AVL Trees

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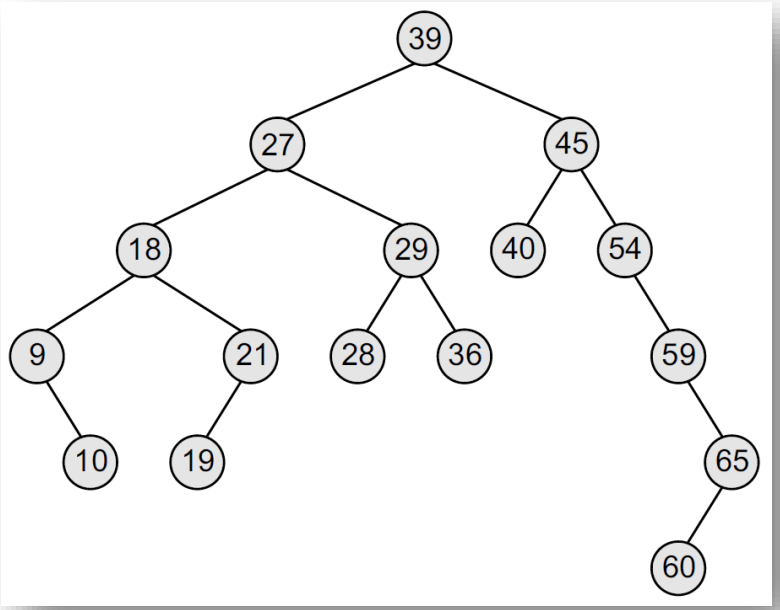
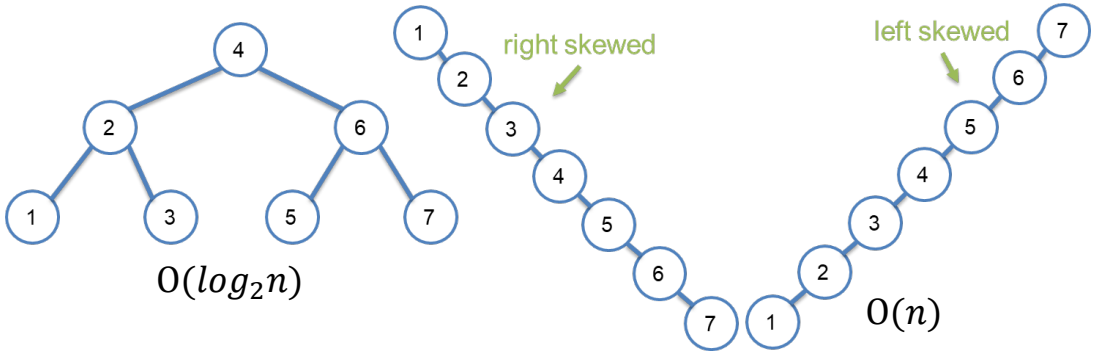
Review

- Binary search tree is also known as **ordered binary tree**
  - All the nodes in the **left sub-tree** have a value **less** than that of the root node
  - All the nodes in the **right sub-tree** have a value either **equal to or greater** than the root node



AVL Trees.

- AVL tree is a self-balancing binary search tree
  - AVL tree is designed by G.M. Adelson-Velsky and E.M. Landis in 1962
  - The heights of the two sub-trees of a node may differ by at most one

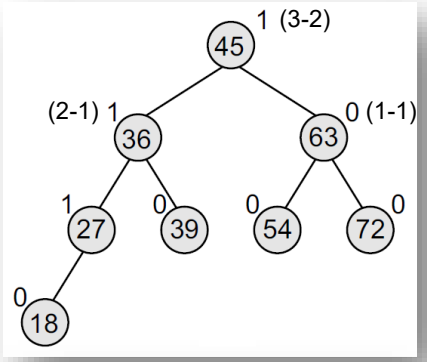


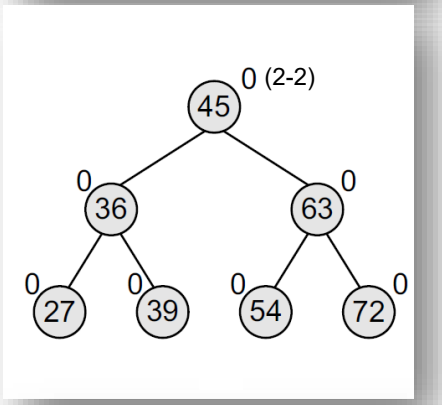
- The structure of an AVL tree stores an additional variable called the **Balance Factor**
  - Every node has a balance factor
  - The balance factor of a node is calculated by subtracting the height of its right sub-tree from the height of its left sub-tree
  - Every node has a balance factor of −1, 0, or 1

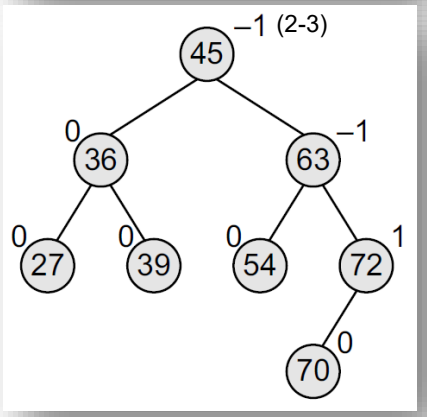
The height of the leaf node is 1

AVL Trees..

- If the balance factor of root is 1, then it means that the left sub-tree of the tree is one level higher than that of the right sub-tree
  - Left-heavy tree
- If the balance factor of root is 0, then it means that the height of the left sub-tree is equal to the height of the right sub-tree
  - Balance tree
- If the balance factor of root is −1, then it means that the left sub-tree of the tree is one level lower than that of the right sub-tree
  - Right-heavy tree



