2-3 Trees

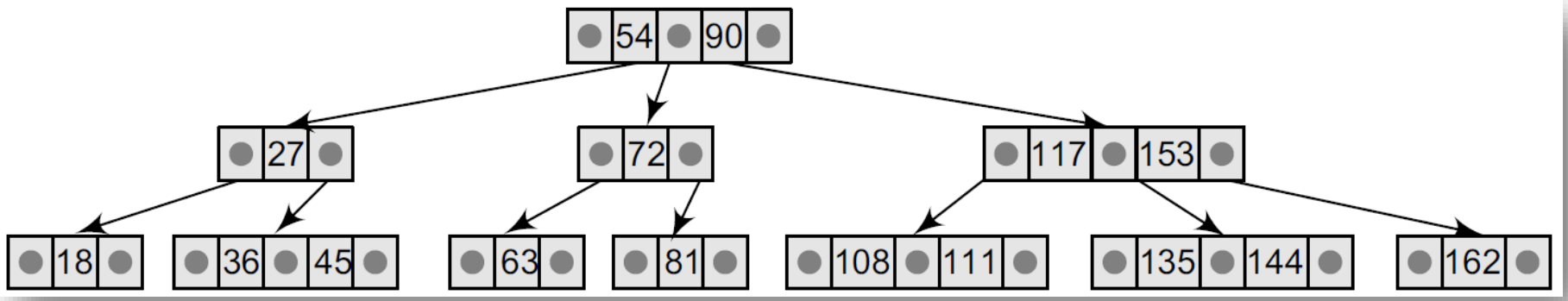
Kuan-Yu Chen (陳冠宇)

Review

- A B tree of order m is a tree with all the properties of an M-way search tree and has additional properties
  - The root node has at least two children
  - Every node in the B tree has at most (maximum) m children
  - Every node in the B tree except the root node has at least (minimum) $\left\lceil \frac{m}{2} \right\rceil$ children
    - Degree=4, at least 2 children, at least 1 key
    - Degree=5, at least 3 children, at least 2 key
  - All leaf nodes are at the same level

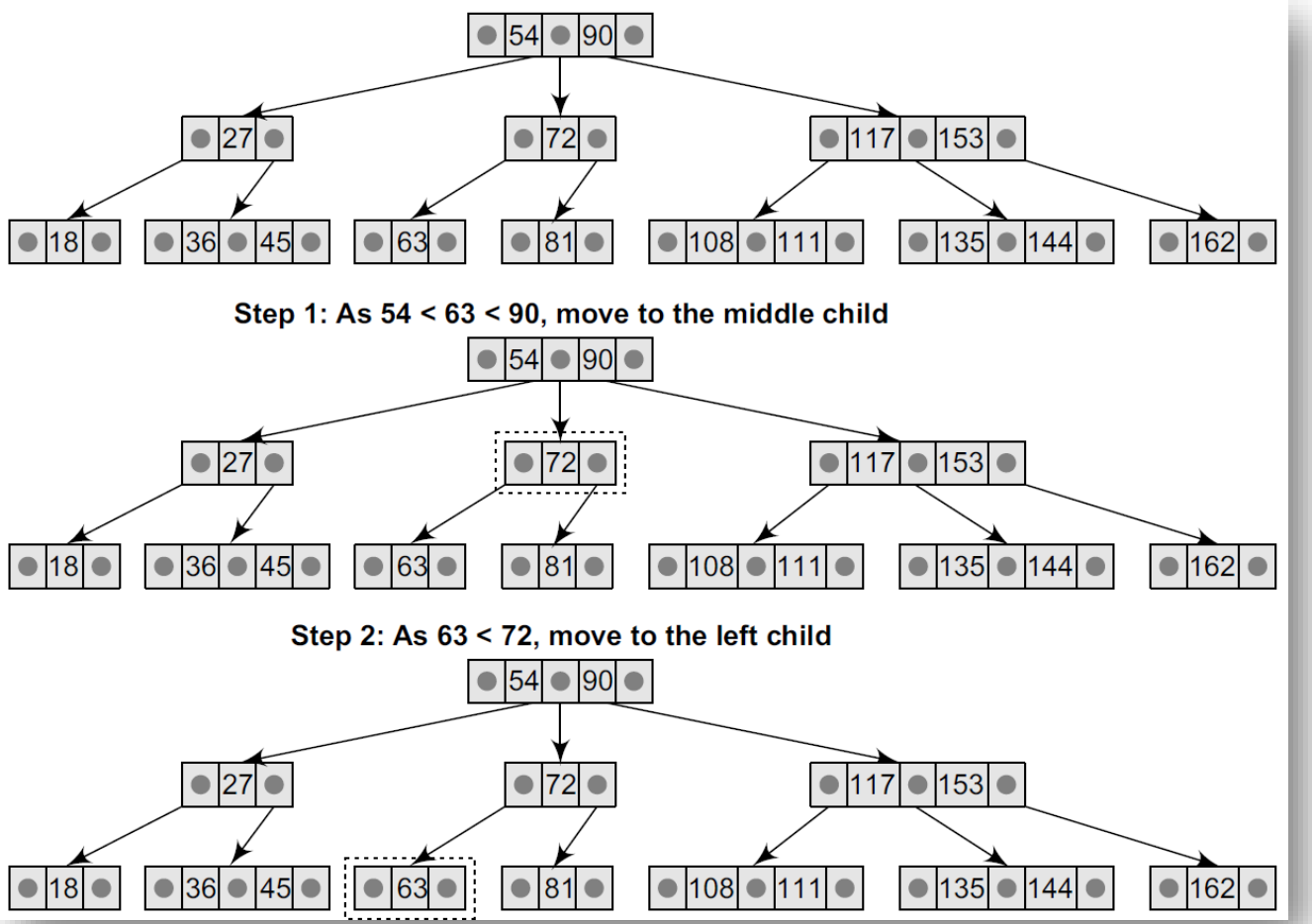
2-3 Trees

- In a 2-3 tree, proposed by John Hopcroft in 1970, each interior node has either two or three children
  - **2-3 tree is a B-tree of order 3**
  - Nodes with two children are called 2-nodes
    - The 2-nodes have one data value and two children
  - Nodes with three children are called 3-nodes
    - The 3-nodes have two data values and three children
  - All the leaf nodes are at the same level



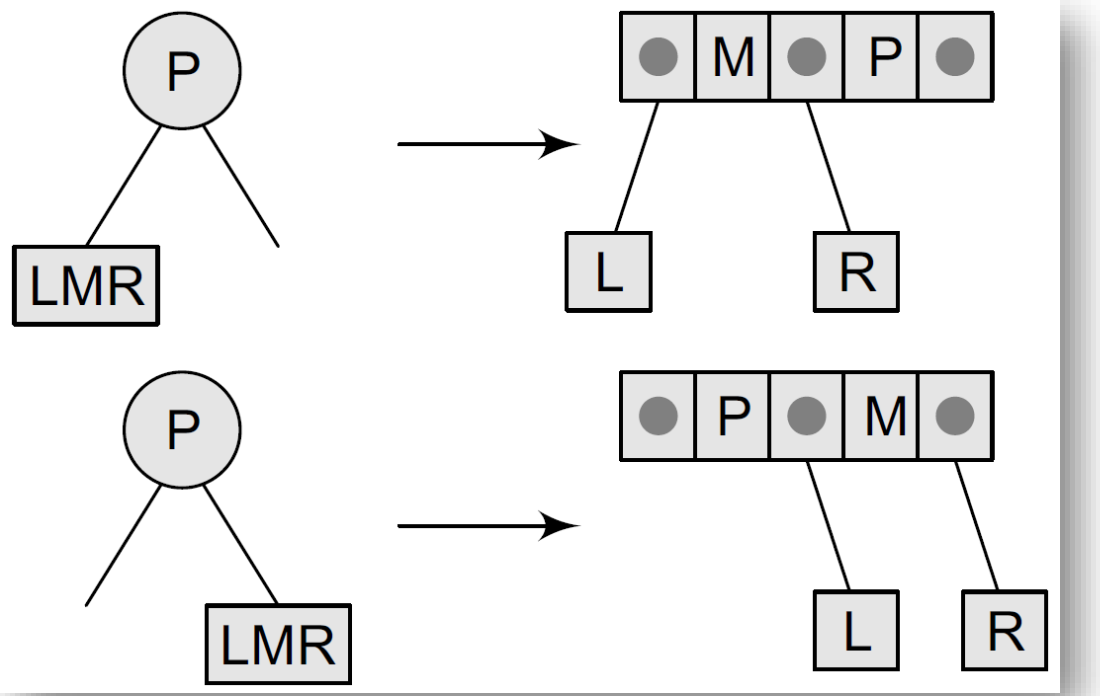
2-3 Trees – Searching

- The search operation is used to determine whether a data value is present in a 2-3 tree
  - Search for 63



