540 \\ -5.9910 \\ -1.9520 \end{Bmatrix} \mathrm{km\,s^{-1}}$$

We then implemented a MATLAB function to receive vectors $\vec{r}_1$ and $\vec{V}_1$ as inputs and return the Keplerian elements of the corresponding orbit. The outcomes were as follows:

$$a_1 = 9723.689\,\mathrm{km}; \quad e_1 = 0.086\,527 \quad i_1 = 0.461\,526\,\mathrm{rad}$$

$$\Omega_1 = 1.175\,699\,\mathrm{rad} \quad \omega_1 = 0.982\,554\,\mathrm{rad} \quad \theta_1 = 1.406\,683\,\mathrm{rad}$$

2.1.2 Discuss the result, evaluate other relevant orbit data

From these results, we were able to determine the specific energy ϵ_1, the semilatus rectum p_1, the orbital period T_1, and the periapsis and apoapsis altitudes r_{p1} and r_{a1}.

$$\epsilon_1 = \frac{-\mu}{2 * a_1} = -20.4963\,\mathrm{km^2\,s^{-2}}; \qquad p_1 = a_1 * (1 - e_1^2) = 9650.889\,\mathrm{km} \qquad T_1 = 2 * \pi * \sqrt{\frac{a_1^3}{\mu}} = 9542.4\,\mathrm{s};$$

$$h_{p1} = a_1 * (1 - e_1) - R_0 = 2504.327\,\mathrm{km}; \qquad h_{a1} = a_1 * (1 + e_1) - R_0 = 4187.051\,\mathrm{km};$$

2.2 Final orbit

2.2.1 Determine final orbit position and velocity at set point given keplerian parameters

We were also given the finishing point of the final orbit we had to reach in Keplerian elements.

$$a_2 = 16\,720.00\,\mathrm{km}; \quad e_2 = 0.2502 \quad i_2 = 1.1190\,\mathrm{rad}$$

$$\Omega_2 = 0.6245\,\mathrm{rad} \quad \omega_2 = 3.1350\,\mathrm{rad} \quad \theta_2 = 3.1000\,\mathrm{rad}$$

The second MATLAB function we utilized was *orbitalToCar*, which, given a vector containing an orbit's keplerian elements, provides the position and velocity vectors corresponding to that orbit. Our findings are as follows:

$$\vec{r}_2 = \begin{Bmatrix} 17190.0324 \\ 11847.8031 \\ -905.5606 \end{Bmatrix} \mathrm{km} \qquad \vec{V}_2 = \begin{Bmatrix} -0.7734 \\ 1.4743 \\ 3.3966 \end{Bmatrix} \mathrm{km\,s^{-1}}$$

2.2.2 Discuss the result, evaluate other relevant orbit data

Once again, we were able to determine the specific energy ϵ_1, the semilatus rectum p_1, the orbital period T_1, and the periapsis and apoapsis altitudes r_{p1} and r_{a1}.

$$\epsilon_2 = \frac{-\mu}{2 * a_2} = -11.9199\,\mathrm{km^2\,s^{-2}}; \qquad p_2 = a_2 * (1 - e_2^2) = 15\,673.327\,\mathrm{km} \qquad T_2 = 2 * \pi * \sqrt{\frac{a_2^3}{\mu}} = 21\,516.2\,\mathrm{s};$$

$$h_{p2} = a_2 * (1 - e_2) - R_0 = 6158.656\,\mathrm{km}; \qquad h_{a2} = a_2 * (1 + e_2) - R_0 = 14\,525.344\,\mathrm{km};$$

