Linked List & Binary Tree Traversal

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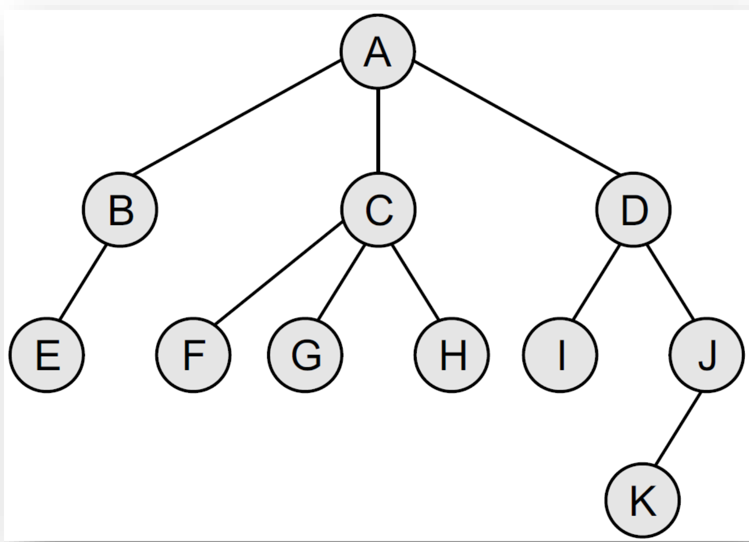
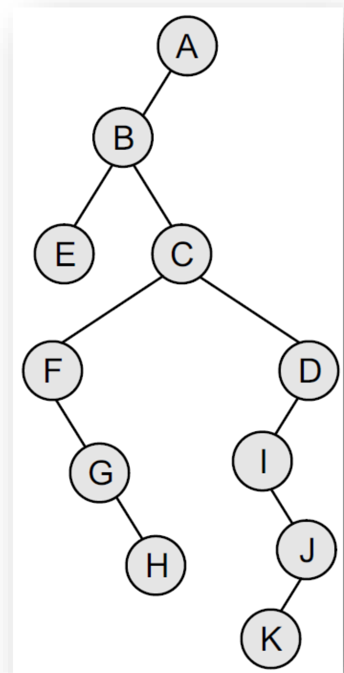
Review

- A tree is a **non-linear** data structure, which is mainly used to store data that is **hierarchical** in nature
  - General Trees
  - Forests
  - Binary Trees
  - Expression Trees
  - Tournament Trees

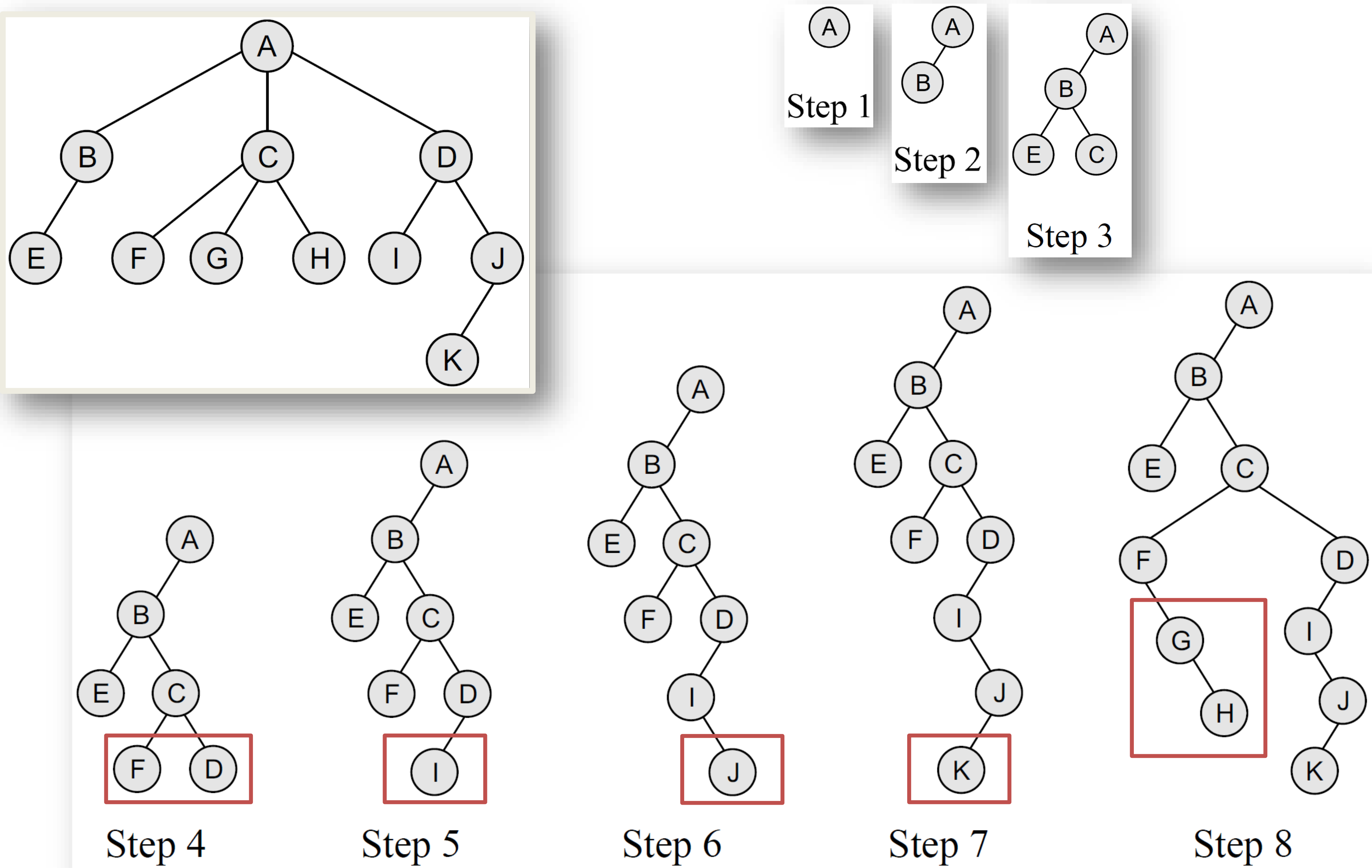
From a General Tree to a Binary Tree.

