GEOPHYSICS, VOL. 71, NO. 2 (MARCH-APRIL 2006); P. L13–L16, 1 FIG.
10.1190/1.2184367

Understanding 3D analytic signal amplitude

Xiong Li[1]

INTRODUCTION

The concept of the analytic signal goes back at least to Ville (1948). The analytic signal $\mathbf{a}(x)$ of function $f(x)$ is a complex quantity defined as

$$\mathbf{a}(x) = f(x) - i\mathrm{H}[f(x)], \tag{1}$$

where $\mathrm{H}[f(x)]$ represents the Hilbert transform of $f(x)$. Nabighian (1972, 1974) applies the analytic signal concept to potential-field data in two dimensions. For a potential field $\phi(x)$ measured along the x-axis at a constant observation height z and generated by a 2D source aligned parallel to the y-axis, the horizontal derivative ϕ_x and the vertical derivative ϕ_z are a Hilbert transform pair. We could thus write the analytic signal of potential-field data in two dimensions as

$$\mathbf{A}(x) = \phi_x + i\phi_z, \tag{2}$$

where the 2D analytic signal amplitude (ASA) of potential-field data is

$$|\mathbf{A}(x)| = \sqrt{\phi_x^2 + \phi_z^2}. \tag{3}$$

One can show that, under this 2D assumption, the ASA is independent of magnetization direction.

Nabighian (1984) generalizes the Hilbert transform and the analytic signal from two to three dimensions but does not give an expression for its amplitude which, as in the 2D case, would be independent of magnetization direction. Craig (1996), in a highly mathematical paper, evaluates the analytic signal for multivariate data. Suitably specialized, the bivariate case reduces to Nabighian's 3D expression. Roest et al. (1992) write the analytic signal in three dimensions as a vector encompassing the horizontal derivatives and their Hilbert transform, and the 3D ASA of the potential field $\phi(x, y)$ measured on a horizontal plane as

$$|\mathbf{A}(x, y)| = \sqrt{\phi_x^2 + \phi_y^2 + \phi_z^2}. \tag{4}$$

W. R. Roest and his collaborators use the 3D ASA to estimate magnetic source depth (Roest et al., 1992) and to identify remanent magnetization (Roest and Pilkington, 1993). However, their applications and many conclusions are based on a 2D vertical-magnetic-contact model assumption.

Unfortunately, many researchers and practitioners have ignored this critical 2D assumption when they interpret, extend characteristics of, and use the 3D ASA. In this note, I correct misunderstandings and demonstrate by citing references and using examples, that in three dimensions and in general, the ASA is dependent on everything that the total magnetic intensity (TMI) itself may depend: the direction of the inducing field, the direction of the remanent magnetization, the dipping angle of the source body, and the depths to the top and bottom of the source body. Nevertheless, the 3D ASA may complement the reduction-to-the-pole (RTP) and the horizontal-gradient technique for edge detection, particularly when sources of interest are shallow or very regional, the magnetic latitude is low, and the remanent magnetization is significant, yet its parameters are unknown.

SOME MISUNDERSTANDINGS

The misunderstanding and abuse of the 3D ASA may be among the most devastating practices in potential-field geophysics over the past decade. An early misconception, e.g., pointed out by Li and Götze (2001) was that the height (incorrectly called free air) correction was to relocate the gravity anomaly from the observation position to sea level.

A first obvious mistake is that most works purporting to treat analytic signals really mean the ASA. As defined above, the analytic signal is a complex or vector quantity (e.g., equation 2 for 2D cases), but the ASA (equation 3 for 2D and equation 4 for 3D cases) is a nonnegative real. In actuality, the ASA is nothing more than the total gradient as used in potential-field geophysics under that name for many decades.

