Distribuciones continuas de probabilidad

1. $f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otro caso} \end{cases}$

a) i) $f(x) \geq 0 \quad \forall\, x \in \mathbb{R}$

$f(x,y) \geq 0 \quad \forall x,y \in \mathbb{R} \longrightarrow$

$\frac{2}{3}(0+2(0)) = 0$ → caso mínimo

$\frac{2}{3}(1+2(1)) = 2$ → caso máximo

0 → cualquier otro caso

Siempre será mayor o igual a 0.

ii) $\int_0^1 f(x)\,dx = 1 \longrightarrow$ Se debe integrar en su dominio.

$$\frac{2}{3}\int_0^1\int_0^1 (x+2y)\,dy\,dx \to \frac{2}{3}\int_0^1 yx + y^2\Big|_0^1 dx \to \frac{2}{3}\int_0^1 (x+1)\,dx \to \frac{2}{3}\left(\frac{x^2}{2}+x\Big|_0^1\right) \to \frac{2}{3}\left(\frac{1}{2}+1\right) = 1$$

b) $g(x) = \int_0^1 \frac{2}{3}(x+2y)\,dy = \frac{2}{3}x + \frac{2}{3}$

$$\to \frac{2}{3}\left(xy + y^2\Big|_0^1\right) \to \frac{2}{3}(x+1) \to \frac{2}{3}x + \frac{2}{3}$$

$h(y) = \int_0^1 \frac{2}{3}(x+2y)\,dx = \frac{4}{3}y + \frac{1}{3}$

$$\to \frac{2}{3}\left(\frac{x^2}{2} + 2xy\Big|_0^1\right) \to \frac{2}{3}\left(\frac{1}{2}+2y\right) \to \frac{4}{3}y + \frac{1}{3}$$

c) $E(x) = \int_0^1 x \cdot g(x)\,dx = \frac{10}{18}$

$$\to \int_0^1 x \cdot \frac{2}{3}(x+1)\,dx \to \frac{2}{3}\int_0^1 (x^2+x)\,dx \to \frac{2}{3}\left(\frac{x^3}{3}+\frac{x^2}{2}\Big|_0^1\right) \to \frac{2}{3}\left(\frac{1}{3}+\frac{1}{2}\right)$$

$$\to \frac{2}{3}\left(\frac{5}{6}\right) = \frac{10}{18}$$

d) $E(y) = \int_0^1 y \cdot h(y)\,dy = \frac{11}{18}$

$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

$$\to \int_0^1 y \cdot \frac{2}{3}\left(2y+\frac{1}{2}\right)dy \to \frac{2}{3}\int_0^1 \left(2y^2+\frac{y}{2}\right)dy \to \frac{2}{3}\left(\frac{2y^3}{3}+\frac{y^2}{4}\Big|_0^1\right) \to \frac{2}{3}\left(\frac{2}{3}+\frac{1}{4}\right)$$

$$\to \frac{2}{3}\left(\frac{11}{12}\right) = \frac{11}{18}$$

e) $\sigma_{xy} = \mathbb{E}(xy) - \mathbb{E}(x) \cdot \mathbb{E}(y) = -0.00617$

$$\mathbb{E}(xy) = \int_0^1 \int_0^1 x \cdot y \cdot f(x,y)\, dy\, dx = \frac{1}{3}$$

$$\rightarrow \int_0^1 \int_0^1 x \cdot y \cdot \frac{2}{3}(x+2y)\, dy\, dx \rightarrow \frac{2}{3} \int_0^1 \int_0^1 (x^2 y + 2xy^2)\, dy\, dx \rightarrow \frac{2}{3} \int_0^1 \left(\frac{x^2 y^2}{2} + \frac{2xy^3}{3} \Big|_0^1 \right) dx$$

$$\rightarrow \frac{2}{3} \int_0^1 \left(\frac{x^2}{2} + \frac{2x}{3} \right) dx \rightarrow \frac{2}{3} \left(\frac{x^3}{6} + \frac{x^2}{3} \Big|_0^1 \right) \rightarrow \frac{2}{3} \left(\frac{1}{6} + \frac{1}{3} \right) = \frac{2}{3} \left(\frac{1}{2} \right) = \frac{1}{3}$$

$$\sigma_{xy} = \frac{1}{3} - \frac{10}{18} \cdot \frac{11}{18} = -\frac{1}{162} \approx -0.00617$$

f) $\sigma_{xy} = \mathbb{E}((x - \mu_x) \cdot (y - \mu_y)) = -0.00617$

$$\rightarrow \int_0^1 \int_0^1 (x - \mathbb{E}(x)) \cdot (y - \mathbb{E}(y)) \cdot f(x,y)\, dy\, dx \rightarrow \int_0^1 \int_0^1 \left(x - \frac{10}{18}\right)\left(y - \frac{11}{18}\right) \cdot \frac{2}{3}(x+2y)\, dy\, dx$$

$$\rightarrow \int_0^1 \left(-\frac{2}{27} x^2 + \frac{19}{243} x - \frac{5}{243} \right) dx = -\frac{1}{162} \approx -0.00617.$$

e) Para que las variables sean independientes la covarianza debe ser $= 0$.
Como la covarianza $= -0.00617$, podemos concluir que NO son independientes.