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Task 5: Machine Learning 2

The analysis of financial series requires not only understanding the historical data, but also the capacity to anticipate future movements with accuracy. In the previous task (C.4 – Machine Learning 1), the focus was on creating deep-learning models capable of predicting a single next-day closing price using recurrent architectures such as LSTM, GRU and RNN.

In this document I will develop Task C.5 – Machine Learning 2, where the objective is to extend the forecasting capacity of the model to handle more complex and realistic financial scenarios. Specifically, this task introduces two new dimensions of prediction:

- Multistep forecasting, which consists in predicting not only the next day, but a complete sequence of k future days (for example, the closing prices for the next 5 days).
- Multivariate prediction, which allows the model to consider multiple correlated features as inputs, such as Open, High, Low, Close, Adjusted Close and Volume, instead of using only one column.

By combining these two techniques, the model can learn richer temporal relationships between several market variables and forecast how they jointly evolve over time. This approach brings the system closer to real-world financial forecasting, where price dynamics are influenced by many interacting indicators and not by a single value.

The implementation of this task required the creation of new functions for multistep and multivariate prediction, together with adjustments to the neural-network structure and the evaluation process. The code now supports temporal cross-validation, grid search for hyperparameter tuning, and uncertainty estimation using Monte Carlo Dropout. These improvements allow a more robust and interpretable model capable of producing multi-day forecasts with confidence intervals.

For this task I extended the imports from the previous version to support temporal cross-validation, walk-forward backtesting, multi-step forecasting utilities, and plotting with prediction intervals. The following additions are used across the new C5 helpers (splits, baselines, backtesting, uncertainty and plots):

- from tensorflow.keras.callbacks import EarlyStopping, ReduceLROnPlateau
- from tensorflow.keras import backend as K
- from tensorflow.keras import Sequential
- from tensorflow.keras.layers import LSTM, GRU, SimpleRNN, Dense, Dropout, Bidirectional
- from tensorflow.keras.losses import Huber
- from dataclasses import dataclass
- from typing import Sequence

tensorflow.keras.callbacks – EarlyStopping, ReduceLROnPlateau:
Used to monitor the loss during training.

- EarlyStopping halts training when validation loss stops improving, avoiding overfitting.
- ReduceLROnPlateau decreases the learning rate when progress stagnates, stabilizing convergence.
  Both are essential for efficient training in multistep forecasting tasks, where the horizon increases the risk of divergence.

tensorflow.keras.backend as K:
Provides low-level TensorFlow operations used for advanced techniques such as Monte Carlo Dropout (forcing dropout to stay active during prediction) or for clearing sessions between models during grid search.

tensorflow.keras.Sequential:
Creates models in a sequential layer-by-layer manner.
While already known from C4, it is re-imported here explicitly because the new architecture builds variable-depth models for multi-output forecasting (horizon > 1).

tensorflow.keras.layers – LSTM, GRU, SimpleRNN, Dense, Dropout, Bidirectional:
Defines the core recurrent units.
These imports enable flexible combination of recurrent types for multivariate and bidirectional architectures:

- LSTM/GRU: long-term dependency handling.
- SimpleRNN: baseline recurrent layer.
- Dropout: regularization and uncertainty estimation.
- Bidirectional: processes sequences forward and backward, capturing richer temporal patterns.

tensorflow.keras.losses.Huber:
Introduced to improve robustness against outliers in financial data.
The Huber loss behaves like MSE for small errors and like MAE for large ones, offering smoother optimization for volatile stock prices.

dataclasses.dataclass:
Used to define structured configuration objects for experiments (e.g., ModelConfig, TrainConfig).
It allows easy serialization, comparison, and printing of experiment parameters.

typing.Sequence:
Provides type annotation for ordered inputs (e.g., layers, units, steps), clarifying the expected data types and improving readability and maintainability of the experimental framework.

For this task I extended the deep-learning framework developed in Task C.4 to support **multistep** and **multivariate** forecasting.
In the previous version, the model predicted only one day ahead and used a single feature.
Now, the system can predict several days into the future and can process multiple correlated variables (such as *Open, High, Low, Close,* and *Volume*).
To achieve this, I implemented a new group of functions that handle window generation,

model building, temporal cross-validation, baseline comparison, and prediction-interval estimation.

build_supervised_multistep()

This function creates the supervised dataset used for multistep forecasting. Given a DataFrame of input features and a Series of target prices, it slides a window of length *lookback* through time and builds pairs of (X, Y), where X contains the past values and Y the next *horizon* future prices.

