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ON THE FORMULATION OF POWER DISTRIBUTION FACTORS FOR LINEAR LOAD FLOW METHODS

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ABSTRACT

Fast linear load flows used in contingency analysis, interchange studies and many optimization schemes often utilize power distribution factors. The success of many of these methods has hinged greatly on the use of the Z matrix reference to swing or an arbitrary ground tie. This paper furnishes a mathematical basis for these heuristic methods.

INTRODUCTION

Under constant power load assumptions, the analysis of power systems is nonlinear. Aproximate linear models are often employed to reduce computation effort or facilitate optimization. The use of distribution factors was probably first presented by Mac Arthur [1], followed by the mathematical settings of El-Abiad, Stagg [2] and Limmer [3,4]. The distribution factors essentially assume linear load and generation representations so that the substitution methods of linear circuits may be applied. Under the assumption that the redistribution of line power flow followed that of current redistribution the method was applied extensively [5-12]. The essence of each method was to assume that an incremental change in base case injected power at bus k (ΔS_k) resulted in a corresponding incremental change in each base case line power flow (ΔS_{ij}) as,

$$\Delta S_{ij} = \sum_{k=1}^{n} T^*_{ij,k} \Delta S_k \ , \tag{1}$$

where T is a rectangular matrix of current distribution factors and * denotes conjugation. The methods used to formulate T determine the accuracy of the linear load flow methods. A mathematical derivation of these distribution factors in a load flow setting is presented below and compared to traditional methods.

CURRENT DISTRIBUTION FACTORS

Consider an n bus plus ground power system with one or more impedance ties to ground (reactive shunts, line charging etc.). For a given schedule of constant power bus loads and swing bus #1, a base case A solution satisfies

$$\begin{bmatrix} V_1^A \\ V_2^A \\ \vdots \\ V_n^A \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1^A \\ I_2^A \\ \vdots \\ I_n^A \end{bmatrix} \tag{2}$$

and

$$S_i^A = V_i^A I_i^{A*} \quad i = 1,2,\ldots,n \ , \tag{3}$$

where Z_{ij} is the ij^{th} entry of the bus impedance matrix referenced to ground, V_i^A is the i^{th} bus voltage referenced to ground in the base case, and I_i^A is the i^{th} bus total injection current entering bus i through a branch not included in Z. As a minimum these include the swing or slack bus (constant voltage) and load bus (constant power) currents. The solution (2) is the result of an iterative scheme. Additional base case quantities such as line currents or line power flows can be computed from the case A voltage vector as,

$$I_{ij}^A = \frac{V_i^A - V_j^A}{\bar{z}_{ij}} \qquad S_{ij}^A = V_j^A I_{ij}^{A*} \qquad S_{ji}^A = -V_i^A I_{ij}^{A*} \ , \tag{4}$$

where $\bar{z}_{ij}$ is the primitive line impedance and line powers are calculated at the receiving end. From (2) it is also clear that,

$$I_{ij}^A = \sum_{\ell=1}^{n} T_{ij,\ell} I_\ell^A \tag{5}$$

where

$$T_{ij,\ell} = \frac{Z_{i\ell} - Z_{j\ell}}{\bar{z}_{ij}} \ . \tag{6}$$

This current distribution factor gives the fraction of each bus injection current that results in each line. Collectively, the distribution factors can be used in a linear load flow to approximate the change in line flow as a result of a change in bus injection. Early uses found that the error was significant and two alternatives were proposed. First, the current distribution factors were modified to be

$$T_{ij,\ell}^S = \frac{Z_{i\ell}^S - Z_{j\ell}^S}{\bar{z}_{ij}} \ , \tag{7}$$

where Z_{ij}^S is the ij^{th} entry of the bus impedance matrix referenced to the swing bus. Alternatively, an arbitrary ground tie of 1+j1 per unit was added at a bus and the distribution factor of equation (6) was used. These two methods both gave improved results. In the following sections, it will be shown that all of these methods are approximations to the current distribution factors with constant swing bus voltage.