- The rules for converting a general tree to a binary tree are given below
  - **Rule 1**: Root of the binary tree = Root of the general tree
  - **Rule 2**: Left child of a node in the binary tree = Leftmost child of the node in the general tree
  - **Rule 3**: Right child of a node in the binary tree = Right sibling of the node in the general tree



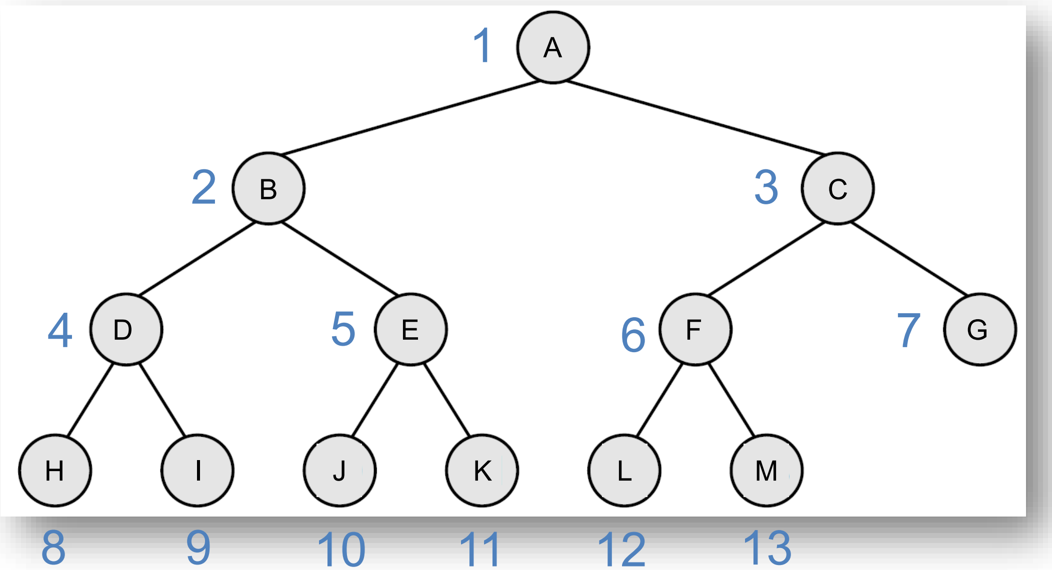


From a General Tree to a Binary Tree..



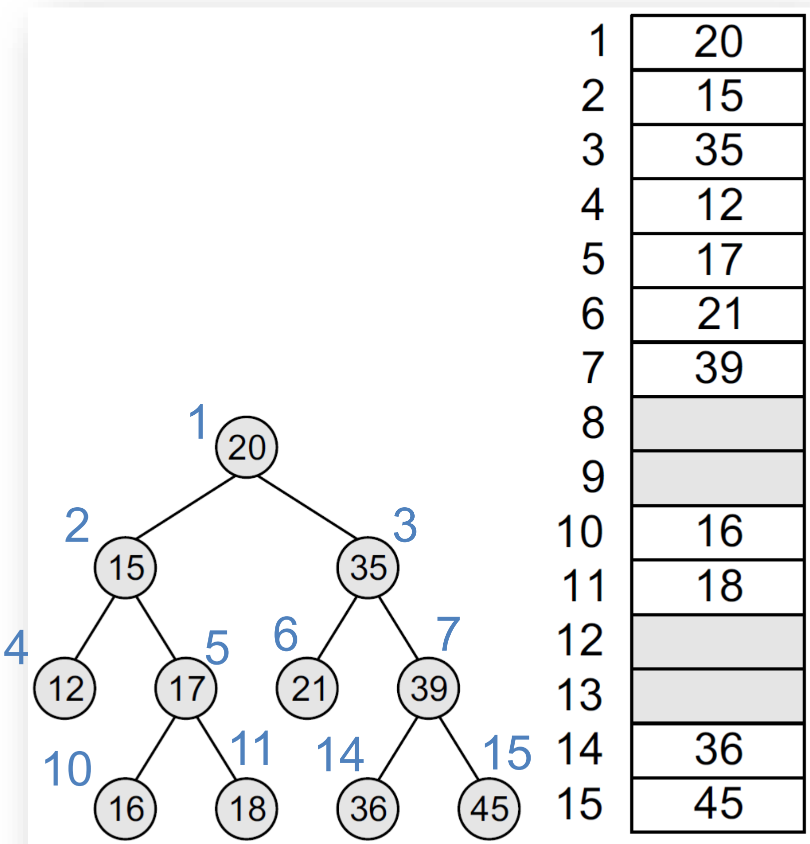
Array Implementation for Binary Tree.

- For a binary tree, we can number all of the nodes ordered
  - If K is a parent node, then its left child can be calculated as $2 \times K$ and its right child can be calculated as $2 \times K + 1$
    - The children of the node 4 are 8 and 9
  - The parent of the node K can be calculated as $\left\lfloor \frac{K}{2} \right\rfloor$
    - The parent of the node 5 is 2



Array Implementation for Binary Tree..

- Sequential representation of trees is done using single or one-dimensional arrays
  - Though it is the simplest technique for memory representation, it is inefficient as it requires a lot of memory space

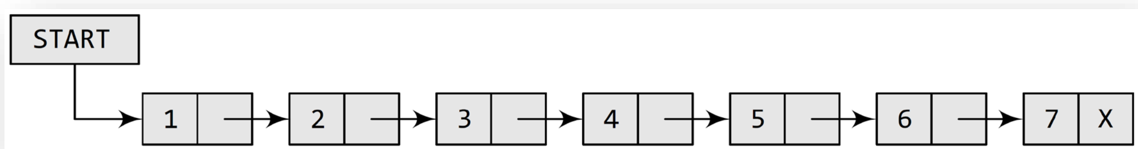


Hight= 4
Number of nodes= $2^4 - 1 = 15$

Linked List.

- A linked list, in simple terms, is a linear collection of data elements
  - Data elements are called **nodes**
  - Each node contains **one or more data fields** and a **pointer** to the next node

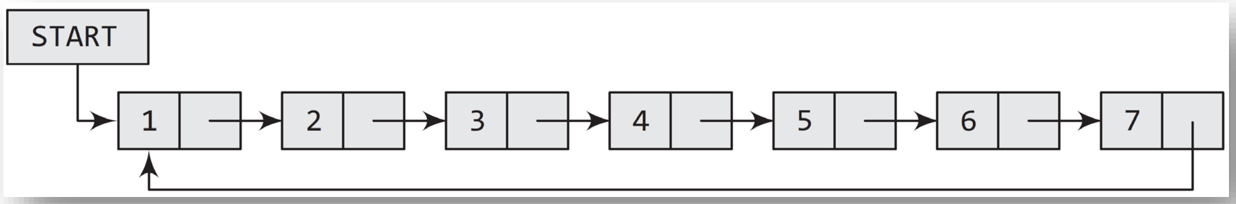
- **Singly linked list** is the simplest type of linked list in which every node contains some data and a pointer to the next node



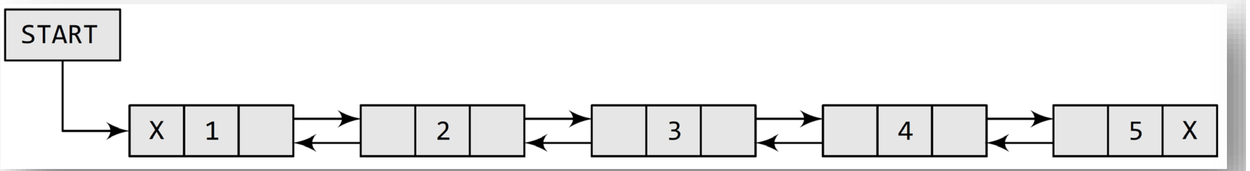

```
struct node
{
        int data;
        struct node *next;
};
```

Linked List..