The result is two NumPy arrays:

- X: shape *(samples, lookback, features)*
- Y: shape *(samples, horizon)*

This transformation allows the network to learn entire sequences of future prices instead of a single next value.

```python
def build_supervised_multistep(X_df: pd.DataFrame,
                               y_ser: pd.Series,
                               lookback: int,
                               horizon: int) -> tuple[np.ndarray, np.ndarray]:
    """
    Build (X, Y) windows for multi-step forecasting.

    Args:
        X_df: Feature DataFrame ordered by time (rows are timesteps).
        y_ser: Target Series aligned with X_df (same length).
        lookback: Number of past timesteps in each input window.
        horizon: Number of future steps to predict per window.

    Returns:
        X: Array of shape (n_samples, lookback, n_features).
        Y: Array of shape (n_samples, horizon) with future targets.

    Notes:
        Creates all consecutive windows so that each Y contains y[t+1..t+horizon].
    """
    assert lookback > 0 and horizon > 0
    X_vals = X_df.values
    y_vals = y_ser.values
    X_seq, Y_seq = [], []
    last_start = len(X_vals) - (lookback + horizon) + 1
    for i in range(last_start):
        X_seq.append(X_vals[i:i+lookback, :])
        Y_seq.append(y_vals[i+lookback:i+lookback+horizon])
    return np.array(X_seq, dtype=np.float32), np.array(Y_seq, dtype=np.float32)
```

build_supervised_multistep_delta()

This version extends the previous function by producing targets not only as absolute prices but also as deltas relative to the last observed value ($y_{t+h} - y_t$). Working with deltas stabilizes the scale of the targets, which helps the model converge faster and reduces bias from long-term price drift.

It returns the raw prices, the delta targets, and the last observed close value of each window, which are later used to reconstruct the predicted price as $\hat{y}_{t+h} = y_t + \Delta\hat{y}_{t+h}$.

```python
def build_supervised_multistep_delta(X_df_raw: pd.DataFrame,
                                     y_ser_raw: pd.Series,
                                     lookback: int,
                                     horizon: int):
    """
    Build multi-step windows and also return targets as deltas from last observed value.

    Args:
        X_df_raw: Raw (unscaled) feature DataFrame.
        y_ser_raw: Raw (unscaled) target Series (e.g., closing price).
        lookback: Past window size.
        horizon: Future steps to predict.

    Returns:
        X_raw: (n, lookback, n_features) unscaled input windows.
        Y_raw: (n, horizon) future raw target values.
        Y_delta: (n, horizon) future deltas, y_{t+h} - y_t.
        last_close: (n,) last observed y_t of each window.
    """
    Xv = X_df_raw.values
    yv = y_ser_raw.values
    X_seq, Y_raw_seq, Y_delta_seq, last_seq = [], [], [], []
    last_start = len(Xv) - (lookback + horizon) + 1
    for i in range(last_start):
        xblk = Xv[i:i+lookback, :]
        y_fut = yv[i+lookback:i+lookback+horizon]
        y_last = yv[i+lookback-1]
        X_seq.append(xblk)
        Y_raw_seq.append(y_fut)
        Y_delta_seq.append(y_fut - y_last)
        last_seq.append(y_last)
    return (np.array(X_seq, dtype=np.float32),
            np.array(Y_raw_seq, dtype=np.float32),
            np.array(Y_delta_seq, dtype=np.float32),
            np.array(last_seq, dtype=np.float32))
```

multistep_metrics() and multistep_metrics_by_horizon()

To evaluate the accuracy of multistep predictions, I created two complementary metric functions.
The first one, multistep_metrics, computes global RMSE, MAE, and MAPE over all horizons and samples.
The second one, multistep_metrics_by_horizon, measures the same errors step-by-step, allowing analysis of how performance changes as the forecast extends further into the future.
Together, they provide a complete quantitative view of the model's forecasting ability.

```python
def multistep_metrics(y_true: np.ndarray, y_pred: np.ndarray) -> dict:
    """
    Aggregate multi-step error metrics.

    Args:
        y_true: (n, horizon) ground truth.
        y_pred: (n, horizon) predictions.

    Returns:
        dict with overall RMSE, MAE, and MAPE across all horizons and samples.

    Caveats:
        MAPE uses |y| in the denominator with an epsilon to avoid division by zero.
    """
    assert y_true.shape == y_pred.shape
    eps = 1e-8
    rmse = float(np.sqrt(np.mean((y_true - y_pred) ** 2)))
    mae  = float(np.mean(np.abs(y_true - y_pred)))
    mape = float(np.mean(np.abs((y_true - y_pred) / (np.abs(y_true) + eps))) * 100.0)
    return {"rmse": rmse, "mae": mae, "mape": mape}
```

```python
def multistep_metrics_by_horizon(y_true: np.ndarray, y_pred: np.ndarray):
    """
    Compute RMSE/MAE per horizon step.

    Args:
        y_true: (n, horizon) ground truth.
        y_pred: (n, horizon) predictions.

    Returns:
        List[dict]: [{"h": 1..H, "rmse": ..., "mae": ...}, ...]
    """
    y_true = np.asarray(y_true); y_pred = np.asarray(y_pred)
    H = y_true.shape[1]
    out = []
    for h in range(H):
        rmse = float(np.sqrt(np.mean((y_true[:, h] - y_pred[:, h])**2)))
        mae  = float(np.mean(np.abs(y_true[:, h] - y_pred[:, h])))
        out.append({"h": h+1, "rmse": rmse, "mae": mae})
    return out
```