CURRENT DISTRIBUTION FACTORS WITH CONSTANT SWING BUS VOLTAGE

Consider again the system described above with a base case A solution satisfying equations (2) and (3). If one (or more) loads are changed to

$$S_k^B = S_k^A + \Delta S_k \tag{8}$$

then the entire problem must be resolved iteratively to obtain the new case B voltage vector, line currents and line powers. As an alternative, assume that:

$$\begin{aligned} \Delta I_k &= I_k^B - I_k^A \approx \Delta S_k^* \\ \Delta I_\ell &= I_\ell^B - I_\ell^A \approx 0 \qquad \ell = 2,3,\ldots n \\ & \qquad\qquad\qquad\qquad\quad \neq k \end{aligned} \tag{9}$$

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Notice that ΔI_1 is not zero since this would imply that the change in load is not met by a simultaneous change in generation. More importantly, if only ΔI_k is changed, the voltage V_1 at the swing bus would not remain constant under the change. In fact, since V_1 is a constant voltage source, it may be eliminated from equation (1) to yield,

$$\begin{bmatrix} V_2^A \\ V_3^A \\ \vdots \\ V_n^A \end{bmatrix} = \begin{bmatrix} Z'_{22} & Z'_{23} & \cdots & Z'_{2n} \\ Z'_{32} & Z'_{33} & \cdots & Z'_{3n} \\ \vdots & \vdots & & \vdots \\ Z'_{n2} & Z'_{n3} & \cdots & Z'_{nn} \end{bmatrix} \begin{bmatrix} I_2^A \\ I_3^A \\ \vdots \\ I_n^A \end{bmatrix} + \begin{bmatrix} Z_{21} \\ Z_{31} \\ \vdots \\ Z_{n1} \end{bmatrix} \frac{V_1^A}{Z_{11}} , \tag{10}$$

where
$$Z'_{ij} = Z_{ij} - \frac{Z_{i1} Z_{1j}}{Z_{11}} . \tag{11}$$

The current distribution is,

$$I_{ij}^A = \sum_{\ell=2}^{n} T'_{ij,\ell} I_\ell^A + \left(\frac{Z_{i1} - Z_{j1}}{\bar{z}_{ij}} \right) \frac{V_1^A}{Z_{11}} \tag{12}$$

where,

$$T'_{ij,\ell} = \left(\frac{Z_{i\ell} - Z_{j\ell}}{\bar{z}_{ij}}\right) - \frac{Z_{1\ell}}{Z_{11}} \left(\frac{Z_{i1} - Z_{j1}}{\bar{z}_{ij}}\right) . \tag{13}$$

Clearly, from equation (6)

$$T'_{ij,\ell} = T_{ij,\ell} - \frac{Z_{1\ell}}{Z_{11}} T_{ij,1} . \tag{14}$$

Returning to the changed case described by equations (9) - (11) the linear system is solved easily for the case B solution,

$$V_i^B = V_i^A + Z'_{ik} \Delta I_k, \quad i = 2,3,\ldots,n \tag{15}$$

and

$$I_{ij}^B = I_{ij}^A + T'_{ij,k} \Delta I_k \tag{16}$$

When the swing bus is represented as a voltage source with loads and other generation represented as current sources, the solutions given by equations (15) and (16) are exact. The solution is approximate when constant power loads or additional voltage controlled buses are present. For purposes of the following analysis, let equation (13) be the "exact" distribution factor for the constant current linear load flow. Clearly the ratio

$$r_\ell = - \frac{Z_{1\ell}}{Z_{11}} = \left.\frac{\Delta I_1}{\Delta I_\ell}\right|_{\Delta I_k = 0} \qquad k = 2,3,\ldots n \neq \ell \tag{17}$$

is the fraction of ΔI_ℓ which is returned to ground through the swing bus. Assuming that the current distribution of the changed case is identical to that of the base case (Equation (6)) is equivalent to assuming $r_\ell = 0$. The frequently used factor of Equation (7) is equivalent to assuming $r_\ell = -1$. The use of Equation (6) in conjunction with an artitrary $1+j1$ ground tie is a similar approximation. These methods will be shown in the following sections.

