EJERCICIOS – ÁLGEBRA LINEAL

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(1)

(a) $x_{n+1} = 4x_n - x_n^2$

$x_0 = 4\,\text{sen}^2\theta$

- $n = -1 \rightarrow x_0 = 4\,\text{sen}^2\theta$
- $n = 0 \rightarrow x_1 = 4x_0 - x_0^2$

$x_1 = 4(4\,\text{sen}^2\theta) - (4\,\text{sen}^2\theta)^2$

$x_1 = 16\,\text{sen}^2\theta - 16\,\text{sen}^4\theta$

$x_1 = 16 \cdot \text{sen}^2\theta\,(1 - \text{sen}^2\theta)$

Utilizando propiedades trigonométricas se tiene que:

$x_1 = 16\,\text{sen}^2\theta\,\cos^2\theta$

$x_1 = 4 \cdot 2\,\text{sen}\,\theta \cdot \cos\theta \cdot 2\,\text{sen}\,\theta \cdot \cos\theta$

$x_1 = 4\,\text{sen}(2\theta) \cdot \text{sen}(2\theta)$

$x_1 = 4\,\text{sen}^2(\theta)$

- $n = 1 \rightarrow x_2 = 4x_1 - x_1^2$

$x_2 = 4(4\,\text{sen}^2(2\theta)) - (4\,\text{sen}^2(2\theta))^2$

$x_2 = 16\,\text{sen}^2(2\theta) - 16\,\text{sen}^4(2\theta)$

$x_2 = 16\,\text{sen}^2(2\theta)\,(1 - \text{sen}^2(2\theta))$

Usando propiedades trigonométricas, se tiene que:

$x_2 = 16\,\text{sen}^2(2\theta)\,\cos^2(2\theta)$

$x_2 = 4 \cdot 2\,\text{sen}(2\theta)\cos(2\theta) \cdot 2\,\text{sen}(2\theta)\cos(2\theta)$

$X_2 = 4 \cdot \text{sen}(4\theta) \cdot \text{sen}(4\theta)$

$X_2 = 4 \cdot \text{sen}^2(4\theta)$

Ahora, hallando el patrón

$n=-1$

$X_0 = 4\,\text{sen}^2(\theta)$

$= 4\,\text{sen}^2(2^0\theta)$

$= 4\,\text{sen}^2(2^{-1+1}\theta)$

$n=0$

$X_1 = 4\,\text{sen}^2(2\theta)$

$= 4\,\text{sen}^2(2^1\theta)$

$= 4\,\text{sen}^2(2^{0+1}\theta)$

$n=1$

$X_2 = 4\,\text{sen}^2(4\theta)$

$= 4\,\text{sen}^2(2^2\theta)$

$= 4\,\text{sen}^2(2^{1+1}\theta)$

$$x_{n+1} = 4\,\text{sen}^2(2^{n+1}\theta); \theta \in \left[0, \tfrac{\pi}{2}\right]$$

b) $x_{n+1} = 4x_n - 4x_n^2$

$X_0 = \text{sen}^2\theta$

- $n = -1 \rightarrow X_0 = \text{sen}^2\theta$
- $n = 0 \rightarrow X_1 = 4X_0 - 4X_0^2$

$X_1 = 4\,\text{sen}^2\theta - 4(\text{sen}^2\theta)^2$

$X_1 = 4\,\text{sen}^2\theta - 4\,\text{sen}^4\theta$

$X_1 = 4\,\text{sen}^2\theta \cdot (1 - \text{sen}^2\theta)$

Usando propiedades trigonométricas, se tiene que:

$X_1 = 4\,\text{sen}^2\theta \cdot \cos^2\theta$

$X_1 = 2\,\text{sen}\,\theta \cdot \cos\theta \cdot 2\,\text{sen}\,\theta \cdot \cos\theta$

$X_1 = \text{sen}(2\theta) \cdot \text{sen}(2\theta)$

$X_1 = \text{sen}^2(2\theta)$

• $n=1 \rightarrow$

$$x_2 = 4x_1 - 4x_1^2$$

$$x_2 = 4\,sen^2(2\theta) - 4\,(sen^2(2\theta))^2$$

$$x_2 = 4\,sen^2(2\theta) - 4\,sen^4(2\theta)$$

$$x_2 = 4\,sen^2(2\theta)(1 - sen^2(2\theta))$$

· Usando propiedades trigonométricas, tenemos

$$x_2 = 4\,sen^2(2\theta)\cdot cos^2(2\theta)$$

$$x_2 = 2\,sen(2\theta)cos(2\theta)\cdot 2\,sen(2\theta)\cdot cos(2\theta)$$

$$x_2 = sen(4\theta)\cdot sen(4\theta)$$

$$x_2 = sen^2(4\theta)$$

Ahora, hallando el patrón:

$n=-1 \downarrow$

$$x_0 = sen^2(\theta) = sen^2(2^0\theta) = sen^2(2^{-1+1}\theta)$$

$n=0 \downarrow$

$$x_1 = sen^2(2\theta) = sen^2(2^1\theta) = sen^2(2^{0+1}\theta)$$

$n=1 \downarrow$

$$x_2 = sen^2(4\theta) = sen^2(2^2\theta) = sen^2(2^{1+1}\cdot\theta)$$

$$x_{n+1} = sen^2(2^{n+1}\theta)\ ;\ \theta \in [0, \pi/2]$$