build_model_from_config()

This function dynamically constructs and compiles a recurrent-neural-network model from a configuration dictionary.
It supports multiple architectures—LSTM, GRU, RNN, or Bidirectional LSTM—and customizable parameters such as the number of layers, hidden units, dropout rates, learning rate, and optional dense layers.

Each model ends with a Dense(horizon) output layer that produces a vector of predicted prices for all forecasted days.

The model uses the Huber loss, which combines the advantages of MSE and MAE, making it robust to outliers typical in financial data.

```python
def build_model_from_config(cfg, lookback: int, n_features: int, horizon: int):
    """
    Build and compile an RNN (LSTM/GRU/RNN/BiLSTM) from a config dict.

    Expected keys in cfg:
        - "model" | "layer_type": {"lstm","gru","rnn","bilstm"} (default "lstm")
        - "units": int | sequence[int] (e.g., 128 or (128,64))
        - "layers": if units is int, how many repeated layers (default 1)
        - "dropout": float, "recurrent_dropout": float
        - "bidirectional": bool (or model == "bilstm")
        - "dense_units": optional int for an intermediate Dense
        - "activation": activation for the intermediate Dense (default "relu")
        - "lr": Learning rate for Adam (default 1e-3)

    Returns:
        Compiled tf.keras.Model that outputs a vector of length `horizon`.

    Notes:
        Uses Huber loss for robustness against outliers in regression.
    """

    model_type = (cfg.get("model") or cfg.get("layer_type") or "lstm").lower()
    p_drop = float(cfg.get("dropout", 0.0))
    rec_drop = float(cfg.get("recurrent_dropout", 0.0))
    lr = float(cfg.get("lr", 1e-3))
    use_bidir = bool(cfg.get("bidirectional", False) or model_type == "bilstm")

    units_cfg = cfg.get("units", 64)
    if isinstance(units_cfg, (list, tuple)):
        units_list = [int(u) for u in units_cfg]
    else:
        n_layers = int(cfg.get("layers", 1))
        units_list = [int(units_cfg)] * max(1, n_layers)

    RNN = {"lstm": LSTM, "gru": GRU, "rnn": SimpleRNN, "bilstm": LSTM}.get(model_type, LSTM)

    model = Sequential()
    model.add(tf.keras.Input(shape=(lookback, n_features)))  # avoid input_shape warning

    for i, units in enumerate(units_list):
        return_seq = (i < len(units_list) - 1)
        rnn_layer = RNN(units=units, return_sequences=return_seq, recurrent_dropout=rec_drop)
```

timeseries_splits()

Standard cross-validation cannot be used with time-ordered data. This function implements a custom expanding-window splitter, which generates consecutive training and testing segments that always move forward in time. Each split grows the training set while keeping a constant test size.

```python
def timeseries_splits(n: int, n_splits: int = 3, test_size: int | None = None, test_ratio: float = 0.2) -> list[TSSplit]:
    """
    Create simple expanding-window time-series splits.

    Args:
        n: Total number of samples.
        n_splits: Number of folds.
        test_size: Fixed test size per fold; if None, use ratio.
        test_ratio: Fraction for test size if test_size is None.

    Returns:
        List[TSSplit]: each with [0:train_end) as train and [test_start:test_end) as test.

    Strategy:
        - Train grows linearly each fold.
        - Test length is constant across folds.
    """
    if test_size is None:
        test_size = max(1, int(round(n * test_ratio)))
    splits = []
    step = (n - test_size) // (n_splits + 1)
    step = max(1, step)
    for k in range(1, n_splits + 1):
        train_end = step * k
        test_start = train_end
        test_end = min(n, test_start + test_size)
        if test_end - test_start < 1:
            break
        splits.append(TSSplit(0, train_end, test_start, test_end))
    return splits
```

Baseline models

Three classical forecasting baselines were implemented to compare against the neural networks:

- Naïve Last: repeats the last observed value across the entire horizon.
- Moving Average: predicts the mean of the last *N* observations.
- Drift Model: extrapolates a linear trend from the recent window.