2.3 Representation

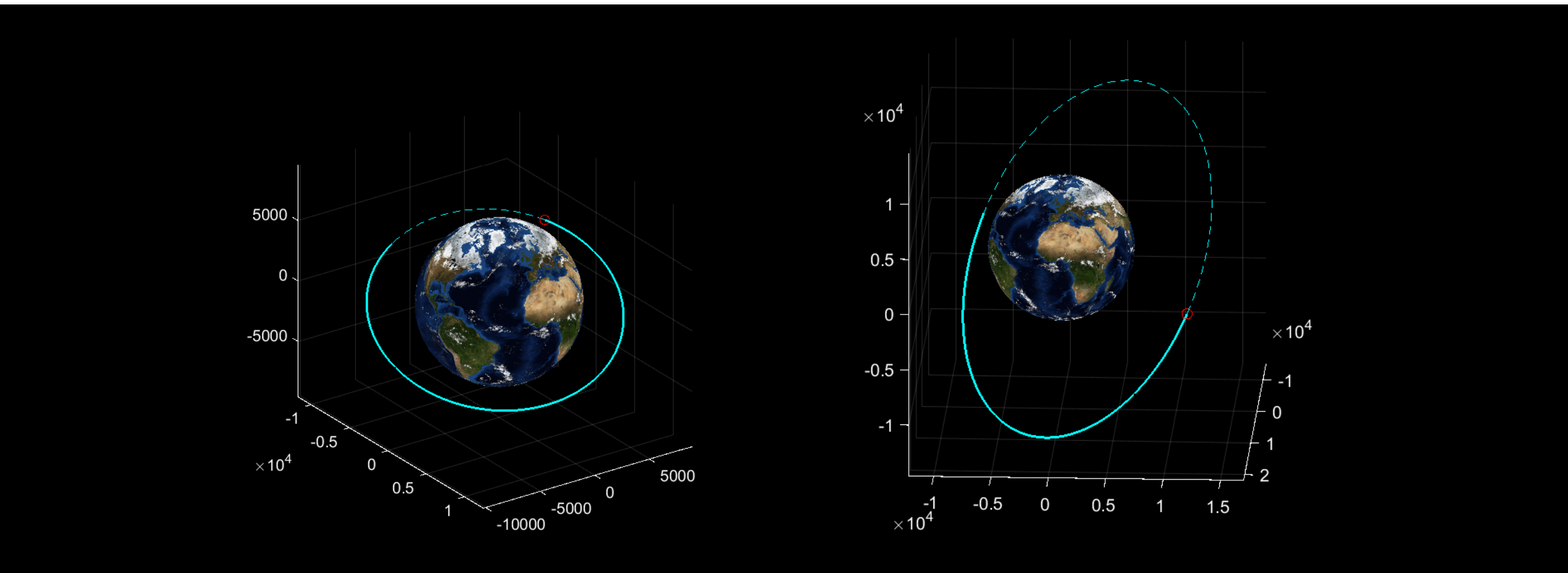


Fig. 1: Initial and final orbits respectively

3 Transfer trajectory definition and analysis

3.1 Standard strategy

3.1.1 Characterization

The standard maneuver, as it name implies, is composed of a set number of impulses in a set order. Not counting the coasting phase between one maneuver point and the other, we have:

- plane change
- periapsis argument change
- perigee-apogee transfer

The orbit's plane change is performed through a single impulse maneuver. This impulse is executed at the first intersection that the satellite encounters from its initial position, which, for the given orbits, happens to be the most convenient both in terms of fuel cost and time taken.

The easiest way to change the periapsis argument is to perform a single impulse secant maneuver when the satellite reaches the closer of the two intersection points between the current orbit and the target orbit (whose apse line is rotated by $\Delta\omega$).
By doing this, we are able to minimize the time taken to complete the maneuver while keeping the fuel cost required to perform the change constant between intersections.

Once the current orbit as the same orientation as the final orbit, we can proceed to modify the eccentricity and the semi-major axis. This will be accomplished using a bi-tangent transfer in this strategy.
There are two ways to perform a bi-tangent transfer: we could start at the periapsis and end at the apoapsis, or the other way around. Once we are in the final orbit, the satellite will simply move to the final point without any further action being required.

3.1.2 Execution

Using the functions developed throughout the course we write a MATLAB script to perform the presented maneuver. For completeness, we perform the standard strategy twice: once with a perigee-apogee transfer and other with an apogee-perigee transfer.