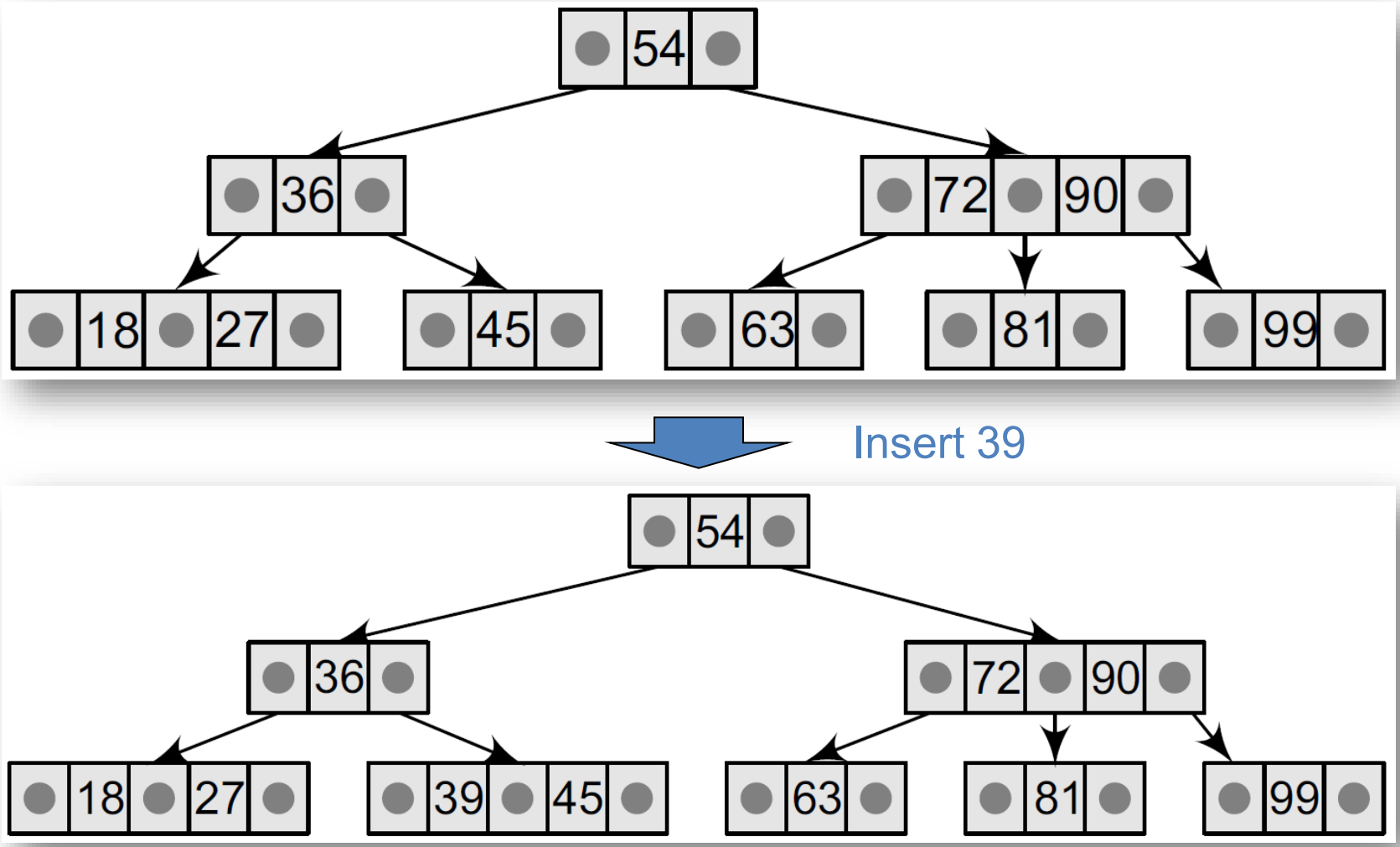
2-3 Trees – Insertion.

- To insert a new value in the 2-3 tree, an appropriate position of the value is located in one of the leaf nodes
  - If after insertion of the new value, the properties of the 2-3 tree do not get violated then insertion is over
  - If any property is violated then the violating node must be split
    - A node is split when it has three data values and four children



2-3 Trees – Insertion..

- Given a 2-3 tree, please insert the data values 39, 37, 42, 47 into the tree



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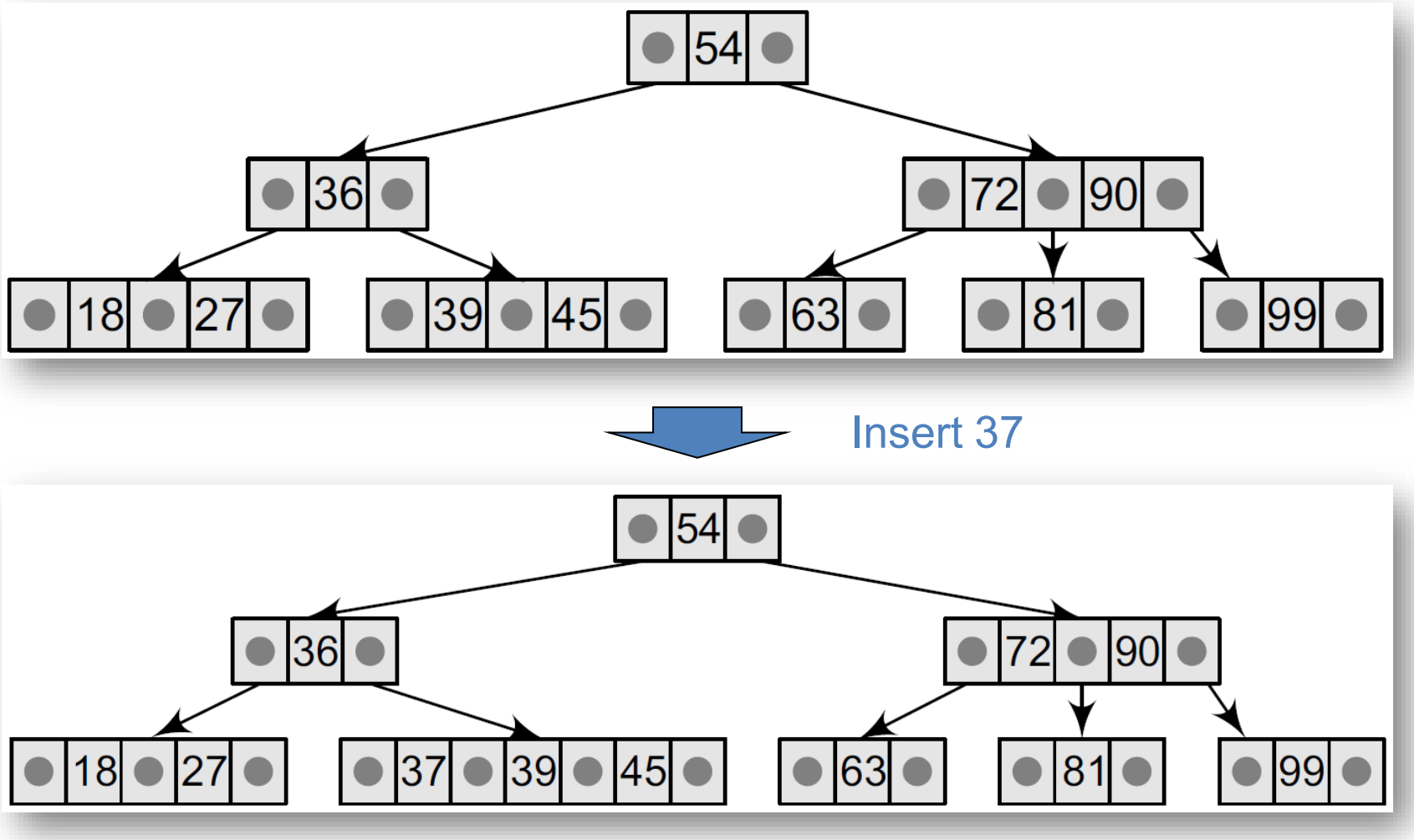