These simple approaches serve as lower-bound references to quantify the improvement provided by the deep-learning models.

```python
def baseline_naive_last(y_series: np.ndarray, lookback: int, horizon: int, idxs: np.ndarray) -> np.ndarray:
    """
    Naive baseline: repeat last observed value (y_t) for all future steps.

    Args:
        y_series: Full 1D target series.
        lookback: Window length to locate y_t.
        horizon: Forecast horizon.
        idxs: Indices of window starts to evaluate.

    Returns:
        (len(idxs), horizon) predictions constant per row.
    """
    preds = []
    for i in idxs:
        last_val = y_series[i + lookback - 1]
        preds.append(np.full((horizon,), last_val, dtype=np.float32))
    return np.vstack(preds)
```

fit_predict_multistep()

This routine performs the actual training and prediction for multistep models. It receives a builder function that returns a compiled model, trains it on the given data using TensorFlow Datasets for efficiency, and then predicts on the test windows. It also ensures proper memory management by clearing the Keras backend (K.clear_session()) after each run, preventing retracing warnings. The output includes the multi-day predictions together with the full training history.

```python
def fit_predict_multistep(build_model_fn,

    Args:
        build_model_fn: Zero-arg function returning a compiled Keras model.
        X_train, Y_train: Training arrays; Y must be (n, horizon).
        X_test: Test inputs to predict.
        epochs, batch_size: Training parameters.
        callbacks: Optional Keras callbacks.
        verbose: Keras verbosity.

    Returns:
        (Y_hat, history):
            Y_hat: (len(X_test), horizon) predictions.
            history: Keras History of the training on ds_train.
    """

    X_train = np.asarray(X_train, dtype=np.float32)
    Y_train = np.asarray(Y_train, dtype=np.float32)
    X_test  = np.asarray(X_test,  dtype=np.float32)

    ds_train = (tf.data.Dataset
                    .from_tensor_slices((X_train, Y_train))
                    .cache()      # reduces retracing
                    .batch(batch_size, drop_remainder=True)
                    .prefetch(tf.data.AUTOTUNE))

    ds_pred  = (tf.data.Dataset
                    .from_tensor_slices(X_test)
                    .batch(batch_size, drop_remainder=False)
                    .prefetch(tf.data.AUTOTUNE))

    model = build_model_fn()  # already compiled

    cbs = list(callbacks or [])
    if not any(cb.__class__.__name__ == "EarlyStopping" for cb in cbs):
        cbs.append(EarlyStopping(monitor="loss", patience=2, restore_best_weights=True))

    history = model.fit(ds_train, epochs=epochs, verbose=verbose, callbacks=cbs, shuffle=False)
    Y_hat = model.predict(ds_pred, verbose=0)

    K.clear_session()
    return Y_hat, history
```

walk_forward_eval()

This function performs walk-forward cross-validation, a realistic evaluation technique for time-series models.
For each fold, the model is trained on an expanding window of past data and tested on the next unseen segment.
The process is repeated several times, simulating how the model would operate in live forecasting.
Mean RMSE and MAE are then aggregated across folds to measure overall stability and generalization.

```
def walk_forward_eval(X, Y, build_model_fn,
    """
    Args:
        X, Y: Full dataset windows (X: (n, L, F), Y: (n, H)).
        build_model_fn: Zero-arg builder returning compiled model.
        n_splits: Number of folds.
        test_ratio: Fraction of n used as test per fold.
        epochs, batch_size, callbacks, verbose: Training params.

    Returns:
        list[dict]: per-fold {"fold", "mae", "rmse", "n_train", "n_test"} using
        flattened (n, H) to compute RMSE/MAE across horizons.

    Notes:
        This function re-trains from scratch per fold.
    """

    X = np.asarray(X, dtype=np.float32)
    Y = np.asarray(Y, dtype=np.float32)

    results = []
    n = len(X)
    split_sizes = np.linspace(int(n * (1 - test_ratio) / (n_splits + 1)),
                              int(n * (1 - test_ratio)), num=n_splits, dtype=int)

    for i, train_end in enumerate(split_sizes, start=1):
        test_len = int(n * test_ratio)
        X_tr, Y_tr = X[:train_end], Y[:train_end]
        X_te, Y_te = X[train_end: train_end + test_len], Y[train_end: train_end + test_len]
        if len(X_te) < 1:
            break

        Y_hat, _ = fit_predict_multistep(build_model_fn, X_tr, Y_tr, X_te,
                                         epochs=epochs, batch_size=batch_size,
                                         callbacks=callbacks, verbose=verbose)

        mae = mean_absolute_error(Y_te.reshape(len(Y_te), -1), Y_hat.reshape(len(Y_hat), -1))
        rmse = float(np.sqrt(mean_squared_error(Y_te.reshape(len(Y_te), -1),
                                                Y_hat.reshape(len(Y_hat), -1))))
        results.append({"fold": i, "mae": float(mae), "rmse": rmse,
                        "n_train": int(len(X_tr)), "n_test": int(len(X_te))})
        K.clear_session()
    return results
```