3.2 Sternfeld strategy

3.2.1 Characterization

The purpose of this strategy is to create a maneuver that optimizes the ΔV required. Generally, plane and apse line changes are most expensive from a ΔV perspective, but they get "cheaper" as the velocity decreases; in particular the ΔV for a (pure) plane change depends on the transversal velocity while for a (pure) apse line change it depends on the radial. For this reason, the following strategy is defined:

- Circularization of the initial orbit: $\boldsymbol{\{a_1, e_1\}} \quad \Rightarrow \quad \boldsymbol{\{r_{a1}, 0\}}$;
- Transfer to a higher orbit: $\boldsymbol{\{r_{a1}, 0, \omega_1\}} \quad \Rightarrow \quad \boldsymbol{\{a_t, e_t, \omega_t\}}$;
- Plane change: $\boldsymbol{\{a_t, e_t, i_1, \Omega_1, \omega_t\}} \quad \Rightarrow \quad \boldsymbol{\{a_{t2}, e_{t2}, i_2, \Omega_2, \omega_{t2}\}}$;
- Transfer to lower orbit: $\boldsymbol{\{a_{t2}, e_{t2}\}} \quad \Rightarrow \quad \boldsymbol{\{r_{a2}, 0\}}$;
- Transfer to final orbit: $\boldsymbol{\{r_{a2}, 0, \omega_{t2}\}} \quad \Rightarrow \quad \boldsymbol{\{a_2, e_2, \omega_2\}}$.

The first maneuver is a circularization, performed in order to have a free apse line change. By doing so the new ω can be chosen so that θ_1, the angle at which the plane change will performed, is exactly π, thus minimizing the ΔV required for the plane change[1].

After that the orbit is raised, and, once the intersection point is reached, the plane change is performed. Again, to save on fuel, the plane change is accompanied by a shape change where the transfer orbit periapsis is raised to the final orbit apoapsis.
Since the sum of the lengths of any two sides of a triangle must be greater than the length of the third side the periapsis change accompanied by the plane change is cheaper than when it follows/precedes the other.

[1]The true anomaly of the intersection θ_1 is given by the difference between u_1, which depends on $\Delta i = i_2 - i_1$ and $\Delta\Omega = \Omega_2 - \Omega_1$, and ω_1; in order to get $\theta_1 = \pi$ we need to manipulate ω_1 via an apse line change

Once in the right plane and in an orbit with the right periapsis (the final orbit apoapsis), the apoapsis and apse line must be changed. Like before, the orbit is firstly lowered until circular, there the apse line is rotated (which will result in just a true anomaly change).

Finally the shaped is matched to the final orbit's with a simple single impulse.

3.2.2 Execution

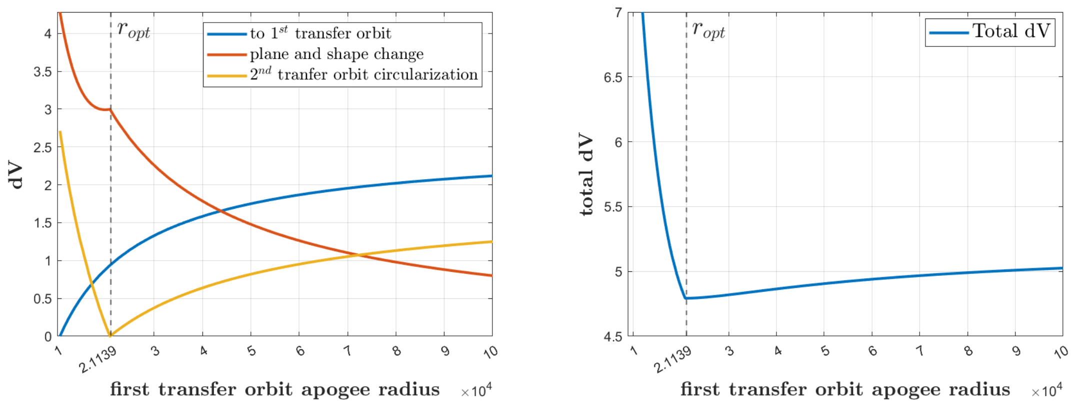
The only degree of freedom of the proposed strategy is the apogee radius of the first transfer orbit. This value is heavily dependent on the starting and target orbits given and very difficult to find in an analytic way. For this reason the ΔV_{tot} is computed as functions of this radius and in the end its minumum is found using the MATLAB function $fminbnd$. ΔV_{tot} is evaluated for different values of r_{at}: starting from r_{a1} up to 1 000 000 km.