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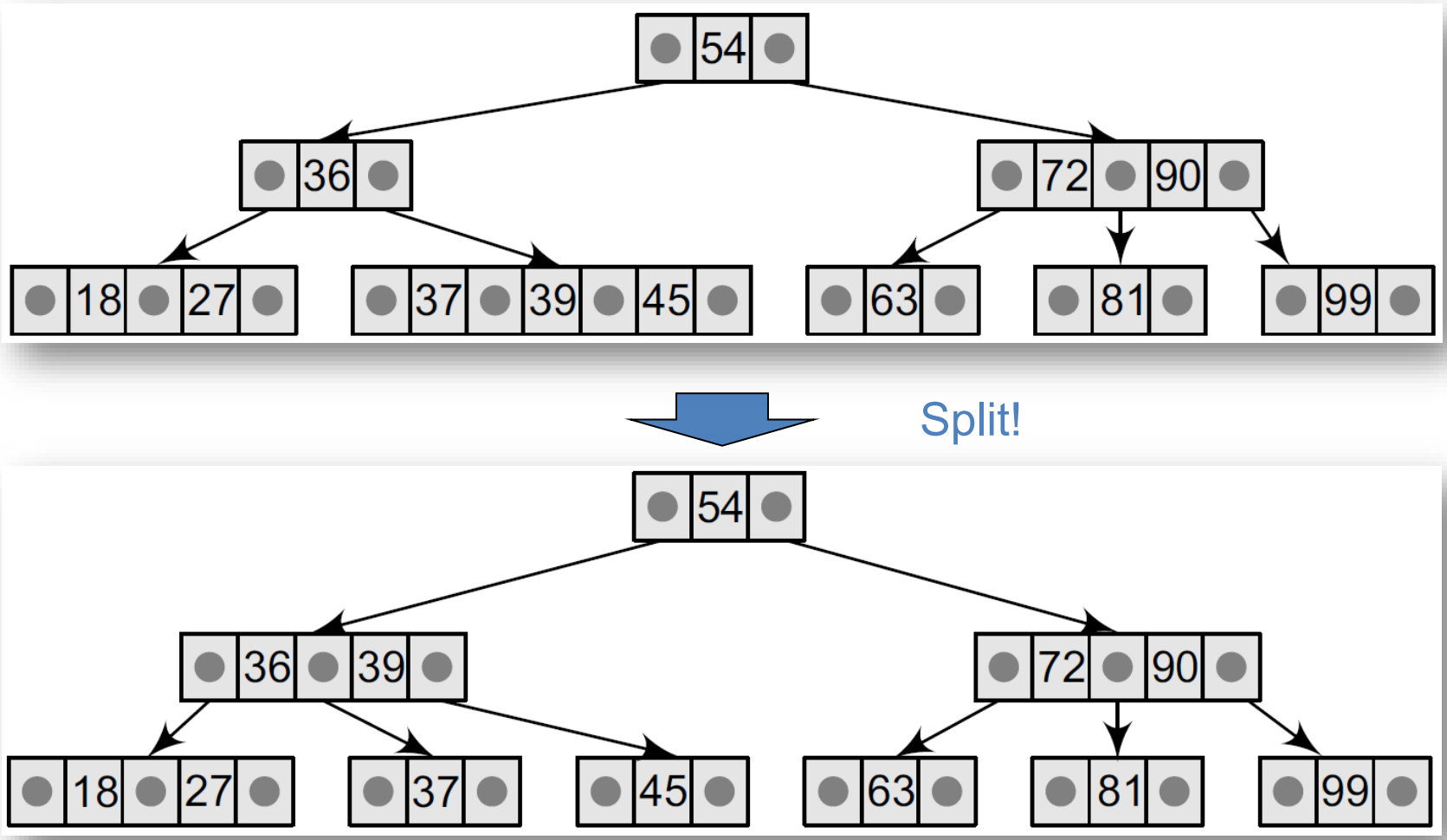

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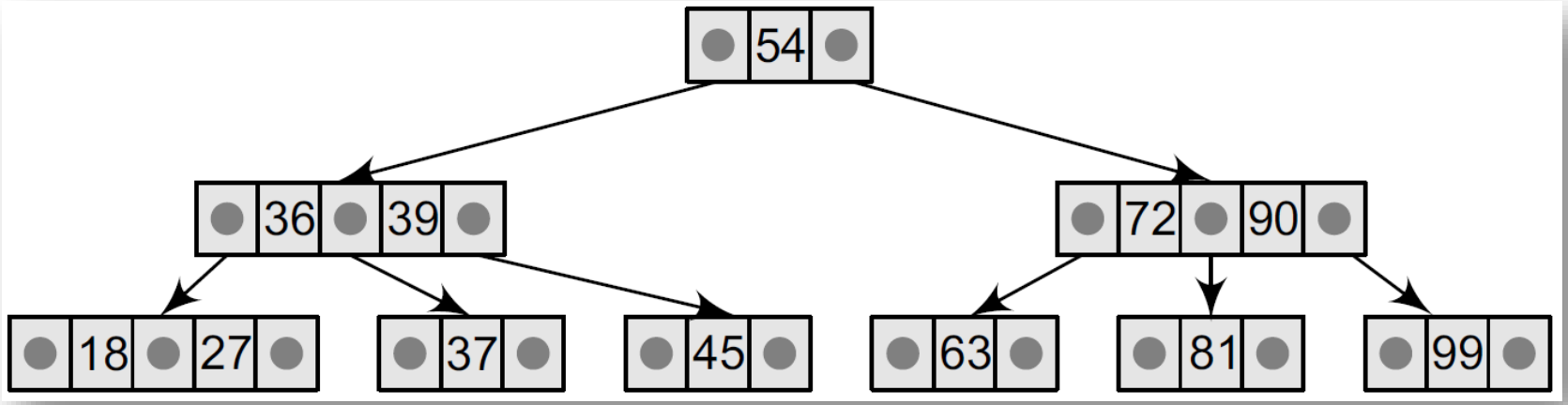
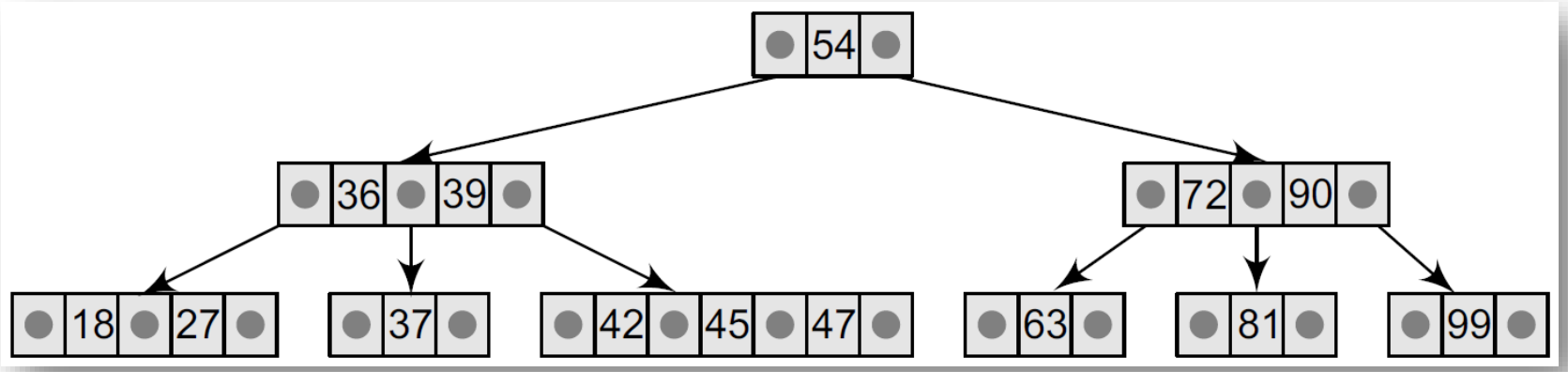


