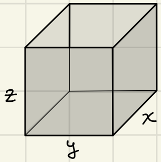
Optimización

3. a)



(Caja sin tapa)

b) Área 1 cara = wl

$$\left.\begin{array}{l}\text{Área caras frontales: } 2yz \\ \text{Área caras laterales: } 2xz \\ \text{Área cara inferior: } xy\end{array}\right\} \text{Área total} = xy + 2yz + 2xz - 12 = 0$$

e) $f: V(x,y,z) = xyz$

$d: A(x,y,z) = \{(x,y,z) \in \mathbb{R}^3 \mid xy + 2yz + 2xz = 12\}$

Lagrange:

$g: xy + 2yz + 2xz - 12 = 0$

$\nabla f = (yz, xz, xy)$; $\nabla g = (y+2z, x+2z, 2y+2x)$

$\nabla f = \lambda \nabla g$

$\therefore$ (i) $yz = \lambda(y+2z)$

(ii) $xz = \lambda(x+2z)$

(iii) $xy = \lambda(2y+2x)$

(iv) $xy + 2yz + 2xz = 12$

$$\left.\begin{array}{l} \text{(i) } \lambda = \frac{yz}{(y+2z)} \\ \text{(ii) } \lambda = \frac{xz}{(x+2z)} \\ \text{(iii) } \lambda = \frac{xy}{(2y+2x)} \end{array}\right\} \begin{array}{l} \frac{yz}{(y+2z)} = \frac{xz}{(x+2z)} \rightarrow \cancel{xyz} + 2yz^{\cancel{2}} = \cancel{xyz} + 2xz^{\cancel{2}} \rightarrow x = y \\ \frac{xz}{(x+2z)} = \frac{xy}{(2y+2x)} \rightarrow \cancel{2xyz} + 2x^{\cancel{2}}z = x^{\cancel{2}}y + \cancel{2xyz} \rightarrow 2z = y \end{array} \Bigg\} x = y = 2z$$

(iv) $(2z)(2z) + 2(2z)(z) + 2(2z)(z) = 12$

$4z^2 + 4z^2 + 4z^2 = 12$

$12z^2 = 12$

$z^2 = 1$

$z = \pm 1$

Si $z = 1$ $\therefore$ $x = 2(1) = 2$ $y = 2(1) = 2$	Si $z = -1$ $\therefore$ $x = 2(-1) = -2$ $y = 2(-1) = -2$

Puntos críticos: $(2,2,1)$; $(-2,-2,-1)$

Reviso indeterminaciones:

- $\lambda = \frac{yz}{(y+2z)} \longrightarrow$ Indeterminación en: $y + 2z = 0 \longrightarrow y = -2z$

  ⓘ $-2z^2 = \lambda(-2z+2z) \longrightarrow -2z^2 = 0$

  ⓘⓘ $xz = \lambda(x+2z)$

  ⓘⓘⓘ $-2xz = \lambda(-4z+2x)$

  ⓘⓥ $\cancel{-2xz} + 2(\overset{0}{\cancel{-2z^2}}) + \cancel{2xz} = 12$

  $0 = 12 \rightarrow$ Incongruencia: no hay punto crítico.

- $\lambda = \frac{xz}{(x+2z)} \longrightarrow$ Indeterminación en: $x + 2z = 0 \longrightarrow x = -2z$

  ⓘ $yz = \lambda(y+2z)$

  ⓘⓘ $-2z^2 = \lambda(-2z+2z) \longrightarrow -2z^2 = 0$

  ⓘⓘⓘ $-2yz = \lambda(2y-4z)$

  ⓘⓥ $-\cancel{2yz} + \cancel{2yz} + 2(\overset{0}{\cancel{-2z^2}}) = 12$

  $0 = 12 \rightarrow$ Incongruencia: no hay punto crítico.

- $\lambda = \frac{xy}{(2y+2x)} \longrightarrow$ Indeterminación en: $2y + 2x = 0 \longrightarrow y = -x$

  ⓘ $-xz = \lambda(-x+2z)$

  ⓘⓘ $xz = \lambda(x+2z)$

  ⓘⓘⓘ $-x^2 = \lambda(-2x+2x) \longrightarrow -x^2 = 0$

  ⓘⓥ $\overset{0}{\cancel{-x^2}} - \cancel{2xz} + \cancel{2xz} = 12$

  $0 = 12 \rightarrow$ Incongruencia: no hay punto crítico.

Evalúo puntos críticos en $f(x,y,z)$

$f(2,2,1) = 2 \cdot 2 \cdot 1 = 4 \; \}$ máx.

$f(-2,-2,-1) = (-2)\cdot(-2)\cdot(-1) = -4 \; \}$ mín.

Hemos verificado que el volumen máximo que puede almacenar un usuario en una caja (sin tapa) tal que su área superficial sea $12\,cm^2$ es: Volumen $= 4\,cm^3$