Binary Search Trees

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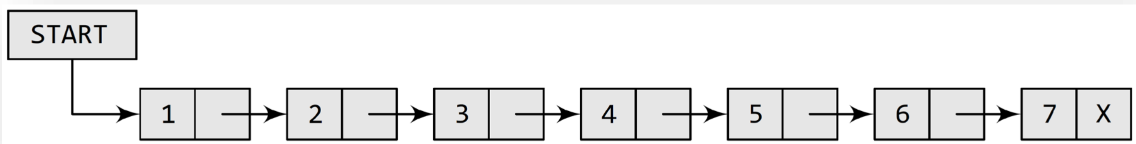
Review

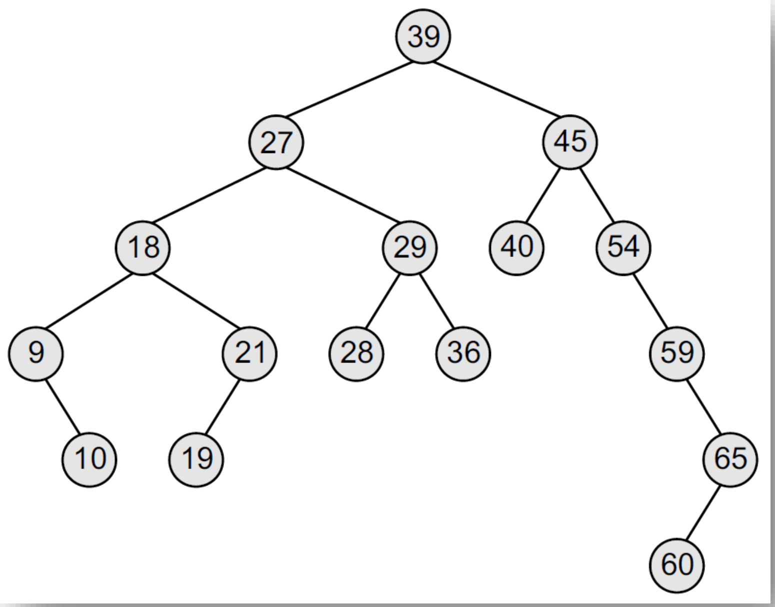
- A linked list, in simple terms, is a linear collection of data elements
  - Data elements are called **nodes**
  - Each node contains **one or more data fields** and a **pointer** to the next node



- Traversing a binary tree is the process of visiting each node in the tree exactly once in a systematic way
  - Pre-order Traversal
  - Post-order Traversal
  - In-order Traversal
  - Level-order Traversal

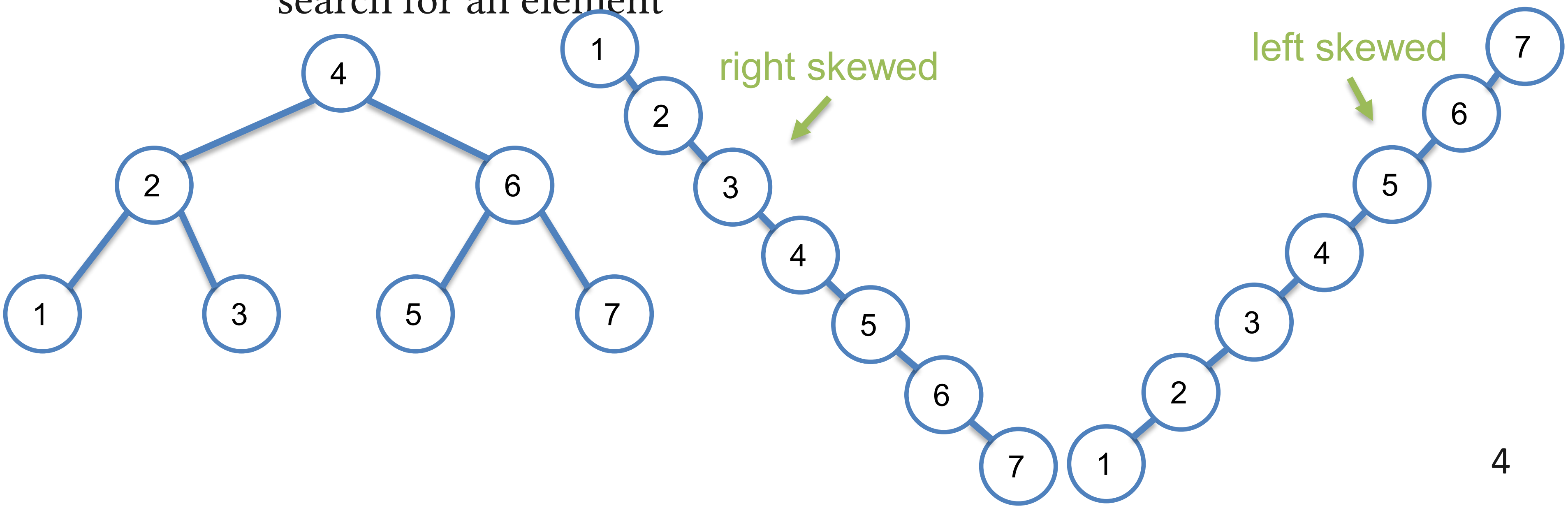
Binary Search Trees.

- A binary search tree, also known as an **ordered binary tree**, is a variant of binary trees in which the nodes are arranged in an order
  - All the nodes in the **left sub-tree** have a value **less** than that of the root node
  - All the nodes in the **right sub-tree** have a value either **equal to or greater** than the root node



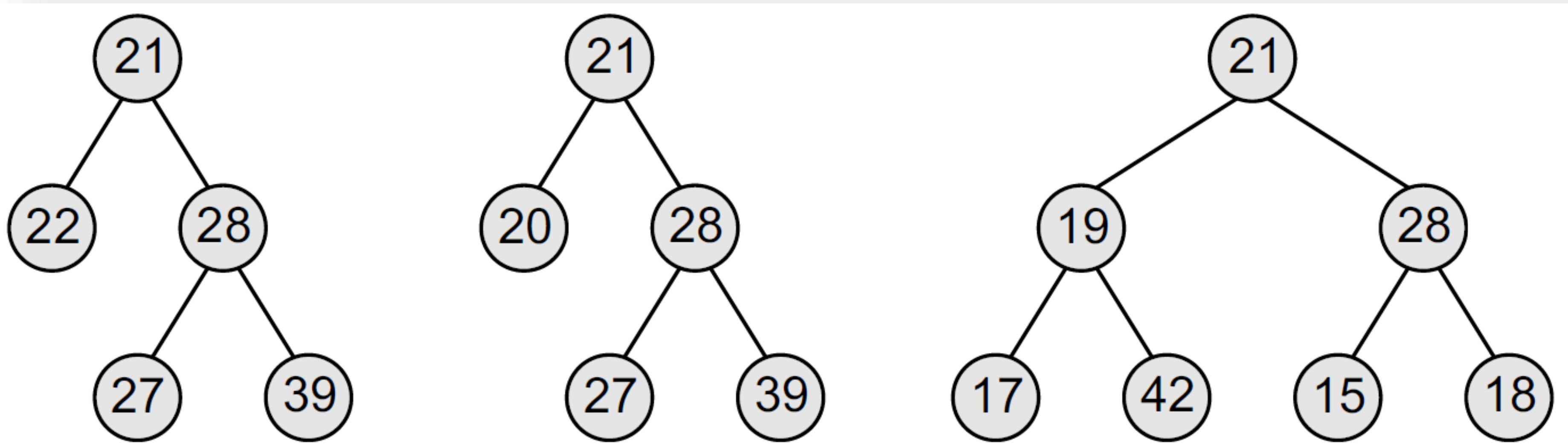
Binary Search Trees..