Insert 42 & 47



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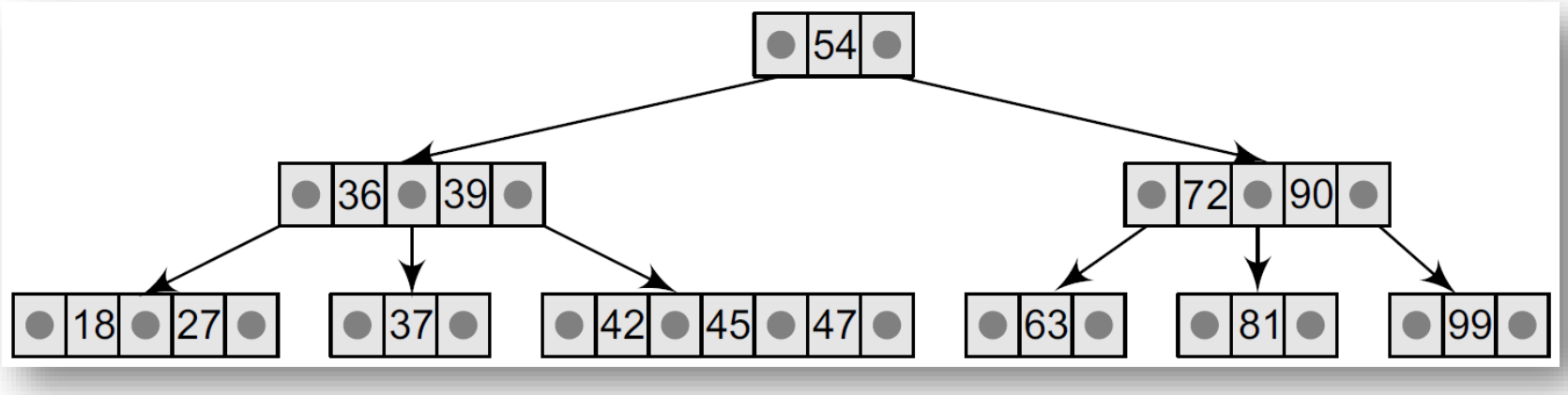
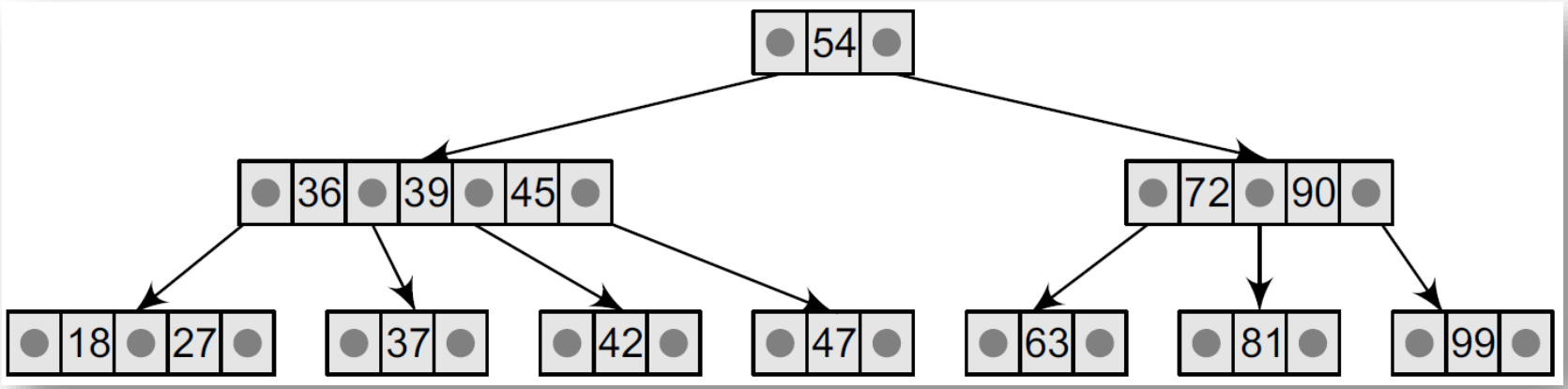


Split!



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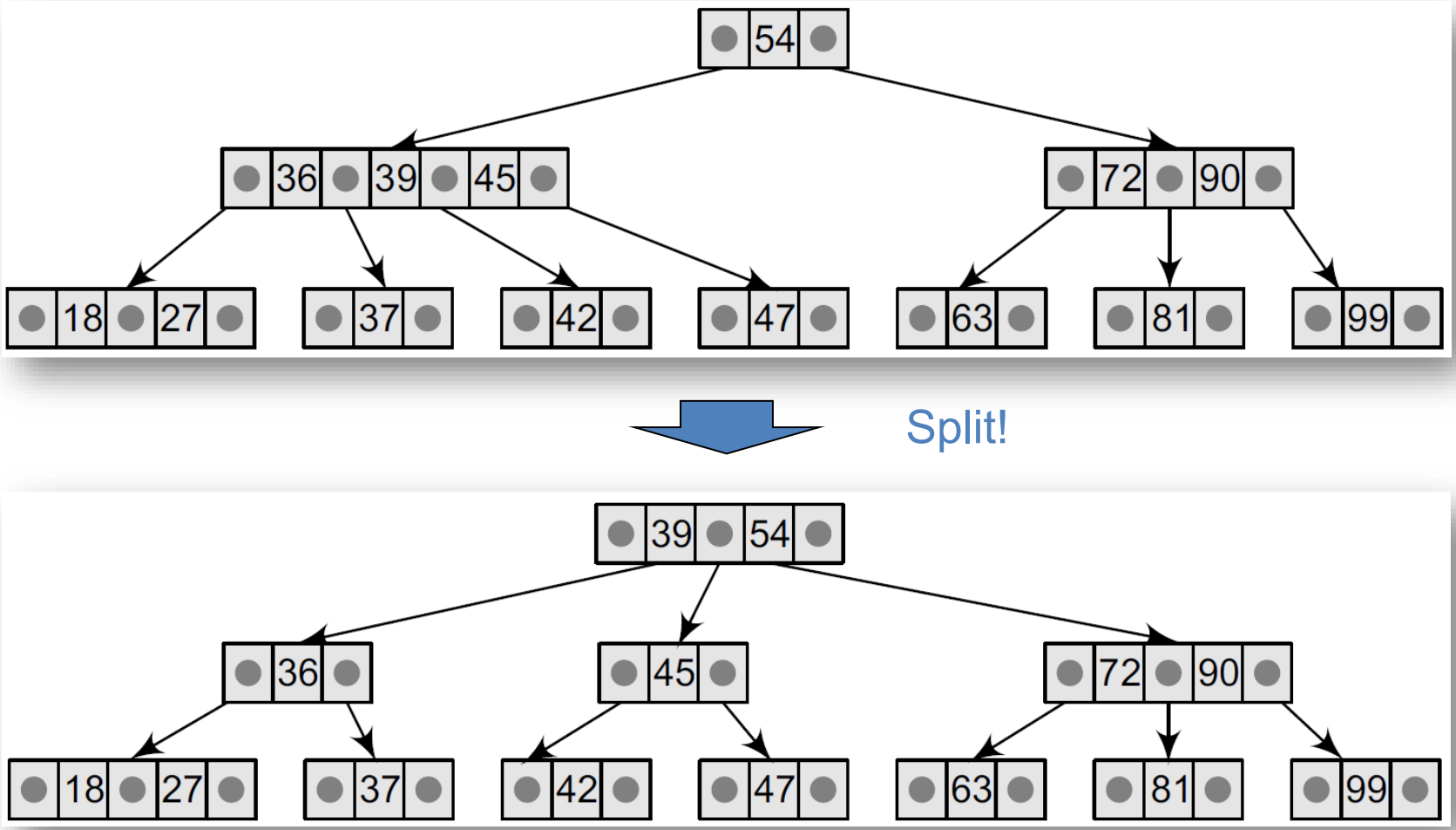
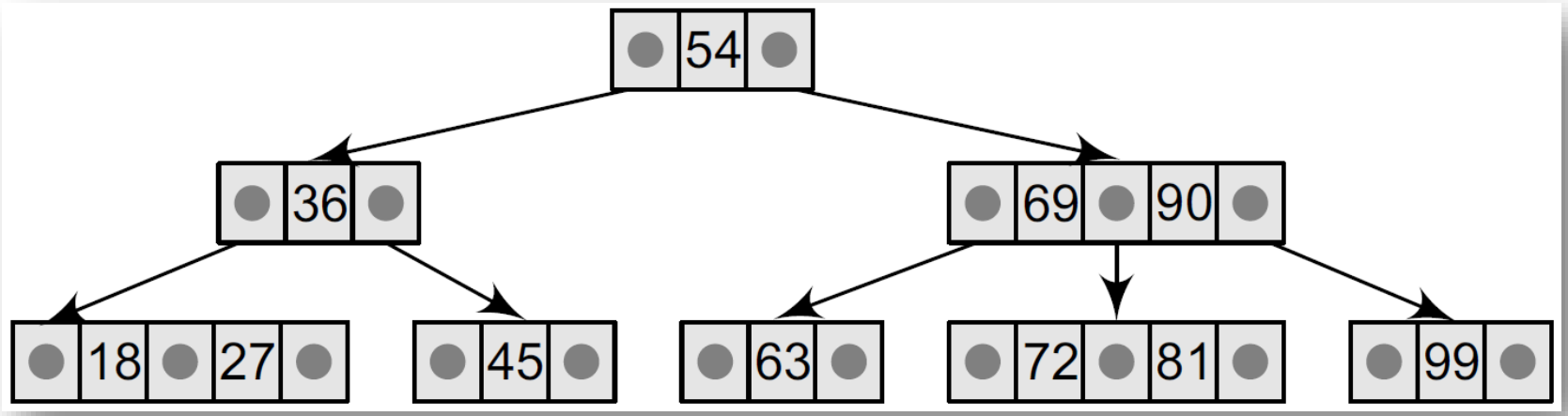