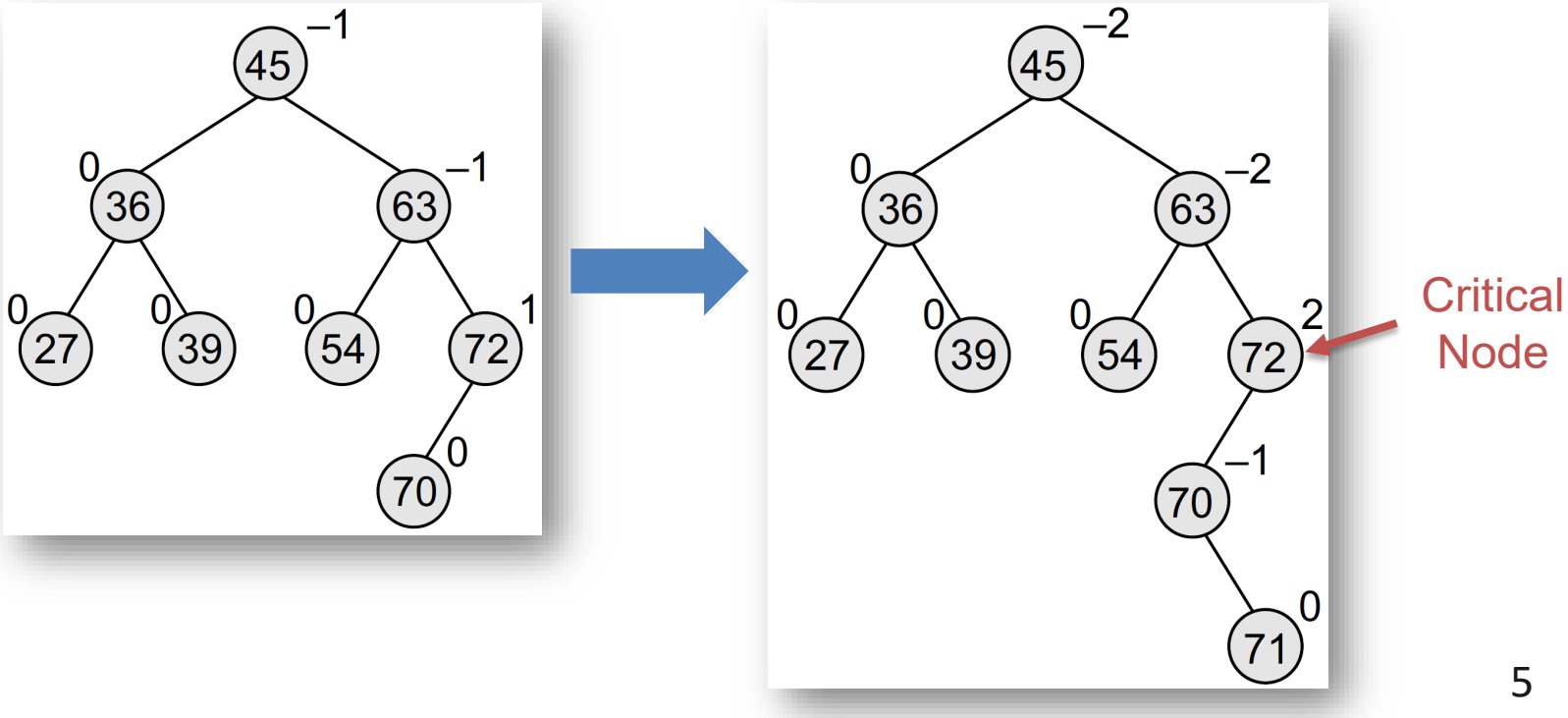




Insertion

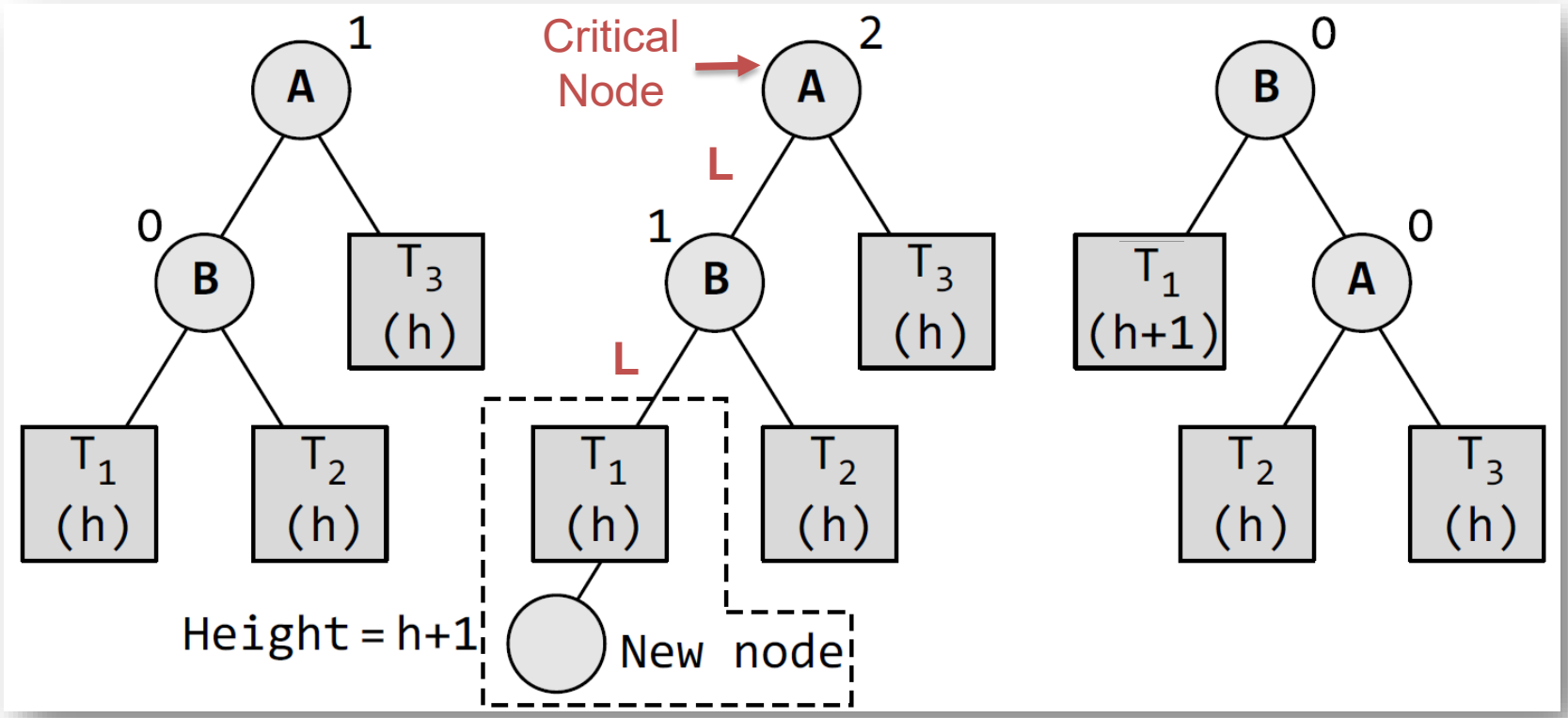
- In the AVL tree, the step of insertion is usually followed by an additional step of **rotation**
  - Rotation is done to restore the balance of the tree

- Insert a node with value 71 in a given AVL tree



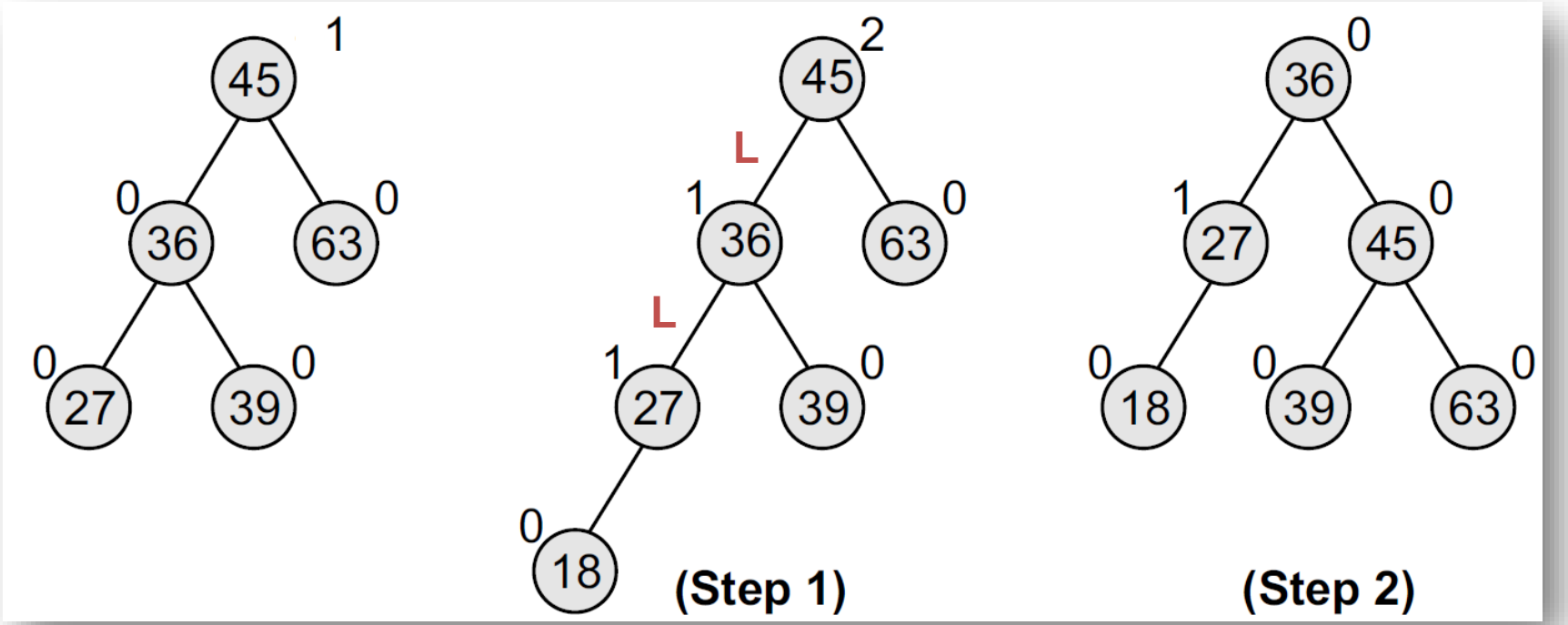
LL Rotation.