- Since the nodes in a binary search tree are ordered, the time needed to search an element in the tree is greatly reduced
  - We do not need to traverse the entire tree
  - At every node, we get a hint regarding which sub-tree to search in
    - The average running time of a search operation is $O(log_2 n)$
    - In the worst case, a binary search tree will take $O(n)$ time to search for an element



Binary Search Trees or not?.

- Which trees are binary search trees?



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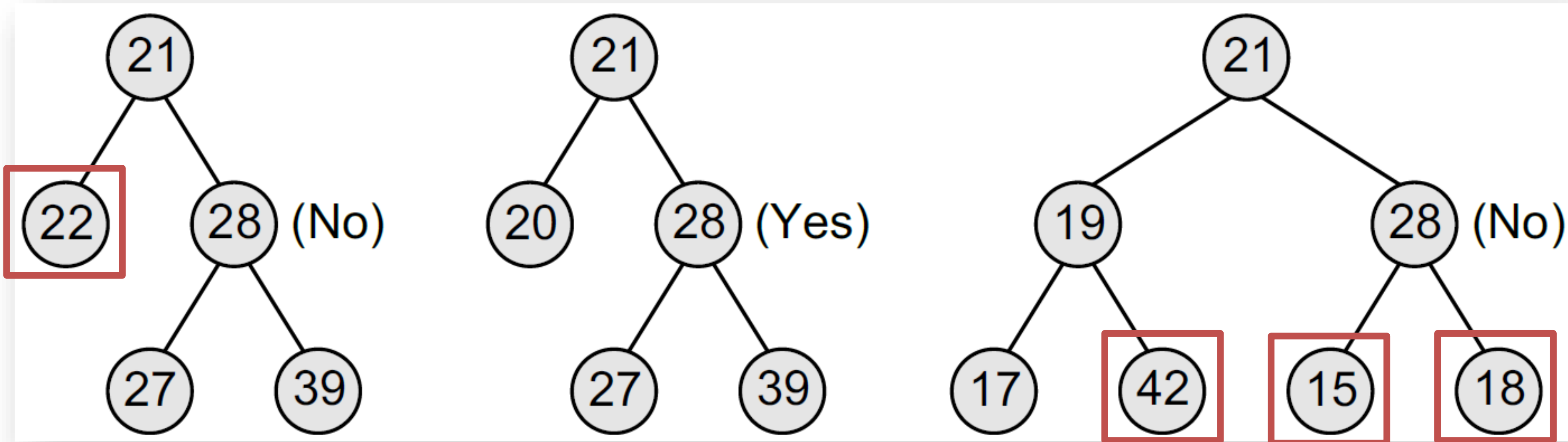


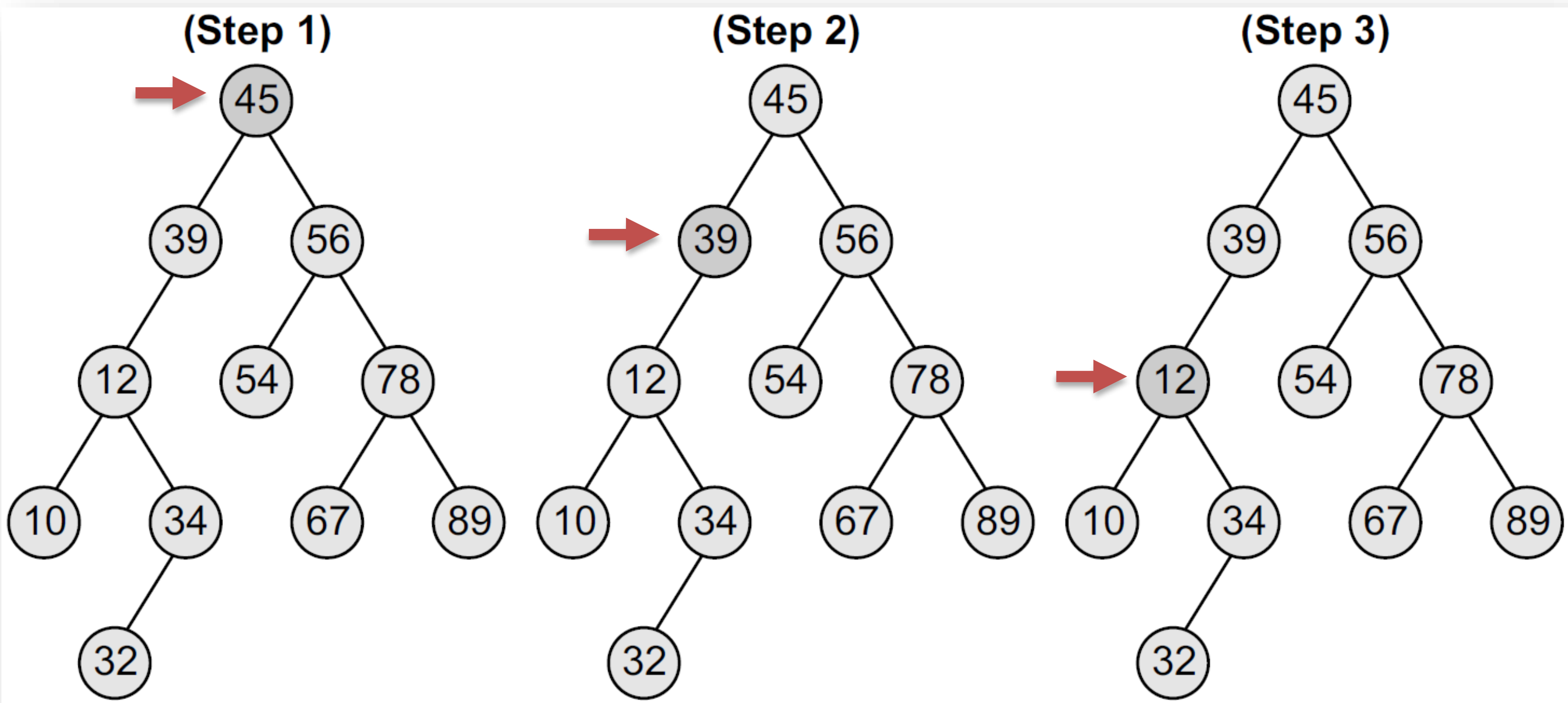
Searching a Node.