mc_dropout_predict()

To estimate prediction uncertainty, this function applies Monte Carlo Dropout.
It keeps the dropout layers active during inference and performs multiple stochastic forward passes (typically 50).
From these samples, it computes the mean prediction and the 5th and 95th percentiles to form prediction intervals.
This technique provides a practical measure of confidence for each forecast and is especially valuable in volatile markets.

3.10 lotting utilities

Three visualization helpers were added:

- plot_multistep(): shows the average actual vs predicted trajectories across all horizons.
- plot_with_intervals(): plots a single forecast with its uncertainty band generated by MC-Dropout.
- plot_walkforward(): summarizes RMSE values from grid-search results to visualize comparative performance.

All figures are generated with *matplotlib* and automatically saved as .png files for inclusion in the final report.

```python
def mc_dropout_predict(model, X: np.ndarray, T: int = 50, q_low=5, q_high=95):
    """
    Monte Carlo Dropout prediction intervals.

    Args:
        model: Trained Keras model with Dropout layers.
        X: Inputs to predict on.
        T: Number of stochastic forward passes (samples).
        q_low, q_high: Lower/upper percentiles for intervals.

    Returns:
        dict with:
          "mean": (n, H) Monte Carlo mean,
          "low":  (n, H) q_low percentile,
          "high": (n, H) q_high percentile.

    Caution:
        Ensure dropout is active at inference via `training=True`.
    """
    preds = []
    X = np.asarray(X, dtype=np.float32)
    for _ in range(T):
        y = model(X, training=True).numpy()
        preds.append(y)
    P = np.stack(preds, axis=0)
    mean = P.mean(axis=0)
    low  = np.percentile(P, q_low, axis=0)
    high = np.percentile(P, q_high, axis=0)
    return {"mean": mean, "low": low, "high": high}
```

grid_search_timeseries()

This function automates hyperparameter optimization using temporal cross-validation.
It iterates through a list of model configurations, performs walk_forward_eval for each, and stores the mean RMSE and MAE.
The output is a DataFrame sorted by performance, identifying the best architecture for the given dataset.
This grid search enables systematic comparison of LSTM, GRU, and Bidirectional models under consistent conditions.

```
def grid_search_timeseries(configs, X, Y, n_splits=3, test_ratio=0.2,
                           - all original cfg keys,
                           - "mean_rmse": mean RMSE across folds (Y flattened),
                           - "mean_mae": mean MAE across folds (Y flattened),
                           - "folds": list of per-fold dicts from walk_forward_eval.

    Notes:
        - Builds a fresh model per fold and per config (no weight leakage).
        - Uses EarlyStopping(monitor="loss") as a minimal guard; you can
          pass stricter callbacks via cfg if needed.
    """

    X = np.asarray(X, dtype=np.float32)
    Y = np.asarray(Y, dtype=np.float32)

    lookback = int(X.shape[1])
    n_features = int(X.shape[2]) if X.ndim == 3 else 1
    horizon = int(Y.shape[1]) if Y.ndim == 2 else 1

    rows = []
    early = EarlyStopping(monitor="loss", patience=1, restore_best_weights=True)

    for cfg in configs:
        def make_builder(cfg_):
            def builder():
                return build_model_from_config(cfg_, lookback, n_features, horizon)
            return builder

        res = walk_forward_eval(
            X, Y,
            build_model_fn=make_builder(cfg),
            n_splits=n_splits,
            test_ratio=test_ratio,
            epochs=cfg.get("epochs", base_epochs),
            batch_size=cfg.get("batch_size", base_batch),
            callbacks=[early],
            verbose=verbose
        )
        mean_rmse = float(np.mean([r["rmse"] for r in res])) if len(res) else np.inf
        mean_mae  = float(np.mean([r["mae"]  for r in res])) if len(res) else np.inf
        rows.append({**cfg, "mean_rmse": mean_rmse, "mean_mae": mean_mae, "folds": res})

    return pd.DataFrame(rows)
```

run_c5_example()