Z REFERENCED TO SWING DISTRIBUTION FACTORS

The $n + 1$ dimension indefinite admittance matrix is a combined description of network topology and primitive branch admittances irrespective of bus sources. Considering the selection of ground (bus 0) as reference in Equation (2), the ground axis may be appended to the bus impedance matrix as,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ V_0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & Z_{10} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & Z_{20} \\ \vdots & \vdots & & \vdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & Z_{n0} \\ Z_{01} & Z_{02} & \cdots & Z_{0n} & Z_{00} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_0 \end{bmatrix} \tag{18}$$

where V_i is the voltage difference between buses i and 0. Clearly, with zero as reference,

$$V_0 = Z_{0i} = Z_{i0} = 0, \tag{19}$$

and for a power system with all I_i leaving node 0,

$$I_0 = - \sum_{i=1}^{n} I_i \tag{20}$$

Subtracting equation 1 from the remaining n equations in (18) and substituting from (20)

$$I_1 = - \sum_{\substack{i=0 \\ \neq 1}}^{n} I_i , \tag{21}$$

the equations written referenced to the swing bus are,

$$\begin{bmatrix} V_2^s \\ V_3^s \\ \vdots \\ V_n^s \\ V_0^s \end{bmatrix} = \begin{bmatrix} Z_{22}^s & Z_{23}^s & \cdots & Z_{2n}^s & Z_{20}^s \\ Z_{32}^s & Z_{33}^s & \cdots & Z_{3n}^s & Z_{30}^s \\ \vdots & & & & \vdots \\ Z_{n2}^s & Z_{n3}^s & \cdots & Z_{nn}^s & Z_{n0}^s \\ Z_{02}^s & Z_{03}^s & \cdots & Z_{0n}^s & Z_{00}^s \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ \vdots \\ I_n \\ I_0 \end{bmatrix} \tag{22}$$

where
$$V_i^s = V_i - V_1 \tag{23}$$

and
$$Z_{ij}^s = (Z_{ij} - Z_{1j}) - (Z_{i1} - Z_{11}). \tag{24}$$

The assertion of the last section is clear since from Equations (5) and (24), the current distribution factor referenced to swing is,

$$T_{ij,\ell}^S = \left(\frac{Z_{i\ell} - Z_{j\ell}}{\bar{z}_{ij}}\right) - \left(\frac{Z_{i1} - Z_{j1}}{\bar{z}_{ij}}\right) , \tag{25}$$

which from Equations (13) and (17) gives,

$$T_{ij,\ell}^S = \left. T'_{ij,\ell} \right|_{r_\ell = -1} . \tag{26}$$

Thus the use of current distribution factors referenced to swing is equivalent to assuming that the entire change in bus injection current is matched by an equal change in swing current. The error is small when the system has weak ties to ground. Furthermore, when there are no impedance ties to ground in the network model, the impedance matrix referenced to ground does not exist, and Equation (7) would give exact current redistributions. It is important to note that when all shunt ties are omitted, the selection of the reference bus for the bus admittance matrix is not arbitrary. If the voltage at the swing bus is to remain constant under the bus injection changes, the current distribution factors must be formed from Equation (7). Recent advances in optimization have employed the D.C. load flow equations. When applied to the linear computation of line power flows, the D. C. load flow equations are equivalent to the current distribution factors discussed above [13].

ARBITRARY GROUND TIE DISTRIBUTION FACTORS

Early use of Z matrix methods for contingency analysis and interchange calculations used the ground referenced distribution of Equation (6) in conjunction with constant impedance load representations. The results were found to be improved by the addition of a 1+j1 ground tie at "any" bus. The success was attributed to the assertion that one lumped load representation performed equally well if not better than complete constant impedance load representation. In light of the above analysis, the arbitrary ground tie can be considered an approximation to the "exact" method discussed above. As such it is viewed more as a means to maintain constant swing voltage rather than load representation. Let the arbitrary bus be the swing bus. The addition of a ground tie at the swing bus would normally (under constant voltage) have no affect on the circuit response. The addition of a ground tie of $\bar{z}_{tie}$ at bus 1 in the system of Equation (2) would result in a base case current distribution of

$$I^{A}_{ij} = \sum_{\ell=1}^{n} T^{tie}_{ij,\ell}\, I^{A}_{\ell}, \tag{27}$$

where

$$T^{tie}_{ij,\ell} = \left(\frac{Z_{i\ell} - Z_{j\ell}}{\bar{z}_{ij}}\right) - \frac{Z_{i\ell}}{Z_{11} + \bar{z}_{tie}} \left(\frac{Z_{i1} - Z_{j1}}{\bar{z}_{ij}}\right), \tag{28}$$