- The search function is used to find whether a given value is present in the tree or not
  - Checks if the binary search tree is empty
  - Compare the value
    - Find
    - Go left
    - Go right

```
searchElement (TREE, VAL)

Step 1: IF TREE -> DATA = VAL OR TREE = NULL
          Return TREE
        ELSE
         IF VAL < TREE -> DATA
           Return searchElement(TREE -> LEFT, VAL)
         ELSE
           Return searchElement(TREE -> RIGHT, VAL)
         [END OF IF]
        [END OF IF]
Step 2: END
```

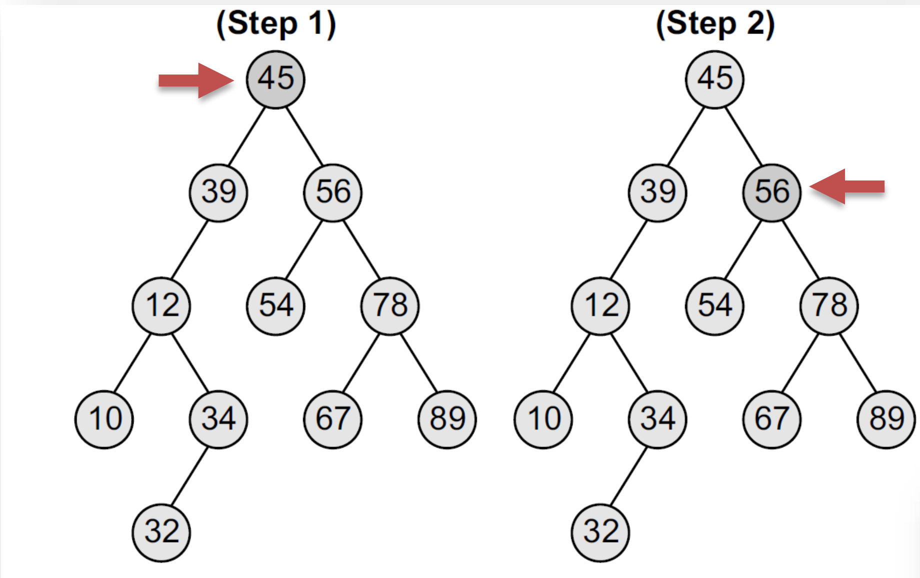
Searching a Node..

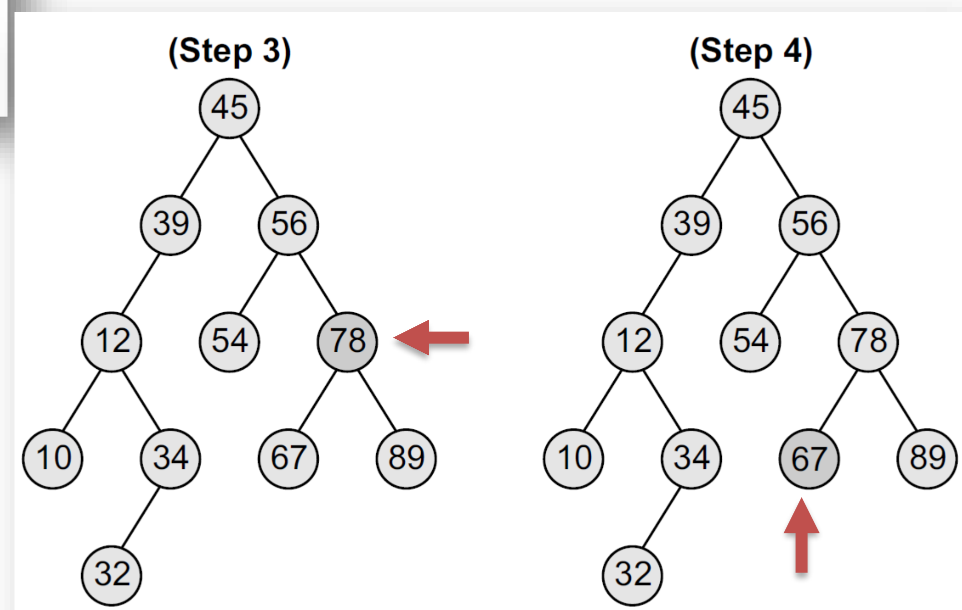
- Searching a node with value 12 in the given binary search tree



Searching a Node…

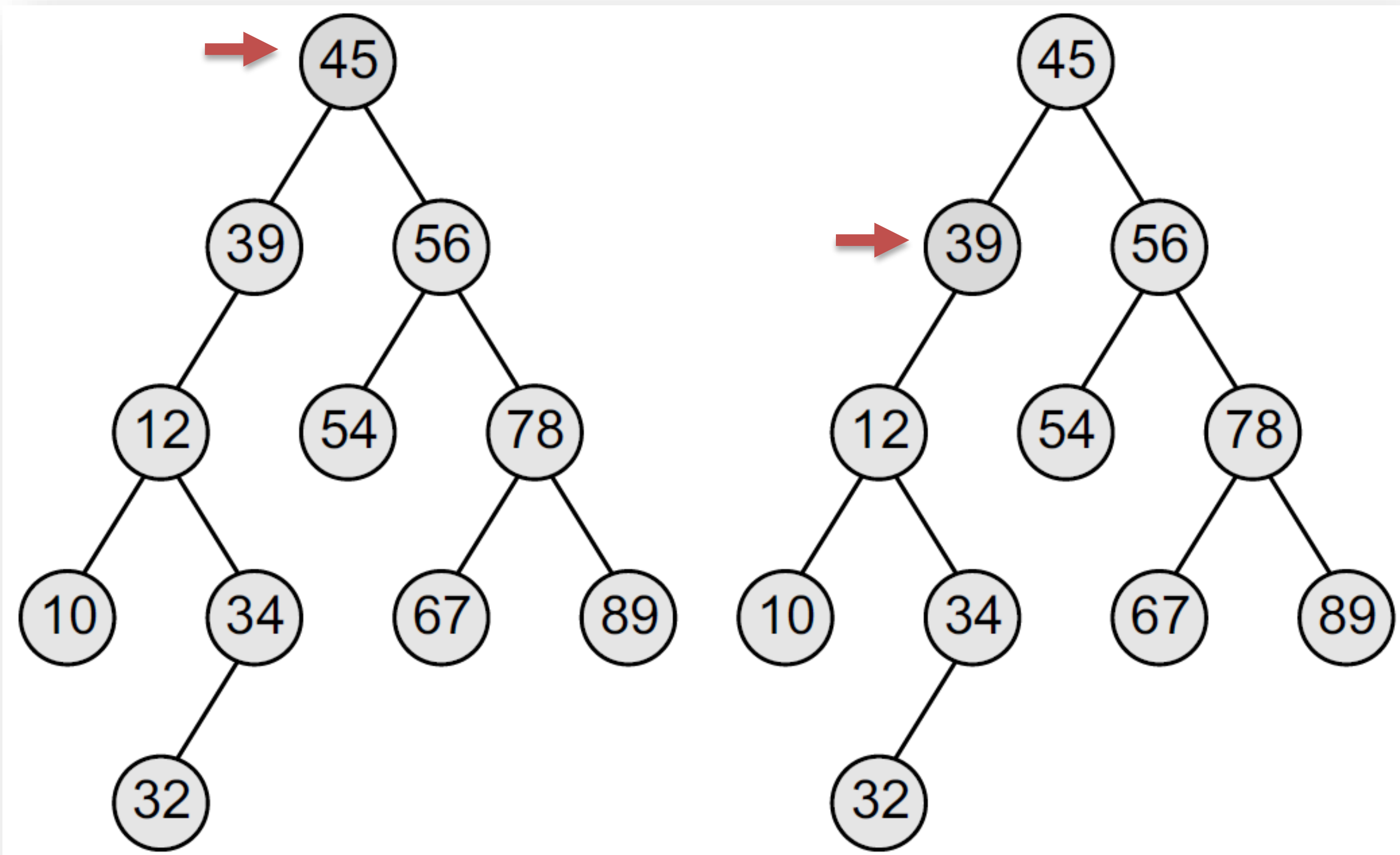
- Searching a node with value 67





Searching a Node….

