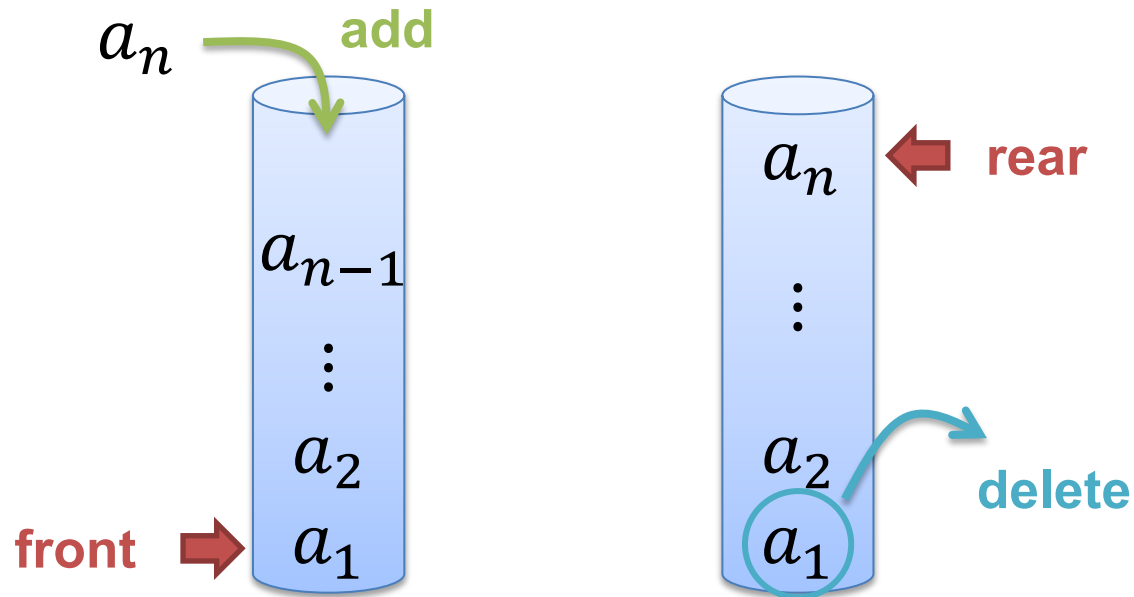


Trees

Kuan-Yu Chen (陳冠宇)

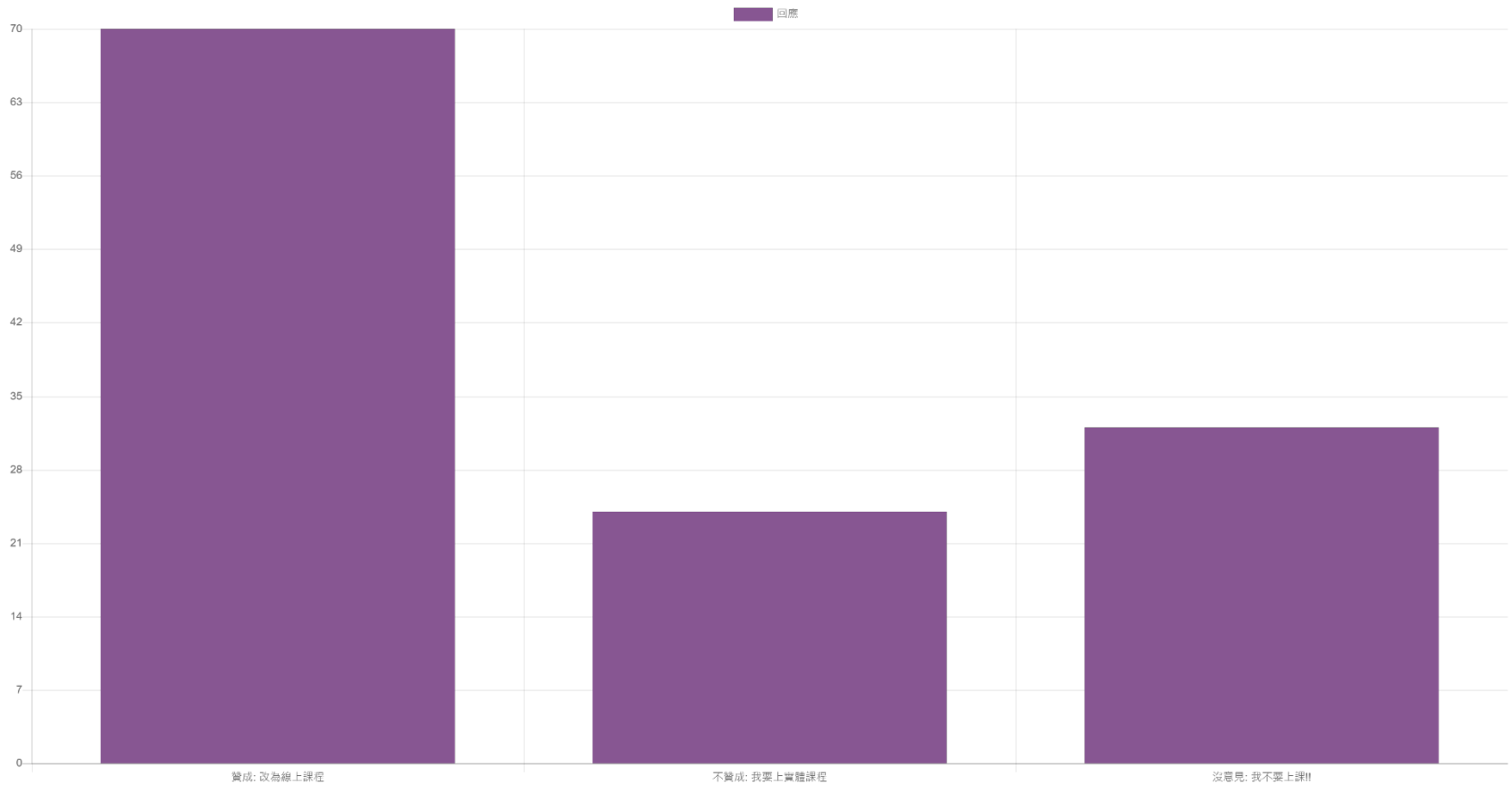
Review

- Queue
 - FIFO
 - Variations
 - Circular Queue
 - Deque
 - Priority Queue
 - Multiple Queue