3.2.3 Results

The function is executed and the optimal values found to be:

$$r_{at} = 21\,135.048\,\mathrm{km} \xrightarrow{\text{corresponds to}} dV_{tot} = 4.7931\,\mathrm{km\,s^{-1}}$$

As can be seen from table 5 the strategy is made of 5 different impulses. The first and last impulses are constant no matter the apogee radius of the first transfer orbit. Differently, the second, third and fourth vary, and so the total delta velocity. Below their value is displayed in function of the chosen apogee radius:



3.2.4 Representation

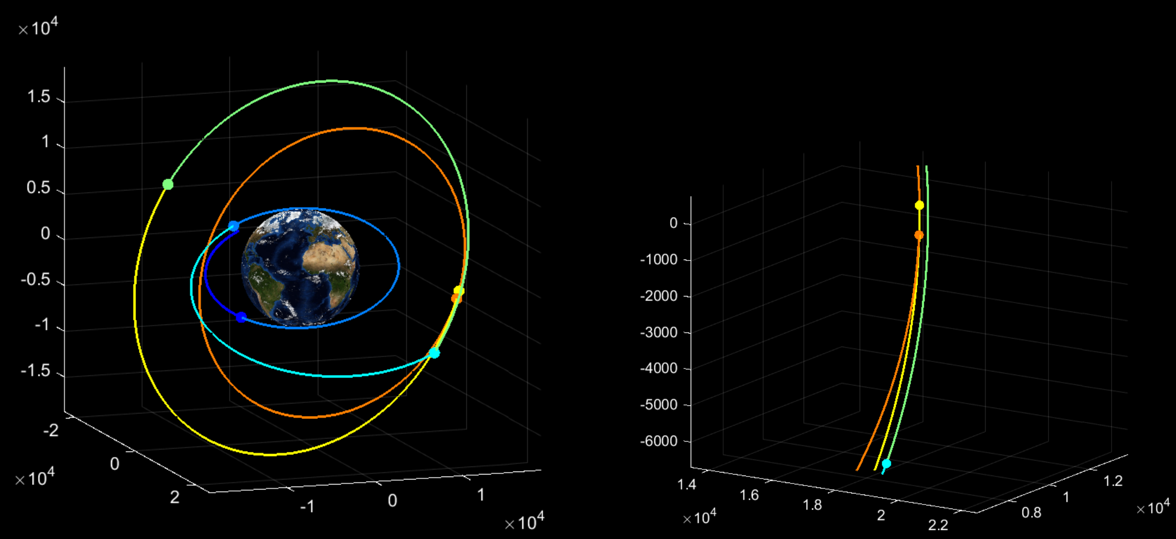


Fig. 3: Plot of Sternfeld maneuver and detail

3.3 Tree impulses strategy

3.3.1 Characterization

The following strategy is the result of a tradeoff between travel time and ΔV, since, depending on the design of the real mission, number of burns may be limited, the quantity of impulses was also kept as low as possible. The strategy is defined as follows:

- Circularization at apoapsis: $\{\boldsymbol{a_1, e_1}\} \quad \Rightarrow \quad \{\boldsymbol{r_{a1}, 0}\}$;
- Change of semi-major apsis at plane intersection: $\{\boldsymbol{r_{a1}, 0}\} \quad \Rightarrow \quad \{\boldsymbol{a_t, e_t}\}$;
- Change of orbital plane, shape and periapis anomaly: $\{\boldsymbol{a_t, e_t, i_i, \Omega_t, \omega_t}\} \quad \Rightarrow \quad \{\boldsymbol{a_f, e_f, i_f, \Omega_f, \omega_f}\}$.

The first manoeuvre is executed at apogee of the initial orbit: this allows to reduce travel time from the initial node to the 2n̂d manoeuvre node, as much as reducing the amount of ΔV required for the plane-change part of it.

The second impulse allows to change orbit shape and size with a tangent manouever and is executed at the fist available node on orbital plane intersection apsis, this way second transfer orbit and final orbit intersection node and apogee of second transfer orbit will coincide.

Third impulse allows to enter final orbit.

3.3.2 Execution

While the calculation for first and second manouvers' ΔV is trivial, the third can be obtained with a vectorial sum. It is possible to derive the total ΔV as follows:

$$\Delta V_{tot} = \|\Delta V_{plane} + \Delta V_{shape}\|$$

As pointed previously it is more efficient to execute multiple manoeuvers at the same time.

