

# IceCube Neutrino Classification

## Bayesian Mixture Models for Astrophysical Source Detection

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~880k  
Neutrino Events

3  
Bayesian Models

110  
Candidate Sources

12k  
MCMC Samples

### Overview

We develop a sequence of **Bayesian mixture models** to classify ~880,000 high-energy neutrino events from the IceCube Observatory (IC86, 2011–2018) as either *astrophysical signal* or *atmospheric background*. Events are characterized by reconstructed energy (modeled as truncated power laws) and directional information (spatial compatibility weights with 110 candidate point sources). The core challenge is severe class imbalance; energy-based discrimination is ultimately limited by the flexibility of the spectral model, shifting the classification burden onto directional information.

### Data

**Source:** IceCube all-sky point-source dataset (public). Track-like events for sub-degree angular precision.

**Preprocessing:** restricted to fully deployed IC86 detector period; events with angular error > 4° removed.

#### Key observables:

- $\log_{10}(E/\text{GeV})$ : reconstructed energy, main spectral discriminant
- $w_i$ : spatial compatibility weight with  $J = 110$  candidate sources, computed as a sum of Gaussian kernels combining detector resolution  $\sigma_i$  and source extent  $\sigma_{\text{src}}$

### Models

Three nested Bayesian models, from most to least complete:

- **Conceptual Model:** full joint spatial-energetic likelihood. Astrophysical component is a mixture of  $J$  Gaussian kernels on the sky; atmospheric component is uniform. Computationally infeasible at this scale.
- **Model 1:** pre-computed spatial weight  $w_i$  enters the likelihood directly:

$$p(E_i, \mathbf{x}_i) = \pi w_i p(E_i | \text{astro}) + (1-\pi) p(E_i | \text{atmo})$$

Closest tractable approximation to the full generative framework.

- **Models 2 & 3:** energy-only likelihood; spatial information applied *a posteriori* via Bayesian odds reweighting:

$$\gamma_i^* = \frac{\mathcal{O}_{E,i} w_i}{1 + \mathcal{O}_{E,i} w_i}$$

Models 2 and 3 differ only in prior specification (see table).

	$\alpha_{\text{astro}}$	$\alpha_{\text{atmo}}$	$\pi_{\text{astro}}$
Model 1	$\mathcal{N}(2.2, 0.5^2)$	$\mathcal{N}(3.7, 0.3^2)$	Beta(1,99)
Model 2	$\mathcal{N}(2.2, 0.5^2)$	$\mathcal{N}(3.7, 0.3^2)$	Beta(2,35)
Model 3	Unif(1,4)	Unif(1,5)	Beta(2,3)

Inference via **Stan/RStan**: 4 chains × 4,500 iterations (1,500 warm-up) = 12,000 posterior samples.

### Results

**MCMC convergence:** excellent across all models. Gelman-Rubin  $\hat{R} \approx 1$ , rapid ACF decay, consistent traceplots.

**Key finding: spectral degeneracy.** Inferred power-law indices collapse to nearly identical values ( $\alpha_{\text{astro}} \approx \alpha_{\text{atmo}} \approx 1.12$ ), indicating the model is **degenerate**: the two components become empirically indistinguishable and the mixture reduces to a single effective distribution.

**Consequence for  $\pi_{\text{astro}}$ :** the global astrophysical fraction is **prior-dominated** in Models 2–3; Model 1's spatial likelihood allows partial data constraint.

Model	$\hat{\pi}_{\text{astro}}$ (posterior mean)
Model 1	0.0025
Model 2	0.0417
Model 3	0.3732

**Event-level classification:** posterior probabilities  $\gamma_i^*$  are nearly perfectly correlated across all three models, and strongly correlated with  $w_i$ . Spatial proximity to known sources, and not energy, is the primary driver of classification.

**Spatial validation:** despite the degeneracy, top 5% events by  $\gamma_i^*$  do cluster around the 110 candidate source positions, suggesting the spatial weighting scheme is geometrically coherent, even if the overall model is misspecified.

### Limitations & Lessons

- **Spectral degeneracy:** the truncated power-law parameterization is too rigid: both components collapse to the same index, making the model effectively single-component. Posterior predictive checks confirm it cannot reproduce the observed bimodal energy distribution.
- **Energy is uninformative here:** not a finding, but a modeling failure. Broken power laws or non-parametric spectral models would be needed to recover discriminative power.
- **$\pi_{\text{astro}}$  is non-identifiable** from energy alone; classification reduces to spatial reweighting under all three models.

### Technologies

**Languages & tools:** R, RStan, tidyverse, ggplot2.

**Data:** IceCube all-sky point-source dataset (IC86, public).