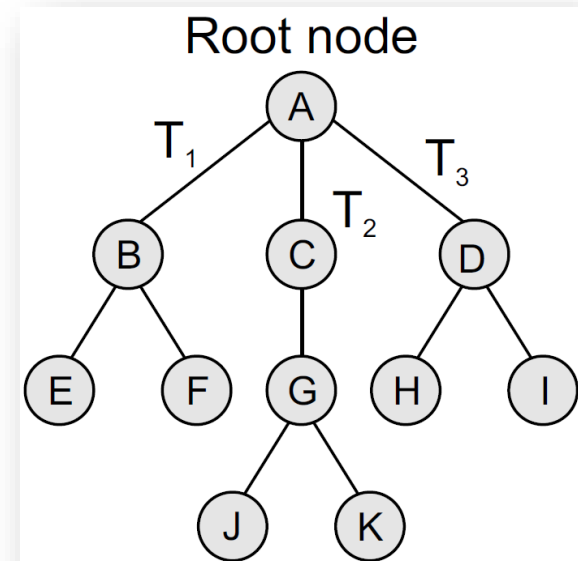
Wednesday Online!

- <https://ctld.webex.com/meet/kychen>



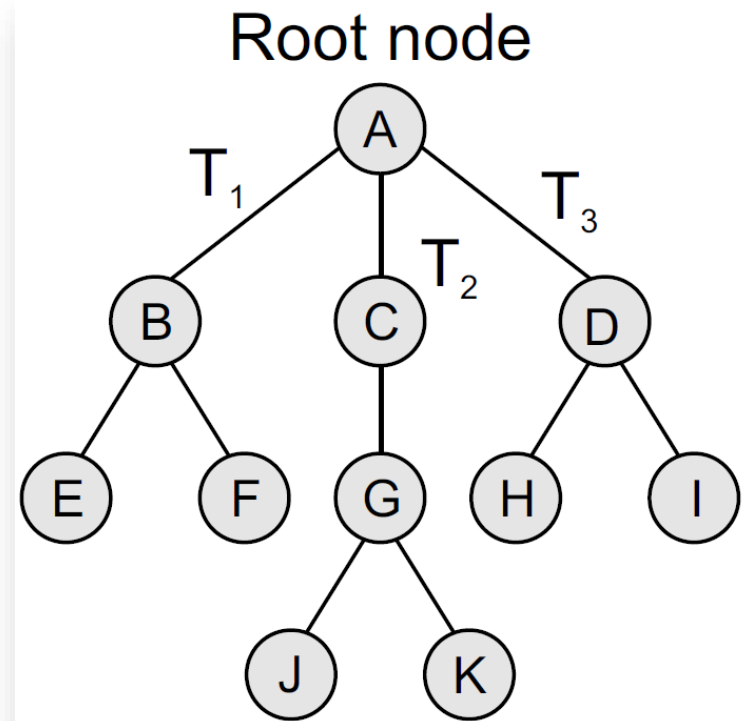
Trees

- So far, we have discussed **linear** data structures
 - Arrays
 - Stacks
 - Queues
- A tree is a **non-linear** data structure, which is mainly used to store data that is **hierarchical** in nature
 - Nodes B, C, and D are **children** of node A
 - Node A is **parent** of nodes B, C, and D



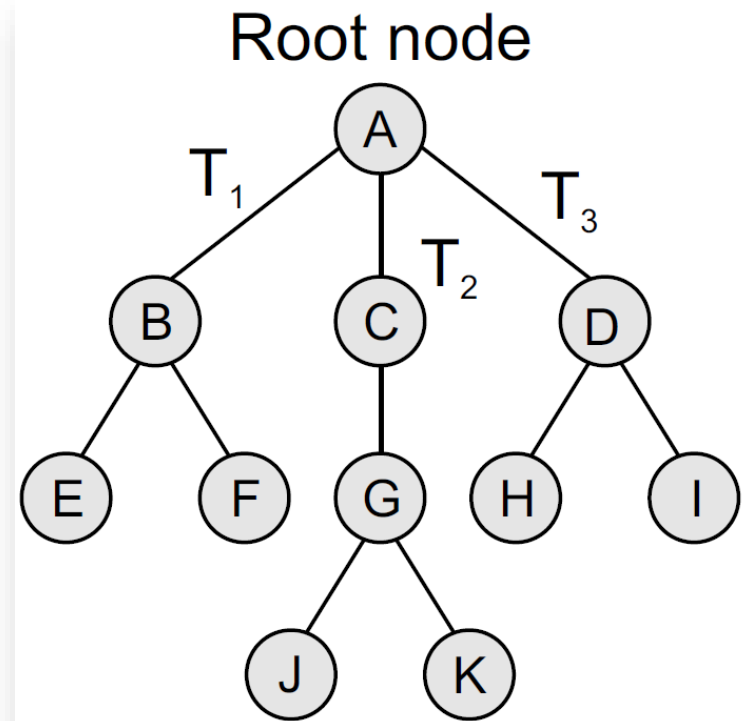
Basic Terminology.

- Root node
 - The root node R is the topmost node in the tree
 - If $R = NULL$, then it means the tree is empty
- Sub-trees
 - The trees T_1, T_2 and T_3 are called the sub-trees
- Leaf node
 - A node that has no children is called the leaf node or the terminal node
 - E, F, J, K, H, I
- Path
 - A sequence of consecutive edges is called a path
 - The path from the node A to node I is A, D, I



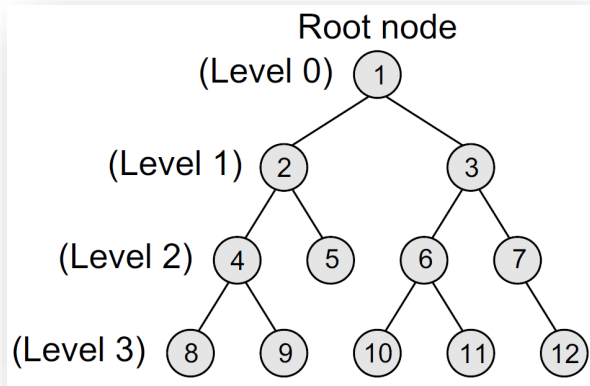
Basic Terminology..

