

Laboratorio #3

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Ejercicio #1

Utilizando la definicion de suma (\oplus) para los números naturales unarios, llevar a cabo la suma entre tres [s(s(s(0)))] y cuatro [s(s(s(s(0))))]. Debe elaborar todos los pasos de forma explícita. Como referencia, se presenta nuevamente la definición de suma para numeros naturales unarios:

$$n \oplus m := \begin{cases} m & \text{si } n = o \\ n & \text{si } m = o \\ \sigma(i \oplus m) & \text{si } n = \sigma(i) \end{cases}$$

Demostración:

$$\sigma(\sigma(\sigma(0))) \oplus \sigma(\sigma(\sigma(0)))) \qquad \qquad n = \sigma(\sigma(\sigma(0))), \ i = \sigma(\sigma(0))$$
 (1)

$$\sigma[\sigma(\sigma(0)) \oplus \sigma(\sigma(\sigma(0)))] \qquad \qquad n = \sigma(\sigma(0)), \quad i = \sigma(0)$$

$$\sigma[\sigma[\sigma(0) \oplus \sigma(\sigma(\sigma(\sigma(0))))]] \qquad \qquad n = \sigma(0), \quad i = 0$$

$$\sigma[\sigma[\sigma[0 \oplus \sigma(\sigma(\sigma(\sigma(0))))]] \qquad \qquad n = 0 \quad \therefore 0 \oplus m = m$$

$$(4)$$

$$\sigma[\sigma[\sigma(0) \oplus \sigma(\sigma(\sigma(0))))]] \qquad \qquad n = \sigma(0), \ i = 0$$
(3)

$$\sigma[\sigma[\sigma[0 \oplus \sigma(\sigma(\sigma(0))))]]] \qquad \qquad \mathbf{n} = 0 \quad \therefore 0 \oplus m = m$$
 (4)

$$\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0))))))) \qquad \qquad 3+4=7 \tag{5}$$

Ejercicio #2

Definir inductivamente una función para multiplicar (⊗) números naturales unarios. Consejo: Puede apoyarse de la definición de suma estudiada durante la clase.

Definición:

$$a \otimes b := \begin{cases} 0 & \text{si } a = 0 \lor b = 0 \\ a & \text{si } b = \sigma(0) \\ b & \text{si } a = \sigma(0) \\ (a \otimes i) \oplus a & \text{si } b = \sigma(i) \end{cases}$$

Ejercicio #3

Verifique que su definición de multiplicación es correcta multiplicando los siguientes valores:

1. $\sigma(\sigma(\sigma(0))) \otimes 0$

$$\sigma(\sigma(\sigma(0))) \otimes 0$$
 $a = \sigma(\sigma(\sigma(0))), b = 0$ (1)

 $\therefore \sigma(\sigma(\sigma(0))) \otimes 0 = 0 \tag{2}$

2. $\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$

$$\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$$
 $a = \sigma(\sigma(\sigma(0))), b = \sigma(0)$ (1)

$$\sigma(\sigma(\sigma(0))) \qquad \qquad \therefore \ \sigma(\sigma(\sigma(0))) \otimes \sigma(0) = \sigma(\sigma(\sigma(0))) \tag{2}$$

3. $\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$

$$\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0)) \qquad \qquad \mathbf{a} = \sigma(\sigma(\sigma(0))), \, \mathbf{b} = \sigma(\sigma(0)) \tag{1}$$

$$(\sigma(\sigma(\sigma(0))) \otimes \sigma(0)) \oplus \sigma(\sigma(\sigma(0))) \qquad \qquad a \otimes \sigma(0) = a$$
 (2)

$$\sigma(\sigma(\sigma(0))) \oplus \sigma(\sigma(\sigma(0)))$$
 Definición de la suma. (3)

$$\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0)))))) \qquad \therefore 3 \times 2 = 6 \tag{4}$$

Ejercicio #4

Demostrar utilizando inducción:

1.
$$a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$$

Caso Base:

• a = 0

Demostración.

$$0 \oplus \sigma(\sigma(0)) = \sigma(\sigma(0))$$
$$\sigma(\sigma(0)) = \sigma(\sigma(0))$$

Caso Inductivo:

• $a = \sigma(a)$

• Hipótesis Inductiva: $a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$

• Demostrar que: $\sigma(a) \oplus \sigma(\sigma(0)) = \sigma(\sigma(\sigma(a)))$

Demostración.