The wrong assertion about the 3D ASA can be seen in the original paper that defines equation 4, the 3D ASA:

"The absolute value of the analytic signal in three dimensions is also the envelope over all possible directions of the magnetization and inducing field. Graphically, this cannot be shown as easily as in the 2D case. In addition, demonstration would involve a large number of calculations, because one would have to vary four parameters, i.e., the inclinations

Manuscript received by the Editor November 26, 2003; revised manuscript received August 5, 2005; published online February 23, 2006.
[1]Fugro Robertson Inc., 6100 Hillcroft, Fifth Floor, Houston, Texas 77081. E-mail: xli@fugro.com.

and declinations of the earth's magnetic field and of the source body magnetization" (Roest et al., 1992, 119).

Here, I reproduce a few more examples of false or misleading statements about the 3D ASA:

- Qin (1994) defines the ASA in the wavenumber domain and proves mathematically that "the shape of the AS [analytic signal] over 3D bodies is also independent of the inclination" (666) and "the inclination of magnetization only affects the amplitude and not the overall characteristic shape of AS" (673).
- "It is clear... that the magnitude of the analytic signal is independent of the direction of magnetization for the two-dimensional case, and this can similarly be shown to be true for the three-dimensional case" (Blakely, 1995, 355).
- "While this function [the ASA] is not a measurable parameter, it is extremely interesting in the context of interpretation, as it is completely independent of the direction of magnetization and the direction of the Earth's field" (Milligan and Gunn, 1997, 65).
- "It is well known (MacLeod et al., 1993) that the analytic signal [equation 4 in this note] is insensitive to the direction of magnetization. Therefore calculating the analytic signal for TMI data which contains both induced and remanent components will transform both components into a single coherent response" (Paine et al., 2001, 239).

After the explanation and demonstration of many researchers, it is hardly surprising that, in practice, users widely accept — without theoretical proof and numerical verification — and hence exercise the following fallacy in general 3D cases: The shape and magnitude of the 3D ASA are independent of magnetic inclination and declination.

CORRECT INTERPRETATION

The quotations above are valid only for the vertical contact model. However, for this model the 3D ASA degenerates to the 2D ASA. Several workers have challenged the works containing incorrect or inappropriate understanding and interpretation of the ASA in three dimensions.

Agarwal and Shaw (1996) and Huang et al. (1997) point out that Qin's (1994) definition of the 3D ASA in the wavenumber domain is incorrect. Qin (1997) agrees, noting that the "theoretical derivations of the mathematics in the paper cannot therefore prove that the analytic signal over a 3D causative body is independent of magnetization [direction]" (883).

Agarwal and Shaw (1996) and Salem et al. (2002) prove mathematically that the shape of the ASA over dipolar sources is dependent on magnetization direction. Haney et al. (2003) prove that, in general 3D cases, the ASA is not the envelope over all directions, as claimed in Roest et al. (1992), and there is more than one envelope in three dimensions. As a result, the 3D ASA is not independent of the inclinations and declinations of the body's magnetization and the earth's magnetic field.

One application of the 3D ASA is estimating magnetic source depth. It works accurately only for a vertical contact model. Using a prism model, Zhang (2001, his Figure 1) calculates the relative errors of estimates in horizontal position and depth as functions of the ratio of the half-width to the top depth of the prism. The estimate (particularly of the horizontal position) is very inaccurate when the ratio is small.

EDGE DETECTION

The horizontal gradient amplitude (HGA) $\sqrt{\phi_x^2 + \phi_y^2}$ of the gravity anomaly tends to have maxima located over the edges of gravity sources, a fact widely used to detect edges or boundaries. As explained by Cordell and Grauch (1985), when the HGA is applied to locate boundaries from magnetic anomalies, we first must transform the latter to pseudogravity anomalies. The pseudogravity transformation involves a magnetic RTP process, but the RTP suffers from two difficulties. First, the RTP at low magnetic latitudes is notoriously unstable. Second, we often lack information about the remanent magnetization, although it is needed in the RTP calculation. The prospect of an alternative, direct route to edge detection might have been the driving force for different groups that suggested using the ASA of the total magnetic intensity (MacLeod et al., 1993; Milligan and Gunn, 1997; Rajagopalan, 2003). The ASA can be used as an edge-detection tool, but only with great caution. Grauch and Cordell (1987) explain that the HGA maxima of gravity or pseudogravity anomalies are not directly over the top edges of structures when the boundaries are not near vertical, several boundaries are close together, and the observation surface is undulating. The ASA has only more and stronger limitations.