- By LL Rotation
  - Node B becomes the root, with T1 and A as its left and right child
  - T2 and T3 become the left and right sub-trees of A



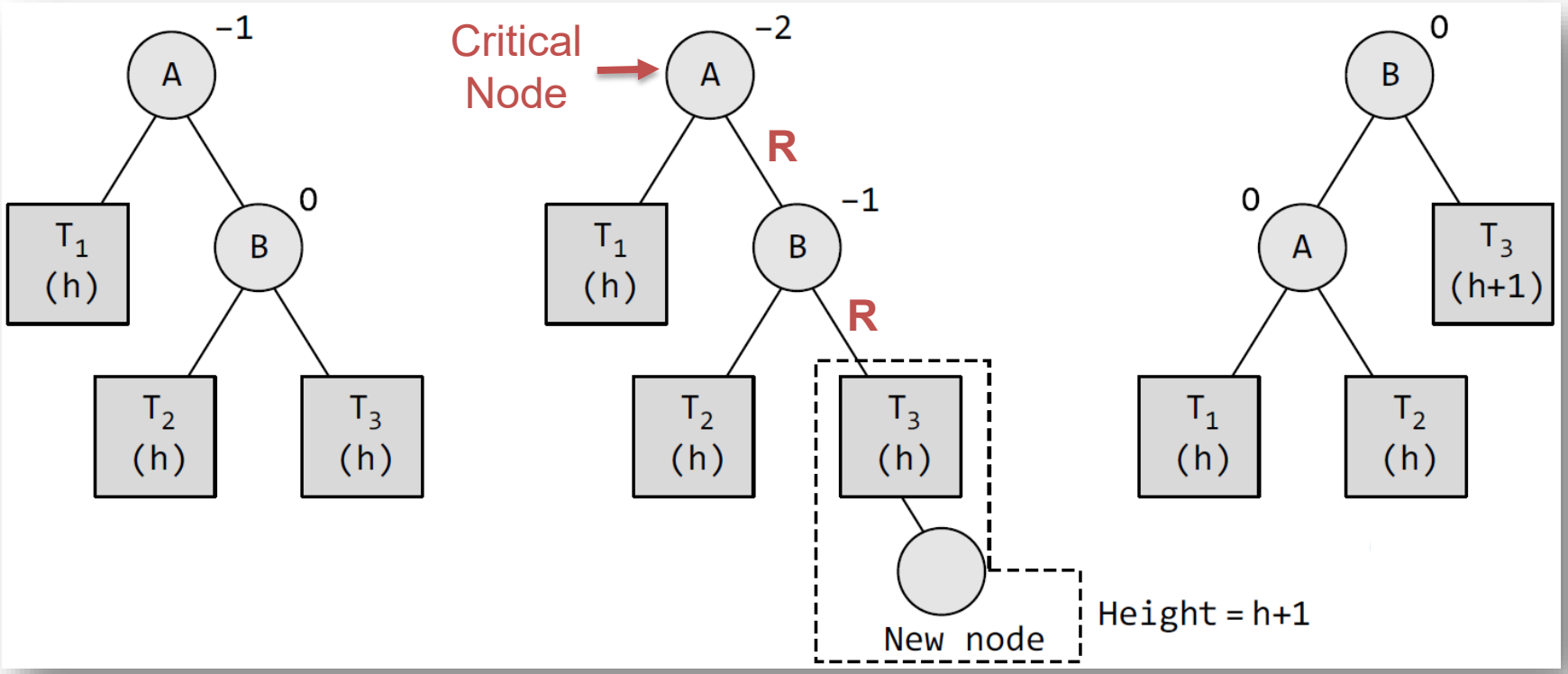
LL Rotation..

- Example for LL Rotation
  - Insert 18 in a given AVL tree



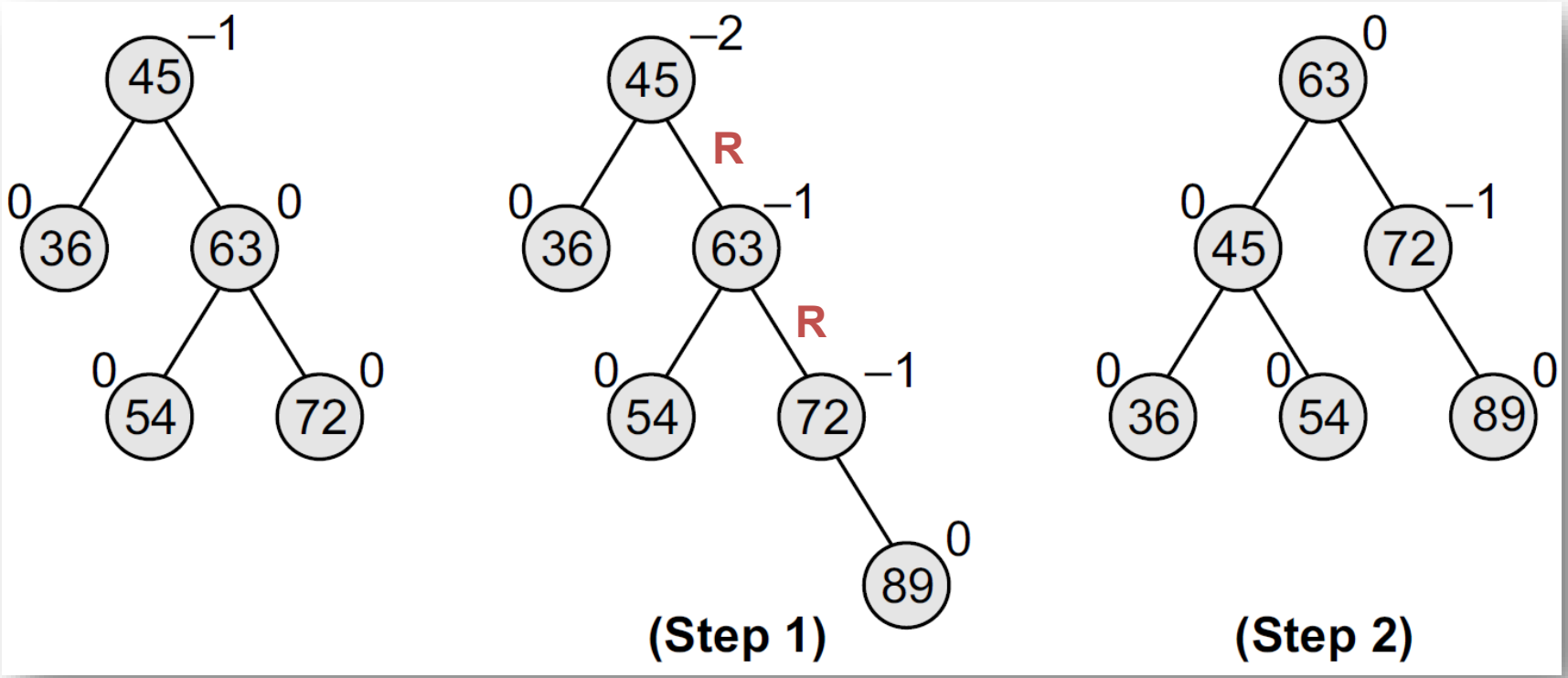
RR Rotation.

- In the context of RR rotation
  - Node B becomes the root, with A and T3 as its left and right child
  - T1 and T2 become the left and right sub-trees of A