$$\sigma(a) \oplus \sigma(\sigma(0)) = \sigma(\sigma(\sigma(a))) \qquad \text{Definición de la suma.} \qquad (1)$$

$$\sigma[a \oplus \sigma(\sigma(0))] = \sigma(\sigma(\sigma(a))) \qquad \text{La suma es conmutativa.} \qquad (2)$$

$$\sigma[\sigma(\sigma(0)) \oplus a] = \sigma(\sigma(\sigma(a))) \qquad \text{Definición de la suma.} \qquad (3)$$

$$\sigma[\sigma[\sigma(0) \oplus a]] = \sigma(\sigma(\sigma(a))) \qquad \text{Definición de la suma.} \qquad (4)$$

$$\sigma[\sigma[\sigma[0 \oplus a]]] = \sigma(\sigma(\sigma(a))) \qquad 0 + a = a \qquad (5)$$

$$\sigma(\sigma(\sigma(a))) = \sigma(\sigma(\sigma(a))) \qquad (6)$$

2. $a \otimes b = b \otimes a$

Caso Base:

• a = 0

Demostración.

$$0 \otimes b = b \otimes 0$$
$$0 = 0$$

Caso Inductivo:

- $a = \sigma(a)$
- Hipótesis Inductiva: $a \otimes b = b \otimes a$
- Demostrar que: $\sigma(a) \otimes b = b \otimes \sigma(a)$

Demostraci'on.

$$\sigma(a) \otimes b = b \otimes \sigma(a) \qquad \qquad \text{Definición de la multiplicación.} \qquad (1)$$

$$\sigma(a) \otimes b = (b \otimes a) \oplus b \qquad \qquad \text{Por hipotesis inductiva.} \qquad (2)$$

$$\sigma(a) \otimes b = (a \otimes b) \oplus b \qquad \qquad \text{Definición de la multiplicación.} \qquad (3)$$

$$(a \otimes b) \oplus b = (a \otimes b) \oplus b \qquad \qquad (4)$$

3. $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

Caso Base:

• c = 0

Demostración.

$$a \otimes (b \otimes 0) = (a \otimes b) \otimes 0$$
 i = a \otimes b
 $a \otimes 0 = i \otimes 0$
 $0 = 0$

Caso Inductivo:

- $c = \sigma(c)$
- Hipótesis Inductiva: $a \otimes (b \otimes c) = (a \otimes b) \otimes c$
- Demostrar que: $a \otimes (b \otimes \sigma(c)) = (a \otimes b) \otimes \sigma(c)$

Demostración.

$$a\otimes (b\otimes \sigma(c))=(a\otimes b)\otimes \sigma(c) \qquad \text{Definición de la multiplicación.} \qquad (1)$$

$$a\otimes (b\otimes \sigma(c))=((a\otimes b)\otimes c)\oplus (a\otimes b) \qquad \text{Por hipotesis inductiva.} \qquad (2)$$

$$a\otimes (b\otimes \sigma(c))=(a\otimes (b\otimes c))\oplus (a\otimes b) \qquad \text{La suma es conmutativa.} \qquad (3)$$

$$a\otimes (b\otimes \sigma(c))=(a\otimes b)\oplus (a\otimes (b\otimes c)) \qquad \text{La suma es distributiva.} \qquad (4)$$

$$a\otimes (b\otimes \sigma(c))=a\otimes (b\oplus (b\otimes c)) \qquad \text{La suma es conmutativa.} \qquad (5)$$

$$a\otimes (b\otimes \sigma(c))=a\otimes ((b\otimes c)\oplus b) \qquad \text{Definición de la multiplicación.} \qquad (6)$$

$$a\otimes (b\otimes \sigma(c))=a\otimes (b\otimes \sigma(c)) \qquad (7)$$

4.
$$(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$$

Caso Base:

c = 0

Demostración.

$$(a \oplus b) \otimes 0 = (a \otimes 0) \oplus (b \otimes 0)$$
 i = a \oplus b
 $i \otimes 0 = 0 \oplus 0$
 $0 = 0$

Caso Inductivo:

- $c = \sigma(c)$
- Hipótesis Inductiva: $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
- Demostrar que: $(a \oplus b) \otimes \sigma(c) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$

Demostración.

1) $(a \oplus b) \otimes \sigma(c) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$

Definición de la multiplicación.

2) $(a \oplus b) \otimes c \oplus (a \oplus b) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$

Por hipótesis inductiva.

3) $(a \otimes c) \oplus (b \otimes c) \oplus (a \oplus b) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$

La suma es conmutativa.

4) $((a \otimes c) \oplus a) \oplus ((b \otimes c) \oplus b) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$ Definición de la multiplicación.

5) $(a \otimes \sigma(c)) \oplus (b \otimes \sigma(c)) = (a \otimes \sigma(c)) \oplus (b \otimes \sigma(c))$