Huang and Guan (1998) examine the ASA using model tests. Their findings are as follows:

1) Even for a 2D thick dike with infinite strike and bottom depth, "the locations of the analytic signal's maxima are not directly over both edges of the thick dike.... Only for a very shallow depth of the top surface ($h \to 0$), the locations of the maxima are essentially over both edges of the thick dike" (668).
2) For a 2D finite dipping step model, the locations of ASA maxima vary with both burial depth and dipping angle (their Figure 2).
3) The shape and magnitude of the ASA are dependent on the actual observation surface (their Figure 5).

This last point is important. The analytic signal is defined for observations made on a horizontal level for two dimensions (Nabighian, 1972, 1974) and a horizontal plane for three dimensions (Roest et al., 1992). Many practitioners do not realize this important assumption but calculate the ASA from magnetic anomalies observed on an undulating surface or a draped airborne survey and then interpret the ASA result.

Huang et al. (1997) and Huang and Guan (1998) also test the 3D ASA on upright prism models. Here I present similar test results. Figure 1 shows the total magnetic intensity and ASA anomalies over a cube for different inclinations, declinations, and ratios of the top depth to the width of the cube. Figures 1b and 1d indicate that the shape and magnitude of the ASA are dependent on the inclination of the earth's magnetic field. Figures 1b and 1f show the contrast for two different declinations of the earth's magnetic field. Figures 1a and 1h demonstrate the effects of remanent magnetization.

In Figures 1a–h, the cube is buried at a very shallow depth; 0.1 is the value for the ratio of the top depth to the width.

In such cases, one can argue that the ASA maxima clearly outline the top edges well. I then increase the burial depth of the cube; the results for 0.2 and 0.5 as the depth-to-width ratios are depicted in Figures 1i–k. Changes in magnitude and shape of the ASA become more evident when this ratio increases. As a conclusion, the ASA maxima are (1) over the edges only when the top surface is very shallow (the ratio of the top depth to the width of the prism is less than 0.1), (2) shift from edges when the top depth increases, and (3) eventually become a single point near the center of the prism when the ratio is greater than 1.0.

Figure 1l shows the HGA of gravity responses for a depth-to-width ratio of 0.5. Comparing Figures 1k and 1l, the HGA of the gravity or pseudogravity is clearly a better edge detector than the ASA of total magnetic intensity. This is not surprising because the HGA is proportional to $1/R^3$ but the ASA is proportional to $1/R^4$, where R is the distance between source and observation. Another advantage of HGA is that it involves only the two horizontal derivatives, whereas ASA requires the vertical derivative as well. In general, horizontal derivatives can be calculated accurately in the space domain, but the vertical-derivative calculation has a much lower accuracy. Hsu et al. (1996) calculate the ASA of the first- or higher-order vertical derivative of the magnetic anomaly and name their technique the enhanced analytic signal. In theory, it reduces the effect of the interference between closely spaced edges and