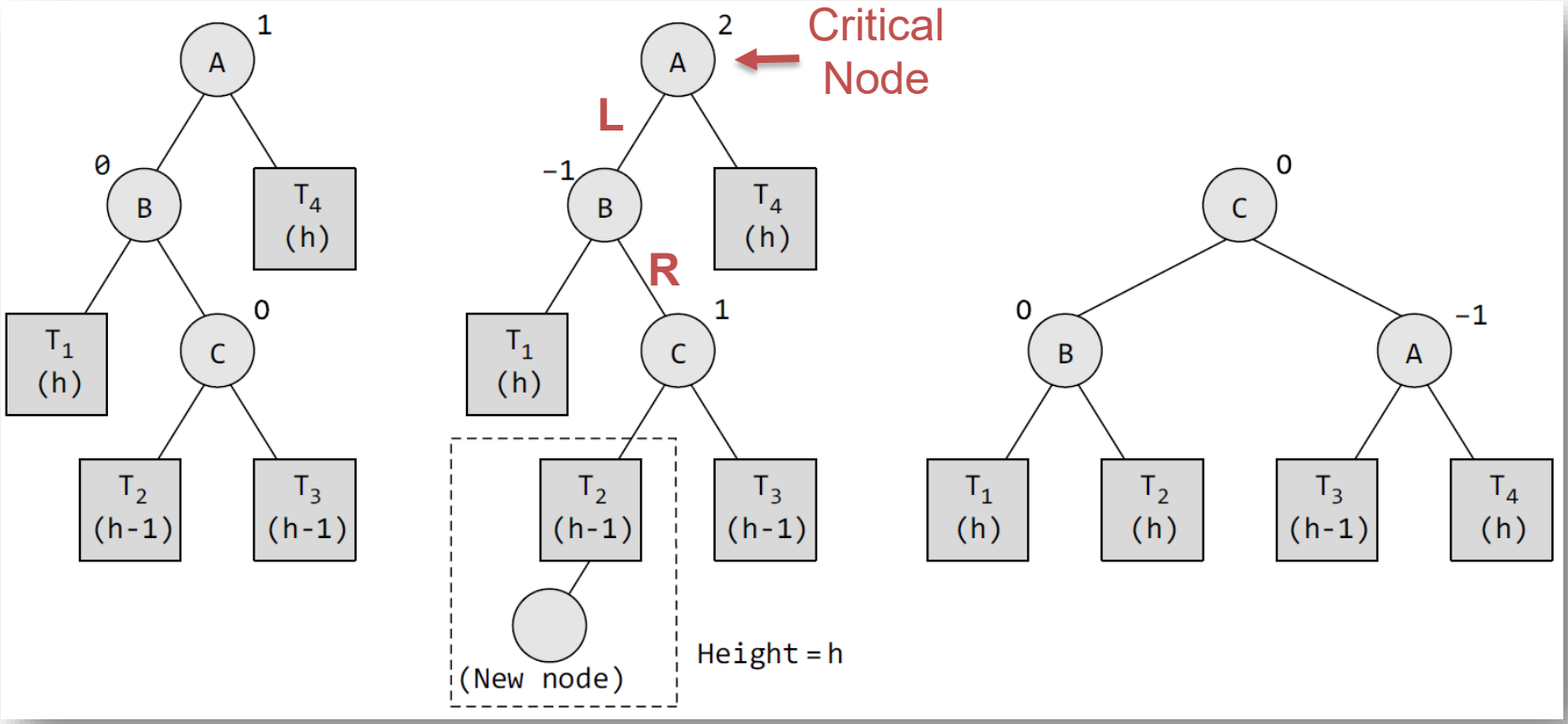
RR Rotation..

- Example for RR Rotation
  - Insert 89 in a given AVL tree



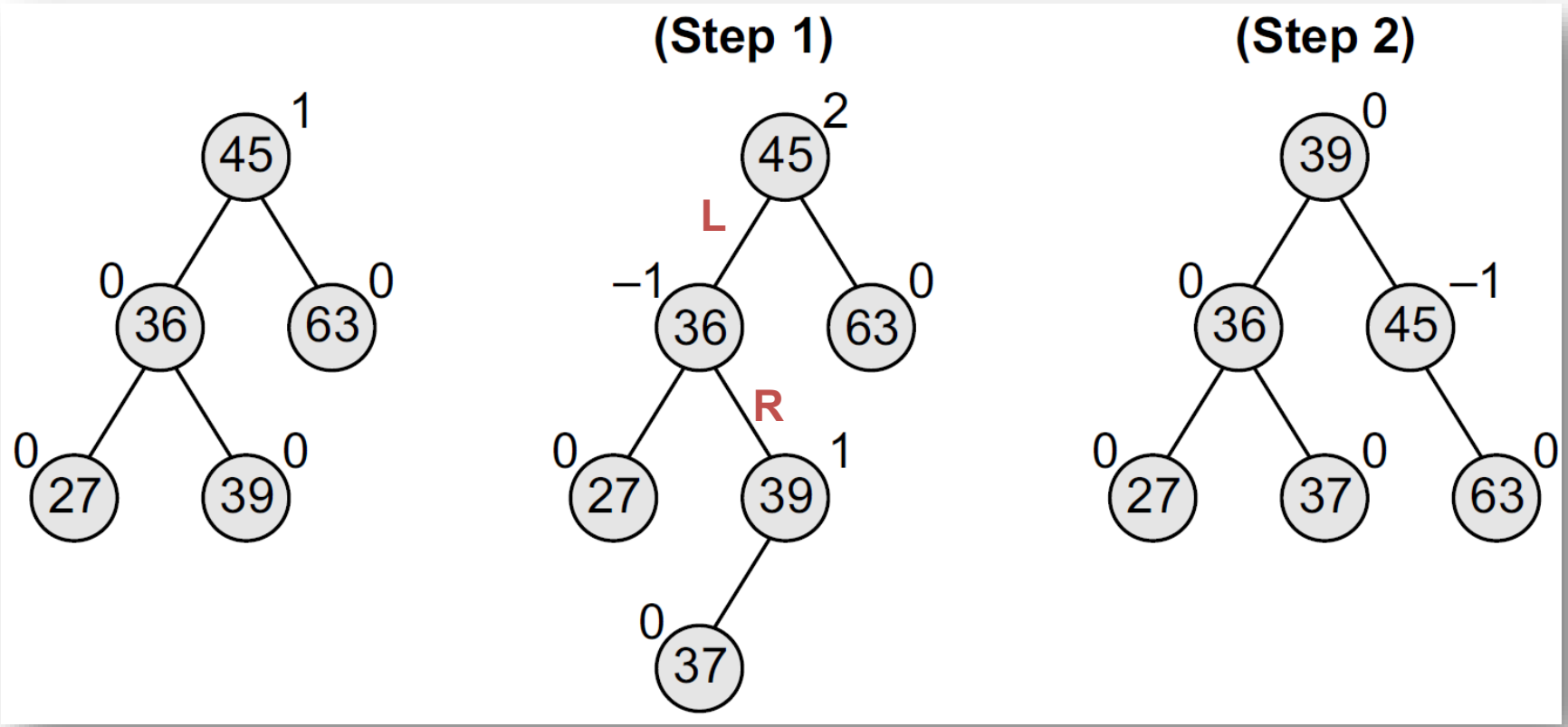
LR Rotation.

- By LR rotation
  - Node C becomes the root, with B and A as its left and right children
  - Node B has T1 and T2 as its left and right sub-trees and T3 and T4 become the left and right sub-trees of node A



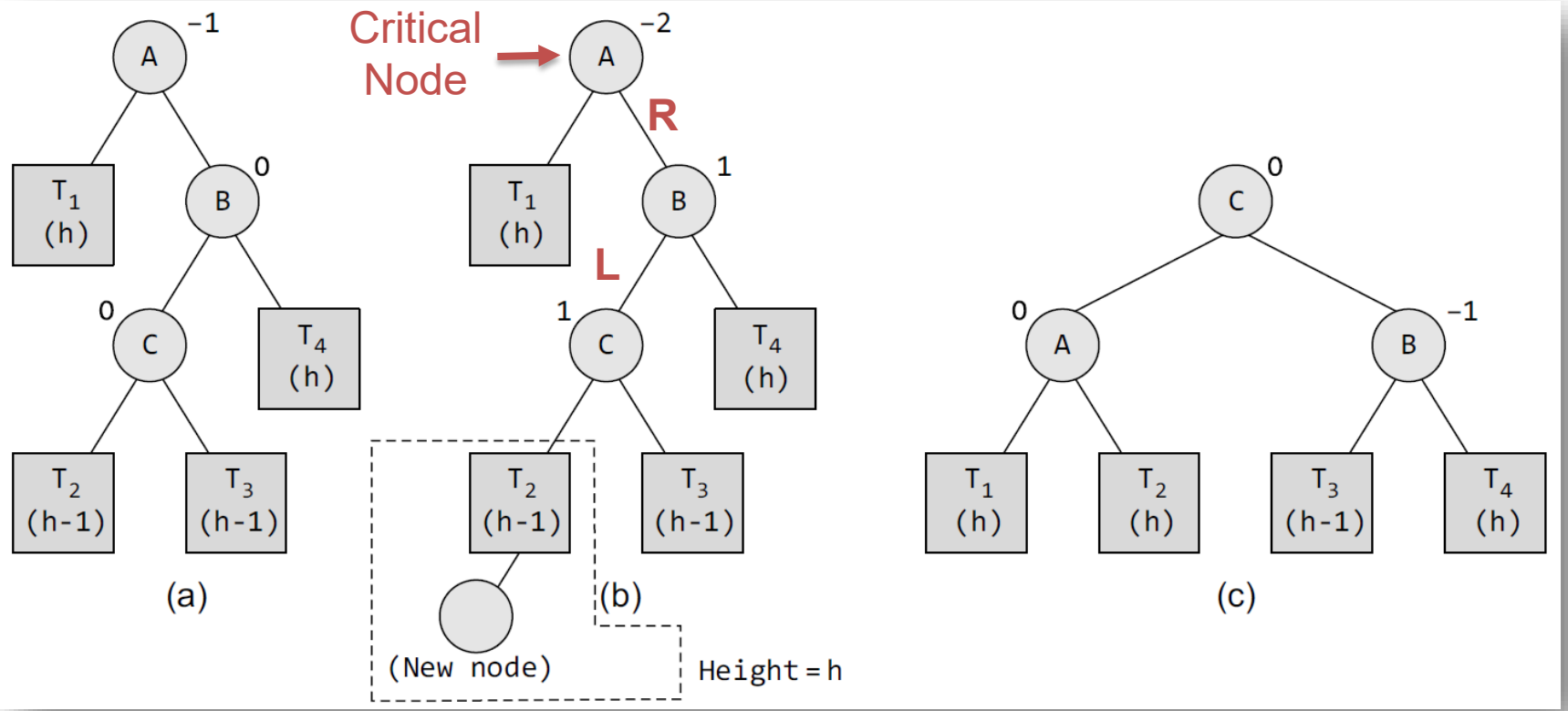
LR Rotation..