3.3.3 Representation

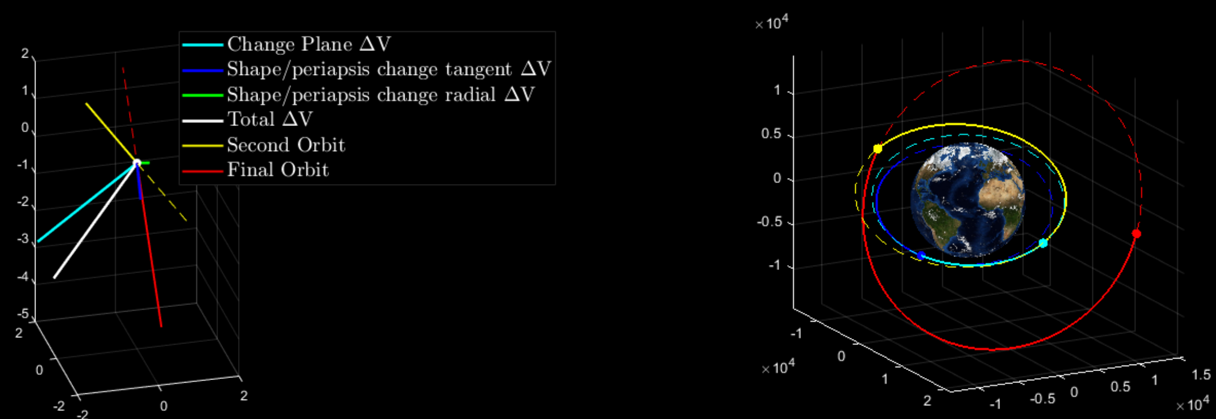


Fig. 4: ΔV graph and ΔV-optimized orbit

3.4 Direct transfer

3.4.1 Characterization

The following strategy has the objective of reducing to a minimum both travel time and ΔV.
The maneuver consists of an optimized direct transfer between two arbitrary nodes, belonging respectively to the initial and final orbit. In order to achieve this, the maneuver has been divided into 3 parts:

- Coasting from the release position to the first maneuver node;
- Direct transfer from initial to final orbit: $\{\boldsymbol{a_i, e_i, i_i, \Omega_i, \omega_i}\} \quad \Rightarrow \quad \{\boldsymbol{a_t, e_t, i_t, \Omega_t, \omega_t}\}$;

- Coasting along the final orbit to arrive point: $\{\boldsymbol{a_t, e_t, i_t, \Omega_t, \omega_t}\} \quad \Rightarrow \quad \{\boldsymbol{a_f, e_f, i_f, \Omega, f\omega_f}\}$.

Real anomalies of both impulses, θ_i and θ_f, can be set to arbitrary values. It is therefore possible to obtain the position vectors of the maneuver nodes: $\theta_i \rightarrow \vec{r}_i$ and $\theta_f \rightarrow \vec{r}_f$.

Inclination i_t and RAAN Ω_t, can be obtained from the direction of the moment of inertia:

$$\hat{h} = \frac{\vec{r}_i \wedge \vec{r}_f}{\|\vec{r}_i\|\|\vec{r}_f\|}$$

Since from 2 given points it is possible to track infinite coplanar orbits one more arbitrary parameter must be established. In our case the chosen parameter is ω_t because it allows for a finite discretization in the MATLAB program.
Given the angular momentum direction unit vector $\hat{h}$, it is possible direction of ascending node unit vector $\hat{N}$ as $\hat{N} = \hat{k} \wedge \hat{h}$. Once $\hat{N}$ has been obtained, the the relative anomaly $\alpha = \omega + \theta_1$ between $\hat{N}$ and r_1 can be derived, and finally the real anomaly of initial and final impulse on the transfer orbit θ_1 and θ_2. It is now possible to define the remaining parameters a and e as follows:

$$e_t = \frac{\|r_1\| - \|r_2\|}{\|r_1\|cos(\theta_1) - \|r_2\|cos(\theta_2)} \quad \text{and} \quad a_t = \frac{\|r_1\|(1 + ecos(\theta_1)}{1 - e^2}$$

3.4.2 Execution

The following are the imputs required to execute the strategy, please note that due to the nature of this kind of manouever the two impulses will both require a considerable amount of ΔV, that depending on the on the real mission design, may or may not be feasible in a real-case scenario.

