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https://gabriel-vzh2vs.github.io/SYS3062-Website/

UVA's Systems and Information Engineering

January 20th, 2026

SYS 3062: Lab 1

Darts on a Board: Monte Carlo

One of the reasons is so that this does not happen to you!

> *"We have developed a novel mathematical model using trapezoids for estimating the area under curves from various metabolic studies", 1993: A Mathematical Model for the Determination of Total Area Under Glucose Tolerance and Other Metabolic Curves.*
>
> Mary M. Tai, MS, EDD

Hmmm, this might sound familiar to some of the audience.

Tai's Formula Is the Trapezoidal Rule

We were disturbed to read the article by M. M. Tai titled "A Mathematical Model for the Determination of Total Area Under Glucose Tolerance and Other Metabolic Curves" (1). The author seems to claim "Tai's formula" as a new method of computing area under a curve. The formula given is simply the trapezoidal rule, published in many beginning calculus texts (for example, see Swokowski [2] or Faires and

Imagine that you are a retirement account manager. Your task is to analyze the total value of an account with a fixed return based of on a 6% change yearly.

Your job is to use a method from this course that allows you to identify future projections for the total value of the account over a period of ten years.

Definition

A **Monte Carlo Method** is a set of numerical techniques used to solve complex systems (deterministic or stochastic) by evaluating them by sampling from random variables.

Applicable Systems:

> Integrals and Differential Equations

> Complex Queuing Networks

Key Mechanics

> **Randomness:** Uses pseudo-random number generation to sample from probability distributions.

> **Convergence:** Relies on the *Law of Large Numbers*, as we increase the number of samples, the result converges to the parameter's value.

1777: The Early Precursor

> **Georges-Louis Leclerc, Comte de Buffon** proposed the famous "Needle Experiment."

> **The Experiment:** Estimating π by dropping needles on a lined surface.

> **Significance:** It was the first demonstration that *random sampling* could be used to solve deterministic mathematical problems.

1946: The Modern Insight

> **Stanislaw Ulam** (mathematician at Los Alamos) was recovering from illness and playing solitaire.

> **The Question:** What is the probability of winning a game?

> **The Realization:** Instead of calculating complex combinatorics, it was easier to simply *play 100 games* and count the wins.
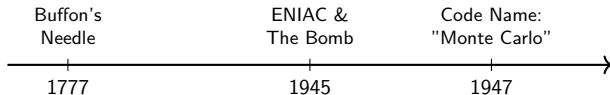
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Catalysts for Development (1945-1947)

Monte Carlo flourished due to two events:

1. The need to model neutron diffusion for the atomic bomb.
2. The invention of the ENIAC (the first electronic computer), which made thousands of simulations possible.

Why "Monte Carlo"?

> - The name was coined by physicist Nicholas Metropolis.
> - It was a code name inspired by Ulam's uncle, who frequently borrowed money to gamble at the Monte Carlo Casino in Monaco.

Buffon's Needle	ENIAC & The Bomb	Code Name: "Monte Carlo"
1777	1945	1947

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Objective

To estimate the value of the mathematical constant π using a stochastic (random) approach rather than traditional deterministic formulas.

Key Concepts:

- **Monte Carlo Method:** Using random sampling to solve numerical problems.
- **Geometric Probability:** Relating area to the likelihood of a random point.
- **Law of Large Numbers:** As we increase the number of trials (N), our estimate converges to the true value.

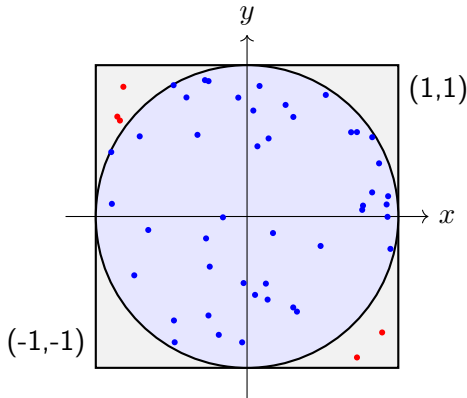*Today, we will simulate throwing thousands of "darts" to find π.*

The Setup:

> Imagine a square with side length $a = 2$.
> Inscribe a circle with radius $r = 1$.
> Area of Square $= 2 \times 2 = 4$.
> Area of Circle $= \pi r^2 = \pi(1)^2 = \pi$.