- Ancestor node
 - An ancestor of a node is any predecessor node on the path from root to that node
 - The root node does not have any ancestors
 - Nodes *A*, *C* and *G* are the ancestors of node *K*
- Descendant node
 - A descendant node is any successor node on any path from the node to a leaf node
 - Leaf nodes do not have any descendants
 - Nodes *E* and *F* are the descendants of node *B*

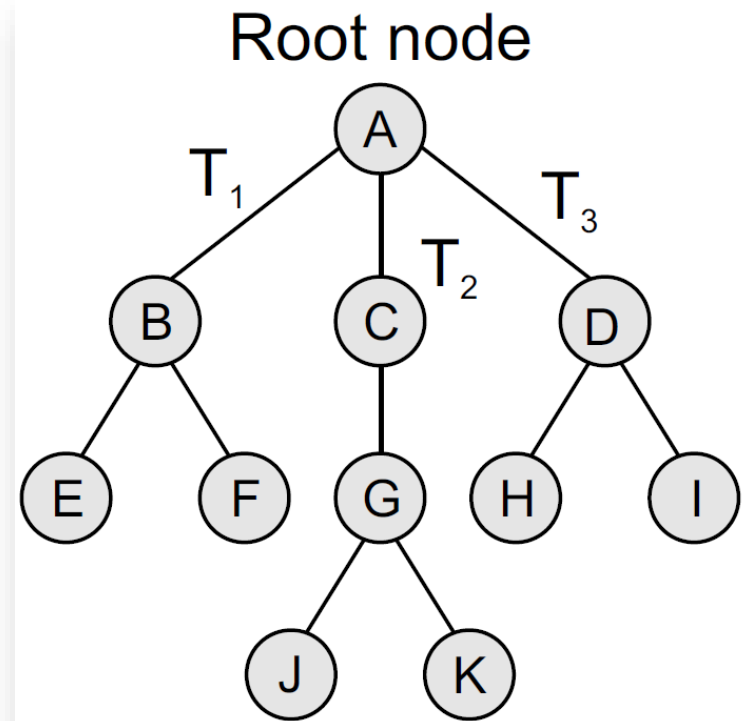


Basic Terminology...

- Level number
 - Every node in the tree is assigned a level number
 - The root node is at level 0
 - Children of the root node (i.e., B , C and D) are at level number 1



- Degree
 - Degree of a node is equal to the number of children that a node has
 - The degree of J is 0
 - The degree of B is 2

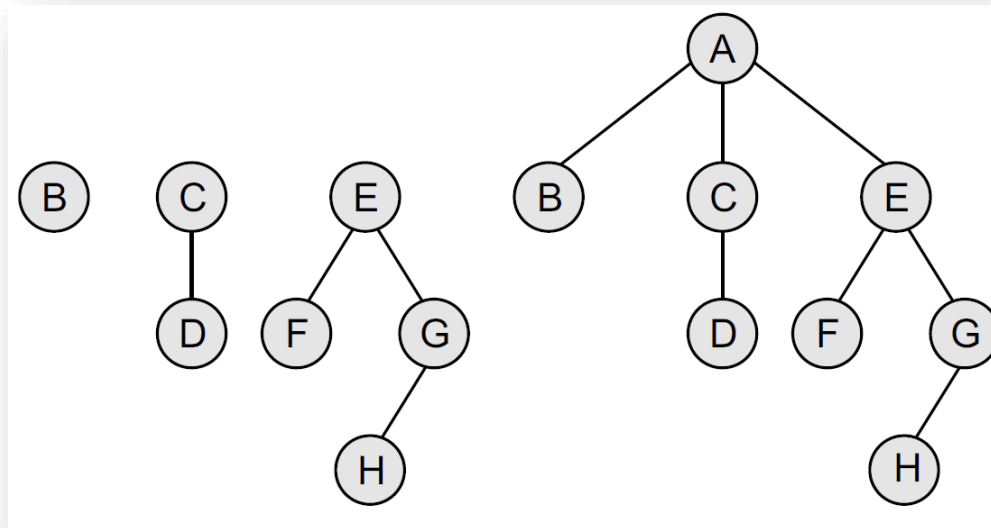


Types of Trees

- Trees can be classified into six classes
 - General Trees
 - Forests
 - Binary Trees
 - **Binary Search Trees**
 - Expression Trees
 - Tournament Trees

Forest

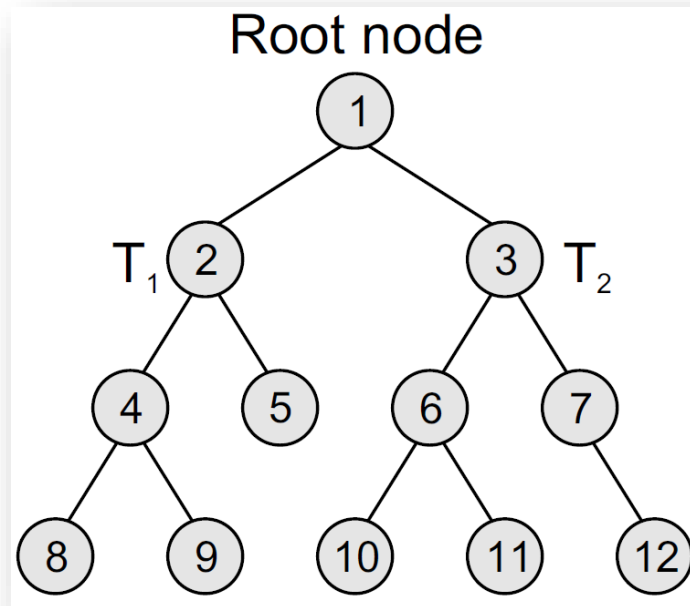
- A forest is a disjoint union of trees
 - We can convert a forest into a tree by adding a single node as the root node of the tree
 - A set of disjoint trees (or forests) is obtained by deleting the root and the edges connecting the root node to nodes at level 1