θ_1 [deg]	θ_2 [deg]	ω [deg]
101.82	156.36	144.72

Table 1: Optimal parameters for direct transfer

3.4.3 Result

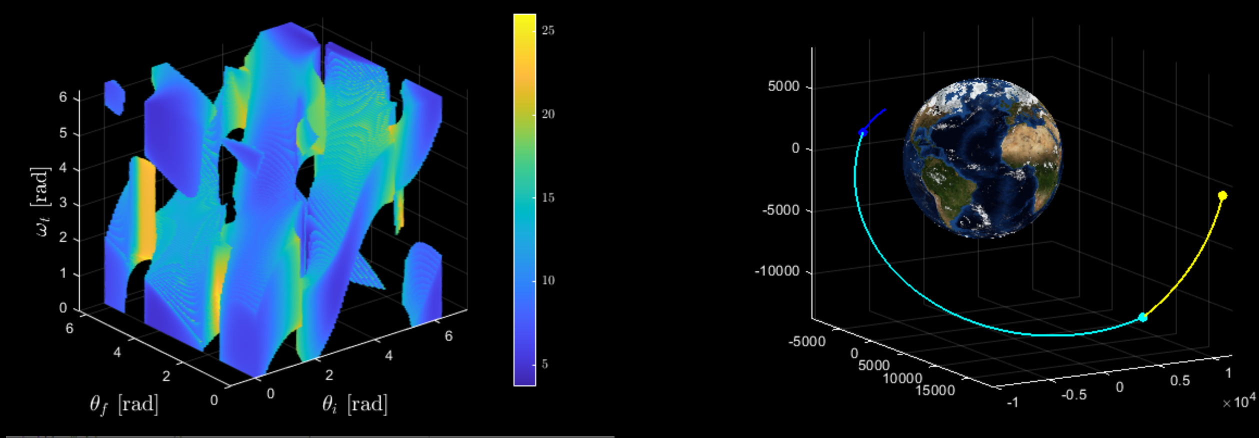


Fig. 5: ΔV graph and ΔV-optimized orbit

3.5 Direct transfer - Lambert

3.5.1 Characterization

This direct transfer is based on Lambert's problem, concerned with the determination of an orbit given two position vectors and the time of flight.

3.5.2 Execution

The suggested textbook[2] is used as a reference to write a MATLAB function that, given two position vectors and the time of flight, solves Lambert's problem. This function is then implemented into a script that finds the minimum time of flight while ensuring that the transfer orbit perigee has an altitude of at least 200 km above the earth's surface.
Once the transfer orbit is found the two necessary ΔVs are computed by means of a simple vectorial difference.

3.5.3 Results

While this strategy optimizes the time of flight it requires a considerable amount of ΔV, sometimes so high it would be almost impossible to perform with current technologies[3]. Because of that the time of flight is evaluated at different values of ΔV_{tot}. The point corresponding to ΔV_{max} of $7\,\mathrm{km\,s^{-1}}$ is highlighted.