- Searching a node with the value 40



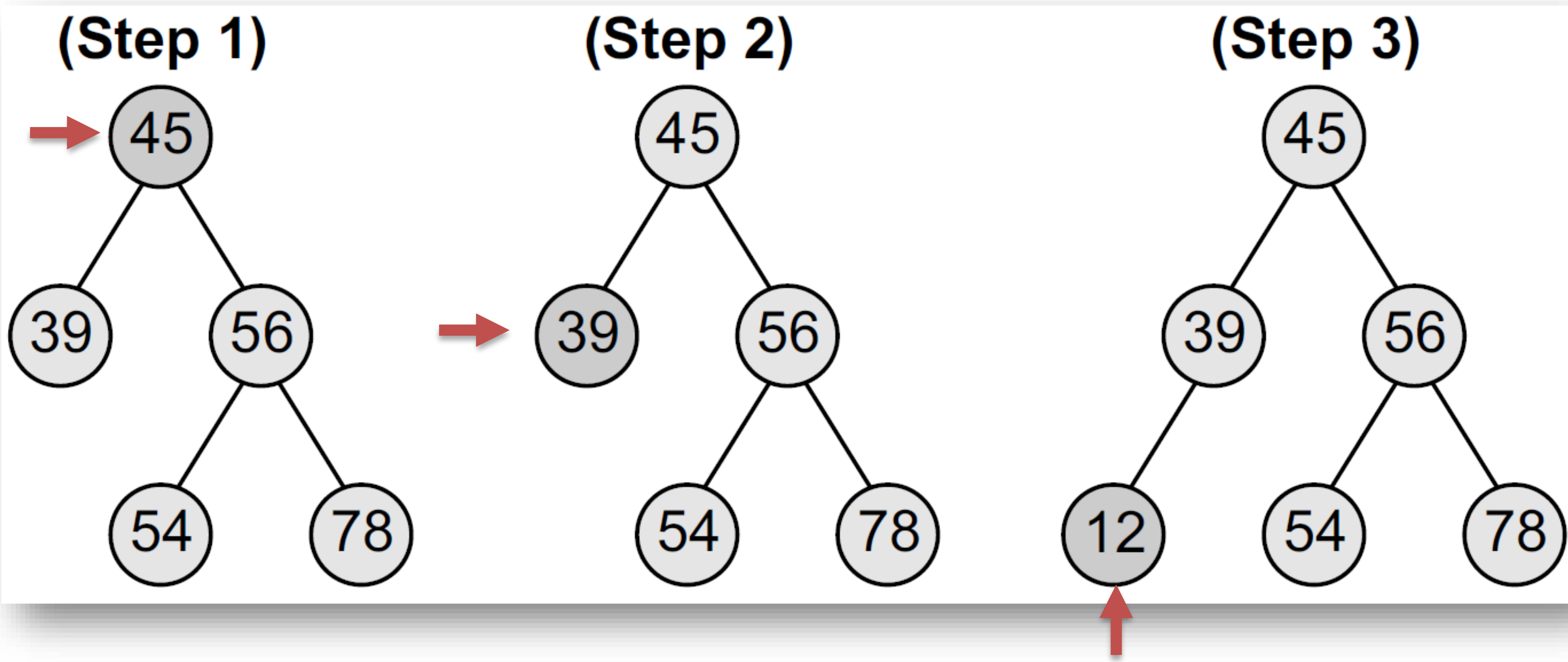
Inserting a Node.

- The insert function is used to add a new node with a given value at the correct position in the binary search tree

```
Insert (TREE, VAL)

Step 1: IF TREE = NULL
            Allocate memory for TREE
            SET TREE -> DATA = VAL
            SET TREE -> LEFT = TREE -> RIGHT = NULL
        ELSE
            IF VAL < TREE -> DATA
                  Insert(TREE -> LEFT, VAL)
            ELSE
                  Insert(TREE -> RIGHT, VAL)
            [END OF IF]
        [END OF IF]
Step 2: END
```

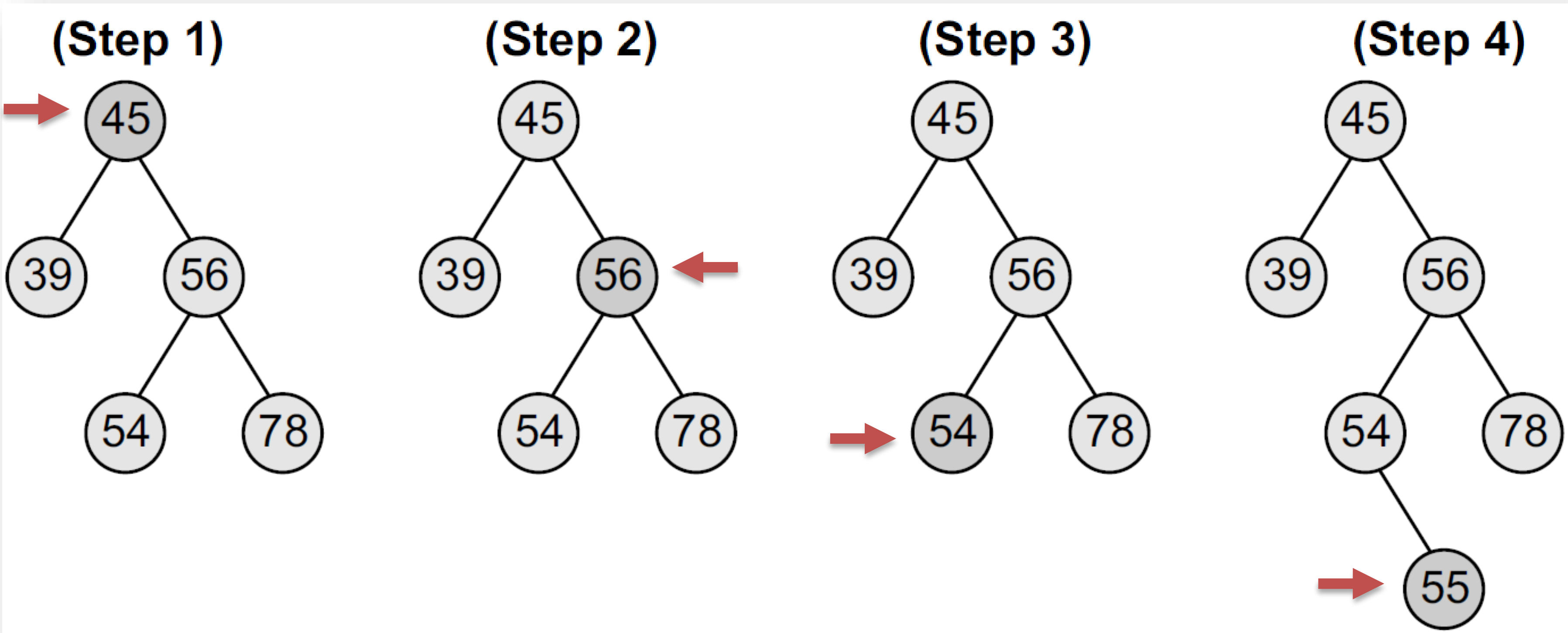
Inserting a Node..