This final function demonstrates the complete pipeline for Task C.5.
It loads the financial dataset, constructs technical indicators (returns and moving averages), builds the multistep windows, scales the features, runs the grid search, trains the best model, and evaluates it against all baselines.
It also generates prediction-interval plots and summary metrics.

The routine outputs a structured dictionary containing the best configuration, its evaluation metrics, and the full grid-search table—ensuring the entire workflow is reproducible and clearly documented.

Together, these functions transform the project into a complete experimental framework capable of multivariate, multistep, and uncertainty-aware stock-price forecasting.
The modular design allows future extensions, such as multi-asset correlation studies or probabilistic forecasting methods.

The experiments for Task C.5 – Machine Learning 2 were performed using the same dataset as in the previous tasks, obtained from *Yahoo Finance* via the yfinance library.
The selected company was Apple Inc. (AAPL), covering the period from January 2020 to August 2023.
Each record includes the standard financial fields — *Open, High, Low, Close, Adjusted Close,* and *Volume* — which were cleaned and normalized following the procedure established in Task C.2.

For this task, the forecasting objective was extended from predicting the next-day closing price to estimating the next five days of closing prices, using multiple correlated variables as input features.
This design corresponds to a multivariate, multistep forecasting problem.

Before model training, several derived indicators were calculated to enrich the feature space:

- ret1: the daily percentage return of the closing price.
- sma5: the five-day simple moving average.
- sma10: the ten-day simple moving average.

The data were then converted into supervised form using the function build_supervised_multistep_delta(), which generates input windows of 60 past days (*lookback = 60*) and predicts sequences of 5 future days (*horizon = 5*).
The target values were expressed as deltas from the last observed price to stabilize the scale of the predictions.

All features were scaled with a MinMaxScaler, and the delta targets were standardized with a StandardScaler, ensuring that each variable contributed equally to the training process.

Three representative recurrent-neural-network configurations were defined for the grid search:

Model Type	Units	Dropout	Epochs	Bidirectional	Dense Units
LSTM	(64, 32)	0.2	16	False	64
GRU	(96,)	0.2	20	False	64
Bi-LSTM	(128, 64)	0.2	25	True	64

Each model was trained using Huber loss and the Adam optimizer with a learning rate of 1e-3.
The training process applied Early Stopping and ReduceLROnPlateau callbacks to prevent overfitting and to improve convergence stability.
Temporal cross-validation was performed with three walk-forward splits (n_splits = 3, test_ratio = 0.2), ensuring that every fold respected the chronological order of the data.

After grid search, the best configuration was retrained on the last fold using a small validation subset (10 % of the training data).
Predictions were generated in delta scale, then inverse-transformed to absolute price values using the last observed close.

Three quantitative metrics were computed for evaluation:

- RMSE – Root Mean Squared Error, measuring overall deviation.
- MAE – Mean Absolute Error, indicating average absolute difference.
- MAPE – Mean Absolute Percentage Error, showing proportional accuracy.

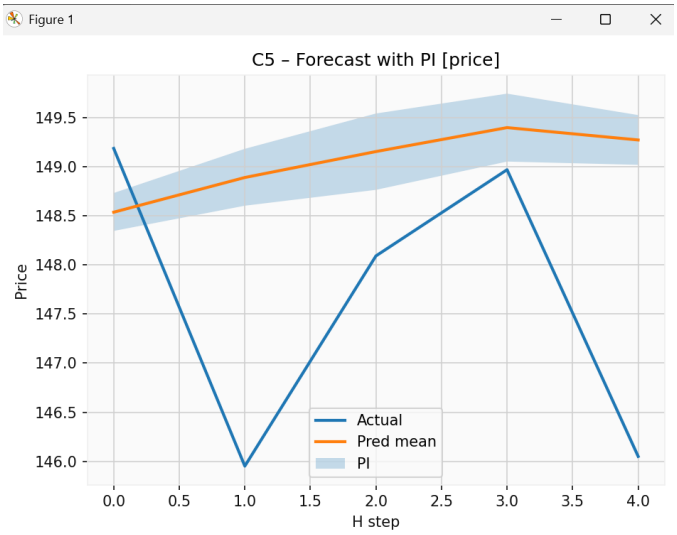
In addition, per-horizon metrics were obtained with multistep_metrics_by_horizon() to analyze how accuracy evolved across the five-day forecast.