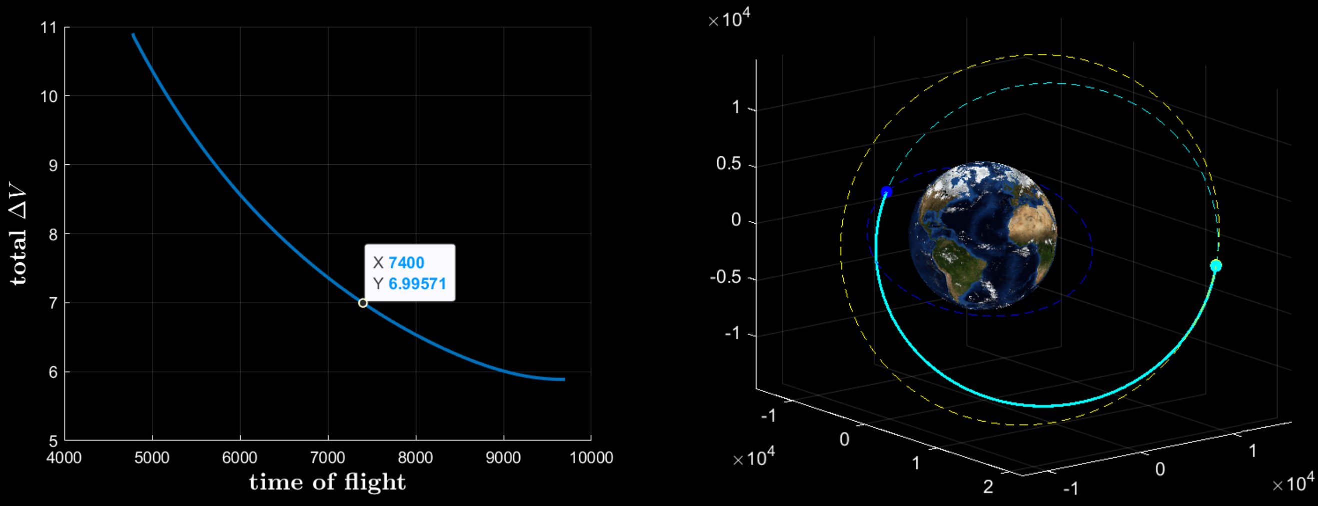


Fig. 6: Lambert strategy: time and velocity relation and orbit plot

4 Conclusion

There are nearly infinite amounts of ways to transfer from one orbit to another, but not all of them are the most efficient or the easiest to use. By varying the points of maneuver in the standard strategy we learned the importance that the shape and dimension hold when changing plane and periapsis argument. From there, we started to develop functions that, using the theory we studied, were able to minimize the cost of the maneuver. This was still not enough, so we decided to try with direct transfers, where, other than theory, we needed to cycle through all of the possible combinations in order to find the ideal one to satisfy our needs. Once we had the results we asked for, we noticed the importance of the starting and target true anomalies when trying to minimize the time and costs. After all these calculations and plots, the main thing we understood was that there is no perfect maneuver, but only the one that best suits your situation.

[2] Orbital Mechanics for Engineering Students - Howard D. Curtis Ph.D. Purdue University Professor (2019, Butterworth-Heinemann)

[3] Considering thermo-chemical engines since lower thrust ones, while capable of high ΔV changes, need a considerable amount of time, deviating even more from the hypothesis of immediate impulses needed for this dissertation

5 Appendix

5.1 Tables

	$T\,[s]$	$a\,[km]$	e	$i\,$[deg]	$\Omega\,$[deg]	$\omega\,$[deg]	$\theta\,$[deg]	$\Delta V\,[m/s]$
initial orbit	0	9723.688	0.08652	26.44	67.36	56.30	80.6	0
plane change	5271.3	9723.688	0.08652	26.44	67.36	56.30	259.93	4632.2
		9723.688	0.08652	64.11	35.78	80.04	259.93	
apse line rotation	87896	9723.688	0.08652	64.11	35.78	80.04	49.79	849.30
		9723.688	0.08652	64.11	35.78	179.62	310.21	
to first transfer orbit	9916.9	9723.688	0.08652	64.11	35.78	179.62	0	953.07
		14892.8	0.40358	64.11	35.78	179.62	0	
to final orbit	18960.6	14892.8	0.40358	64.11	35.78	179.62	180	408.90
		16720	0.25020	64.11	35.78	179.62	180	
final destination	40246.9	16720	0.25020	64.11	35.78	179.62	177.62	0
							TOT	6844.0

Table 2: Standard strategy (complete) with perigee-apogee transfer

	$T\,[s]$	$a\,[km]$	e	$i\,$[deg]	$\Omega\,$[deg]	$\omega\,$[deg]	$\theta\,$[deg]	$\Delta V\,[m^2/s]$
to first transfer orbit	14688.1	9723.688	0.08652	64.11	35.78	179.62	180	528.5
		11550.8	0.08534	64.11	35.78	359.62	0	
to final orbit	20865.4	11550.85	0.08534	64.11	35.78	359.62	180	912.0
		16720	0.25020	64.11	35.78	179.62	360	
final destination	31393.6	16720	0.25020	64.11	35.78	179.62	177.62	0
							TOT	6922.9