2-3 Trees – Deletion.

- To delete a value in internal node, it is replaced by its in-order successor and then removed
  - If a node becomes empty after deleting a value, it is then merged with another node to restore the property of the tree

- Given a 2-3 tree, please delete the values 69, 72, 99, 81



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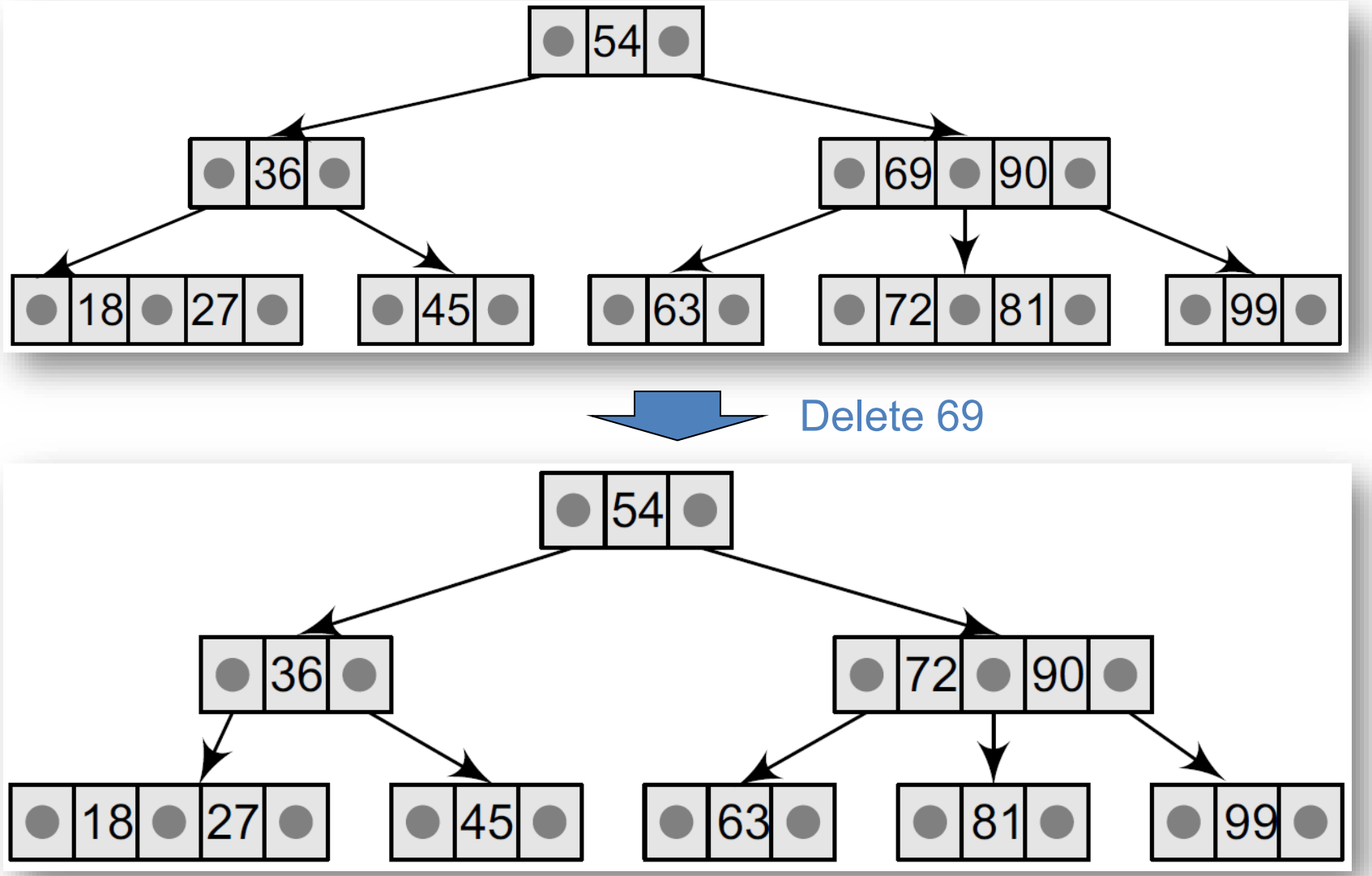

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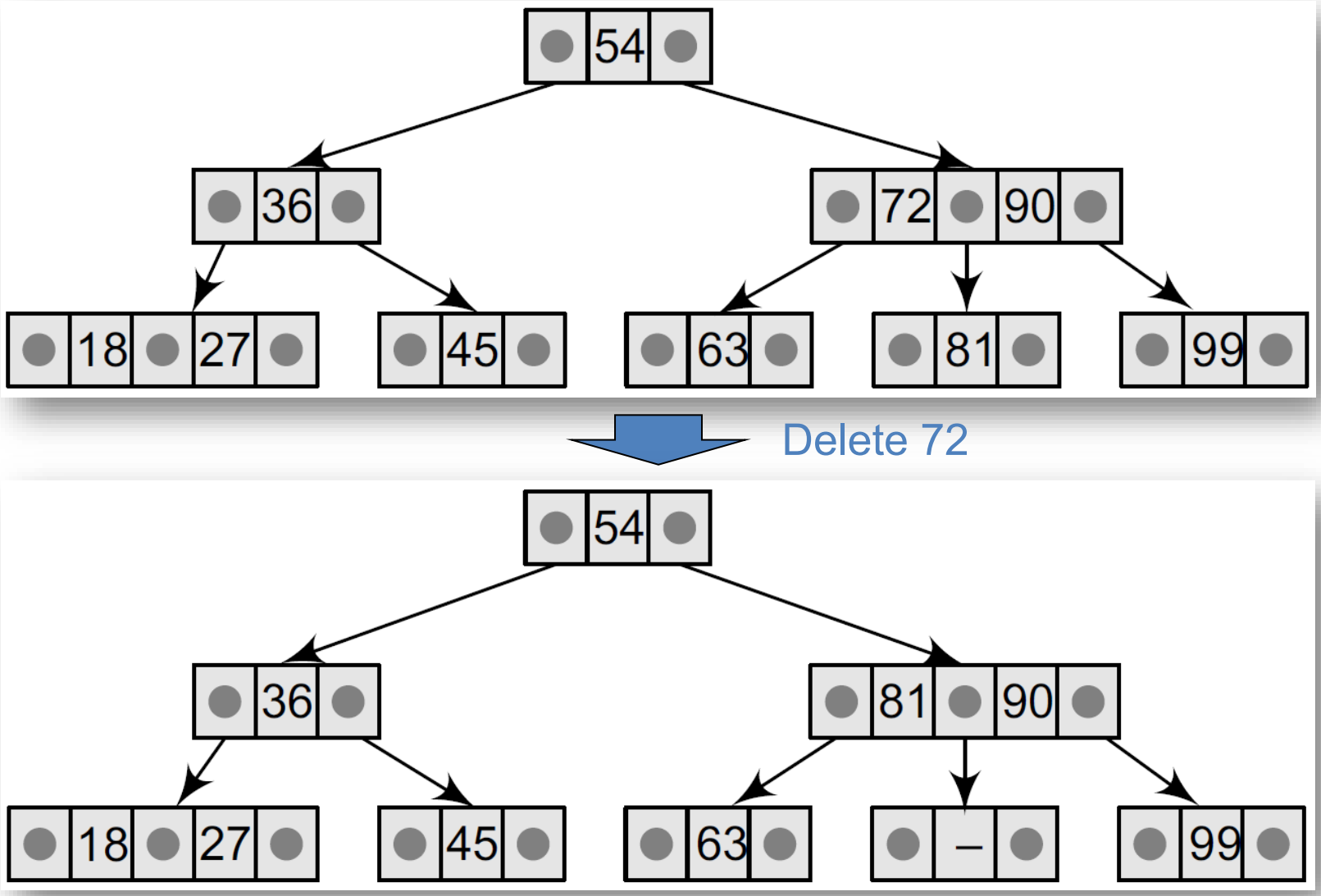