- While a general tree must have a root, **a forest may be empty** because by definition it is a set, and sets can be empty

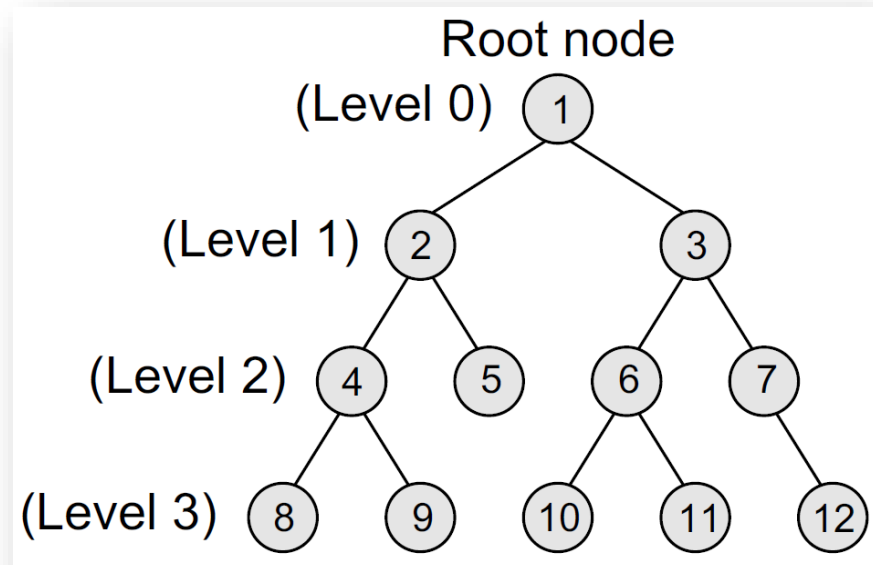
Binary Trees.

- A binary tree is a data structure that is defined as a collection of elements called nodes
- In a binary tree, the topmost element is called the root node, and each node has 0, 1, or at the most 2 children



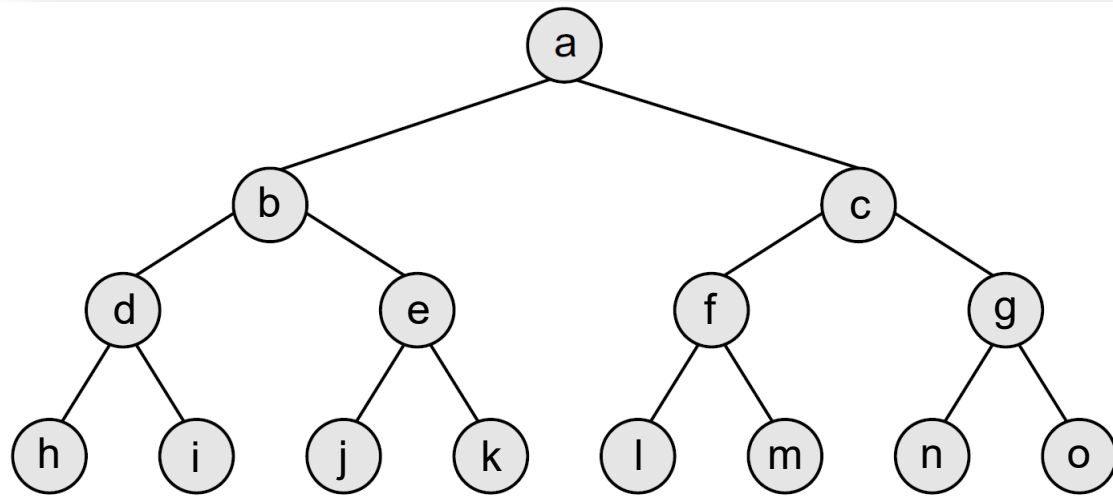
Binary Trees..

- Parent
 - Every node other than the root node has a parent
 - Node 2 is the parent of nodes 4 and 5
- Sibling
 - All nodes that are at the same level and share the same parent are called  (brothers)
 - Nodes 4 and 5 are siblings



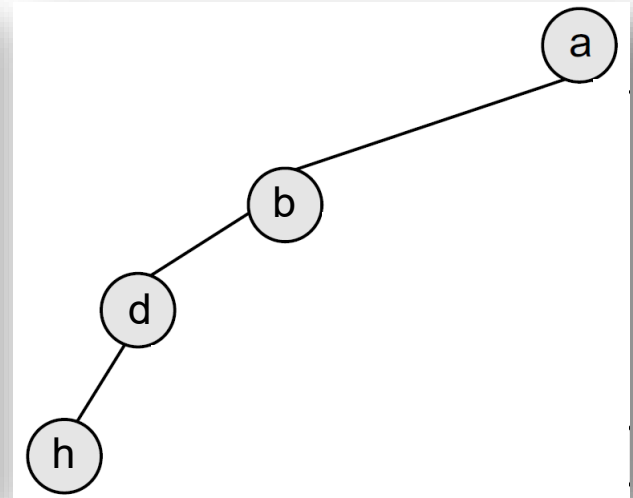
Binary Trees...

- Height of a Tree
 - It is the total number of nodes on the path from the root node to the deepest node in the tree
 - A binary tree of height h has at least h nodes and at most $2^h - 1$ nodes
 - The level of a tree is started from 0
 - Root node is at level 0



Height= 4

Number of nodes= $2^4 - 1 = 15$



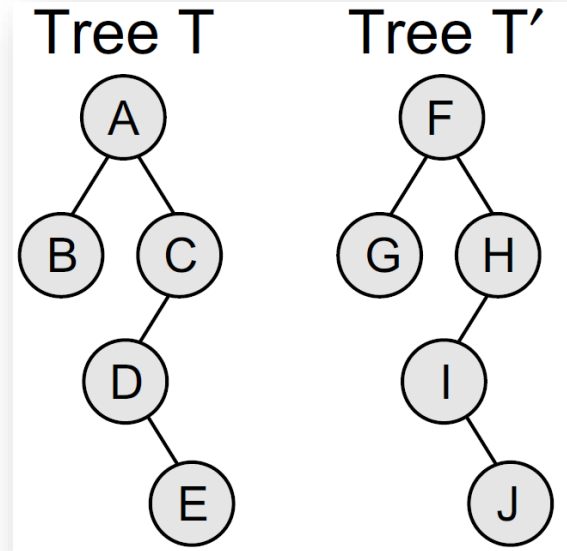
Height= 4

Number of nodes= 4

Binary Trees....