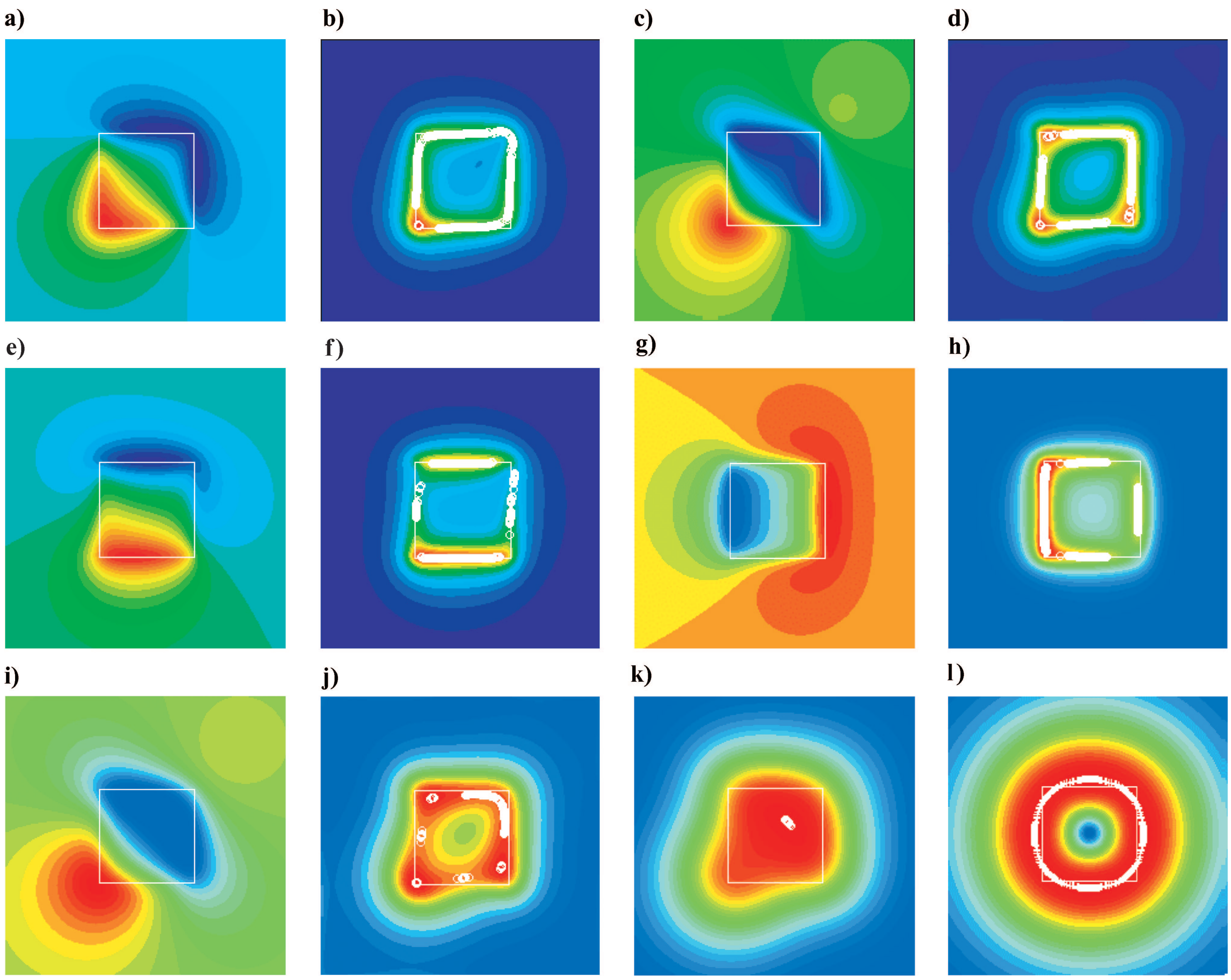

Figure 1. The TMI (a, c, e, g, i) and ASA (b, d, f, h, j, k) responses on a horizontal plane, resulting from a cube for different inclinations I and declinations D of the inducing field, different inclinations i and declinations d of the remanent magnetization, and different ratios r of the depth to the top surface to the width of the cube. (a, b) $I = D = 45°$, $r = 0.1$; (c, d) $I = 15°$, $D = 45°$, $r = 0.1$; (e, f) $I = 45°$, $D = 15°$, $r = 0.1$; (g, h) $I = D = 45°$, $i = d = -45°$, $r = 0.1$; (i, j) $I = 15°$, $D = 45°$, $r = 0.2$; (k) $I = 15°$, $D = 45°$, $r = 0.5$. Except for views (g) and (h), there is no (skew) remanent magnetization. (l) The HGA of the gravity responses for $r = 0.5$. In all views, red is high, blue is low, and color is in linear stretch for peak-to-peak values. The thin white square shows the cube outline in planar view. White circles are boundaries determined from the maxima of the 3D ASA, except that the white crosses in (l) are determined from the HGA maxima of the gravity anomaly, using the method of Blakely and Simpson (1986) for counter $N \geq 2$.

anomalies and enhances shallow sources. In practice, however, calculating second- and higher-order derivatives accurately is always difficult because of noise in observed data. I recommend HGA for edge detection when a pseudogravity transformation of the observed total magnetic intensity can be conducted with confidence.

Nevertheless, Figure 1 does demonstrate that, in the absence of the knowledge of the ambient and particularly remanent field parameters, the ASA does a decent job. It is easier to interpret ASA than total magnetic intensity.

The 3D ASA definition, i.e., equation 4, is popularly accepted and used. Other possible definitions of the 3D ASA exist. For example, Mohan and Anand Babu's (1995) propose a different definition. Similarly, it can be shown that, in general, their 3D ASA definition does not have maxima directly over edges and is not independent of magnetization direction.

CONCLUSIONS

In three dimensions and in general, the ASA is independent of nothing but depends on everything that the gravity or magnetic field itself may depend: the burial depth, extent, dipping angle of a source body, body's magnetization direction, and earth's magnetic-field direction.