- **Circular linked list** is a simple variant, where the last node contains a pointer to the first node of the list



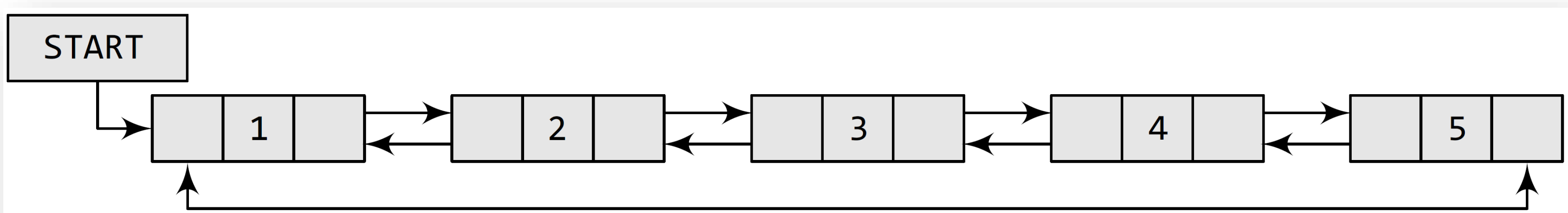
- **Doubly linked list** or a two-way linked list is a more complex type of linked list
  - It contains a pointer to the next as well as the previous node in the sequence
  - The linked list consists of three parts—data, a pointer to the next node, and a pointer to the previous node




```
struct node
{
        struct node *prev;
        int data;
        struct node *next;
};
```

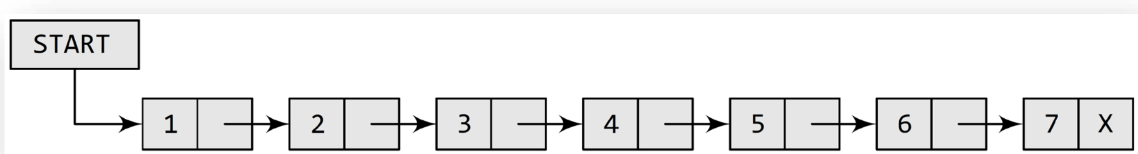
Linked List…

- **Circular doubly linked list** or a circular two-way linked list is a more complex type of linked list
  - It contains a pointer to the next as well as the previous node in the sequence
    - The next field of the last node stores the address of the first node of the list
    - The previous field of the first field stores the address of the last node



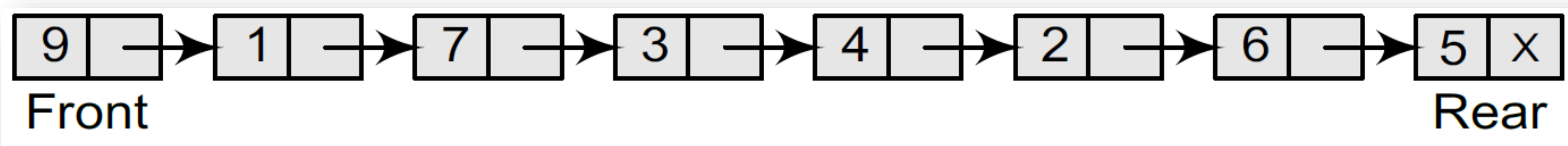
Linked List vs. Array

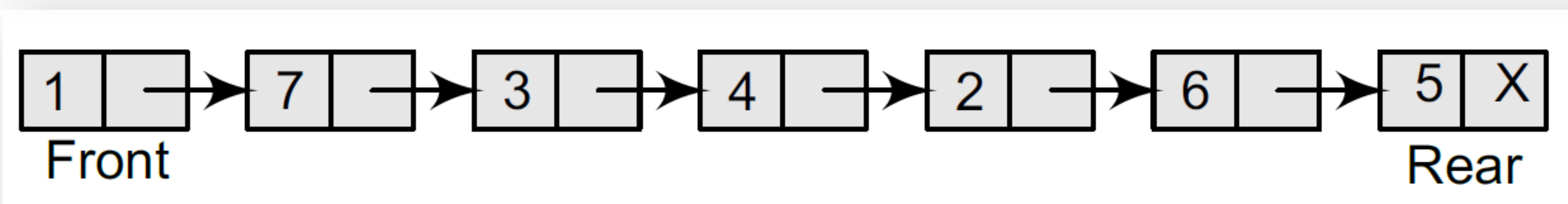
- Both arrays and linked lists are a linear collection of data elements
  - A linked list does not store its nodes in consecutive memory locations
  - A linked list does not allow random access of data
    - Nodes in a linked list can be accessed only in a sequential manner
  - A linked list can add any number of elements in the list
    - This is not possible in case of an array

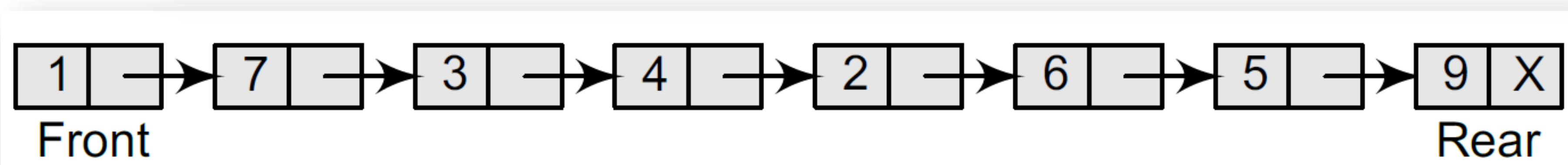


Implementation for Queue by Link List.

- Although creating a queue by an array is easy, its drawback is that the array must be declared to have some fixed size
  - If the array size cannot be determined in advance, the linked representation is used







Implementation for Queue by Link List..

- Declare

```
#include <stdio.h>
#include <conio.h>
#include <malloc.h>
struct node
{
    int data;
    struct node *next;
};
struct queue
{
    struct node *front;
    struct node *rear;
};
struct queue *q;
void create_queue(struct queue *);
struct queue *insert(struct queue *,int);
struct queue *delete_element(struct queue *);
```

Implementation for Queue by Link List...

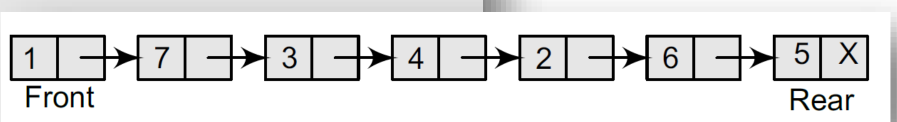
- Create a queue

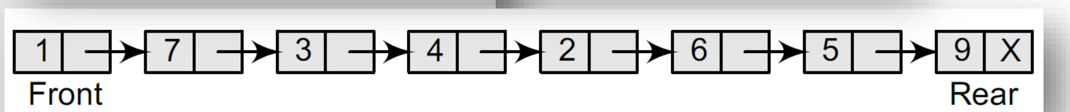
```
void create_queue(struct queue *q)
{
    q -> rear = NULL;
    q -> front = NULL;
}
```

Implementation for Queue by Link List....