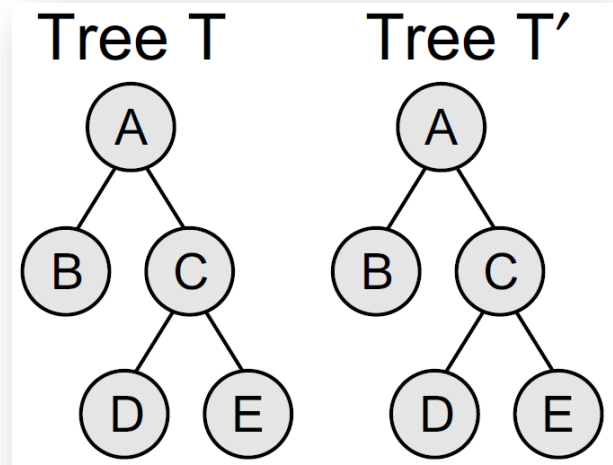
- Similar Binary Trees

- Two binary trees T and T' are said to be similar if both these trees have the **same structure**



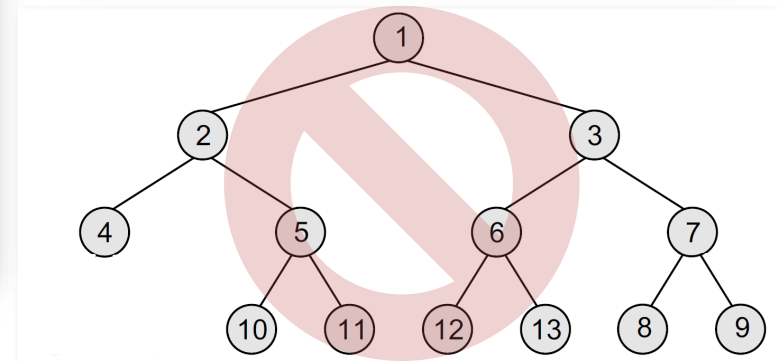
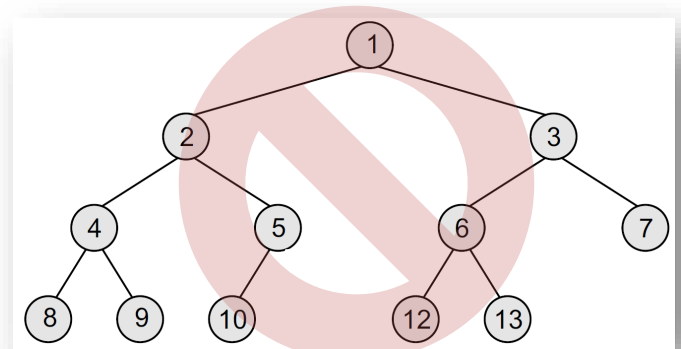
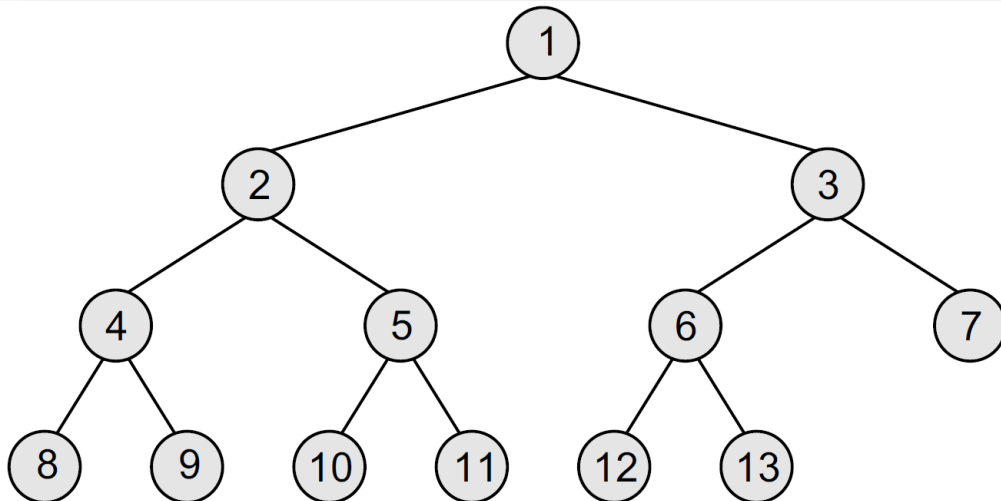
- Copies

- Two binary trees T and T' are said to be copies if they have **similar structure** and if they have **same content** at the corresponding nodes