- Example for LR Rotation
  - Insert 37 in a given AVL tree



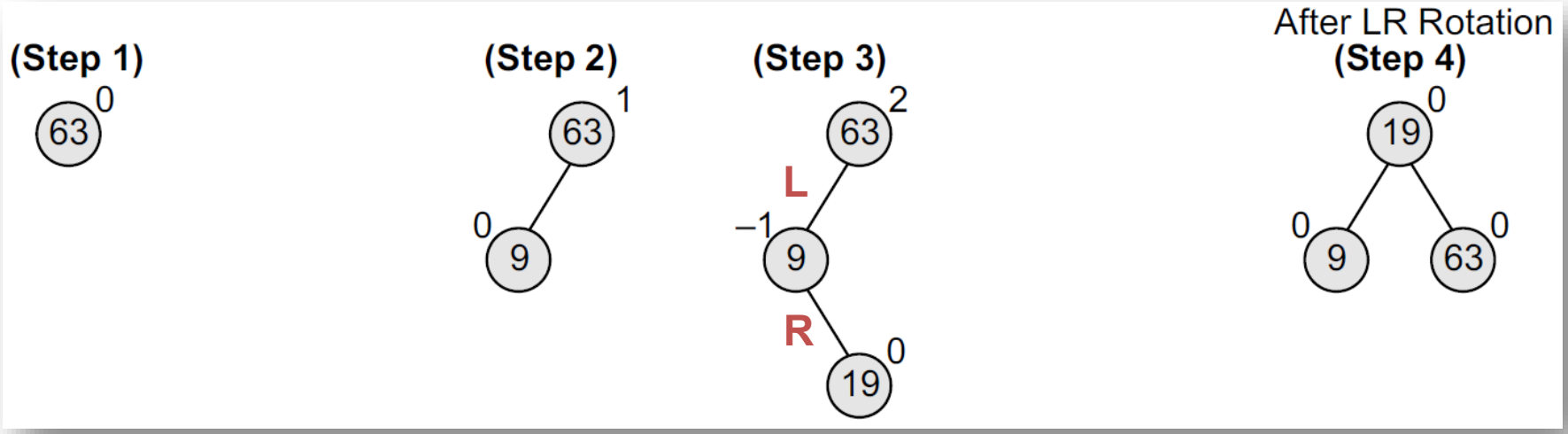
RL Rotation

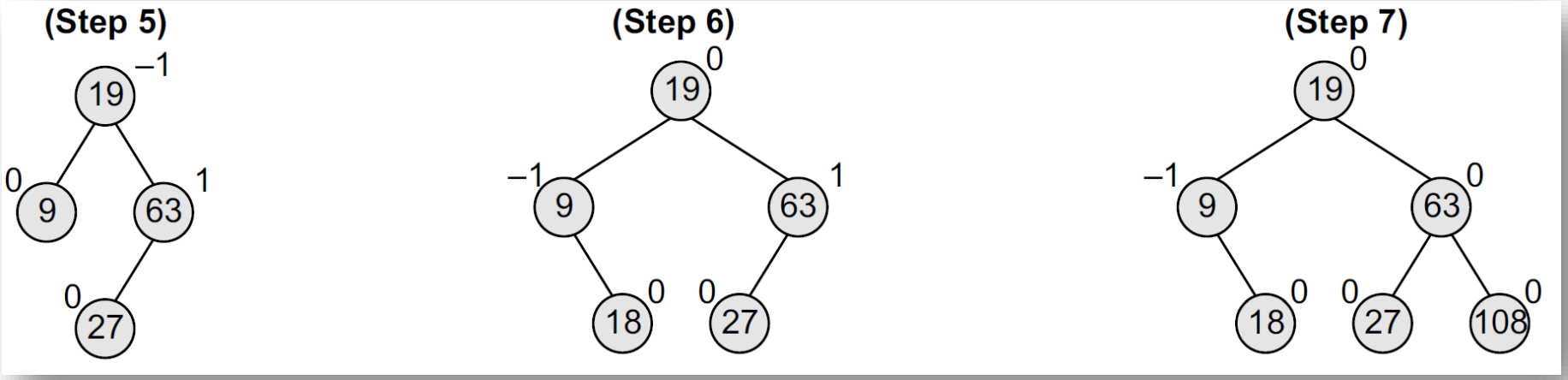
- By RL rotation
  - Node C becomes the root, with A and B as its left and right children
  - Node A has T1 and T2 as its left and right sub-trees and T3 and T4 become the left and right sub-trees of node B



Example.

- Construct an AVL tree by inserting the following elements in the given order: 63, 9, 19, 27, 18, 108, 99, 81





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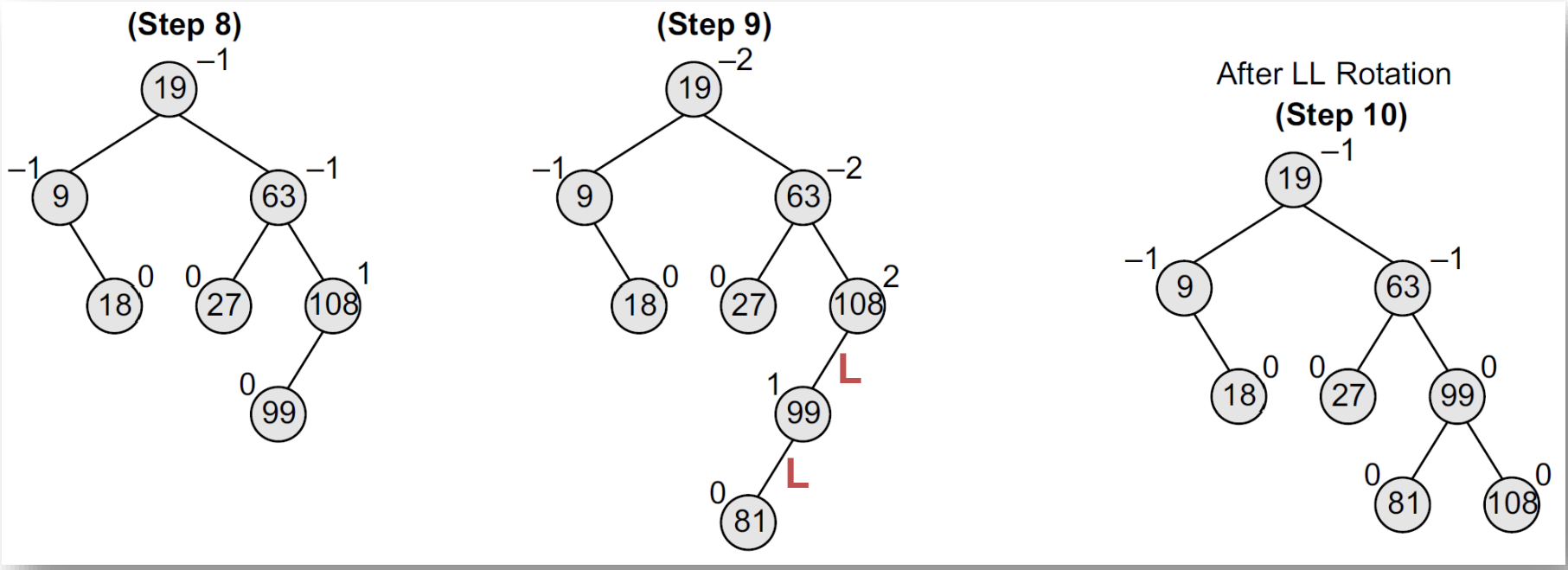
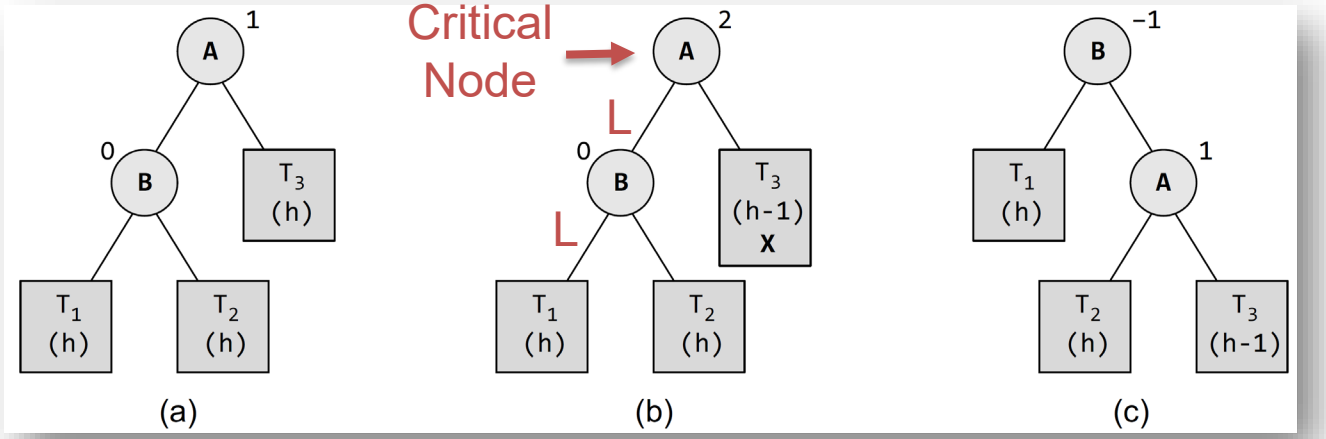