The 3D ASA has been applied to two types of quantitative interpretation: depth estimation and remanent magnetization. The ASA can be used to estimate the depth of a vertical contact. In such cases, a 2D ASA analysis is sufficient. Other techniques, such as Euler deconvolution, can work on anomalies observed on an undulating surface and on more sorts of idealized sources. I do not recommend the ASA as a 3D depth interpretation tool. In remanent magnetization identification, correlating the 3D ASA of the magnetic RTP result and the HGA of pseudogravity works well for a vertical-contact case. In general cases, ASA and HGA have different decay rates. Depending on geometry and source depth, the correlation between ASA and HGA is not necessarily the strongest when the magnetization inclination and declination are chosen correctly. Identification through this correlation analysis thus is no longer viable.

In terms of a qualitative interpretation, the 3D ASA can be advantageous over total magnetic intensity. In the absence of accurate remanent magnetization information and/or at low latitudes, the 3D ASA presents a possibility of enhanced interpretation. However, users must be aware of assumptions, and interpretation of magnetic anomalies and 3D ASA should not be done independently of geologic information and other constraints.

ACKNOWLEDGMENTS

The author thanks Fugro Robertson Inc. for permission to publish this work. Corrections and comments by Maurice Craig, Misac Nabighian, and Alan Reid helped improve the note. He is also grateful to associate editor João B. C. Silva, assistant editor José Carcione, editor Gerard T. Schuster, and four anonymous reviewers for their constructive remarks.

REFERENCES

Agarwal, B. N. P., and R. K. Shaw, 1996, Comment on "An analytic signal approach to the interpretation of total field magnetic anomalies" by Shuang Qin: Geophysical Prospecting, **44**, 911–914.

Blakely, R. J., 1995, Potential theory in gravity and magnetic applications: Cambridge University Press.

Blakely, R. J., and R. W. Simpson, 1986, Approximating edges of source bodies from magnetic or gravity anomalies: Geophysics, **51**, 1494–1498.

Cordell, L., and V. J. S. Grauch, 1985, Mapping basement magnetization zones from aeromagnetic data in the San Juan basin, New Mexico, *in* W. J. Hinze, ed., The utility of regional gravity and magnetic anomaly maps: SEG, 181–197.