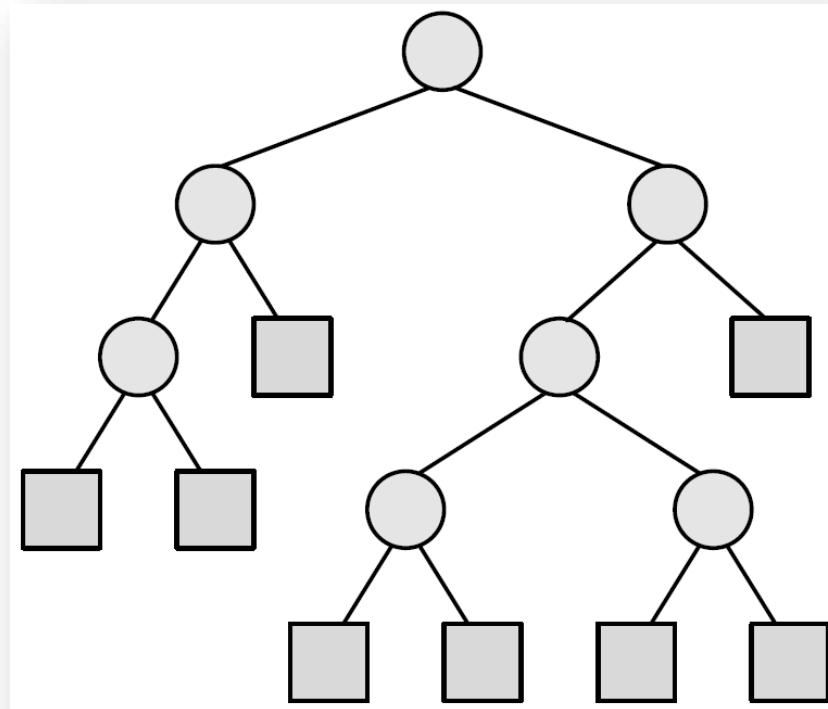
Complete Binary Tree

- Complete Binary Trees
 - A complete binary tree is a binary tree that satisfies two properties
 1. Every level, except possibly the last, is completely filled
 2. All nodes appear as far left as possible



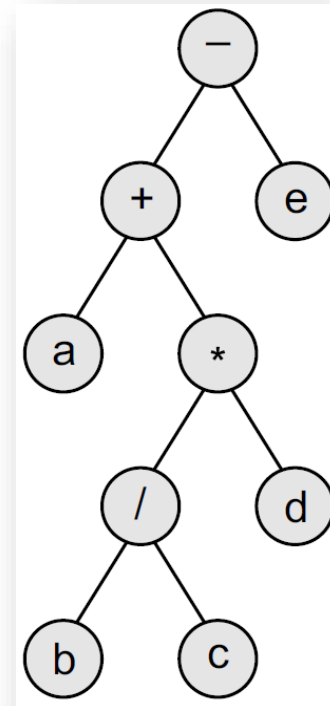
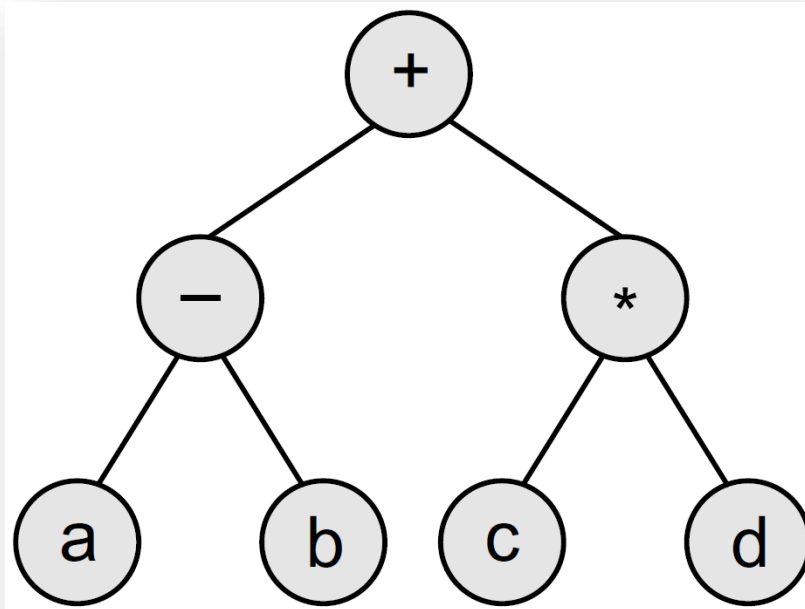
Extended Binary Tree

- A binary tree is said to be an extended binary tree (or a 2-tree) if each node in the tree has **either no child** or **exactly two children**
 - Nodes having two children are called **internal nodes** and nodes having no children are called **external nodes**



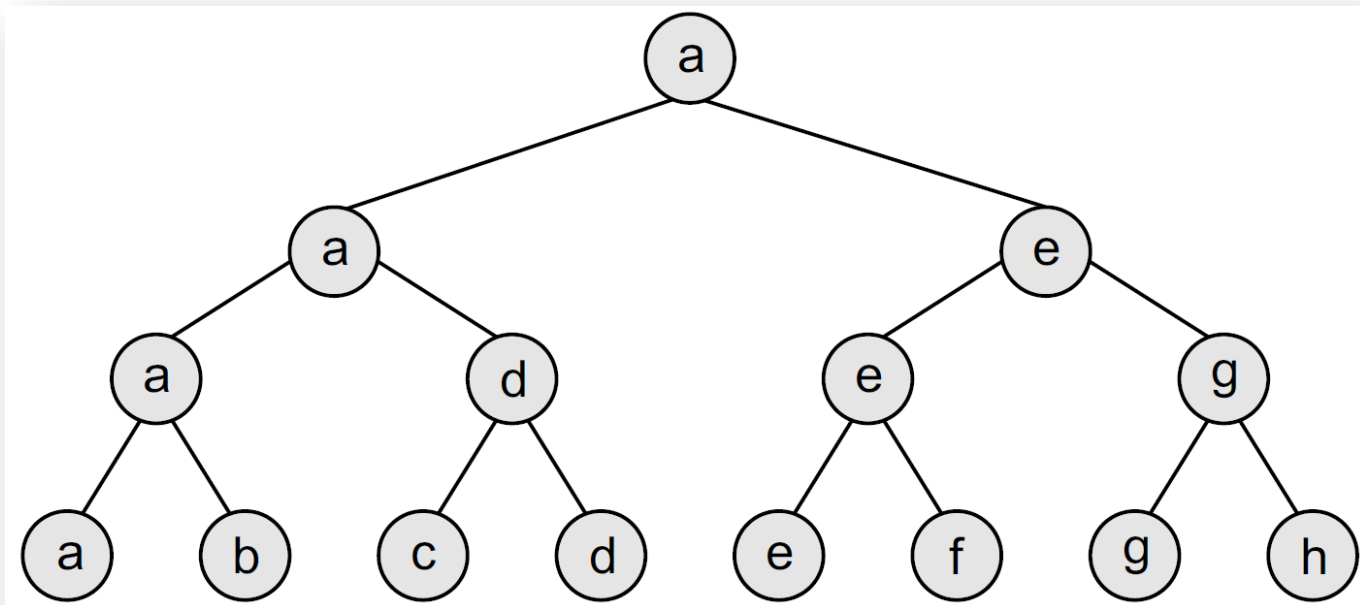
Expression Trees

- Binary trees are widely used to store algebraic expressions (代數運算)
 - Given an algebraic expression $(a - b) + (c \times d)$
 - Given an expression $a + b \div c \times d - e$