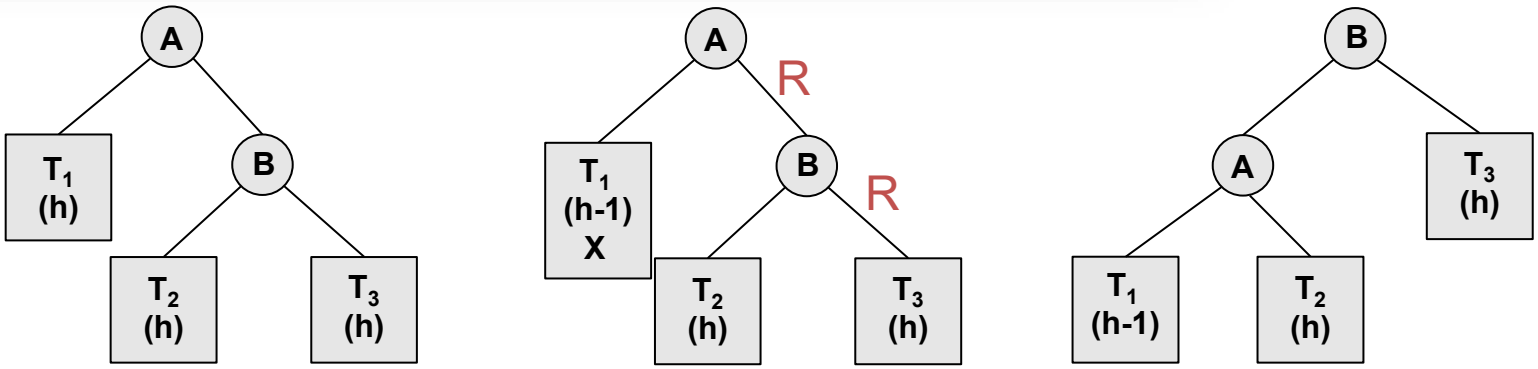
Deletion

- Deletion of a node in an AVL tree may disturb the balance of the tree
  - To rebalance the AVL tree, we need to perform rotations!
  - There are three rotation methods
    - R0 Rotation (LL Rotation or RR Rotation)
    - R1 Rotation (LL Rotation or RL Rotation)
    - R-1 Rotation (RR Rotation or LR Rotation)

Deletion – R0 Rotation

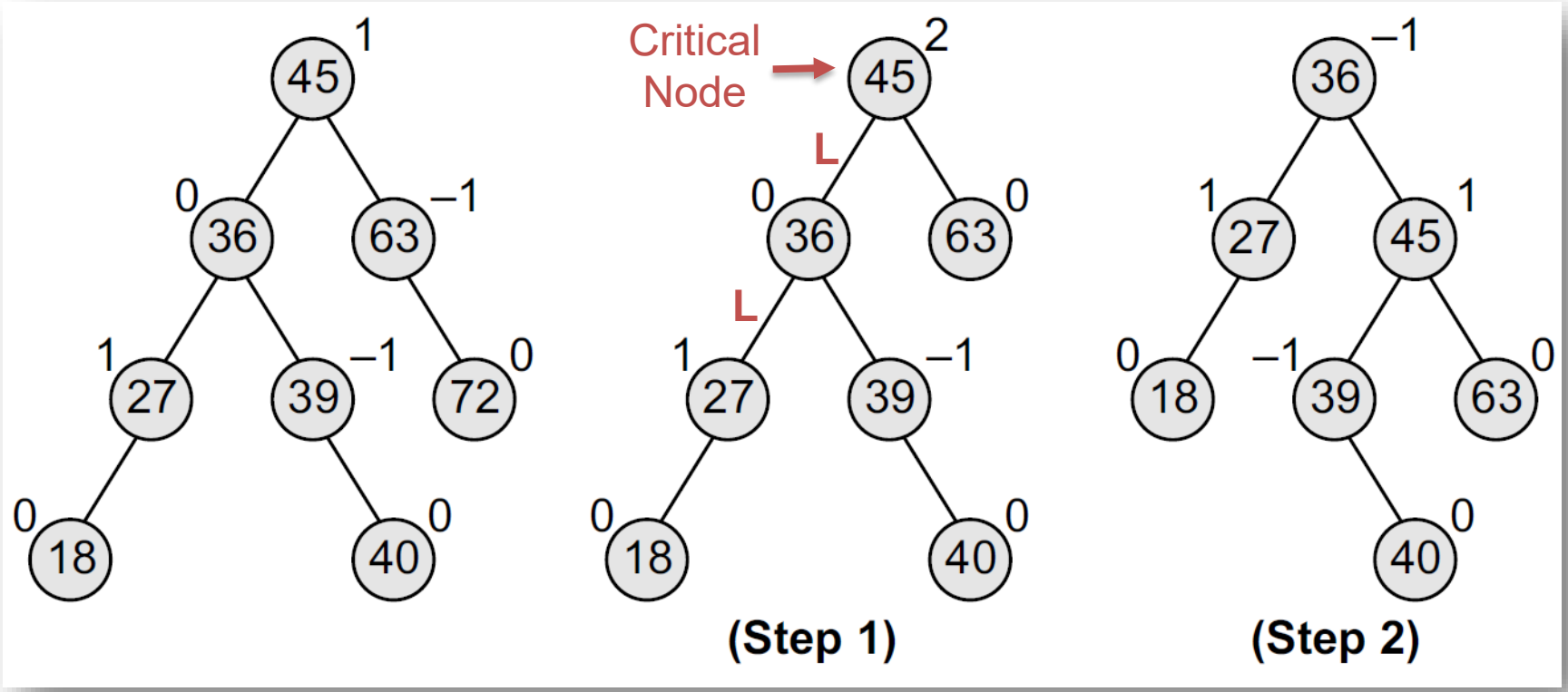
- Let B be the root of the left or right sub-tree of A (critical node)
  - R0 rotation is applied if the balance factor of B is 0
    - RR Rotation or LL Rotation





