



Laboratorio #4

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1. Ejercicio #1

- $(a \equiv d) :\Leftrightarrow a := \{1, 2, 4, 8, 16, 32, 64\} \equiv d := \{n \in \mathbb{N} \mid \exists i \in \mathbb{N} . n = 2^i \wedge n < 100\}$
- $(b \equiv f) :\Leftrightarrow b := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . x = n/5\} \equiv f := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . n = x + x + x + x + x\}$
- $(c \equiv e) :\Leftrightarrow c := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . n = x * x\} \equiv e := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . x = \sqrt{n}\}$

2. Ejercicio #2

1. $A := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . n = x/5\}$

2. Partiendo de:

- $A := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . n = x/5\}$
- $B := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . n = x/4\}$
- $C := (A \cap B)$

3. $D := \{n \in \mathbb{N} \mid \exists x \in \mathbb{N} . 1 < x < n \wedge n/x\}$

4. $E := \{e \subset P(\mathbb{N}) \mid \exists x \in \mathbb{N} \wedge \exists n \in e . n = x/15\}$

5. $F := \{f \subset P(\mathbb{N}) \mid \exists x \in f \wedge \exists n \in f . x + n = 42\}$

3. Ejercicio #3

$$P := \{ \langle a, b, c \rangle . a \in \mathbb{N} . b \in \mathbb{N} . c \in \mathbb{N} \mid \exists x \in \mathbb{N} . x > 1 . x < a \wedge x < b . a/x \wedge b/x . c = a * b \wedge c < 30 \}$$

4. Ejercicio #4

1. $\lambda x \in \mathbb{N} . x + x = \{ \langle x, x + x \rangle \mid x \in \mathbb{N} \}$
2. Partiendo de:
 - $A := \lambda x \in \mathbb{N} . true = \{ \langle x, true \rangle \mid x \in \mathbb{N} \wedge x/5 \}$
 - $B := \lambda x \in \mathbb{N} . false = \{ \langle x, false \rangle \mid x \in \mathbb{N} \wedge \neg(x/5) \}$
 - $C := (A \cap B)$
3. $f \circ g \in P(\mathbb{N})$
4. $\lambda x \in \mathbb{N} . f(g(x)) = \{ \langle x, f(g(x)) \rangle \mid x \in \mathbb{N} \wedge f(x) \in \mathbb{N} \wedge g(x) \subset f(x) \}$

5. Ejercicio #5

1. $f(x) = x^2 \rightarrow$ Surjectiva
2. $g(x) = \frac{1}{\cos(x-1)} \rightarrow$ Injectiva
3. $h(x) = 2x \rightarrow$ Bijectiva
4. $w(x) = x + 1 \rightarrow$ Bijectiva

6. Ejercicio #6

1. $B_1 := \{ \langle a, b \rangle \mid a \in \mathbb{N} . b \in \mathbb{N} . \forall x \in \mathbb{N} . a = 2x \wedge a > 0 \wedge b > 0 \}$
2. $B_{2a} := \{ \langle a, b \rangle \mid a \in \mathbb{N} . b \in \mathbb{N} . \forall x \in \mathbb{N} . a = (2x + 1) \wedge b > 0 \}$
3. $B_2 := \{ \langle a, b \rangle \mid a \in \mathbb{N} . b \in \mathbb{Z} . \forall x \in \mathbb{N} . a = (2x + 1) \wedge b < 0 \}$
4. $B := \{ \langle a, b \rangle \mid a \in \mathbb{N} . b \in \mathbb{Z} \}$