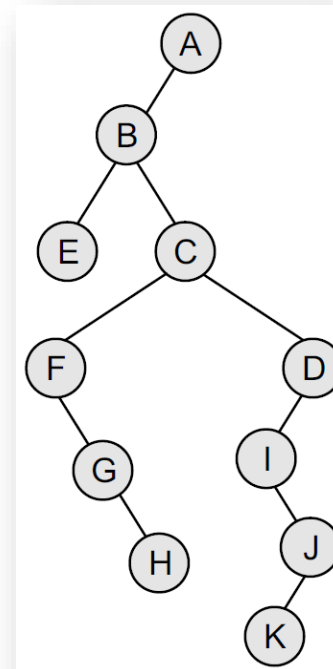
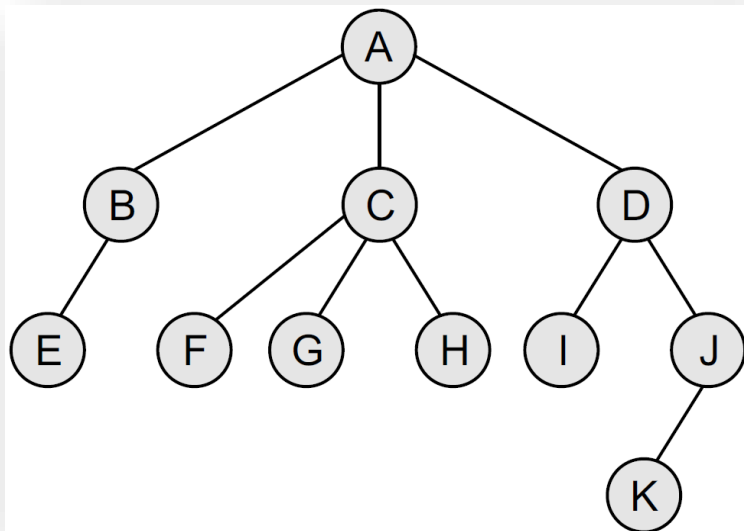
Tournament Trees

- In a tournament tree (also called a **selection tree**), each external node represents a candidate and each internal node represents the selected candidate by its children nodes
 - These tournament trees are also called **winner trees** because they are being used to record the winner at each level
 - We can also have a **loser tree** that records the loser at each level

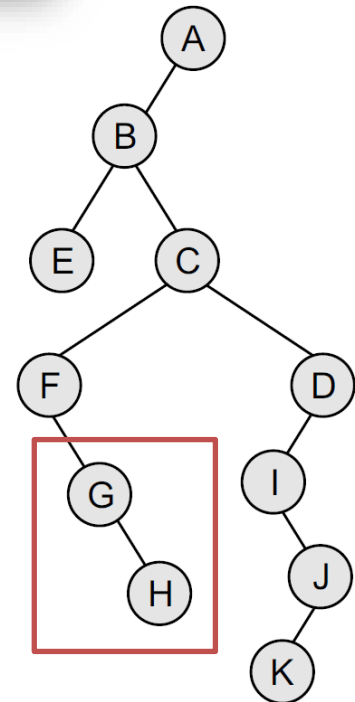
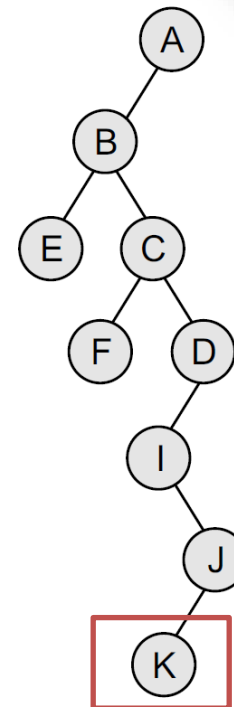
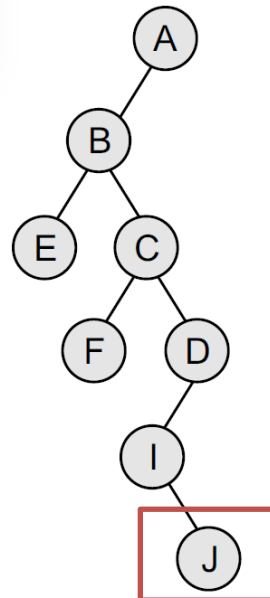
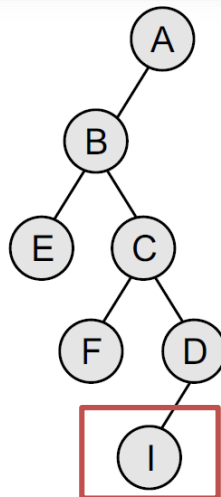
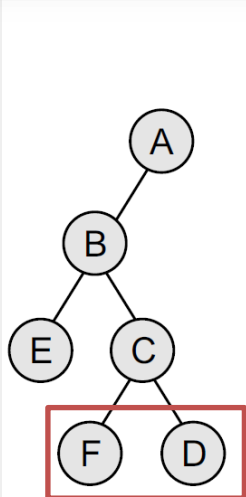
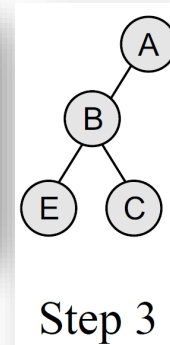
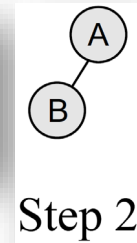
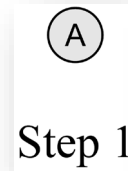
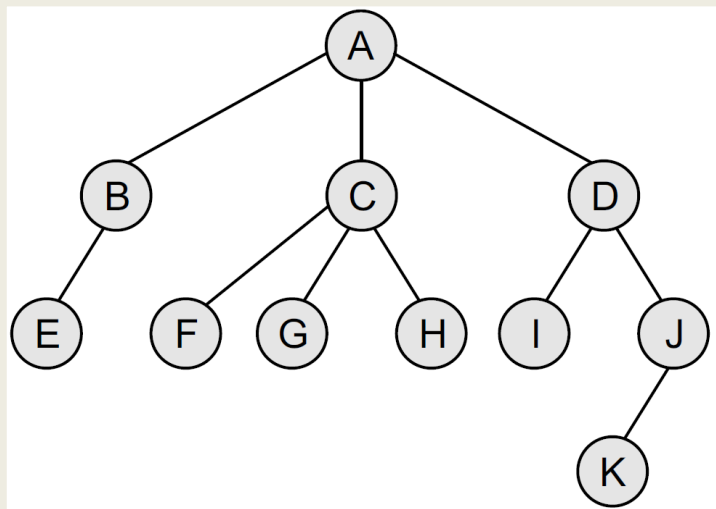