- For insertion

```
struct queue *insert(struct queue *q,int val)
{
    struct node *ptr;
    ptr = (struct node*)malloc(sizeof(struct node));
    ptr -> data = val;
    if(q -> front == NULL)
    {
        q -> front = ptr;
        q -> rear = ptr;
        q -> front -> next = q -> rear -> next = NULL;
    }
    else
    {
        q -> rear -> next = ptr;
        q -> rear = ptr;
        q -> rear -> next = NULL;
    }
    return q;
}
```

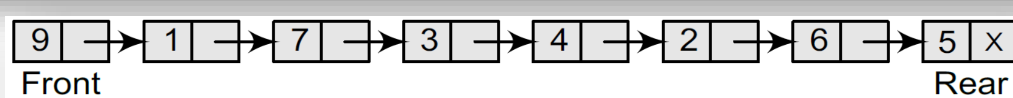


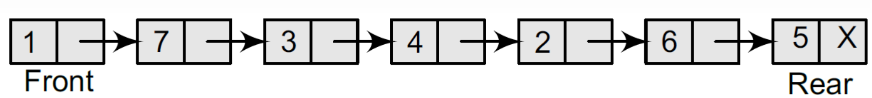


Implementation for Queue by Link List.....

- For deletion

```
struct queue *delete_element(struct queue *q)
{
    struct node *ptr;
    ptr = q -> front;
    if(q -> front == NULL)
        printf("\n UNDERFLOW");
    else
    {
        q -> front = q -> front -> next;
        printf("\n The value being deleted is : %d", ptr -> data);
        free(ptr);
    }
    return q;
}
```



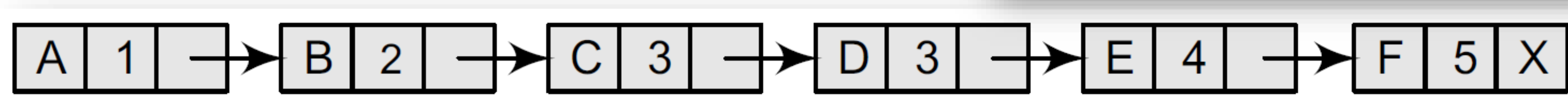


Priority Queue

- Linked Representation of a Priority Queue
  - Every node of the list will have three parts:
    1. the information or data part
    2. the priority number of the element
    3. the address of the next element

FRONT	REAR
3	3
1	3
4	5
4	1

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & & & A & & \\ 2 & B & C & D & & \\ 3 & & & & E & F \\ 4 & I & & & G & H \end{array}$$

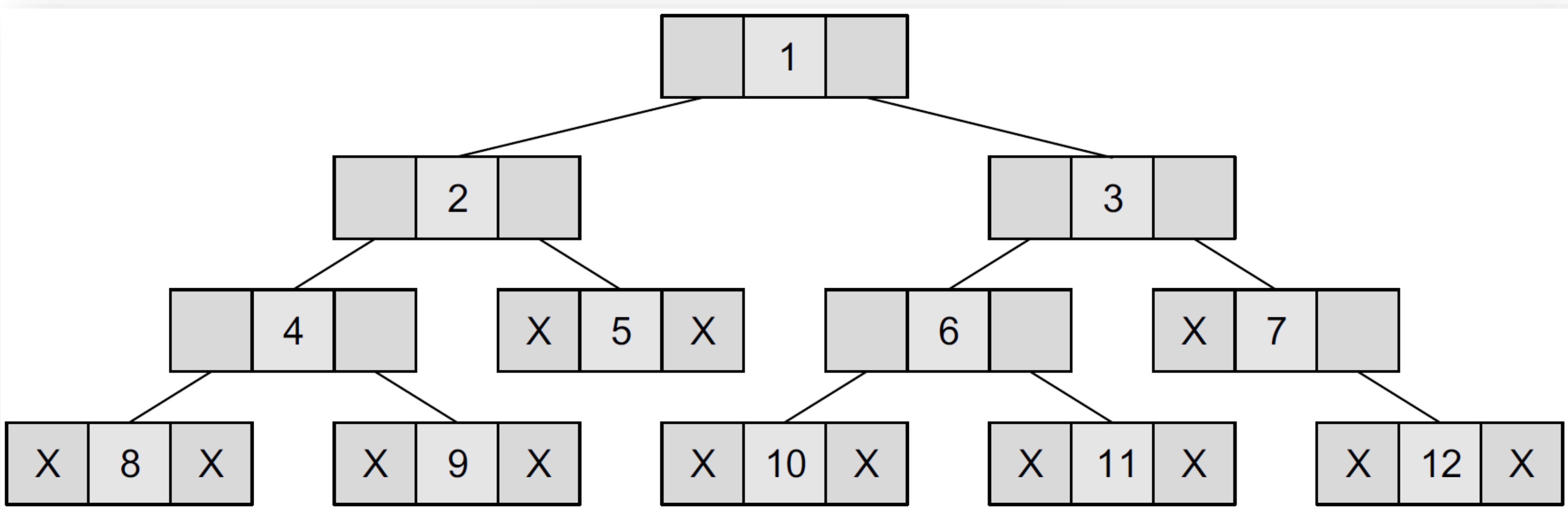


  - From the example
    - Since *A* has a priority number 1 and *B* has a priority number 2, then *A* will be processed before *B* as it has higher priority than *B*
    - We cannot make out whether *A* was inserted before *E* or whether *E* joined the queue before *A*
    - We can definitely say that *C* was inserted in the queue before *D* because when two elements have the same priority

Binary Trees

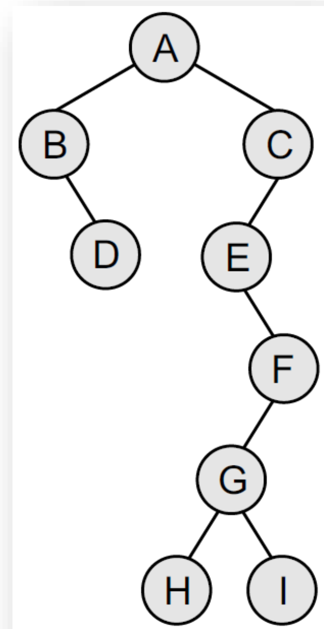
- In the linked representation of a binary tree, every node will have three parts: the data element, a pointer to the left node, and a pointer to the right node

```
struct node {
        struct node *left;
        int data;
        struct node *right;
};
```