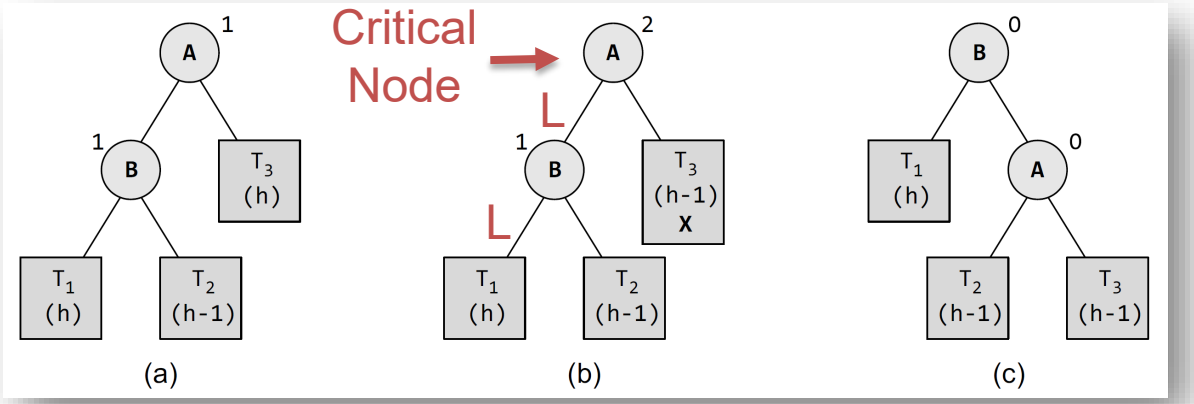
Example

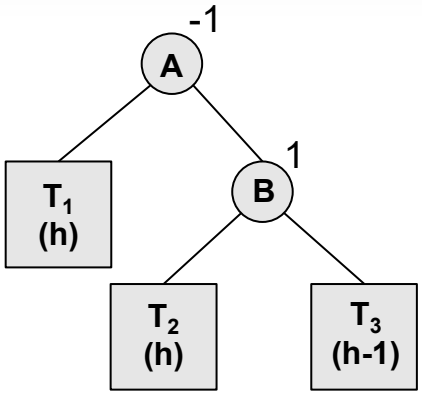
- Delete 72 from a given AVL tree

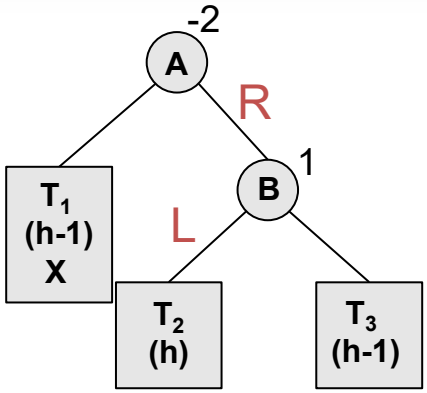


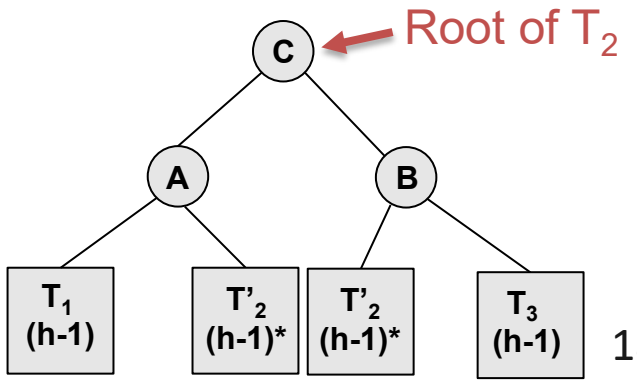
Deletion – R1 Rotation

- Let B be the root of the left or right sub-tree of A (critical node)
  - R1 rotation is applied if the balance factor of B is 1
    - LL Rotation or RL Rotation



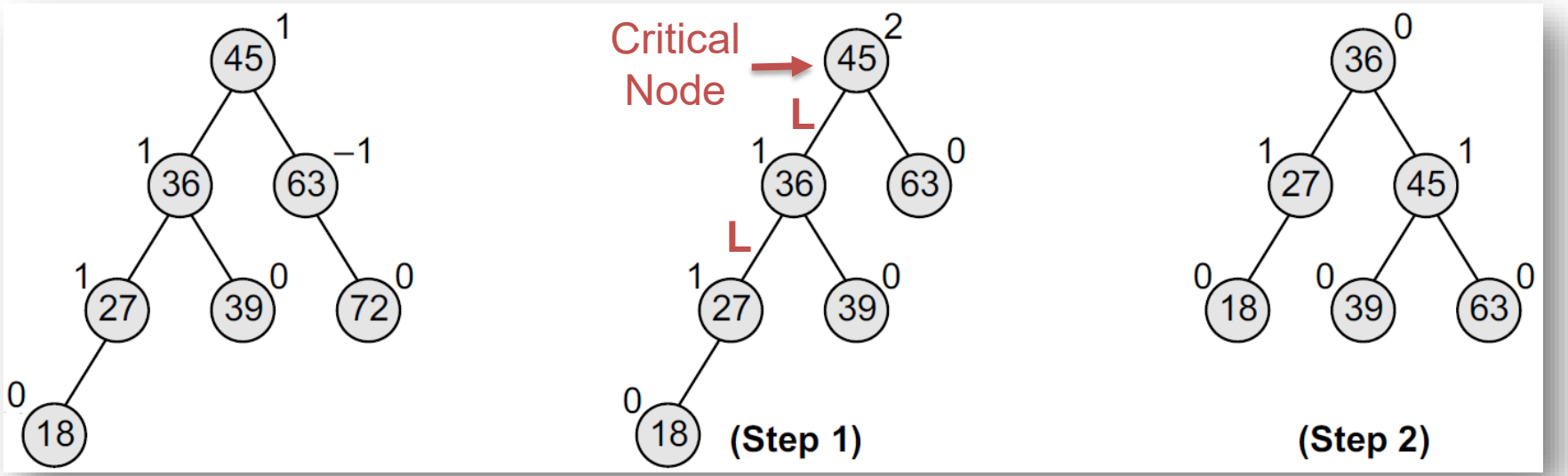






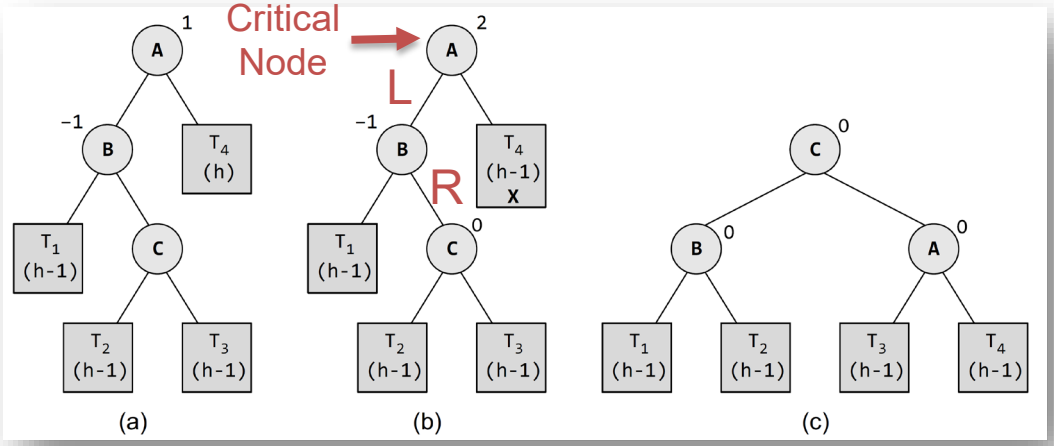
Example

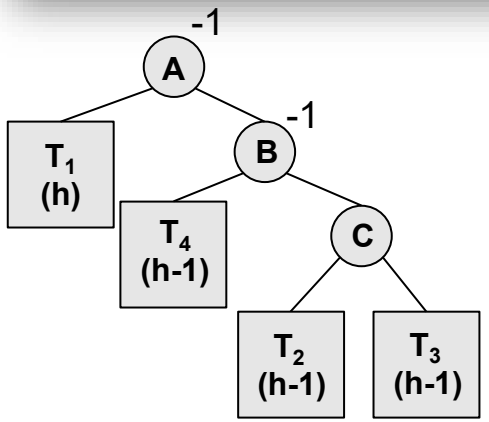
- Delete 72 from a given AVL tree

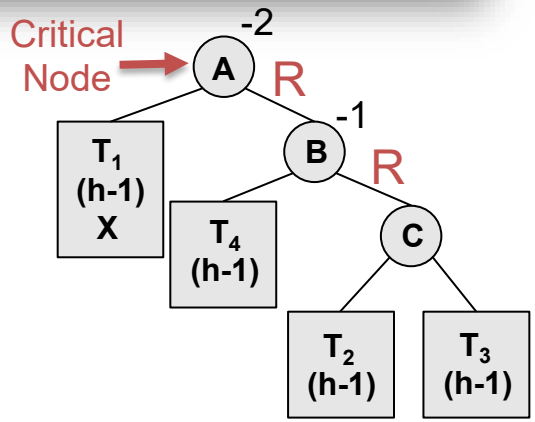


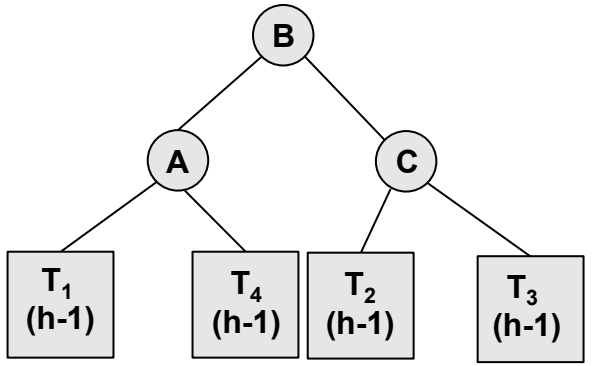
Deletion – R-1 Rotation

- Let B be the root of the left or right sub-tree of A (critical node)
  - R–1 rotation is applied if the balance factor of B is –1
    - RR Rotation or LR Rotation



