Universidad del Istmo de Guatemala
Facultad de Ingeniería
Informática 1

Laboratorio #3

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Ejercicio #1

Utilizando la definicion de suma ($\oplus$) para los números naturales unarios, llevar a cabo la suma entre tres $[s(s(s(0)))]$ y cuatro $[s(s(s(s(0))))]$. Debe elaborar todos los pasos de forma explícita. Como referencia, se presenta nuevamente la definición de suma para numeros natruales unarios:

$$n \oplus m := \begin{cases} m & \text{si } n = o \\ n & \text{si } m = o \\ \sigma(i \oplus m) & \text{si } n = \sigma(i) \end{cases}$$

Demostración:

$$\sigma(\sigma(\sigma(0))) \oplus \sigma(\sigma(\sigma(\sigma(0)))) \qquad \text{n} = \sigma(\sigma(\sigma(0))),\ \ \text{i} = \sigma(\sigma(0)) \tag{1}$$
$$\sigma[\sigma(\sigma(0)) \oplus \sigma(\sigma(\sigma(\sigma(0))))] \qquad \text{n} = \sigma(\sigma(0)),\ \ \text{i} = \sigma(0) \tag{2}$$
$$\sigma[\sigma[\sigma(0) \oplus \sigma(\sigma(\sigma(\sigma(0))))]] \qquad \text{n} = \sigma(0),\ \ \text{i} = 0 \tag{3}$$
$$\sigma[\sigma[\sigma[0 \oplus \sigma(\sigma(\sigma(\sigma(0))))]]] \qquad \text{n} = 0 \quad \therefore 0 \oplus m = m \tag{4}$$
$$\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0))))))) \qquad 3 + 4 = 7 \tag{5}$$

Ejercicio #2

Definir inductivamente una función para multiplicar ($\otimes$) números naturales unarios.
Consejo: Puede apoyarse de la definición de suma estudiada durante la clase.

Definición:

$$a \otimes b := \begin{cases} 0 & \text{si } a = 0 \ \vee \ b = 0 \\ a & \text{si } b = \sigma(0) \\ b & \text{si } a = \sigma(0) \\ (a \otimes i) \oplus a & \text{si } b = \sigma(i) \end{cases}$$

Ejercicio #3

Verifique que su definición de multiplicación es correcta multiplicando los siguientes valores:

1. $\sigma(\sigma(\sigma(0))) \otimes 0$

$$
\begin{aligned}
&\sigma(\sigma(\sigma(0))) \otimes 0 && \text{a} = \sigma(\sigma(\sigma(0))),\ \text{b} = 0 && (1)\\
&0 && \therefore \sigma(\sigma(\sigma(0))) \otimes 0 = 0 && (2)
\end{aligned}
$$

2. $\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$

$$
\begin{aligned}
&\sigma(\sigma(\sigma(0))) \otimes \sigma(0) && \text{a} = \sigma(\sigma(\sigma(0))),\ \text{b} = \sigma(0) && (1)\\
&\sigma(\sigma(\sigma(0))) && \therefore\ \sigma(\sigma(\sigma(0))) \otimes \sigma(0) = \sigma(\sigma(\sigma(0))) && (2)
\end{aligned}
$$

3. $\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$

$$
\begin{aligned}
&\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0)) && \text{a} = \sigma(\sigma(\sigma(0))),\ \text{b} = \sigma(\sigma(0)) && (1)\\
&(\sigma(\sigma(\sigma(0))) \otimes \sigma(0)) \oplus \sigma(\sigma(\sigma(0))) && \text{a} \otimes \sigma(0) = \text{a} && (2)\\
&\sigma(\sigma(\sigma(0))) \oplus \sigma(\sigma(\sigma(0))) && \text{Definición de la suma.} && (3)\\
&\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0)))))) && \therefore 3 \times 2 = 6 && (4)
\end{aligned}
$$

Ejercicio #4

Demostrar utilizando inducción:

1. $a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$

Caso Base:

- $a = 0$

Demostración.

$$
\begin{aligned}
0 \oplus \sigma(\sigma(0)) &= \sigma(\sigma(0))\\
\sigma(\sigma(0)) &= \sigma(\sigma(0))
\end{aligned}
$$

□

Caso Inductivo:

- $a = \sigma(a)$
- **Hipótesis Inductiva:** $a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$
- *Demostrar que:* $\sigma(a) \oplus \sigma(\sigma(0)) = \sigma(\sigma(\sigma(a)))$

Demostración.