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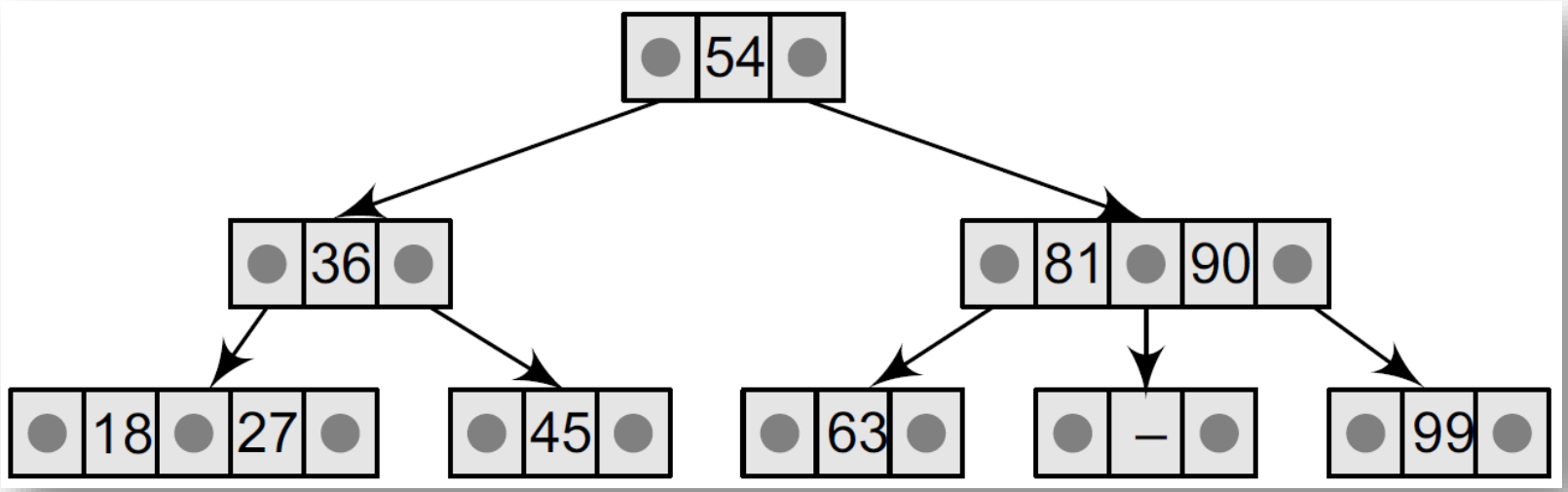
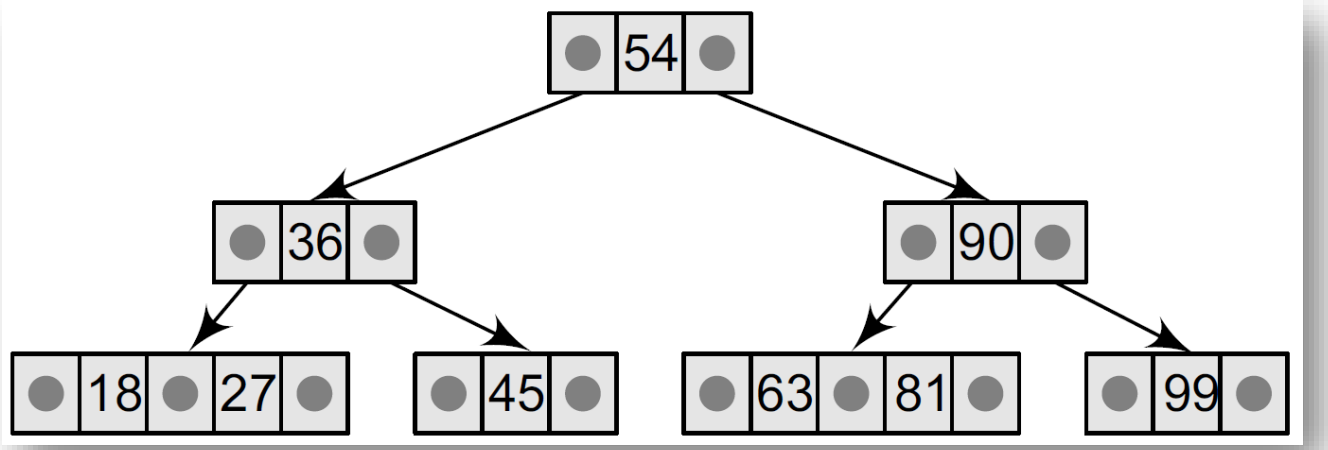


Merge the node with its sibling and the value in between them at their parent node



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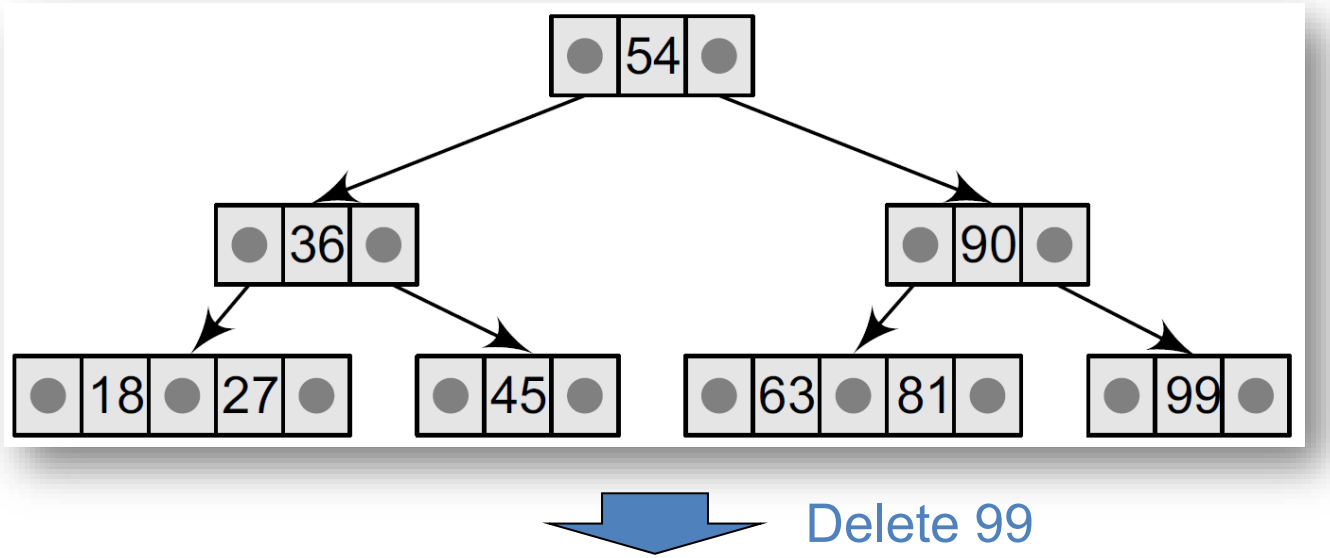