- Inserting a node with values 12



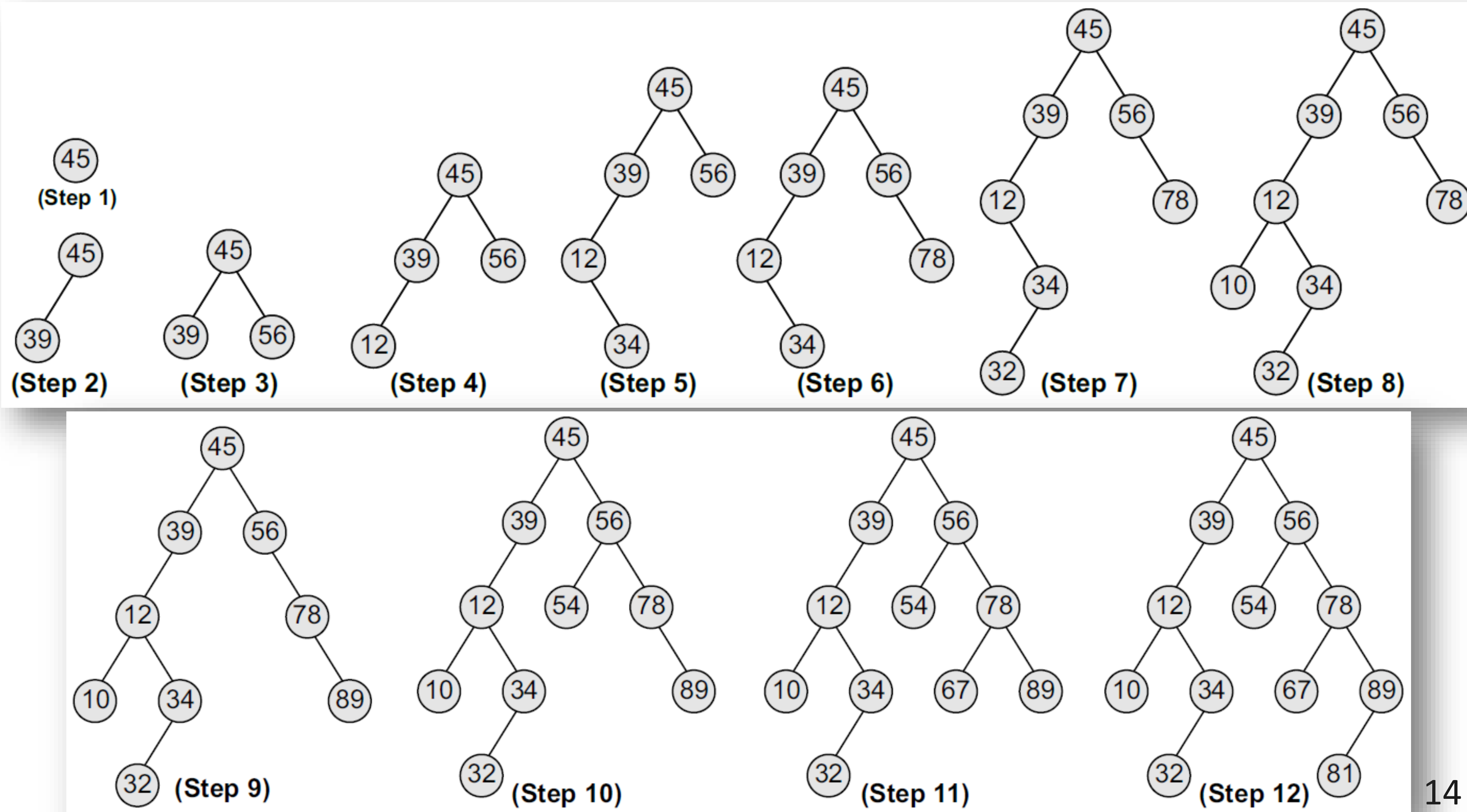
Inserting a Node...

- Inserting a node with values 55



Steps for Creating a Binary Search Tree

- Create a binary search tree using the following data elements:
  45, 39, 56, 12, 34, 78, 32, 10, 89, 54, 67, 81



Deleting a Node.

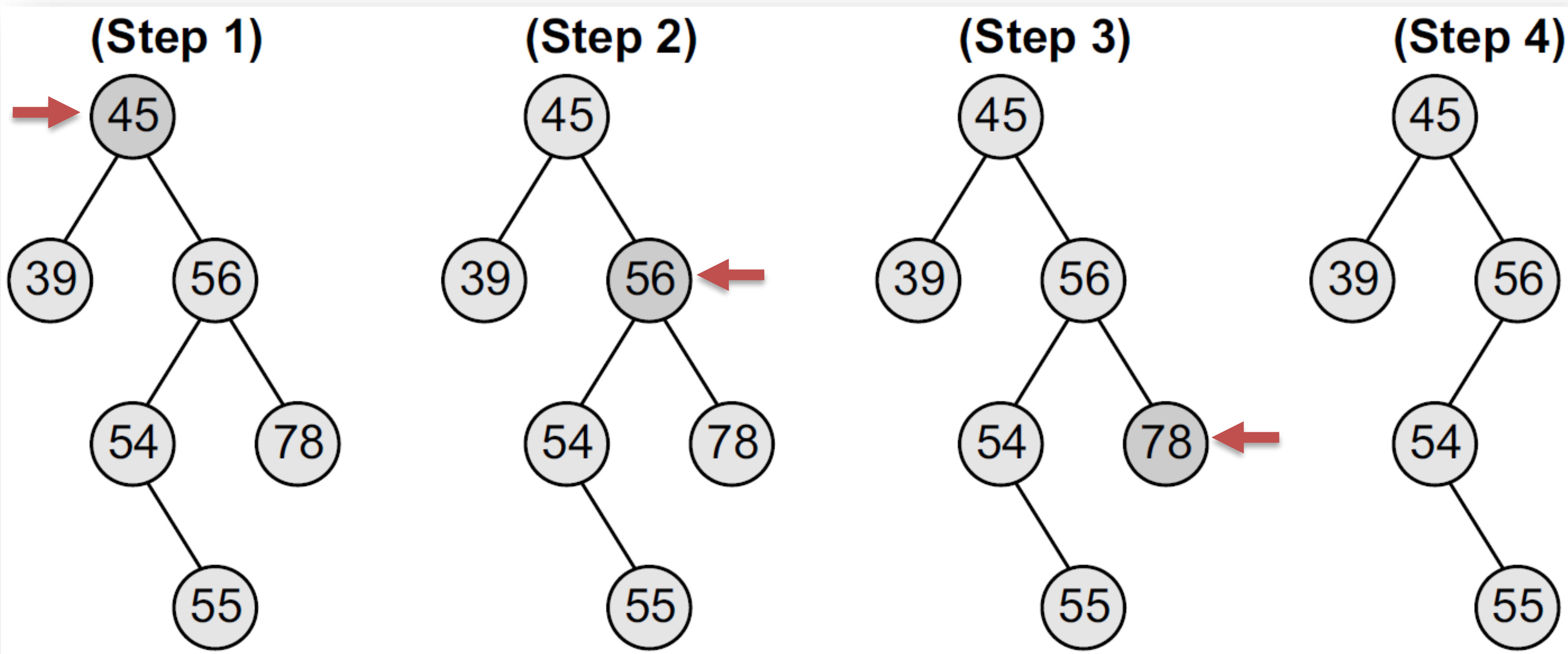
- The delete function deletes a node from the binary search tree
  - In order to take care the properties of binary search tree, we can divide the deleting functions into three categories
    - Deleting a node that has no children
    - Deleting a node with one child
    - Deleing a node with two children

```
Delete (TREE, VAL)

Step 1: IF TREE = NULL
          Write "VAL not found in the tree"
        ELSE IF VAL < TREE -> DATA
          Delete(TREE->LEFT, VAL)
        ELSE IF VAL > TREE -> DATA
          Delete(TREE -> RIGHT, VAL)
        ELSE IF TREE -> LEFT AND TREE -> RIGHT
          SET TEMP = findLargestNode(TREE -> LEFT)
          SET TREE -> DATA = TEMP -> DATA
          Delete(TREE -> LEFT, TEMP -> DATA)
        ELSE
          SET TEMP = TREE
         IF TREE -> LEFT = NULL AND TREE -> RIGHT = NULL
              SET TREE = NULL
          ELSE IF TREE -> LEFT != NULL
               SET TREE = TREE -> LEFT
          ELSE
               SET TREE = TREE -> RIGHT
          [END OF IF]
          FREE TEMP
        [END OF IF]
Step 2: END
```