$$\begin{align}
\sigma(a) \oplus \sigma(\sigma(0)) &= \sigma(\sigma(\sigma(a))) && \text{Definición de la suma.} \tag{1}\\
\sigma[a \oplus \sigma(\sigma(0))] &= \sigma(\sigma(\sigma(a))) && \text{La suma es conmutativa.} \tag{2}\\
\sigma[\sigma(\sigma(0)) \oplus a] &= \sigma(\sigma(\sigma(a))) && \text{Definición de la suma.} \tag{3}\\
\sigma[\sigma[\sigma(0) \oplus a]] &= \sigma(\sigma(\sigma(a))) && \text{Definición de la suma.} \tag{4}\\
\sigma[\sigma[\sigma[0 \oplus a]]] &= \sigma(\sigma(\sigma(a))) && 0 + a = a \tag{5}\\
\sigma(\sigma(\sigma(a))) &= \sigma(\sigma(\sigma(a))) \tag{6}
\end{align}$$

□

2. $a \otimes b = b \otimes a$

Caso Base:

- $a = 0$

Demostración.

$$\begin{aligned}
0 \otimes b &= b \otimes 0 \\
0 &= 0
\end{aligned}$$

□

Caso Inductivo:

- $a = \sigma(a)$
- **Hipótesis Inductiva:** $a \otimes b = b \otimes a$
- *Demostrar que:* $\sigma(a) \otimes b = b \otimes \sigma(a)$

Demostración.

$$\begin{align}
\sigma(a) \otimes b &= b \otimes \sigma(a) && \text{Definición de la multiplicación.} \tag{1}\\
\sigma(a) \otimes b &= (b \otimes a) \oplus b && \text{Por hipotesis inductiva.} \tag{2}\\
\sigma(a) \otimes b &= (a \otimes b) \oplus b && \text{Definición de la multiplicación.} \tag{3}\\
(a \otimes b) \oplus b &= (a \otimes b) \oplus b \tag{4}
\end{align}$$

□

3. $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

Caso Base:

- $c = 0$

Demostración.

$$a \otimes (b \otimes 0) = (a \otimes b) \otimes 0 \qquad \text{i = a} \otimes \text{b}$$
$$a \otimes 0 = i \otimes 0$$
$$0 = 0$$

□

Caso Inductivo:

- $c = \sigma(c)$
- **Hipótesis Inductiva:** $a \otimes (b \otimes c) = (a \otimes b) \otimes c$
- *Demostrar que:* $a \otimes (b \otimes \sigma(c)) = (a \otimes b) \otimes \sigma(c)$

Demostración.

$$a \otimes (b \otimes \sigma(c)) = (a \otimes b) \otimes \sigma(c) \qquad \text{Definición de la multiplicación.} \qquad (1)$$
$$a \otimes (b \otimes \sigma(c)) = ((a \otimes b) \otimes c) \oplus (a \otimes b) \qquad \text{Por hipotesis inductiva.} \qquad (2)$$
$$a \otimes (b \otimes \sigma(c)) = (a \otimes (b \otimes c)) \oplus (a \otimes b) \qquad \text{La suma es conmutativa.} \qquad (3)$$
$$a \otimes (b \otimes \sigma(c)) = (a \otimes b) \oplus (a \otimes (b \otimes c)) \qquad \text{La suma es distributiva.} \qquad (4)$$
$$a \otimes (b \otimes \sigma(c)) = a \otimes (b \oplus (b \otimes c)) \qquad \text{La suma es conmutativa.} \qquad (5)$$
$$a \otimes (b \otimes \sigma(c)) = a \otimes ((b \otimes c) \oplus b) \qquad \text{Definición de la multiplicación.} \qquad (6)$$
$$a \otimes (b \otimes \sigma(c)) = a \otimes (b \otimes \sigma(c)) \qquad (7)$$

□

4. $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$

Caso Base:

- $c = 0$

Demostración.

$$(a \oplus b) \otimes 0 = (a \otimes 0) \oplus (b \otimes 0) \qquad \text{i = a} \oplus \text{b}$$
$$i \otimes 0 = 0 \oplus 0$$
$$0 = 0$$

□

Caso Inductivo:

- $c = \sigma(c)$
- **Hipótesis Inductiva:** $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
- *Demostrar que:* $(a \oplus b) \otimes \sigma(c) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$

Demostración.

1) $(a \oplus b) \otimes \sigma(c) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$	Definición de la multiplicación.
2) $(a \oplus b) \otimes c \oplus (a \oplus b) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$	Por hipótesis inductiva.
3) $(a \otimes c) \oplus (b \otimes c) \oplus (a \oplus b) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$	La suma es conmutativa.
4) $((a \otimes c) \oplus a) \oplus ((b \otimes c) \oplus b) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$	Definición de la multiplicación.
5) $(a \otimes \sigma(c)) \oplus (b \otimes \sigma(c)) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$	

$\square$