or

$$r_{\ell} = - \frac{Z_{i\ell}}{Z_{11} + \bar{z}_{tie}}. \tag{29}$$

Thus if $\bar{z}_{tie} = 0$ is used for subsequent linear analysis, the results would agree identically with Equation (13). Since a tie of 1+j1 is very small relative to Z_{ij}, the success of the arbitrary ground tie is apparent. In the following sections it will be shown that although the swing referenced and arbitrary ground tie distribution factors are approximations to the "exact" constant swing voltage distribution factor, they may produce equivalent results when applied to the non linear load flow problem.

ILLUSTRATIONS

The performance of the current distribution factors in the analysis of nonlinear load flow problems is evaluated below using two typical equivalent systems. The first is taken from reference [5], and is shown in Figure 1 and Table 1. Let this be System 1. The second is taken from reference [14], and is shown in Figure 2 and Table 2. Let this be System 2. The connections to ground for System 1 are minimal, with the ratio of ground connection reactance to line reactance of 5000 - 10000. The connections to ground for System 2 are strong, with the ratio of ground connection reactance to line reactance of 50 - 200. For weak ground connections, the ratio of Equation (17) is near -1.0. This ratio was -1.004 at nearly zero degrees for System 1. This ratio was in contrast -1.1 at nearly zero degrees for System 2. The ratio's are greater than 1.0 (magnitude) since the ground connections are capacitive.

Both systems were subjected to a 10% load increase from the base case at bus 6. The changes in all real power line flows were computed using current distribution factors discussed above. The results are shown in Tables 3 and 4. The base case line flows are shown at each end of each line. The changes are shown as a percentage of base case line flow at each end. Since the current distribution factors above do not account for the change in losses, the computed changes at each end were the same. The changes were also computed using Equation (6) without additional ties to ground. The results showed extremely large error and are not shown. With the exception of lines 8 and 9 of system 1, the percent change computed using equation (13) consistently compares well with that of the exact load flow. These lines were very lightly loaded in the base case, and thus a small megawatt error is magnified. Aside from similar exceptions the approximations of Equations (7) and (28) perform reasonably well.

It was shown that the use of the bus impedance matrix referenced to swing is equivalent to assuming that the entire change in load bus injection current is balanced by an equal change in swing bus generation current. It is interesting to note that in System 1, the ratio of the change in swing bus real power to the change in bus 6 load real power was 1.16. The same ratio for system 2 was 0.981. While these are both near 1.0 so that the approximations appear valid, it is not obvious that the use of a ratio smaller or larger than 1.0 is an improvement. Clearly the increase in load at bus 6 caused an increase in System 1 losses whereas the increase in load at bus 6 caused a decrease in System 2 losses. The analysis is further complicated by noting that the ratio of Equation (17) can be greater than 1.0 (magnitude) even if the system is lossless.

In summary, it should be pointed out that in the analysis presented above although only one bus injection change was specified, the swing bus injection was automatically made to change through the elimination of the swing bus current resulting in Equations (10) through (16). If the swing bus current had been constrained to change with the single bus injection by the ratio of Equation (17), the use of current distribution factors referenced to ground (Equation (6)) and superposition would give the exact same results as Equation (13). Clearly the use of current distribution factors referenced to swing are equivalent to constraining the swing current change to be equal to the bus k load change. When there is a single tie to ground at the swing bus, or no ties to ground, this constraint is correct and the distribution factors referenced to swing would yield exact results in the linear circuit.

CONCLUSIONS

The primary objective of this paper was to give a mathematical basis for the successful use of current distribution factors referenced to swing and arbitrary ground tie modifications. The constant swing bus voltage in the load flow model prohibits the arbitrary selection of all n bus injections, and thus provides this mathematical basis. When multiple ground ties are present the current distribution factors referenced to the swing bus are approximate in the linear circuit. When there are no ground ties or only ties at the swing bus, the current disbribution factors referenced to the swing bus are exact in the linear circuit. Unfortunately the affects of additional voltage controlled buses cannot be accounted for as easily. The success of the arbitrary ground tie is seen more as an attempt to constrain the swing bus current than an equivalent load representation. These methods were considered heuristic methods without mathematical justification. In light of the analysis above, they are clearly an approximation to the exact linear circuit solution method.