Deleting a Node..

- Deleting a node that has no children
  - Simply remove this node without any issue
  - The simplest case of deletion

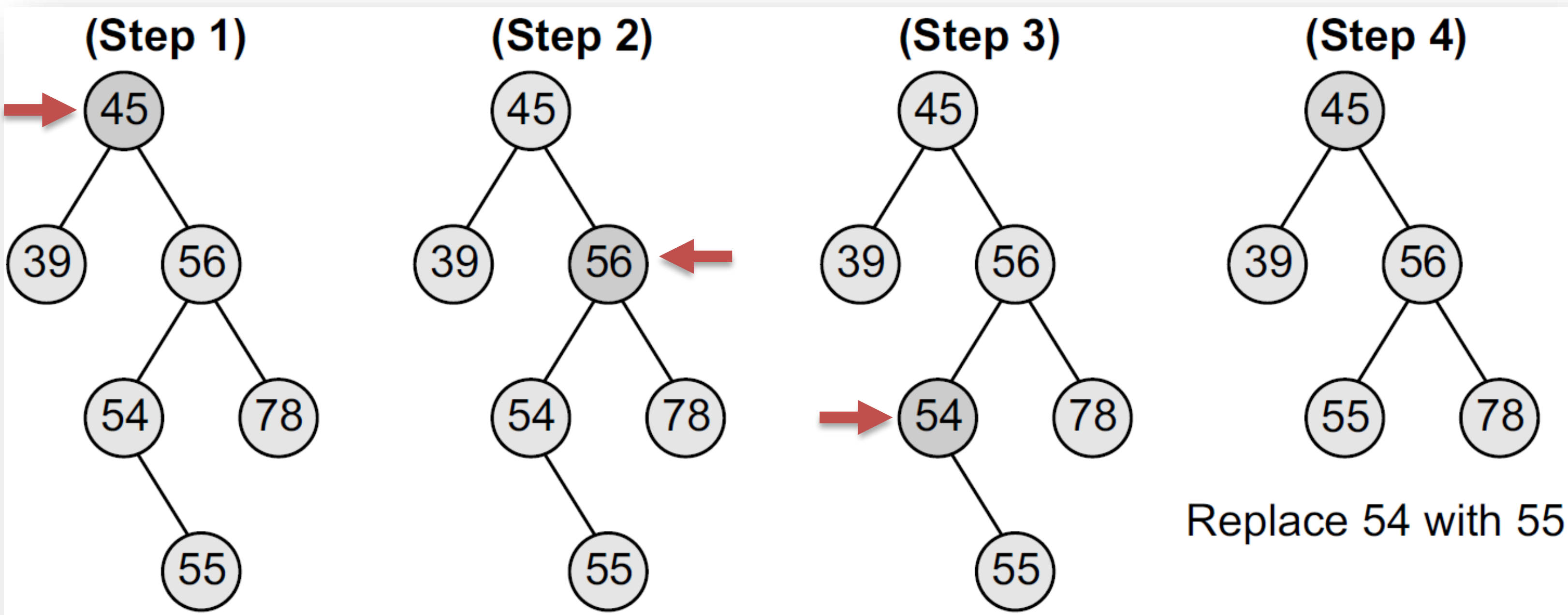
- Deleting node 78 from the given binary search tree



Deleting a Node...

- Deleting a node with one child
  - Replace the node with its child
  - It is also simple!

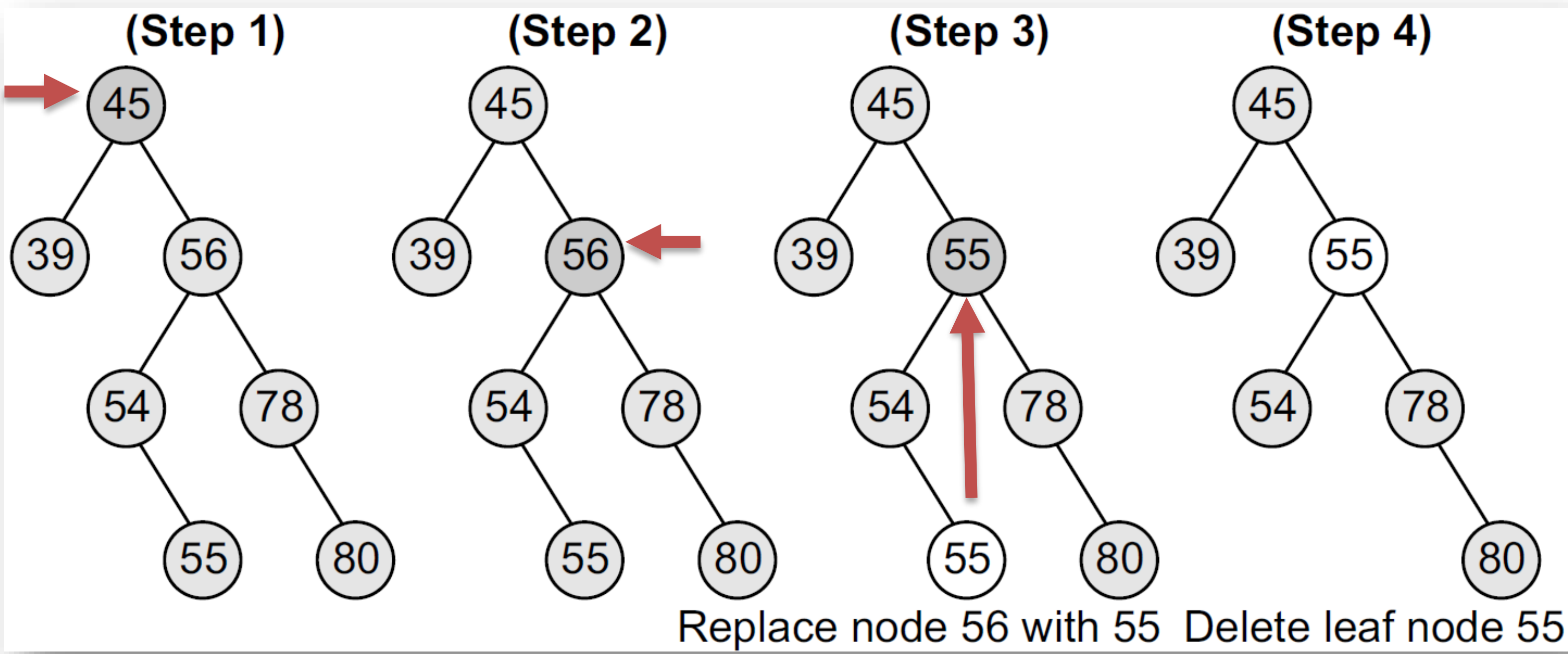
- Deleting node 54 from the given binary search tree