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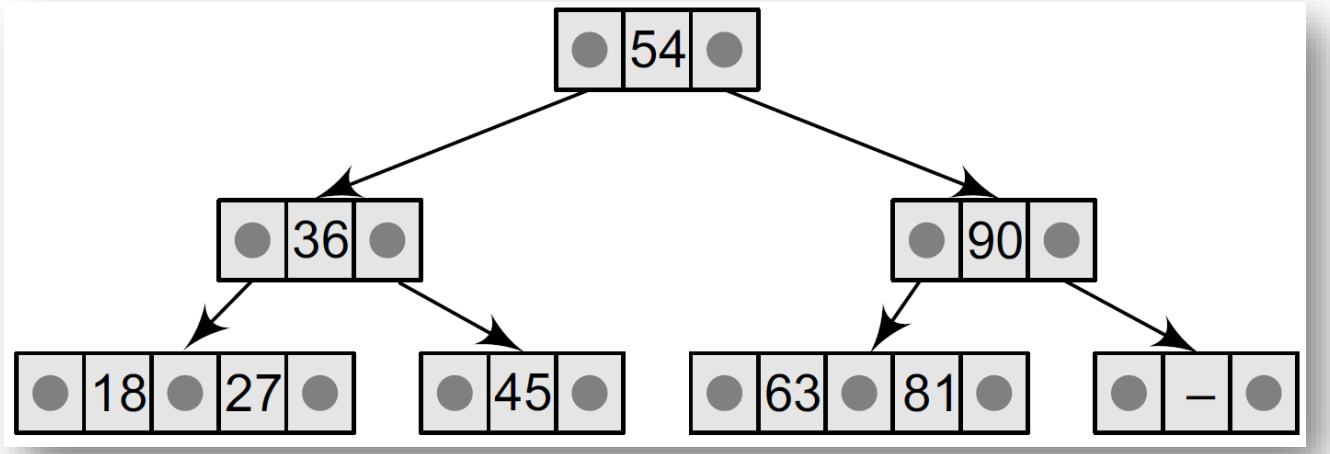
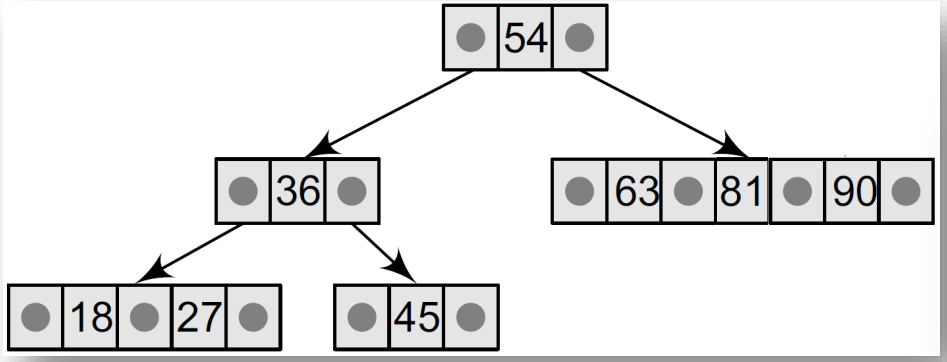
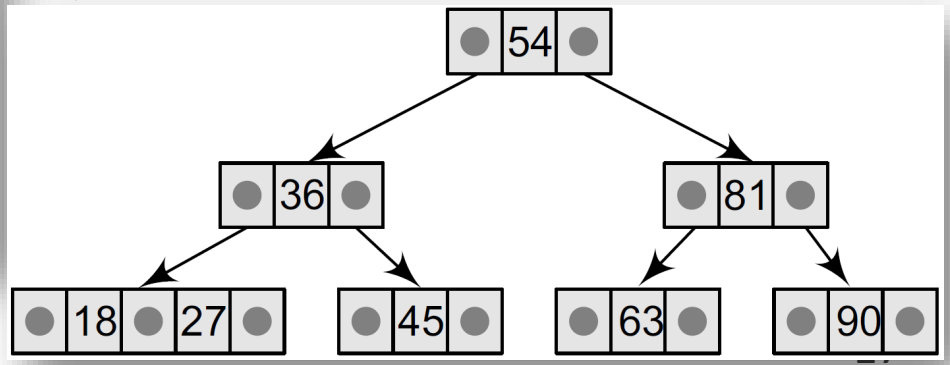


Merge



Split



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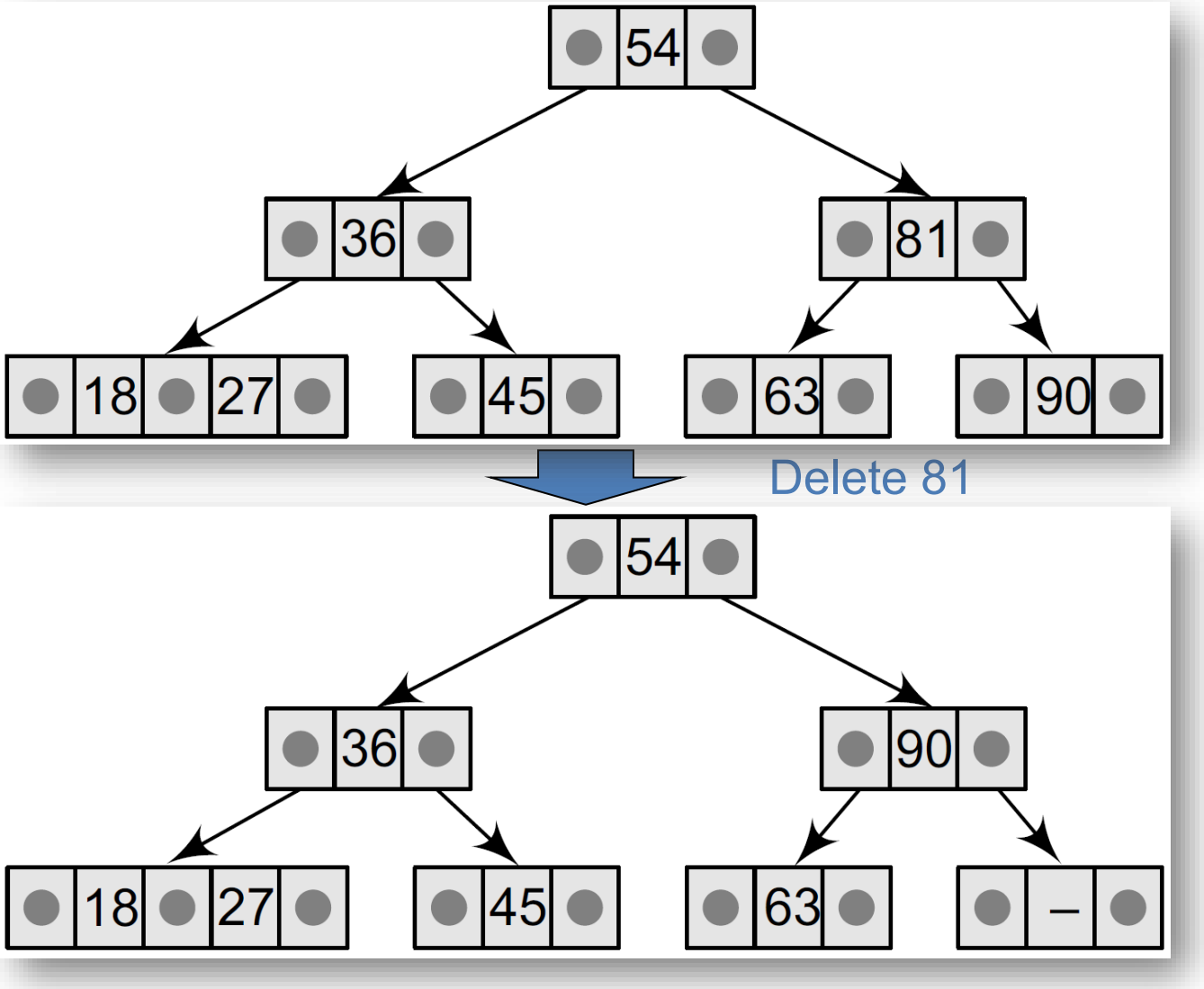