Traversing Binary Tree

- Traversing a binary tree is the process of visiting each node in the tree exactly once in a systematic way
  - There are different algorithms for tree traversals
    - Pre-order Traversal
      - *ABDCEFGHI*
    - Post-order Traversal
      - *DBHIGFECA*
    - In-order Traversal
      - *BDAEHGIFC*
    - Level-order Traversal
      - *ABCDEFGHI*

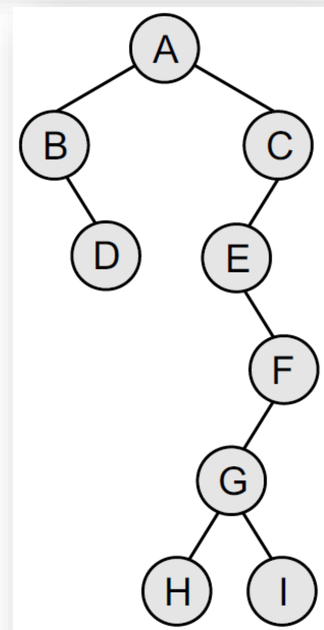


  - Different algorithms differ in the order in which the nodes are visited

In-order

```
Step 1: Repeat Steps 2 to 4 while TREE != NULL
Step 2:             INORDER(TREE -> LEFT)
Step 3:             Write TREE -> DATA
Step 4:             INORDER(TREE -> RIGHT)
        [END OF LOOP]
Step 5: END
```

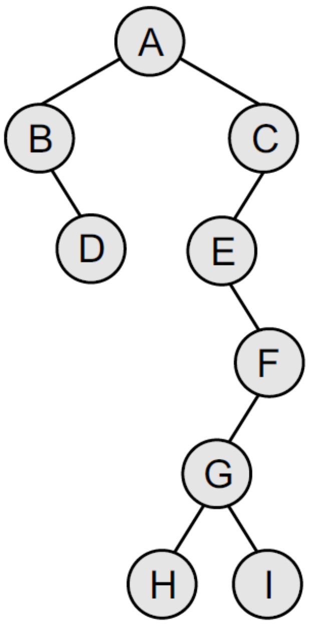
- In-order: BDAEHGIFC



Pre-order

```
Step 1: Repeat Steps 2 to 4 while TREE != NULL
Step 2:                 Write TREE -> DATA
Step 3:                 PREORDER(TREE -> LEFT)
Step 4:                 PREORDER(TREE -> RIGHT)
       [END OF LOOP]
Step 5: END
```

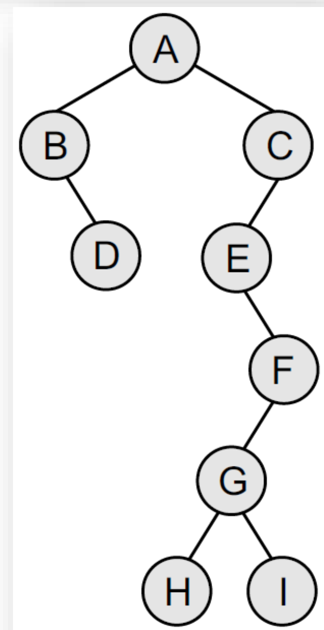
- Pre-order: ABDCEFGHI



Post-order

```
Step 1: Repeat Steps 2 to 4 while TREE != NULL
Step 2:          POSTORDER(TREE -> LEFT)
Step 3:          POSTORDER(TREE -> RIGHT)
Step 4:          Write TREE -> DATA
       [END OF LOOP]
Step 5: END
```

- Post-order: DBHIGFECA



Constructing Binary Tree from Traversal.

- We can construct a binary tree if we are given at least two traversal results
  - In-order traversal
    - The in-order traversal result will be used to **determine the left and the right child nodes**
  - Either pre-order or post-order traversal
    - The pre-order/post-order can be used to **determine the root node**

Constructing Binary Tree from Traversal..

- Take in-order + pre-order for example
  - In-order: $D\ B\ E\ A\ F\ C\ G$
  - Pre-order: $A\ B\ D\ E\ C\ F\ G$

$D\ B\ E\ A\ F\ C\ G$

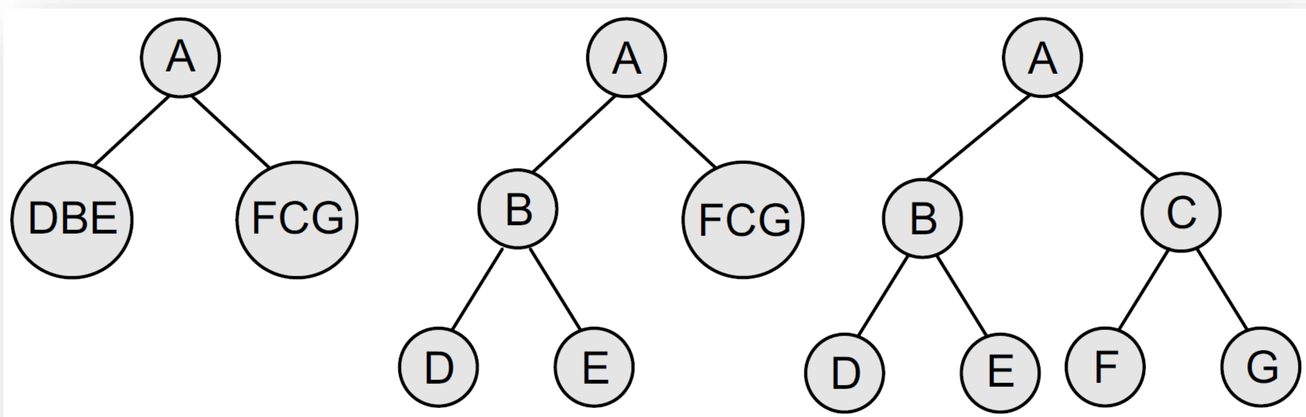
$A\ B\ D\ E\ C\ F\ G$

$D\ B\ E\ A\ F\ C\ G$

$A\ B\ D\ E\ C\ F\ G$

$D\ B\ E\ A\ F\ C\ G$

$A\ B\ D\ E\ C\ F\ G$



Constructing Binary Tree from Traversal…

- Take in-order + post-order for example
  - In-order: $D\ B\ H\ E\ I\ A\ F\ J\ C\ G$
  - Post-order: $D\ H\ I\ E\ B\ J\ F\ G\ C\ A$

