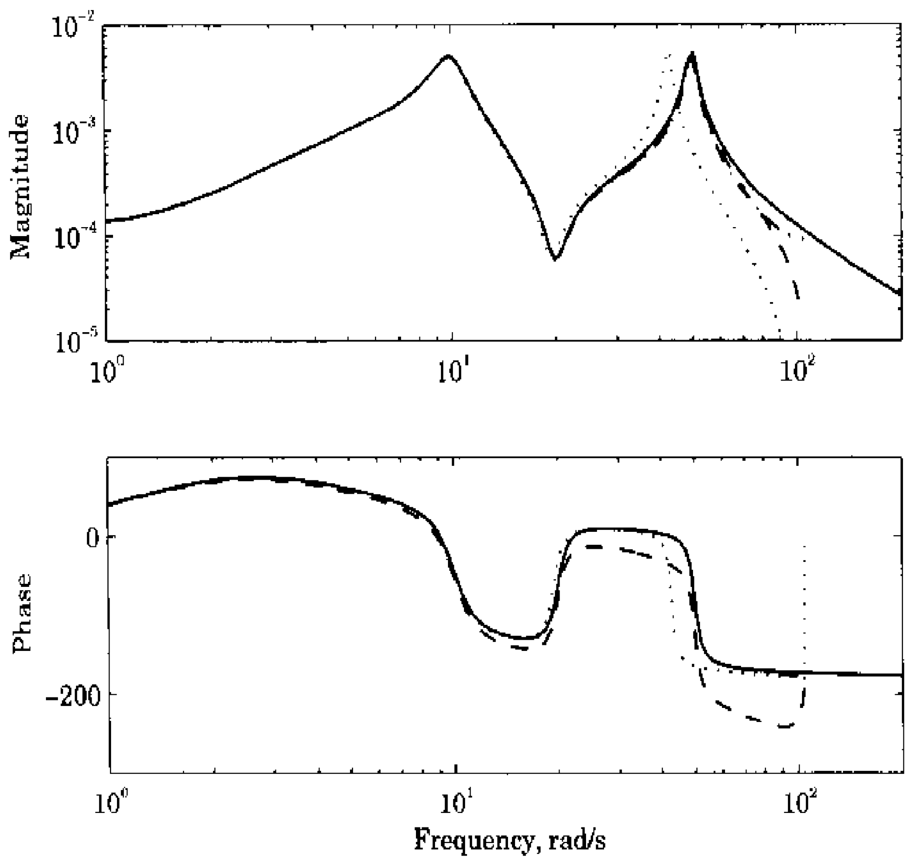

Figure 8.4 Bode diagrams of a continuous-time transfer function $G(s)$ and different sampled approximations $H(e^{sh})$, continuous-time transfer function (solid), ramp invariance (dashed-dotted), step invariance (dashed), and Tustin's approximation (dotted).

Antialiasing Filters

The consequences of aliasing and the importance of antialiasing filters were discussed in Sec. 7.4. Choice of sampling rate and antialiasing filters is important in digital systems that are based on translation of analog design. Some consequences of the selection of sampling rates have been discussed previously. The sampling rate must be so large that the errors due to the approximation are negligible.

The necessity of taking the antialiasing filter into account in the control design can be determined from the results of Sec. 7.4. In general, the antialiasing filter must be taken into consideration when making the design of the controller.

Selection of Sampling Interval

The choice of sampling period depends on many factors. One way to determine the sampling period is to use continuous-time arguments. The sampled system can be approximated by the hold circuit, followed by the continuous-time system. For small sampling periods, the transfer function of the hold circuit can be approximated as

$$\frac{1-e^{-sh}}{sh} \approx \frac{1-1+sh-(sh)^2/2+\cdots}{sh} = 1-\frac{sh}{2}+\cdots$$

The first two terms correspond to the series expansion of $\exp(-sh/2)$. That is, for small h, the hold can be approximated by a time delay of half a sampling interval. Assume that the phase margin can be decreased by 5° to 15°. This gives the following rule of thumb:

$$h\omega_c \approx 0.15 \text{ to } 0.5$$

where ω_c is the crossover frequency (in radians per second) of the continuous-time system. This rule gives quite short sampling periods. The Nyquist frequency will be about 5 to 20 times larger than the crossover frequency.

Example 8.3 Digital redesign of lead compensator

Consider the system in Example A.2, which is a normalized model of a motor. The closed-loop transfer function

$$G_c(s) = \frac{4}{s^2+2s+4}$$

is obtained with the lead compensator

$$G_k(s) = 4\,\frac{s+1}{s+2} \tag{8.9}$$

The closed-loop system has a damping of $\zeta = 0.5$ and a natural frequency of $\omega_0 = 2$ rad/s. The objective is now to find $H(z)$ in Fig. 8.5, which approximates (8.9).

Euler's method gives the approximation

$$H_E(z) = 4\,\frac{z-1+h}{z-1+2h} = 4\,\frac{z-(1-h)}{z-(1-2h)} \tag{8.10}$$

while Tustin's approximation gives

$$H_T(z) = 4\,\frac{(2+h)z-2+h}{(2+2h)z-2+2h} = 4\,\frac{2+h}{2+2h}\cdot\frac{z-(2-h)/(2+h)}{z-(1-h)/(1+h)}$$

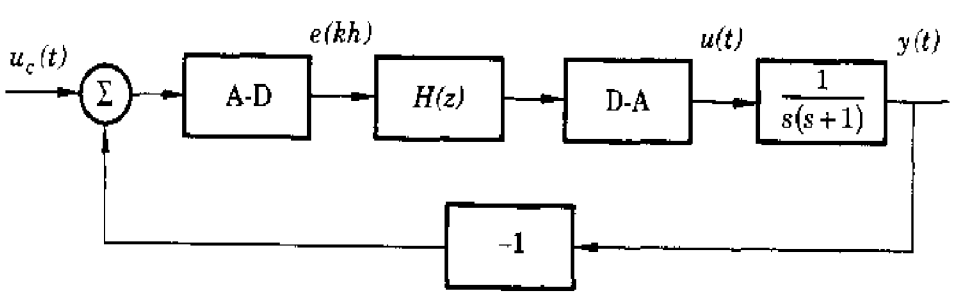

Figure 8.5 Digital control of the motor example.