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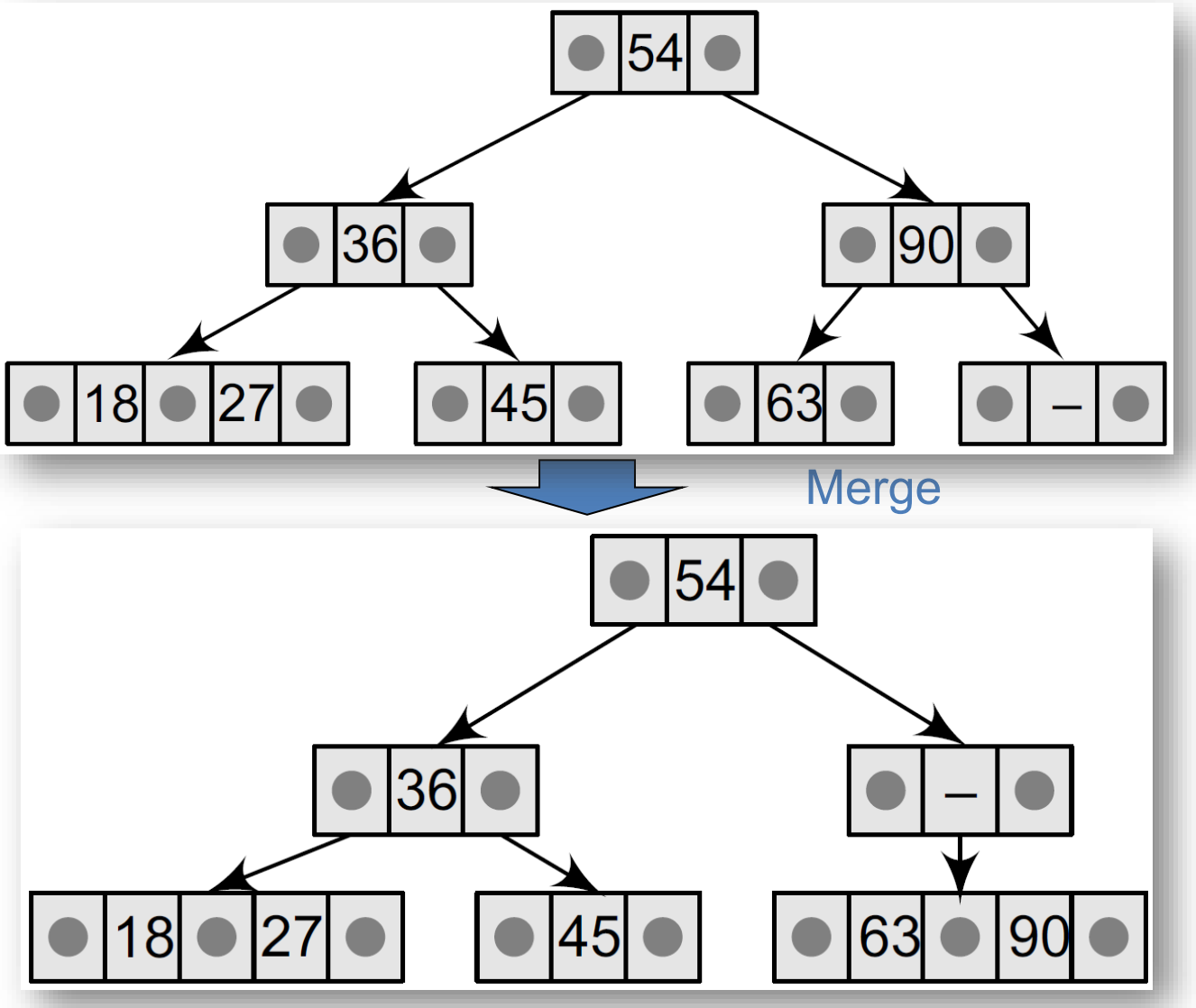

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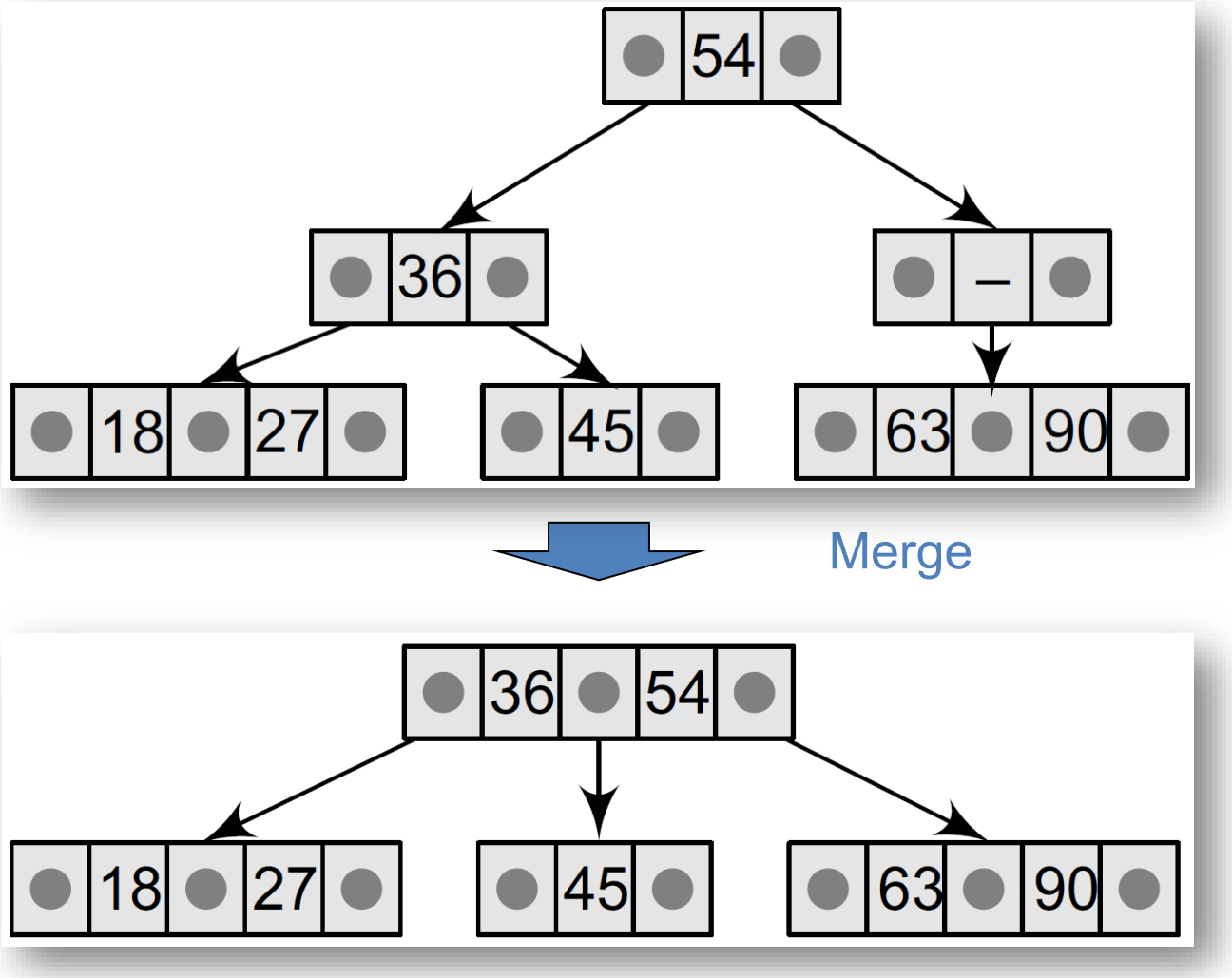

Schedule

- Midterm exam will be held at 11/7 (Mon.) 10:20~12:10

Questions?

kychen@mail.ntust.edu.tw