Université Paris Dauphine - PSL

Bayesian Techniques in Macroeconomics

Final Project: Estimation of a Two-Agent New Keynesian (TANK) Model

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1 Introduction

The objective of this project is to analyze macroeconomic dynamics through the lens of a TANK model. The introduction of "hand-to-mouth" consumers, who consume their full income in every period, allows for a better capture of the aggregate marginal propensity to consume. The model is solved, stationarized, and estimated to match key moments of GDP and consumption growth.

2 The Theoretical Model

The economy is populated by two types of households, a representative firm, and a government.

2.1 Households

2.1.1 Hand-to-Mouth Household (Spenders)

A fraction ω of households are "spenders" (SP) who do not save. They consume their entire disposable income in each period. Their budget constraint is:

$$C_t^{SP} = w_t L_t^{SP} - T_t^{SP} \tag{1}$$

2.1.2 Wealthy Household (Savers)

The remaining fraction $(1-\omega)$ are "savers" (SA) who maximize lifetime utility subject to a budget constraint involving capital K_t and bonds B_t. The Lagrangian for the Saver's problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t^{SA}) - \lambda_t \left[C_t^{SA} + B_{t+1} + K_{t+1} - (1-\delta)K_t - r_t B_t - r_t^K K_t - w_t L_t^{SA} + T_t^{SA} \right] \right\} \tag{2}$$

The First Order Conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial C_t^{SA}} = 0 \Leftrightarrow \frac{1}{C_t^{SA}} = \lambda_t \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \Leftrightarrow \lambda_t = \beta E_t[r_{t+1}\lambda_{t+1}] \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Leftrightarrow \lambda_t = \beta E_t[\lambda_{t+1}(r_{t+1}^K + (1-\delta))] \tag{5}$$

Combining these yields the standard Euler Equations:

$$\frac{1}{C_t^{SA}} = \beta E_t \left[r_{t+1} \frac{1}{C_{t+1}^{SA}} \right] \tag{6}$$

$$\frac{1}{C_t^{SA}} = \beta E_t \left[(r_{t+1}^K + 1 - \delta) \frac{1}{C_{t+1}^{SA}} \right] \tag{7}$$

2.2 Firms

The representative firm operates in a competitive market. It maximizes profit d_t:

$$d_t = Y_t - w_t L_t - r_t^K K_t \tag{8}$$

subject to the Cobb-Douglas production function with stochastic TFP A_t:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{9}$$

The FOCs determine factor prices:

$$\frac{\partial d_t}{\partial L_t} = 0 \Leftrightarrow w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{10}$$

$$\frac{\partial d_t}{\partial K_t} = 0 \Leftrightarrow r_t^K = \alpha \frac{Y_t}{K_t} \tag{11}$$

From these, we derive the capital-labor ratio:

$$K_t = \frac{w_t}{r_t^K} L_t \frac{\alpha}{1-\alpha} \tag{12}$$

The TFP shock follows an AR(1) process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \eta_{A,t} \tag{13}$$

where $\eta_{A,t} \sim \mathcal{N}(0, \sigma_A^2)$.

2.3 Government

The government follows a simple spending rule proportional to output and balances its budget:

$$G_t = \bar{G} \cdot Y_t \quad \text{and} \quad T_t = G_t \tag{14}$$

2.4 Aggregation and Market Clearing

The labor, goods and bonds markets clear as follows:

$$L_t = \omega L_t^{SP} + (1-\omega) L_t^{SA} \tag{15}$$

$$Y_t = C_t + I_t + G_t \tag{16}$$

$$B_t = 0 \tag{17}$$

Aggregate consumption and aggregate taxes are the weighted average of both agents:

$$C_t = \omega C_t^{SP} + (1-\omega) C_t^{SA} \tag{18}$$

$$T_t = \omega T_t^{SP} + (1-\omega) T_t^{SA} \tag{19}$$

We will assume that $T_t^{SP} = T_t^{SA} = T_t$

3 Stationary Equilibrium (Detrended Model)

The economy grows at a rate driven by technology. We define stationary variables (lowercase) by detrending with the trend Γ_t, where $\Gamma_t = \gamma_y \Gamma_{t-1}$.