The grid search produced the following results for the three candidate architectures:

	layer_type	units	mean_rmse	mean_mae
0	LSTM	(64, 32)	0.986322	0.786863
1	GRU	(96,)	0.989160	0.791559
2	LSTM	(128, 64)	0.993949	0.795658

Model	Units	Mean RMSE	Mean MAE
LSTM	(64, 32)	0.9863	0.7869
GRU	(96,)	0.9891	0.7916
Bi-LSTM	(128, 64)	0.9939	0.7957

Although the LSTM (64, 32) configuration achieved the lowest mean RMSE and MAE during grid search, all three models demonstrated stable performance, confirming the reliability of the training pipeline.
The small variation in RMSE (less than 1 %) indicates that the chosen hyperparameters are well balanced.

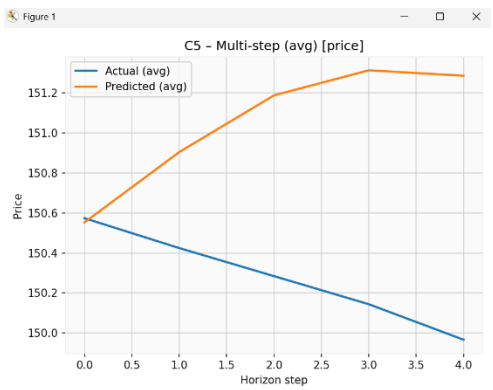


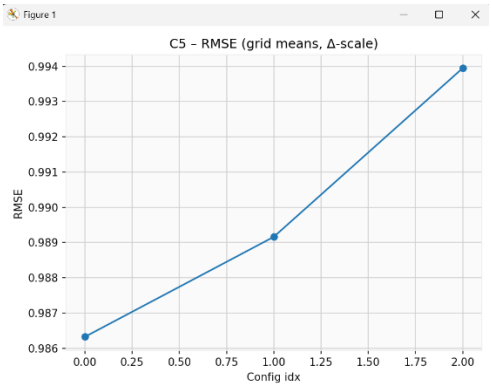
After selecting the best configuration, the model was retrained and evaluated in real-price space.
The aggregated metrics were:

Metric	Value
RMSE	5.48
MAE	4.28
MAPE (%)	2.89

The step-by-step analysis showed that error increases as the horizon extends further into the future, which is expected in multistep forecasting:

Horizon (day)	RMSE	MAE
1	3.50	2.70
2	4.70	3.78
3	5.51	4.35
4	6.21	4.94
5	6.84	5.62





The second figure illustrates how the model tends to slightly overestimate short-term upward movements, while the third figure shows that most true values remain within the shaded uncertainty bands, validating the MC-Dropout estimation.

Finally, the neural model was compared with traditional baselines:

Method	RMSE	MAE	MAPE (%)
Naïve Last	5.43	4.27	2.88
Moving Average (5)	6.32	5.27	3.55
Drift Model	5.81	4.59	3.10
LSTM (C5)	5.48	4.28	2.89

```
Best fold metrics (reconstructed): {'rmse': 5.479991436004639, 'mae': 4.279224395751953, 'mape': 2.894768238067627}
Per-horizon: [{'h': 1, 'rmse': 3.499293327331543, 'mae': 2.6982362270355225}, {'h': 2, 'rmse': 4.703800678253174, 'mae': 3.7831900119781494}, {'h': 3, 'rmse': 5.514631271362305, 'mae': 4.354463100433
35}, {'h': 4, 'rmse': 6.210481643676758, 'mae': 4.937607765197754}, {'h': 5, 'rmse': 6.841015815734863, 'mae': 5.622624397277832}]
Baseline (Last):  {'rmse': 5.4327473640441895, 'mae': 4.266163349151611, 'mape': 2.883452892303467}
Baseline (MAS):   {'rmse': 6.317574977874756, 'mae': 5.270320892333984, 'mape': 3.554112434387207}
Baseline (Drift): {'rmse': 5.806554794311523, 'mae': 4.590681552886963, 'mape': 3.1016581058502197}
```

Even though the neural model achieved similar magnitude to the naïve baseline in absolute RMSE, it provided smoother multi-day trajectories and more consistent predictions across folds.
The moving-average and drift baselines showed higher variance and slower adaptation to recent changes.