$D\ B\ H\ E\ I\ A\ F\ J\ C\ G$

$D\ H\ I\ E\ B\ J\ F\ G\ C\ A$

$D\ B\ H\ E\ I\ A\ F\ J\ C\ G$

$D\ H\ I\ E\ B\ J\ F\ G\ C\ A$

$D\ B\ H\ E\ I\ A\ F\ J\ C\ G$

$D\ H\ I\ E\ B\ J\ F\ G\ C\ A$

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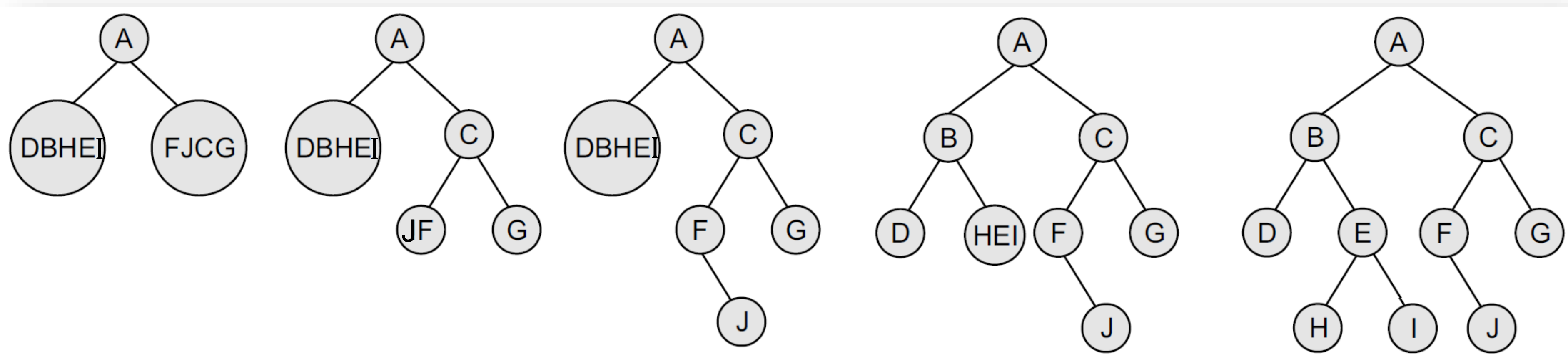
$D\ H\ I\ E\ B\ J\ F\ G\ C\ A$

$D\ B\ H\ E\ I\ A\ F\ J\ C\ G$

$D\ H\ I\ E\ B\ J\ F\ G\ C\ A$

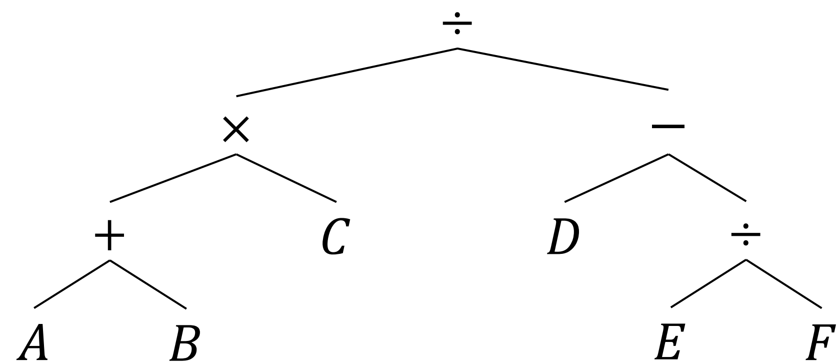


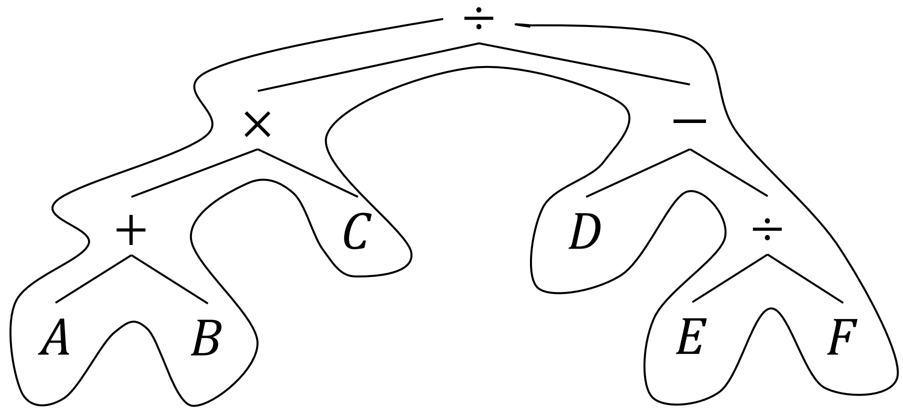
Constructing Binary Tree from Traversal….

- Steps for constructing a binary tree from traversal sequences
  1. Use the pre-order/post-order sequence to determine the root node of the tree
  2. Elements on the left side of the root node in the in-order traversal sequence form the left sub-tree of the root node
  3. Similarly, elements on the right side of the root node in the in-order traversal sequence form the right sub-tree of the root node
  4. Recursively select each element from pre-order/post-order traversal sequence and create its left and right sub-trees from the in-order traversal sequence

By Looking!.

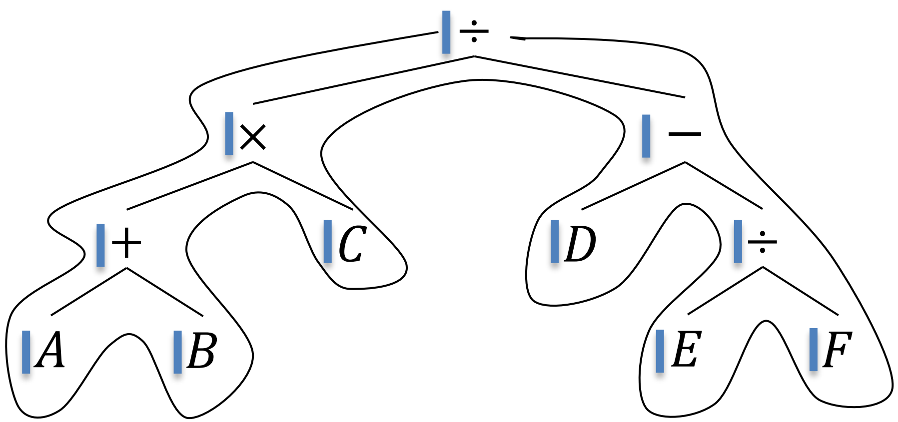
- Given a binary tree, please write down the in-order, pre-order and post-order expressions, respectively





By Looking!..

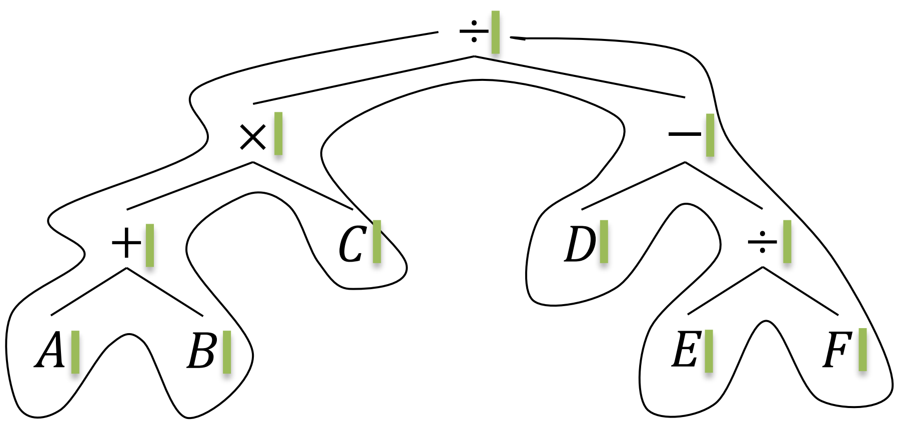
- Given a binary tree, please write down the in-order, pre-order and post-order expressions, respectively
  - $\{[(A+B)\times C]\div[D-(E\div F)]\}$



$\div\times+ABC-D\div EF$

By Looking!...