3.1 Labor Detrending and Aggregation

Aggregate labor depends on the weighted sum of both agents:

$$L_t = \omega L_t^{SP} + (1-\omega) L_t^{SA} \tag{20}$$

$$L_t^{SP} = \Gamma_t l_t^{SP} \quad \text{and} \quad L_t^{SA} = \Gamma_t l_t^{SA} \tag{21}$$

Dividing the aggregate equation by the trend Γ_t gives the stationary aggregation:

$$\frac{L_t}{\Gamma_t} = \omega \frac{\Gamma_t l_t^{SP}}{\Gamma_t} + (1-\omega) \frac{\Gamma_t l_t^{SA}}{\Gamma_t} \tag{22}$$

$$l_t = \omega l_t^{SP} + (1-\omega) l_t^{SA} \tag{23}$$

$$l^{SP} = l^{SA} = \frac{1}{3} \implies l = \frac{1}{3} \tag{24}$$

3.2 Capital Accumulation Law

Starting from the non-stationary accumulation law $K_{t+1} = (1-\delta)K_t + I_t$, dividing by Γ_t yields:

$$\frac{K_{t+1}}{\Gamma_t} = (1-\delta)\frac{K_t}{\Gamma_t} + \frac{I_t}{\Gamma_t} \tag{25}$$

Using $\Gamma_{t+1} = \gamma_y \Gamma_t$, we obtain the stationary law:

$$\gamma_y k_{t+1} = (1-\delta) k_t + i_t \tag{26}$$

3.3 Households (Detrended)

For spenders, the budget constraint becomes:

$$c_t^{SP} = w_t l_t^{SP} - t_t \tag{27}$$

For savers, the Euler equation for capital is detrended using $C_t^{SA} = \Gamma_t c_t^{SA}$:

$$\frac{1}{\Gamma_t c_t^{SA}} = \beta E_t \left[\frac{r_{t+1}^K + 1 - \delta}{\gamma_y \Gamma_t c_{t+1}^{SA}} \right] \implies \frac{1}{c_t^{SA}} = \frac{\beta}{\gamma_y} E_t \left[\frac{r_{t+1}^K + 1 - \delta}{c_{t+1}^{SA}} \right] \tag{28}$$

Similarly for bonds:

$$\frac{1}{c_t^{SA}} = \frac{\beta}{\gamma_y} E_t \left[\frac{r_{t+1}}{c_{t+1}^{SA}} \right] \tag{29}$$

3.4 Firms (Detrended)

The production function $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ is stationarized by dividing by Γ_t. We assume labor supply is stationary ($L_t = \Gamma_t l_t$ in effective terms, or simply constant hours in the stationary model).

$$y_t = A_t k_t^\alpha l_t^{1-\alpha} \tag{30}$$

Factor prices w_t and r_t^K remain stationary ratios of marginal products:

$$w_t = (1-\alpha)\frac{y_t}{l_t}, \quad r_t^K = \alpha \frac{y_t}{k_t} \tag{31}$$

4 Steady State

At the steady state, variables are constant over time.

1. Parameters and Exogenous variables:

$$A = 1, \quad l^{SP} = l^{SA} = l = \frac{1}{3}, \quad B = 0 \tag{32}$$

with $\alpha = 0.33$ (Capital Share) and $\omega = 0.30$ (Share of Spenders)

2. Prices (from Euler):

$$r = \frac{\gamma_y}{\beta} \tag{33}$$

$$r^K = \frac{\gamma_y}{\beta} + \delta - 1 \tag{34}$$

3. Ratios and Levels:

Capital-output ratio from FOC:

$$\frac{k}{y} = \frac{\alpha}{r^K} \tag{35}$$

Output level from production function:

$$y = A\left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}} l \tag{36}$$

4. Consumption and Government:

$$g = 0.2y, \quad t = g \tag{37}$$

$$i = (\gamma_y - 1 + \delta)k \tag{38}$$

$$c = y - i - g \tag{39}$$

Final consumption split:

$$c^{SP} = wl^{SP} - t \tag{40}$$

$$c^{SA} = \frac{c - \omega c^{SP}}{1 - \omega} \tag{41}$$

5 Data Strategy

To estimate the model, we constructed a dataset covering the US economy from 1952:Q1 to 2023:Q4. The raw data was extracted from the Federal Reserve Economic Data (FRED) database.

5.1 Data Selection and Sources

We selected five key time series. Table 1 details the raw variables.

Variable	**FRED Code**	**Description**	**Frequency**
Real GDP	GDPC1	Real Gross Domestic Product	Quarterly
Consumption	PCECC96	Real Personal Consumption Expenditures	Quarterly
Interest Rate	TB3MS	3-Month Treasury Bill Secondary Market Rate	Quarterly
Deflator	GDPDEF	GDP Implicit Price Deflator	Quarterly
Population	CNP16OV	Civilian Noninstitutional Population	Quarterly

Table 1: Raw Data Sources

5.2 Transformations

To ensure consistency with the stationarized theoretical model, we performed the following transformations:

1. **Per Capita Adjustment:** Real GDP and Consumption were divided by the population (Pop_t).