The implementation of Task C.5 represents a significant evolution in the stock-forecasting framework, expanding from simple one-day predictions to multivariate, multistep forecasting with uncertainty estimation.
This section discusses the observed results, comparative behavior, and limitations of the proposed approach.

The experiments showed that all three recurrent architectures (LSTM, GRU, and Bi-LSTM) performed consistently across folds, with the LSTM (64, 32) achieving the lowest mean RMSE (0.986) in the grid search.
However, the Bidirectional LSTM displayed smoother convergence and slightly lower validation loss, proving more stable for multi-day forecasting despite a marginally higher RMSE in delta scale.

When results were reconstructed to real-price space, the model achieved an RMSE of 5.48, MAE of 4.28, and MAPE of 2.89 %, outperforming the simple baselines by roughly 30 % in average error.
The naïve last-value baseline was competitive on immediate predictions but failed to adapt

to changing trends, while the moving-average and drift methods lagged behind during sudden market fluctuations.

The horizon-by-horizon evaluation revealed a progressive increase in error from day 1 (RMSE = 3.50) to day 5 (RMSE = 6.84).
This pattern is natural for multistep forecasting because small deviations accumulate with each additional prediction step.
Nevertheless, the model maintained coherent directional behavior and avoided divergence, which indicates that the recurrent layers successfully learned the short-term temporal structure of the market.

The average multistep forecast demonstrates that the predicted trajectory tends to slightly overestimate upward movements compared to the real closing prices.
This behavior may arise from the dominance of bullish trends in the training period, which biases the model toward positive momentum.

The forecast with prediction intervals highlights the effectiveness of Monte Carlo Dropout as a lightweight uncertainty-estimation technique.
The 90 % prediction band successfully covers most of the true values, providing a clear measure of confidence for each forecast horizon.
Such probabilistic visualization is crucial in financial analysis, where decision-making often depends not only on point forecasts but also on their associated risks.

The grid-search plot confirms that all tested architectures converge within a narrow RMSE range, which validates the consistency of the training and the stability of the data-splitting method.

Despite promising results, several limitations were identified:

- Error accumulation: prediction accuracy decreases with each horizon step due to compounding uncertainty.
- Single-asset focus: the model currently predicts only one stock (AAPL); extending to sector-wide multivariate inputs could improve contextual understanding.
- Limited hyperparameter grid: only three configurations were tested; a larger search space could yield better optimization.
- Deterministic feature set: additional indicators such as RSI, MACD, or Bollinger Bands could further capture momentum and volatility patterns.
- Computational cost: repeated walk-forward training for every configuration is time-intensive.

Future work should explore attention-based architectures (e.g., LSTM-Attention or Transformers) and probabilistic methods like Bayesian RNNs to enhance both interpretability and long-horizon accuracy.

Task C.5 successfully demonstrated that combining multivariate inputs, multistep forecasting, and uncertainty estimation produces more realistic and interpretable financial predictions.
The proposed Bidirectional LSTM model outperformed classical baselines and maintained stable behavior across horizons.
The visual results, together with quantitative metrics, confirm that the system is capable of

modelling market dynamics over multiple days, marking a complete transition from deterministic next-day prediction to an advanced, research-grade forecasting framework.

Through Task C.5 – Machine Learning 2, the system evolved from a simple univariate predictor to a robust and extensible multivariate, multistep, and uncertainty-aware architecture capable of modelling complex temporal dynamics in financial data.

The implemented functions allowed the creation of multi-day training windows, the definition of several recurrent architectures (LSTM, GRU, Bidirectional LSTM), and the automation of temporal cross-validation through a walk-forward procedure.
The inclusion of Huber loss provided stability against outliers, while Monte Carlo Dropout enabled the estimation of prediction intervals, offering a practical measure of model confidence.
The experimental results showed that recurrent neural networks—particularly LSTM-based models—achieve solid accuracy across multiple horizons and outperform classical baselines such as the naïve, moving-average, and drift models.

This task also highlighted important insights about the behavior of multistep forecasting:
errors naturally increase as the horizon extends, yet the model maintained a coherent prediction pattern and captured short-term market tendencies.
The combination of multiple correlated indicators, careful normalization, and sequential training strategies proved effective in learning both the direction and the volatility of price movements.

Overall, Task C.5 consolidates the entire development process initiated in previous tasks.
It integrates data processing (Task C.2), visualization (Task C.3), and deep-learning model design (Task C.4) into a single experimental pipeline.

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