The Logic:

$$\frac{\text{Area}_{\text{circle}}}{\text{Area}_{\text{square}}} = \frac{\pi}{4}$$

$$\therefore \pi = 4 \times \left(\frac{\text{Hits in Circle}}{\text{Total Throws}} \right)$$

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Before coding, we should define the logic flow used to approximate the ratio $\pi/4$.

Algorithm 1 Monte Carlo π Estimation

1: $N \leftarrow$ Total number of darts
2: $count\_inside \leftarrow 0$
3: **for** $i = 1$ **to** N **do**
4:     $x \leftarrow$ random number between -1 and 1
5:     $y \leftarrow$ random number between -1 and 1
6:     $distance \leftarrow \sqrt{x^2 + y^2}$
7:     **if** $distance < 1$ **then**
8:         $count\_inside \leftarrow count\_inside + 1$
9:     **end if**
10: **end for**
11: $Ratio \leftarrow count\_inside/N$
12: $\pi_{estimate} \leftarrow 4 \times Ratio$

First, we need a function to generate random coordinates (x, y) within our square boundaries $[-1, 1]$.

Function: `throw_dart()`

```python
import random

def throw_dart():
    # Scale random(0,1) to range (-1,1)
    # Formula: -1 + 2 * random.random()

    x = -1 + 2 * random.random()
    y = -1 + 2 * random.random()

    return (x, y)
```

Next, we use the distance formula (Pythagorean theorem) to check if the dart landed inside the unit circle.

Function: `is_within_circle()`

```python
from math import sqrt

def is_within_circle(x, y):
    # Calculate distance from origin (0,0)
    dist = sqrt(x**2 + y**2)

    if dist < 1:
        return True
    else:
        return False
```

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We run the experiment N times. As N increases, the error decreases (Law of Large Numbers).
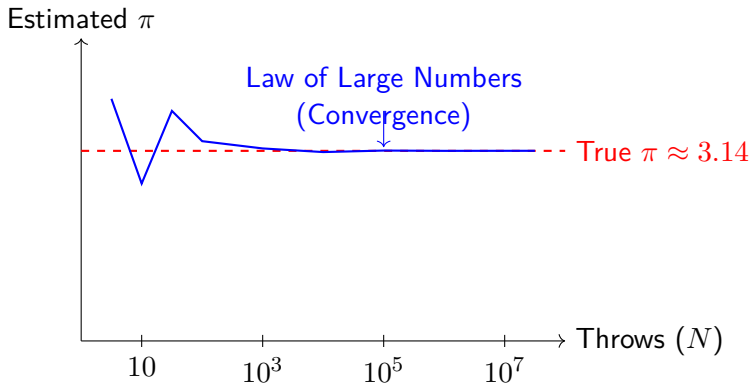
```python
def compute_pi(n_throws):
    count_inside = 0
    for i in range(n_throws):
        r1, r2 = throw_dart()
        if is_within_circle(r1, r2):
            count_inside += 1

    return 4 * (count_inside / n_throws)
```

Example Results:

> $N = 10 \rightarrow \pi \approx 4.0$ (High Error)

> $N = 1,000 \rightarrow \pi \approx 3.148$

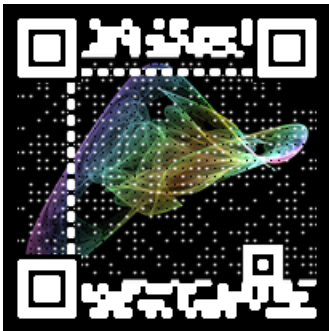> $N = 1,000,000 \rightarrow \pi \approx 3.1415$ (Converged)

The accuracy improves as we increase the number of "darts" (N).

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Now, it's your turn to implement Monte Carlo in Python or using XLRisk! Good Luck!

Throughout the course, I may ask for your feedback on:

> **Course Materials**
> **Lecture & Lab**
> **Assignments**

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First, I will thank you all again for coming to lab at 5 pm on a Tuesday!

And now, you need to do the following tasks:

> **Submit** your Python file to Gradescope by next week's lab!
> **Read** Prelab 2 for next week!

It's going to be an interesting semester!