2. **Stationarization:** We computed the log-difference of the per-capita series to obtain quarterly growth rates:

$$g_{y,t} = \Delta \ln(Y_t/Pop_t) \times 100, \quad g_{c,t} = \Delta \ln(C_t/Pop_t) \times 100$$

3. **Real Interest Rate:** We applied the Fisher equation: $r_t = (i_t - \pi_t) \times 100$.

These transformations yielded the final dataset ‘data2.csv‘.

6 SMM Estimation

The Method of Simulated Moments (SMM) minimizes the distance between empirical moments and simulated moments.

6.1 Identification Issues and The "Beta" Problem

Our initial estimation strategy attempted to estimate the discount factor β. However, since the steady state condition implies $r_{ss} = \gamma_y/\beta$, and both γ_y and r_{ss} are observable in the data, β is structurally identified by the steady state rather than the variance dynamics. Attempting to estimate it freely led to numerical instability. Indeed, this combination of the data and model dynamics gives us a β converging to 1 or even superior to 1, a result that does not make sense from an economic perspective. Consequently, we fixed $\beta = 0.998$ (consistent with the mean real interest rate) and estimated the remaining parameters.

6.2 Estimation Results

We targeted five moments: the means of g_y, g_c, r and the variances of g_y, g_c.

Parameter	Description	Initial Guess	Estimated Value
β	Discount Factor	0.998	*Fixed*
γ_y	Growth Factor	1.0036	**1.0043**
ω	Share of Hand-to-Mouth	0.3000	**0.4084**
σ_A	TFP Shock Volatility	0.0100	**0.0100**
ρ_A	TFP Persistence	0.9000	**0.9000**

Table 2: SMM Estimation Results (Fixed Beta Specification)

The estimation reveals a significant share of constrained agents ($\omega \approx 41\%$).

7 Bayesian Estimation

Building on the structural parameters identified via SMM, we performed a full Bayesian estimation. We introduced a government spending shock to compete with the TFP shock.

7.1 Model Extension and Methodology

To perform the Bayesian estimation, we extend the model by introducing a stochastic shock to government spending. The public spending rule is now defined as:

$$G_t = 0.2 Y_t \epsilon_t^G \tag{42}$$

where 0.2 represents the fixed share of output ($\bar{G}$). The government spending shock ϵ_t^G follows an AR(1) process in logs:

$$\ln(\epsilon_t^G) = \rho_G \ln(\epsilon_{t-1}^G) + \eta_t^G, \quad \eta_t^G \sim \mathcal{N}(0, \sigma_G^2) \tag{43}$$

The structural parameters $(\beta, \gamma_y, \omega)$ are calibrated to the values obtained from the previously conducted SMM estimation. We then use the observed data for GDP growth (g_y) and consumption growth (g_c) to estimate the persistence (ρ_A, ρ_G) and the volatility (σ_A, σ_G) of the TFP and government spending shocks.

7.2 Shock Decomposition

We analyze the historical contribution of shocks to the observed variables.

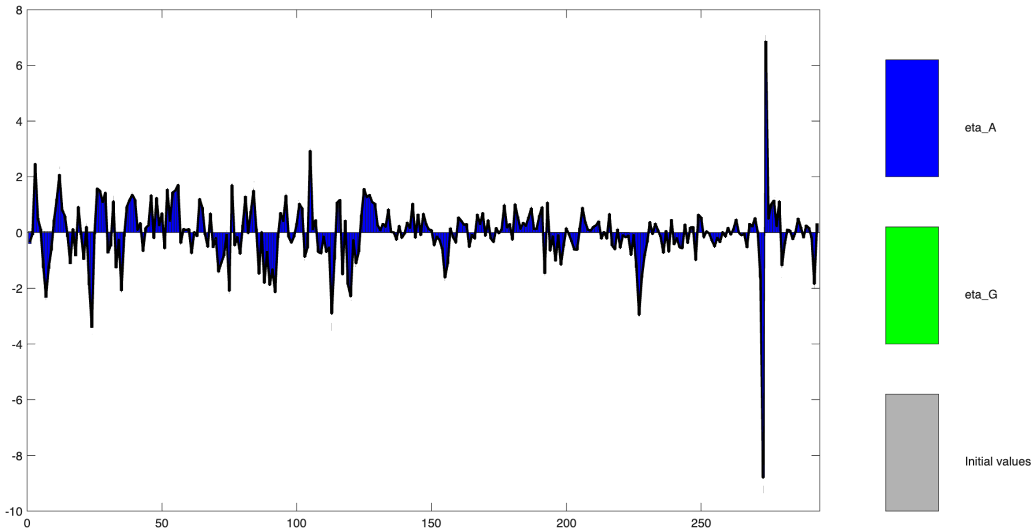

Figure 1: Shock decomposition for GDP growth (g_y)

Figure 1 displays the decomposition for GDP growth. *Interpretation:* Output fluctuations are predominantly driven by TFP shocks (Supply-side). While government spending shocks affect the composition of demand (crowding out investment or boosting consumption for spenders), their net impact on total GDP volatility is secondary compared to technology shocks.