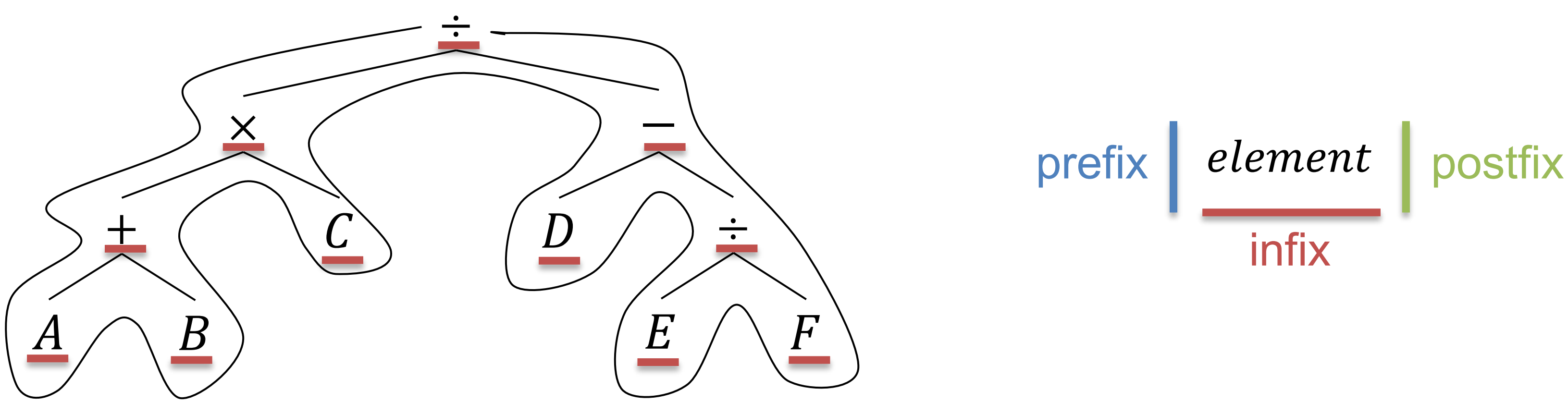
- Given a binary tree, please write down the in-order, pre-order and post-order expressions, respectively
  - $\{[(A+B)\times C]\div[D-(E\div F)]\}$



$$AB + C \times DEF \div - \div$$

By Looking!....

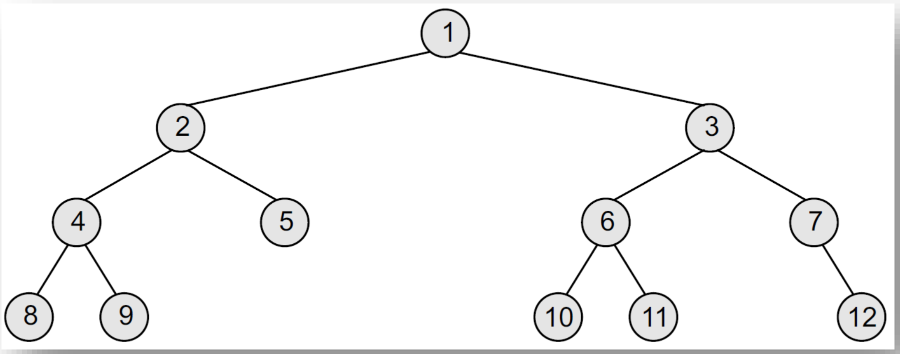
- Given a binary tree, please write down the in-order, pre-order and post-order expressions, respectively
  - $\{[(A+B)\times C]\div[D-(E\div F)]\}$