Many optimization schemes utilizing current distribution factors have assumed a linear relationship between load and generation. Indeed, changes in load are almost always accompanied by a change in generation scheduling. In the linear load flow, approximations to the nonlinear power flow in the network must be made. In providing the mathematical basis for traditional current distribution factors the above analysis has illustrated the need for maintaining the known source voltage. Since this is easy to do in the linear model, it should be done in conjunction with the linear redistribution of load and generation.

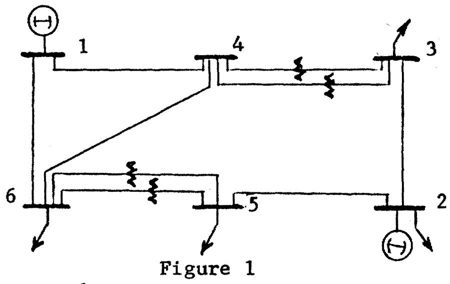

Figure 1
6 Bus system diagram

Table 1.

Six bus system data

Bus	Net injected scheduled power
1	(1) + j (1)
2	0.500 + j (2)
3	-0.550 - j 0.130
4	0.000 + j 0.000
5	-0.300 - j 0.180
6	-0.500 - j 0.050

Line	Bus I	to	Bus J	PU Line Impedance	PU Charging B
1	1		4	0.080 + j 0.370	0.00028
2	1		6	0.123 + j 0.518	0.00042
3	2		3	0.723 + j 1.050	0.00000
4	2		5	0.282 + j 0.640	0.00000
5	4		6	0.097 + j 0.407	0.00030
6	4		3	0.000 + j 0.266 (3)	0.00000
7	4		3	0.000 + j 0.266 (4)	0.00000
8	6		5	0.000 + j 0.428 (5)	0.00000
9	6		5	0.000 + j 1.000 (6)	0.00000

(1) Voltage fixed at 1.05 $\underline{|0^o}$ per unit
(2) Voltage magnitude fixed at 1.1 per unit
(3) Transformer tap = 1.025
(4) Transformer tap = 1.025
(5) Transformer tap = 1.1
(6) Transformer tap = 1.1

Table 3.

Illustration of current distribution factors - System 1.

			$\frac{\Delta P_{IJ} \text{ at } J}{P^o_{IJ} \text{ at } J} \times 100$				$\frac{\Delta P_{IJ} \text{ at } I}{P^o_{IJ} \text{ at } I} \times 100$			
Line No.	Per Unit P^o_{IJ} at J	Per Unit P^o_{IJ} at I	PCT^1	PCT^2	PCT^3	PCT^4	PCT^1	PCT^2	PCT^3	PCT^4
1	.4902	.5124	4.34	4.58	4.70	4.56	4.15	4.38	4.50	4.78
2	.4268	.4524	7.89	7.65	7.54	6.90	7.45	7.22	7.11	7.40
3	.1589	.1824	1.56	-0.78	-1.94	-1.99	1.36	-0.68	-1.69	-1.88
4	.2751	.3176	-0.90	0.45	1.12	0.67	-0.78	0.39	0.97	1.08
5	.0980	.0991	24.10	21.66	20.45	19.10	23.84	21.43	20.23	19.36
6	-.1955	.1955	-8.50	-2.47	0.37	0.80	-8.50	-2.47	0.37	0.80
7	-.1955	.1955	-8.50	-2.47	0.37	0.80	-8.50	-2.47	0.37	0.80
8	-.0174	-.0174	-223.31	-93.29	-33.08	-7.40	-223.31	-93.29	-33.08	-7.40
9	-.0074	-.0074	-223.31	-93.29	-33.08	-7.40	-223.31	-93.29	-33.08	-7.40

PCT^1 = Percent change calculated using equation (7)
PCT^2 = Percent change calculated using equation (28) with $\bar{z}_{tie}$ = 1 + j1
PCT^3 = Percent change calculated using equation (13)
PCT^4 = Percent change calculated using exact load flow