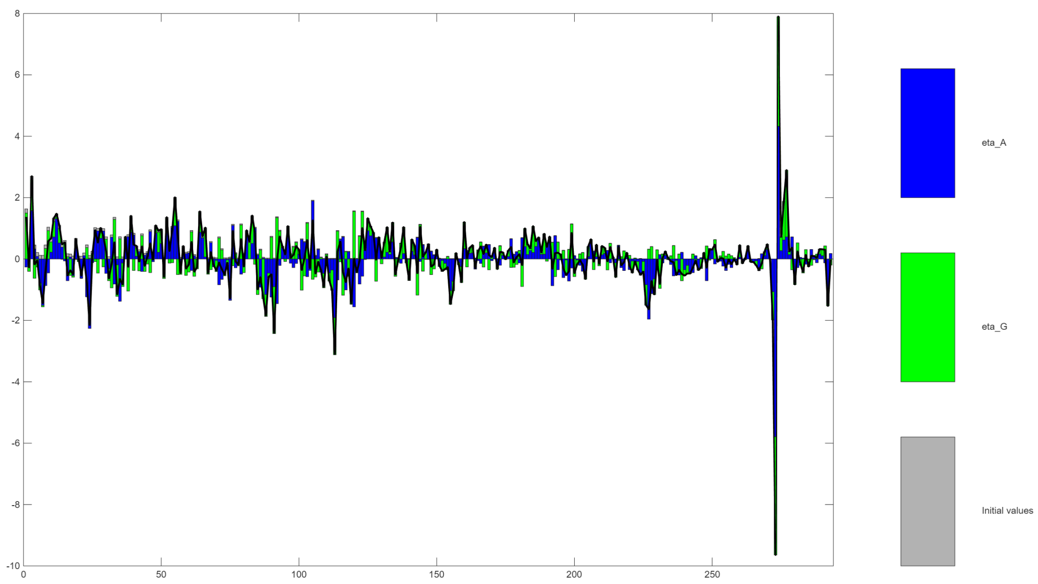

Figure 2: Shock decomposition for consumption growth (g_c)

For consumption (Figure 2), the picture is more mixed. Government spending shocks play a larger role due to their direct impact on the income of hand-to-mouth agents.

7.3 Estimation Results: Shock Parameters

The posterior means confirm that both shocks are highly persistent, with ρ_A and ρ_G exceeding 0.99. This suggests that perturbations to the economy have very long-lasting effects, requiring substantial time for variables to return to their steady state.

Parameter	Prior Mean	Post. Mean	90% HPD Interval	
ρ_A (TFP persistence)	0.989	0.9984	0.9977	0.9990
ρ_G (Govt. persistence)	0.700	0.9947	0.9900	0.9989
σ_A (TFP volatility)	0.010	0.0108	0.0101	0.0117
σ_G (Govt. volatility)	0.010	0.0167	0.0149	0.0189

Table 3: Prior and Posterior Estimates for AR(1) Shock Parameters.

8 Counterfactual Analysis

A key advantage of the TANK model is the ability to analyze the impact of financial inclusion. Using the script 'Test.m', we simulated the economy under different scenarios for ω (the share of hand-to-mouth consumers), ranging from 1% to 90%.

Share of H-t-M (ω)	**Volatility of GDP** ($Var(g_y)$)	**Volatility of Cons.** ($Var(g_c)$)
0.01[a]	0.92	0.18
0.25	0.92	0.25
0.41[b]	0.92	0.32
0.75	0.92	0.54
0.90	0.92	**0.70**

Table 4: Counterfactual Analysis: The impact of ω on Volatility

[a] Representative Agent Limit. [b] Estimated Baseline.

The results in Table 4 demonstrate a clear transmission mechanism:

1. **Output Stability:** The volatility of GDP remains constant (0.92) regardless of ω. This confirms that in this RBC-based framework, production is primarily driven by supply-side technology shocks (TFP) which are independent of the demand-side composition.

2. **Consumption Amplification:** Conversely, consumption volatility increases drastically with ω. When fewer households can smooth consumption through savings, aggregate consumption becomes highly sensitive to current income fluctuations.

9 Conclusion

This project successfully estimated a TANK model for the US economy. The data strategy ensured a consistent mapping between the stationarized model and per-capita observables. The SMM estimation validated a high proportion of non-Ricardian agents ($\omega \approx 41\%$), after resolving identification issues related to the discount factor. Finally, counterfactual analysis highlighted that financial frictions act as a powerful amplifier of consumption volatility, even when output dynamics remain unchanged.