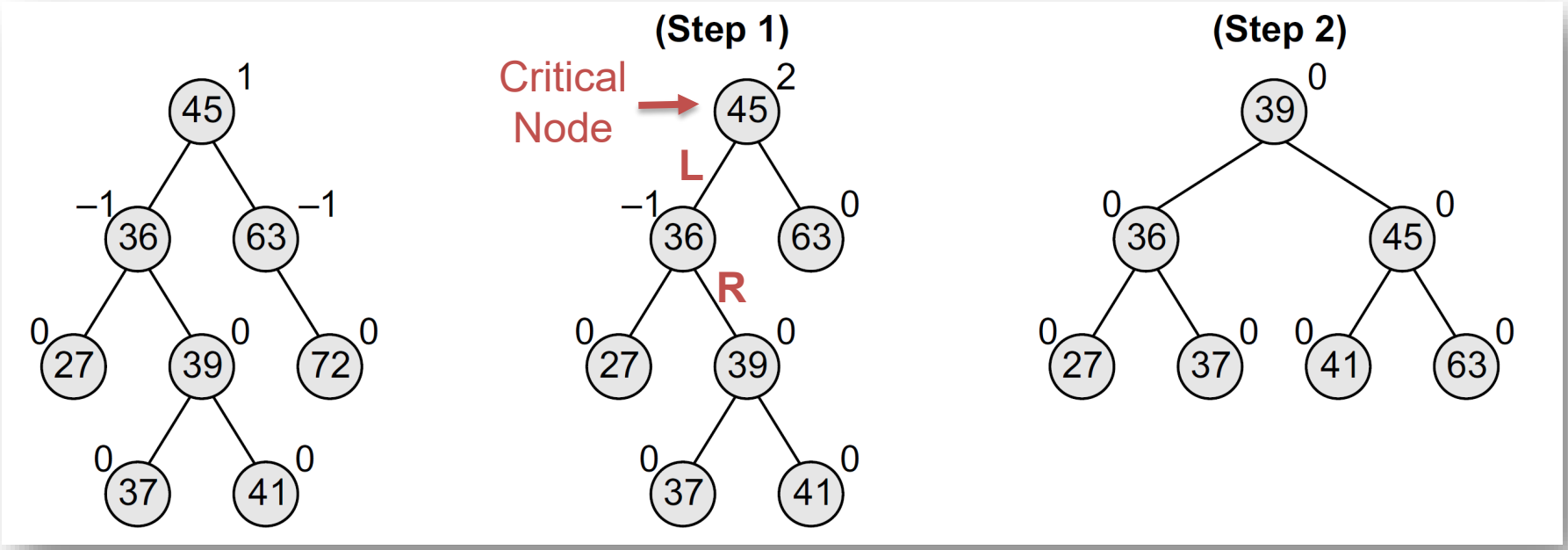






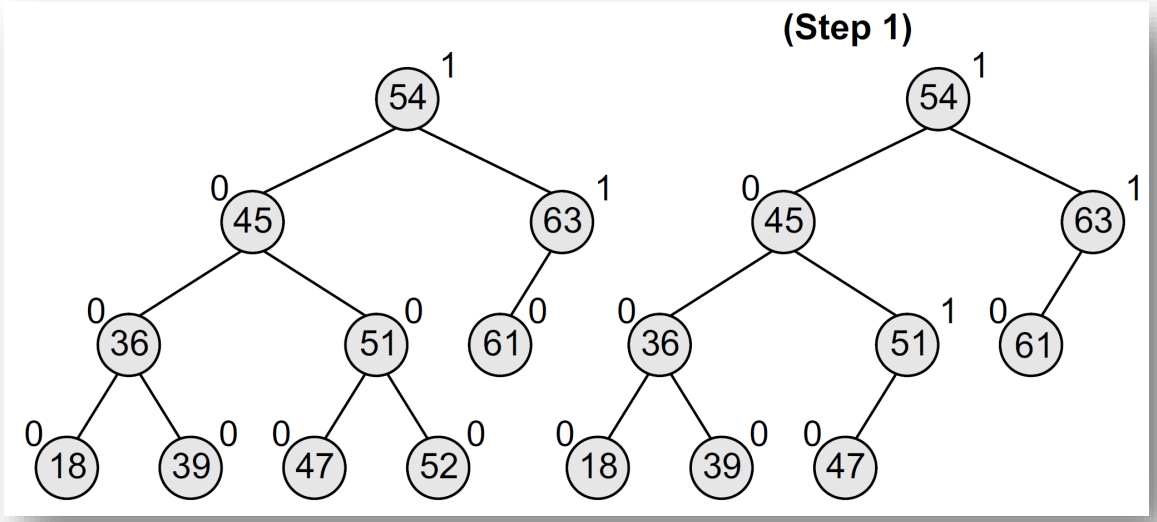
Examples.

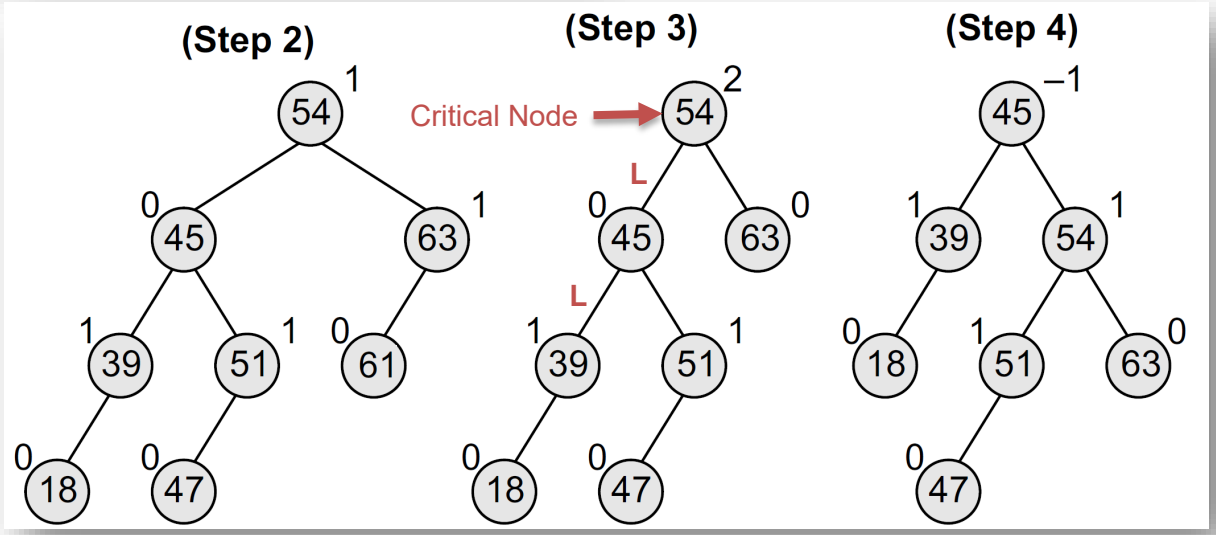
- Delete 72 from a given AVL tree



Examples..

- Delete 52, 36, and 61 from a given AVL tree





Searching in an AVL Tree

- Since AVL tree is a binary search tree, it can be searched using exactly the **same algorithm** as used to search an ordinary binary search tree!

Tentative Schedule

- Midterm will be held on 2022/11/7 (Monday)
  - Please take care of yourself to avoid make-up exam! ! !

Questions?

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