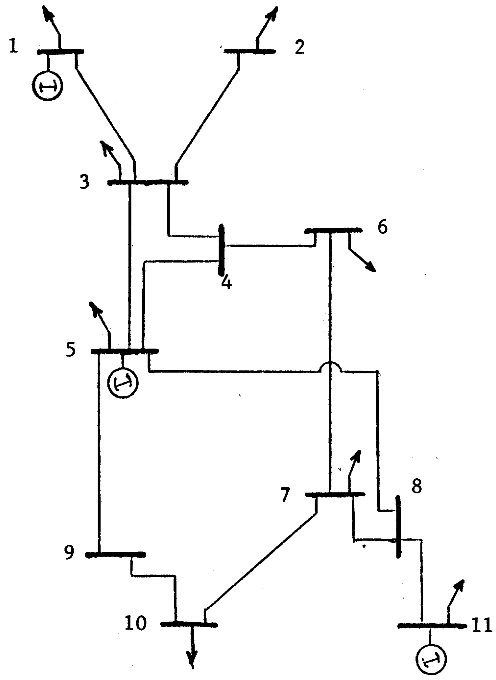

Figure 2.
11 Bus system diagram

Table 2.

Eleven bus system data

Bus	Net injected scheduled power
1	(1) + j (1)
2	-3.720 - j 0.830
3	-1.340 - j 0.157
4	0.000 + j 0.000
5	10.460 + j 1.116
6	-3.700 - j 0.122
7	-1.000 - j 0.067
8	0.000 + j 0.000
9	0.000 + j 0.000
10	-1.370 - j 0.041
11	3.960 - j 0.258

Line	Bus I	to	Bus J	PU Line Impedance	PU Charging B
1	1		3	0.0015 + j 0.0212	0.3840
2	2		3	0.0016 + j 0.0242	0.4263
3	5		3	0.0034 + j 0.0496	0.8260
4	4		3	0.0023 + j 0.0333	0.6023
5	4		5	0.0011 + j 0.0152	0.2737
6	4		6	0.0008 + j 0.0262	0.0000
7	6		7	0.0203 + j 0.1553	0.0722
8	7		10	0.0284 + j 0.1330	0.0306
9	9		10	0.0000 + j 0.0277	0.0000
10	9		5	0.0001 + j 0.0024	0.0433
11	7		8	0.0000 + j 0.0277	0.0000
12	8		11	0.0014 + j 0.0142	0.2658
13	8		5	0.0011 + j 0.0178	0.1842

(1) Voltage fixed at 1.02 $\angle 0^o$ per unit

Table 4.

Illustration of current distribution factors - system 2.

			$\frac{\Delta P_{IJ} \text{ at } J}{P_{IJ}^o \text{ at } J} \times 100$				$\frac{\Delta P_{IJ} \text{ at } I}{P_{IJ}^o \text{ at } I} \times 100$			
Line No.	Per Unit P_{IJ}^o at J	Per Unit P_{IJ}^o at I	PCT^1	PCT^2	PCT^3	PCT^4	PCT^1	PCT^2	PCT^3	PCT^4
1	-3.075	-3.061	-13.31	1.18	-14.58	-11.93	-13.37	1.18	-14.65	-11.89
2	-3.743	-3.720	-0.02	-0.88	0.05	-0.00	-0.02	-0.88	0.05	0.00
3	4.454	4.520	-3.37	-0.92	-3.58	-2.92	-3.32	-0.90	-3.53	-2.96
4	3.705	3.736	-7.14	-3.44	-7.46	-6.40	-7.08	-3.41	-7.40	-6.45
5	-6.548	-6.504	1.38	2.23	1.30	1.27	1.38	2.24	1.31	1.26
6	2.762	2.768	12.76	12.26	12.80	11.63	12.73	12.24	12.78	11.66
7	-0.955	-0.938	5.76	6.62	5.69	5.32	5.87	6.74	5.79	5.21
8	0.102	0.102	-11.78	-7.57	-11.78	-11.20	-11.74	-7.55	-12.12	-11.22
9	1.270	1.270	0.95	0.42	0.99	0.90	0.95	0.42	0.99	0.90
10	-1.270	-1.270	0.95	0.16	1.02	0.90	0.95	0.16	1.02	0.90
11	-2.057	-2.057	2.11	2.32	2.09	1.92	2.11	2.32	2.09	1.92
12	-3.960	-3.940	-0.02	0.51	-0.06	-0.00	-0.02	0.51	0.06	-0.00
13	1.879	1.883	-2.40	0.34	-2.64	-2.09	-2.39	0.35	-2.63	-2.10