$A + B \times C \div D - E \div F$

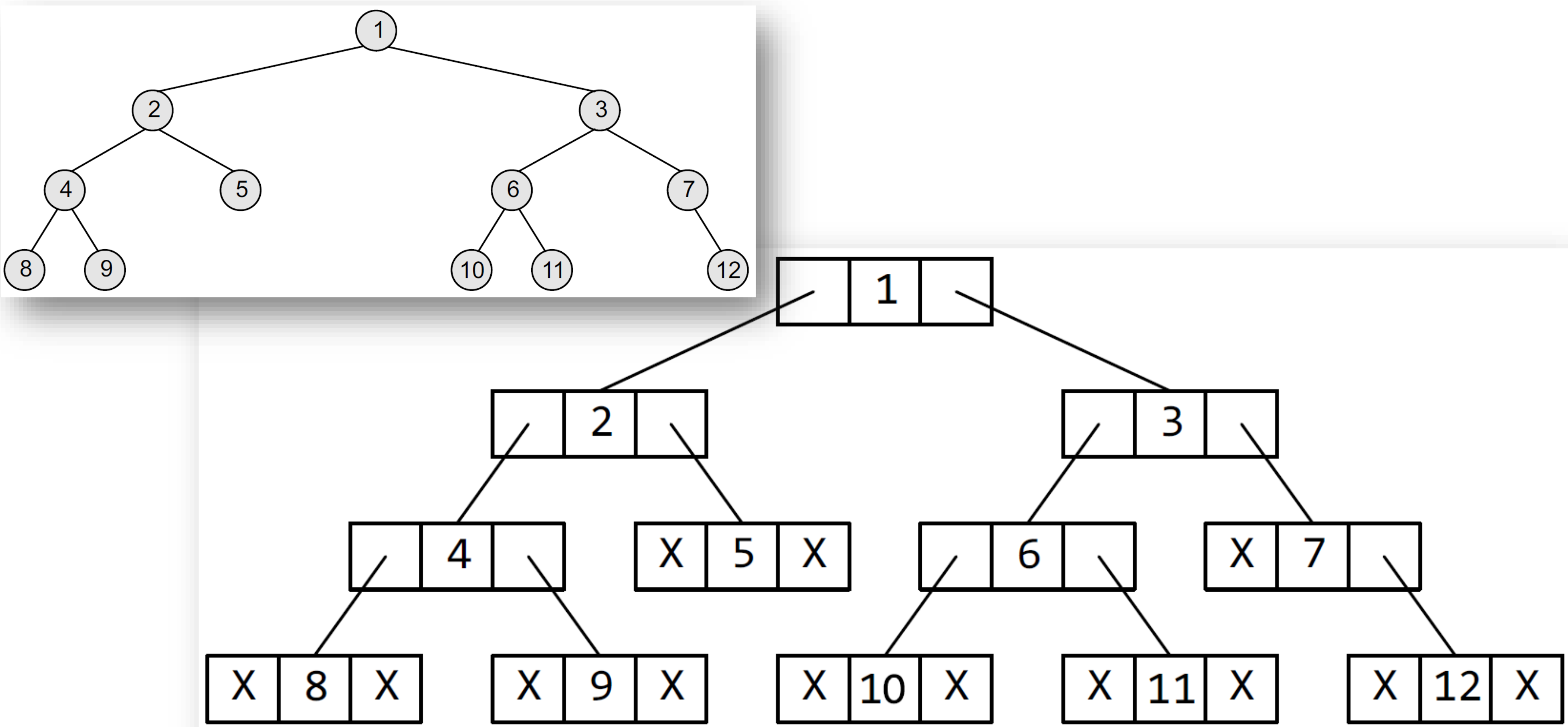
Example

- Given a binary tree, please write down the in-order, pre-order and post-order expressions, respectively
  - Pre-order: 1 2 4 8 9 5 3 6 10 11 7 12
  - Post-order: 8 9 4 5 2 10 11 6 12 7 3 1
  - In-order: 8 4 9 2 5 1 10 6 11 3 7 12



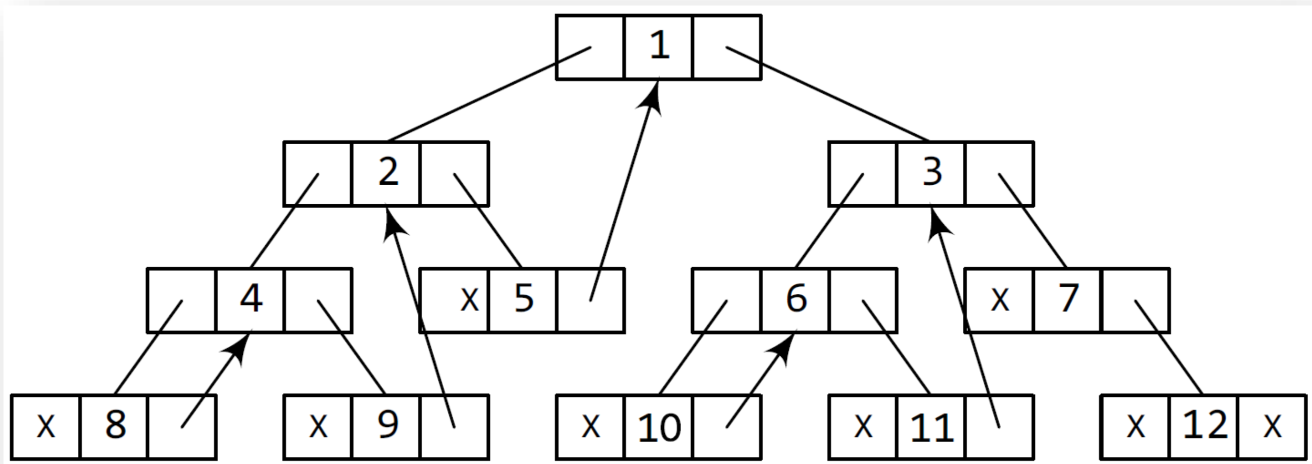
Threaded Binary Trees

- A threaded binary tree is the same as that of a binary tree but with a difference in storing the NULL pointers
  - The space that is wasted in storing a NULL pointer can be efficiently used to store some other useful piece of information



One-way Threaded Trees

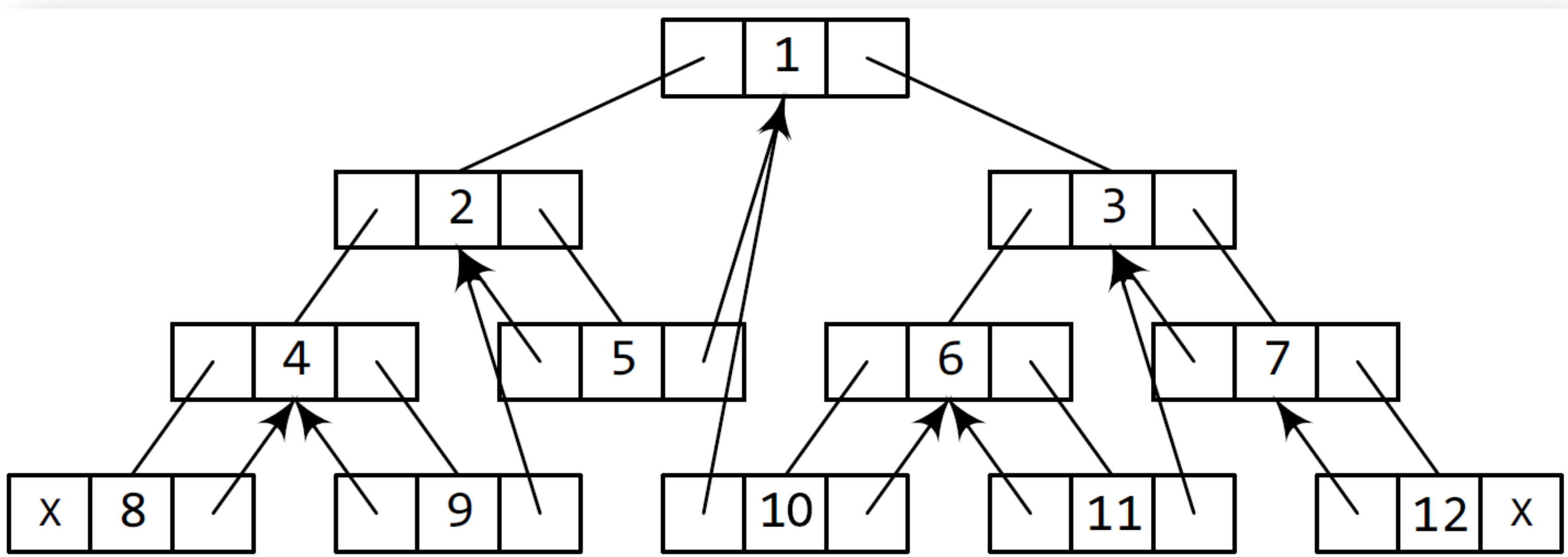
- A **one-way threaded tree** is also called a **single-threaded tree**
  - If the thread appears in the **right field**, then it will point to the **in-order successor** of the node
    - Such a tree is called a **right-threaded binary tree**



  - If the thread appears in the **left field**, then the left field will be made to point to the **in-order predecessor** of the node
    - Such a tree is called a **left-threaded binary tree**

Two-way Threaded Trees

- In a two-way threaded tree, also called a **double-threaded tree**, threads will appear in both the left and the right field of the node
  - The left field will point to the in-order predecessor of the node, and the right field will point to its successor
  - A two-way threaded binary tree is also called a **fully threaded binary tree**



Questions?

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