From a General Tree to a Binary Tree.

- The rules for converting a general tree to a binary tree are given below
 - **Rule 1:** Root of the binary tree = Root of the general tree
 - **Rule 2:** Left child of a node in the binary tree = Leftmost child of the node in the general tree
 - **Rule 3:** Right child of a node in the binary tree = Right sibling of the node in the general tree

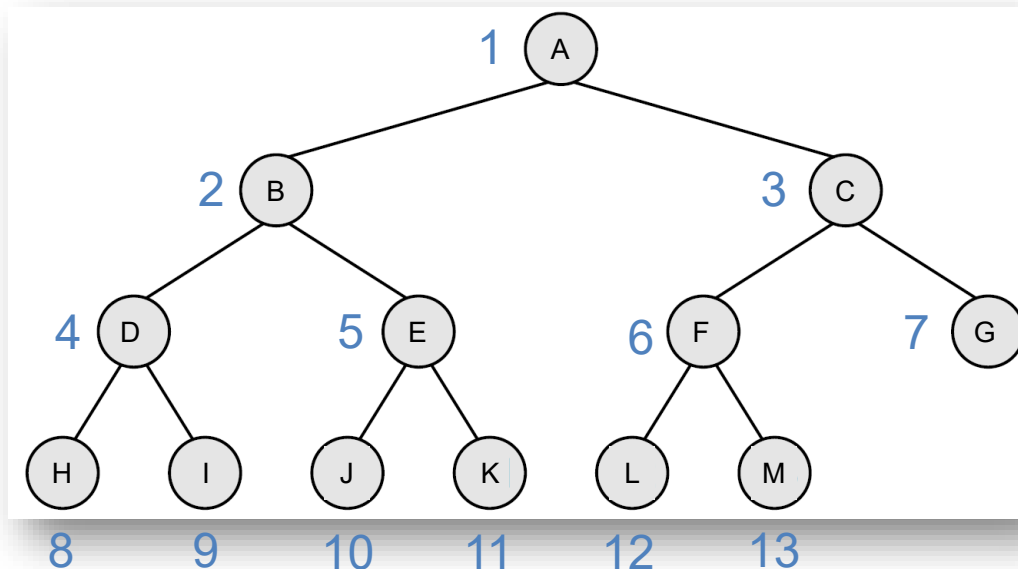


From a General Tree to a Binary Tree..



Array Implementation for Binary Tree.

- For a binary tree, we can number all of the nodes ordered
 - If K is a parent node, then its left child can be calculated as $2 \times K$ and its right child can be calculated as $2 \times K + 1$
 - The children of the node 4 are 8 and 9
 - The parent of the node K can be calculated as $\left\lfloor \frac{K}{2} \right\rfloor$
 - The parent of the node 5 is 2

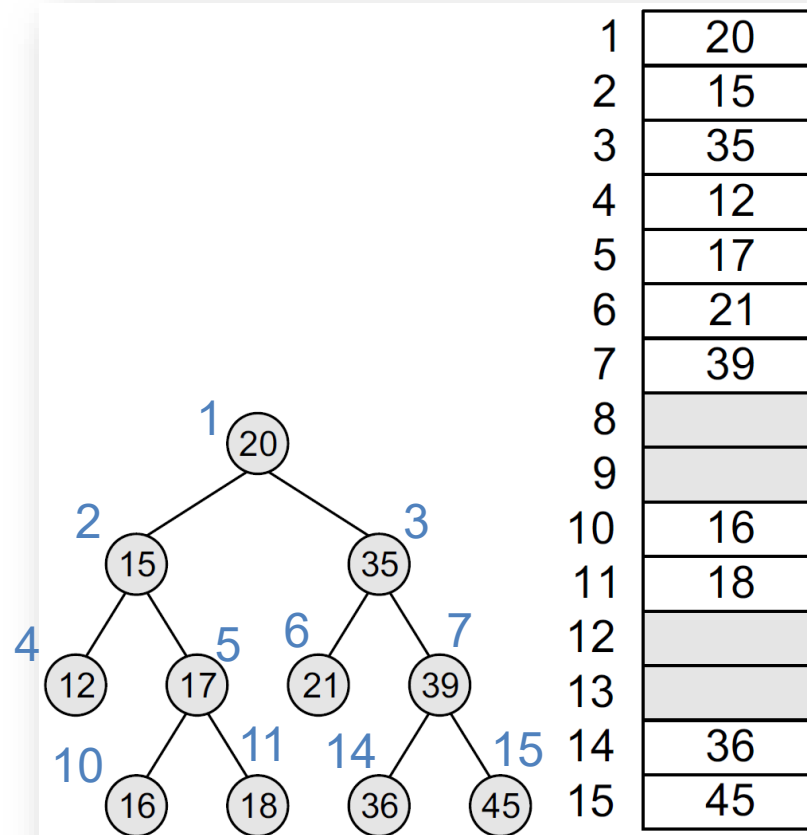


Array Implementation for Binary Tree..

- Sequential representation of trees is done using single or one-dimensional arrays
 - Though it is the simplest technique for memory representation, it is inefficient as it requires a lot of memory space

Height = 4

Number of nodes = $2^4 - 1 = 15$



Questions?



kychen@mail.ntust.edu.tw