Craig, M., 1996, Analytic signals for multivariate data: Mathematical Geology, **28**, 315–330.

Grauch, V. J. S., and L. Cordell, 1987, Limitations of determining density or magnetic boundaries from the horizontal gradient of gravity or pseudogravity data: Geophysics, **52**, 118–121.

Haney, M., C. Johnston, Y. Li, and M. Nabighian, 2003, Envelopes of 2D and 3D magnetic data and their relationship to the analytic signal: Preliminary results: 73rd Annual International Meeting, SEG, Expanded Abstracts, 592–595.

Hsu, S.-K., J.-C. Sibuet, and C.-T. Shyu, 1996, High-resolution detection of geological boundaries from potential-field anomalies: An enhanced analytic signal technique: Geophysics, **61**, 373–386.

Huang, L., Z. Guan, and C. Yao, 1997, Comment on "An analytic signal approach to the interpretation of total field magnetic anomalies" by Shuang Qin: Geophysical Prospecting, **45**, 879–881.

Huang, L., and Z. Guan, 1998, Discussion on "Magnetic interpretation using the 3-D analytic signal" by Walter R. Roest, Jacob Verhoef, and Mark Pilkington: Geophysics, **63,** 667–670.

Li, X., and H.-J. Götze, 2001, Tutorial: Ellipsoid, geoid, gravity, geodesy, and geophysics: Geophysics, **66**, 1660–1668 (Errata in **67**, 997).

MacLeod, I. N., K. Jones, and T. F. Dai, 1993, 3-D analytic signal in the interpretation of total magnetic field data at low magnetic latitudes: Exploration Geophysics, **24**, 679–687.

Milligan, P. R., and P. J. Gunn, 1997, Enhancement and presentation of airborne geophysical data: AGSD Journal of Australian Geology & Geophysics, **17**, no. 2, 63–75.

Mohan, N. L., and L. Anand Babu, 1995, An analysis of 3-D analytic signal: Geophysics, **60**, 531–536.

Nabighian, M. N., 1972, The analytic signal of two-dimensional magnetic bodies with polygonal cross-section — Its properties and use for automated anomaly interpretation: Geophysics, **37**, 507–517.

———, 1974, Additional comments on the analytic signal of two-dimensional magnetic bodies with polygonal cross-section: Geophysics, **39**, 85–92.

———, 1984, Toward a three-dimensional automatic interpretation of potential field data via generalized Hilbert transforms — Fundamental relations: Geophysics, **49,** 780–786.

Paine, J., M. Haederle, and M. Flis, 2001, Using transformed TMI data to invert for remanently magnetized bodies: Exploration Geophysics, **32**, 238–242.

Qin, S., 1994, An analytic signal approach to the interpretation of total field magnetic anomalies: Geophysical Prospecting, **42**, 665–675.

———, 1997, Reply to comment on "An analytic signal approach to the interpretation of total field magnetic anomalies" by Huang, L., Z. Guan, and C. Yao: Geophysical Prospecting, **45**, 883.

Rajagopalan, S., 2003, Analytic signal vs. reduction to pole: Solutions for low magnetic latitudes: Exploration Geophysics, **34**, 257–262.

Roest, W. R., and M. Pilkington, 1993, Identifying remanent magnetization effects in magnetic data: Geophysics, **58**, 653–659.

Roest, W. R., J. Verhoef, and M. Pilkington, 1992, Magnetic interpretation using the 3-D analytic signal: Geophysics, **57**, 116–125.

Salem, A., D. Ravat, T. J. Gamey, and K. Ushijima, 2002, Analytic signal approach and its applicability in environmental magnetic applications: Journal of Applied Geophysics, **49**, 231–244.

Ville, J., 1948, Théorie et applications de la notion de signal analytique: Cables et Transmissions, **2A**, 61–74.

Zhang, J., 2001, An analysis of the accuracy of magnetic source-body geometry determined from the 3D analytic signal: Geophysics, **66**, 579–581.