Deleting a Node….

- Deleing a node with two children
  - Replace the node's value with its **in-order predecessor** (largest value in the left sub-tree) or **in-order successor** (smallest value in the right sub-tree)

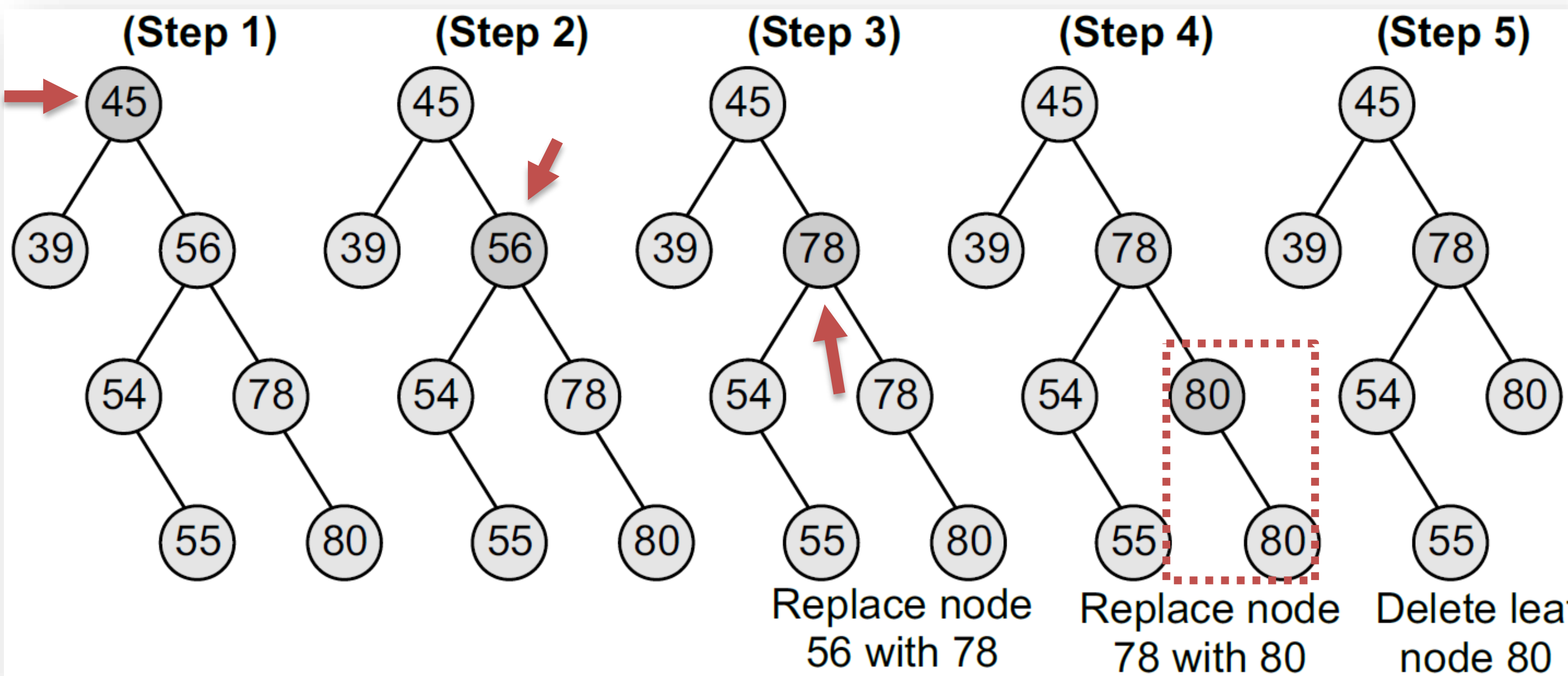
- Deleting node 56 from the given binary search tree



Deleting a Node.....

- Deleing a node with two children
  - Replace the node's value with its **in-order predecessor** (largest value in the left sub-tree) or **in-order successor** (smallest value in the right sub-tree)

- Deleting node 56 from the given binary search tree



Deleting a Node……

```
Delete (TREE, VAL)

Step 1: IF TREE = NULL
          Write "VAL not found in the tree"
        ELSE IF VAL < TREE –> DATA
          Delete(TREE->LEFT, VAL)
        ELSE IF VAL > TREE –> DATA
          Delete(TREE –> RIGHT, VAL)
        ELSE IF TREE –> LEFT AND TREE –> RIGHT
          SET TEMP = findLargestNode(TREE –> LEFT)
          SET TREE –> DATA = TEMP –> DATA
          Delete(TREE –> LEFT, TEMP –> DATA)
        ELSE
          SET TEMP = TREE
          IF TREE –> LEFT = NULL AND TREE –> RIGHT = NULL
              SET TREE = NULL
          ELSE IF TREE –> LEFT != NULL
               SET TREE = TREE –> LEFT
          ELSE
               SET TREE = TREE –> RIGHT
          [END OF IF]
          FREE TEMP
        [END OF IF]
Step 2: END
```

