Parameter Estimation

1. $$f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\mathcal{L}(x,\mu,\sigma) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

$$\ln(\mathcal{L}(x,\mu,\sigma)) = \sum_{i=1}^{n} \ln\left[\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}\right]$$

$$= \sum_{i=1}^{n}\left[\ln\frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(x_i-\mu)^2}{2\sigma^2}\right]$$

i) μ: Ahora hago la derivada parcial de $\ln(\mathcal{L})$ con respecto a μ:

$$\frac{d\ln(\mathcal{L})}{d\mu} = \sum_{i=0}^{n}\left[0 + \frac{\cancel{2}(x_i-\mu)}{\cancel{2}\sigma^2}\right] = \sum_{i=0}^{n}\frac{x_i-\mu}{\sigma^2}$$

Igualo a 0 & resuelvo

$$\sum_{i=0}^{n}\frac{x_i-\mu}{\sigma^2} = 0 \longrightarrow \frac{\sum_{i=0}^{n} x_i - \sum_{i=0}^{n}\mu}{\sum_{i=0}^{n}\sigma^2} = 0 \longrightarrow \sum_{i=0}^{n} x_i - \sum_{i=0}^{n}\mu = 0 \longrightarrow \sum_{i=0}^{n} x_i - \mu n = 0$$

$$\longrightarrow \mu = \frac{1}{n}\sum_{i=0}^{n} x_i$$

ii) σ^2: Ahora hago la derivada parcial de $\ln(\mathcal{L})$ con respecto a σ^2

$$\frac{d\ln(\mathcal{L})}{d\sigma^2} = \sum_{i=0}^{n}\left[\cancel{\sqrt{2\pi\sigma^2}} \cdot \frac{-1}{2\sigma^2\cancel{\sqrt{2\pi\sigma^2}}} + \frac{(x_i-\mu)^2}{2\sigma^4}\right] = \sum_{i=0}^{n}\left[-\frac{1}{2\sigma^2} + \frac{(x_i-\mu)^2}{2\sigma^4}\right]$$

Igualo a 0 & resuelvo.

$$\sum_{i=0}^{n}\left[-\frac{1}{2\sigma^2} + \frac{(x_i-\mu)^2}{2\sigma^4}\right] = 0 \longrightarrow -\sum_{i=0}^{n}\frac{1}{2\sigma^2} + \sum_{i=0}^{n}\frac{(x_i-\mu)^2}{2\sigma^4} = 0 \longrightarrow -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=0}^{n}(x_i-\mu)^2 = 0$$

$$\longrightarrow \frac{1}{2}\left(\frac{-n}{\sigma^2} + \frac{1}{\sigma^4}\sum_{i=0}^{n}(x_i-\mu)^2\right) = 0 \longrightarrow \frac{n}{\cancel{\sigma^2}} = \frac{1}{\sigma^{\cancel{4}\,2}}\sum_{i=0}^{n}(x_i-\mu)^2 \longrightarrow \sigma^2 = \frac{1}{n}\sum_{i=0}^{n}(x_i-\mu)^2$$

μ = media $\rightarrow$ también se representa como $\overline{x}$ entonces obtenemos...

$$\sigma^2 = \frac{1}{n}\sum_{i=0}^{n}(x_i-\overline{x})^2$$