Table 3: Standard strategy (partial) with apogee-perigee transfer

	$T\,[s]$	$a\,[km]$	e	$i\,$[deg]	$\Omega\,$[deg]	$\omega\,$[deg]	$\theta\,$[deg]	$\Delta V\,[m/s]$
initial orbit	0	9723.688	0.08652	26.44	67.36	56.30	80.60	0
initial orbit circularization	2891	9723.688	0.08652	26.44	67.36	56.30	180	271.70
		10565.048	0	26.44	67.36	56.30	180	
to 1^{st} transfer orbit	7803	10565.048	0	26.44	67.36	56.30	79.93	950.70
		15851.866	0.33351	26.44	67.36	136.23	0	
plane and shape change	9931	15851.866	0.33351	26.44	67.36	136.23	180	2972.9
		21021.013	0.00559	64.11	35.78	159.98	180	
2^{nd} tranfer orbit	15165	21021.013	0.00559	64.11	35.78	159.98	360	012.20
		20903.344	0	64.11	35.78	179.62	340.35	
circularization to final orbit	16679	21021.013	0	64.11	35.78	179.62	180	585.50
		16720	0.25020	64.11	35.78	179.62	180	
to final orbit	21286	16720	0.25020	64.11	35.78	179.62	177.62	0
							TOT	4793.1

Table 4: Sternfeld strategy

	$\boldsymbol{T}\,[s]$	$\boldsymbol{a}\,[km]$	$\boldsymbol{e}$	$\boldsymbol{i}\,$[deg]	$\boldsymbol{\Omega}\,$[deg]	$\boldsymbol{\omega}\,$[deg]	$\boldsymbol{\theta}\,$[deg]	$\boldsymbol{\Delta V}\,[m/s]$
Initial orbit	0	9723.688	0.08652	26.44	67.36	56.30	80.60	0
Initial orbit circularization	2891	9723.688	0.08652	26.44	67.36	56.30	180	271.70
		10565.048	0	26.44	67.36	56.30	180	
To 1^{st} transfer orbit	5291	10565.048	0	26.44	67.36	56.30	259.93	273.90
		11624.74	0.0912	26.44	67.36	316.23	0	
to final orbit	11527	11624.74	0.0912	26.44	67.36	316.23	180	4336.20
		16720	0.25020	64.11	35.78	179.62	340.35	
Final orbit	22743	16720	0.25020	64.11	35.78	179.62	177.62	0
							TOT	4881.8

Table 5: Tree impulses strategy

	$\boldsymbol{T}\,[s]$	$\boldsymbol{a}\,[km]$	$\boldsymbol{e}$	$\boldsymbol{i}\,$[deg]	$\boldsymbol{\Omega}\,$[deg]	$\boldsymbol{\omega}\,$[deg]	$\boldsymbol{\theta}\,$[deg]	$\boldsymbol{\Delta V}\,[m/s]$
Initial orbit	0	9723.688	0.08652	26.44	67.36	56.30	80.60	0
Entering transfer orbit	558	9723.688	0.08652	26.44	67.36	56.30	101.82	1309.1
		15071.44	0.35583	32.51	62.89	144.73	17.29	
Entering final orbit	8600	15071.44	0.35583	32.51	62.89	144.73	172.34	2396.6
		16720	0.25020	64.11	35.78	179.62	156.36	
Final orbit	10609	16720	0.25020	64.11	35.78	179.62	177.62	0
							TOT	3705.7

Table 6: Optimized strategy results

	$\boldsymbol{T}\,[s]$	$\boldsymbol{a}\,[km]$	$\boldsymbol{e}$	$\boldsymbol{i}\,$[deg]	$\boldsymbol{\Omega}\,$[deg]	$\boldsymbol{\omega}\,$[deg]	$\boldsymbol{\theta}\,$[deg]	$\boldsymbol{\Delta V}\,[m/s]$
Initial orbit	0	9723.688	0.08652	26.44	67.36	56.30	80.60	0
Entering transfer orbit	0	9723.688	0.08652	26.44	67.36	56.30	80.60	5820.4
		15735.21	0.44786	65.67	35.69	204.55	315.93	
Entering final orbit	7400	15735.21	0.44786	65.67	35.69	204.55	152.72	1175.3
		16720	0.25020	64.11	35.78	179.62	177.62	
Final orbit	7400	16720	0.25020	64.11	35.78	179.62	177.62	0
							TOT	6995.7

Table 7: Lambert strategy