PCT^1 = Percent change calculated using equation (7)
PCT^2 = Percent change calculated using equation (28) with $\bar{z}_{tie} = 1 + j1$
PCT^3 = Percent change calculated using equation (13)
PCT^4 = Percent change calculated using exact load flow

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Discussion

G. T. Heydt (Purdue University, West Lafayette, IN). Dr. Sauer has often chided me for being sloppy in the use of distribution factors – or at least for preferring the use of Z_{bus} referenced to swing rather than Z_{bus} referenced to k, $k \neq$ swing, for the purpose of calculating distribution factors. He has shown that the exact current distribution factors, T, written in terms of Z_{bus} entries from Z_{bus} referenced to bus k are equal to those obtained using Z_{bus} referenced to swing. The fact that this mathematical justification has eluded users of linear power flow studies is surprising considering the simplicity of the derivation. Sauer has explained this important point well, and this discussion will not dwell on the subject. There is a statement in the paper, however, which is enticing and suggestive of a much more lofty goal: Sauer states, "When there are no impedance ties to ground . . . . . Equation (7) would give exact current redistributions". The lofty goal is the ultimate missing component of the power flow study puzzle: the closed form power flow solution.

The questions which I would like to raise relate to the cited statement in the paper. Clearly, if all loads were constant current loads, there would be no effective ground ties and the closed form solution for all line currents is possible. The problem arises as one admits that loads are not constant current entities. Usually, power engineers represent loads by constant power models – the truth is probably somewhere in between. Does the author feel that a modification of the constant current load might yield a closed form power flow solution? For example, one could take the "constant" current at bus k to be a doubly infinite series,

$$I_k = \sum_{i=-\infty}^{\infty} a_i V_k^i .$$

The constant current case is the case where all a_i are zero except a_0. The constant impedance tie case is the case where all a_i are zero except a_1. The consant power case is the case where all a_i are zero except a_{-1}.

Finally, concerning the appearance of multiple voltage controlled busses, I believe that this case can be handled by assuming that all voltage magnitude controlled busses (including the swing bus) should be considered as being tied through a zero impedance phase shifter (of arbitrary angle) to ground. Traditionally, the phase shifter for the swing bus is taken at zero phase shift. Sauer's results are largely the same except that entries of Z_{bus} will change and contain the several arbitrary phase shift angles.

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P. W. Sauer: I would like to thank Dr. Heydt for his comments, and for instilling the desire to seek mathematical justification for this and many other similar methods encountered in power system analysis. The purpose of this paper was to show why swing referenced and arbitrary ground tie distribution factors give markedly improved results over ground referenced distribution factors. Consider a linear circuit. The ground referenced distribution factors do not contain any implicit constraint on the swing bus source current. The "exact" distribution factors implicitly constrain the swing bus current to provide constant swing bus voltage. The swing referenced distribution factors implicitly constrain the swing bus current to be exactly equal to the negative of the change in all other bus injection currents. As such if the system has ground ties, the swing referenced distribution factors give incorrect results even for the linear system. The error however is small in general, which explains their wide use.

I would like to apologize for an omission in equation (12) of the paper copies distributed prior to the conference. Equation (12) should read:

$$I_{ij}^{A} = \sum_{\ell=2}^{n} T'_{ij,\ell}\, I_{\ell}^{A} + \left(\frac{Z_{i1}-Z_{j1}}{\bar{z}_{ij}}\right) \frac{V_1^A}{Z_{11}} .$$

Since the last term is constant, the omission had no affect on the results presented or on the conclusions stated in the paper.

In response to Dr. Heydts' suggestion of a doubly infinite series for the bus k current, I believe, the higher terms will introduce non-linearities which can not be solved using the circuit theory presented in the paper. It is interesting however, to note, that a recently developed power series in power may prove to have near closed form potential for some special, yet needed, load flow applications. Two series have been developed which express all bus voltages as an explicit function of ascending powers of the bus injections. Preliminary results are currently being analyzed and will be made available in the near future.

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