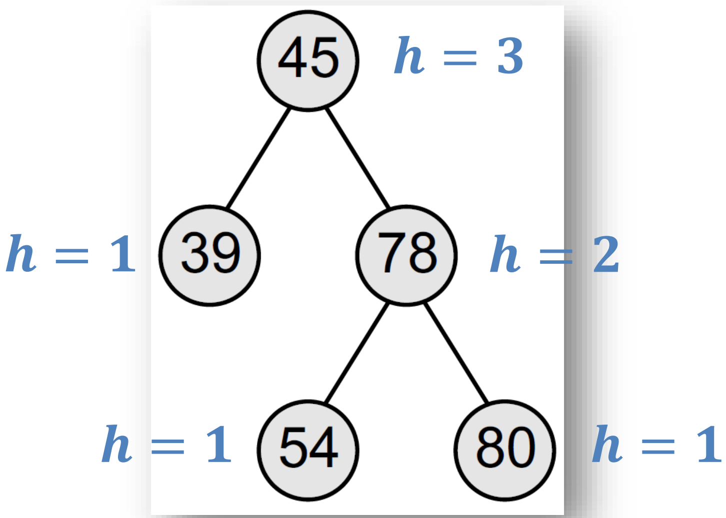
Deleting a node with two children

Deleting a node that has no children

Deleting a node with one child

Height of a Node

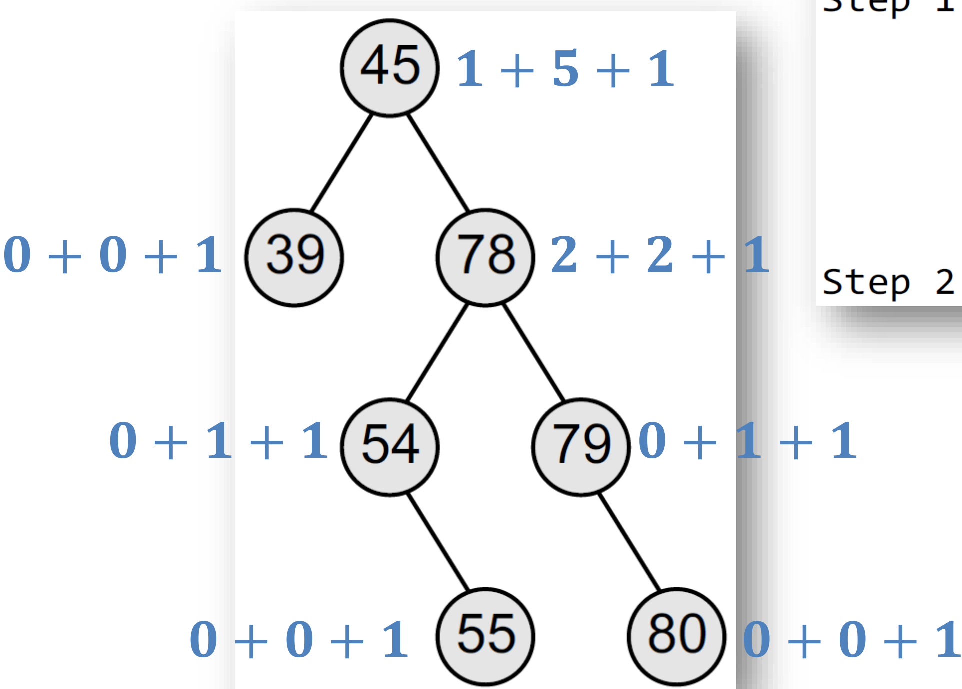
- Height for a node in a binary search tree
  - **The height of the leaf node is 1**
  - In order to determine the height of a node in a binary search tree, we calculate the height of its left sub-tree h_L and the right sub-tree h_R
  - After that, the height of the node is $1 + \max(h_L, h_R)$




```
Height (TREE)

Step 1: IF TREE = NULL
                Return 0
        ELSE
         SET LeftHeight = Height(TREE -> LEFT)
         SET RightHeight = Height(TREE -> RIGHT)
         IF LeftHeight > RightHeight
                Return LeftHeight + 1
         ELSE
                Return RightHeight + 1
         [END OF IF]
        [END OF IF]
Step 2: END
```

Number of Nodes in a Binary Search Tree

- Determining the number of nodes in a binary search tree is similar to determining its height
  - Number of nodes in a binary search tree is the sum of number of nodes in left sub-tree, right sub-tree and 1

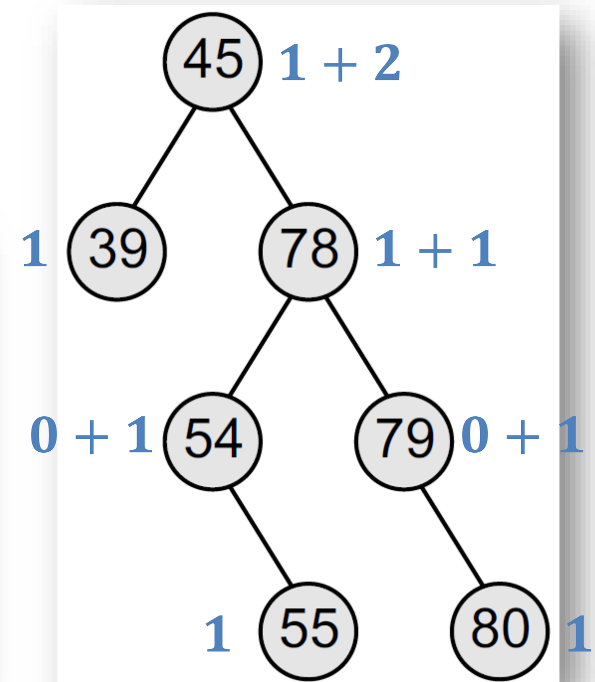



```
totalNodes(TREE)

Step 1: IF TREE = NULL
            Return 0
        ELSE
            Return totalNodes(TREE -> LEFT)
                + totalNodes(TREE -> RIGHT) + 1
        [END OF IF]
Step 2: END
```

Number of External Nodes

- The total number of external nodes or leaf nodes can be calculated by adding the number of external nodes in the left sub-tree and the right sub-tree
  - If the tree is empty, then the number of external nodes will be zero
  - If there is only one node in the tree, then the number of external nodes will be one
  - Number of internal nodes can thus be obtained!

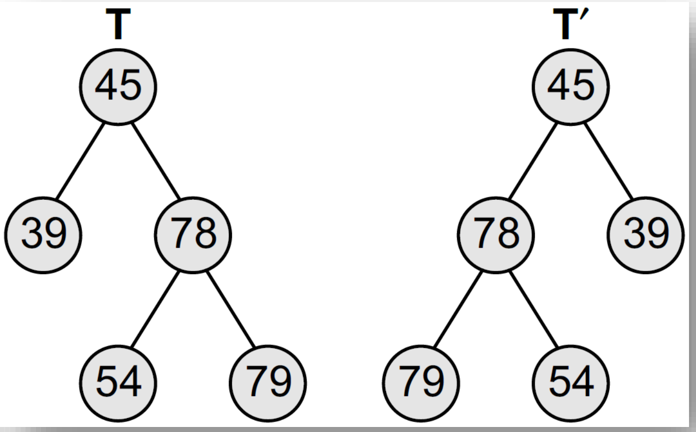
```
totalExternalNodes(TREE)

Step 1: IF TREE = NULL
            Return 0
        ELSE IF TREE -> LEFT = NULL AND TREE -> RIGHT = NULL
            Return 1
        ELSE
            Return totalExternalNodes(TREE -> LEFT) +
            totalExternalNodes(TREE -> RIGHT)
        [END OF IF]
Step 2: END
```



Mirror of a Binary Search Tree

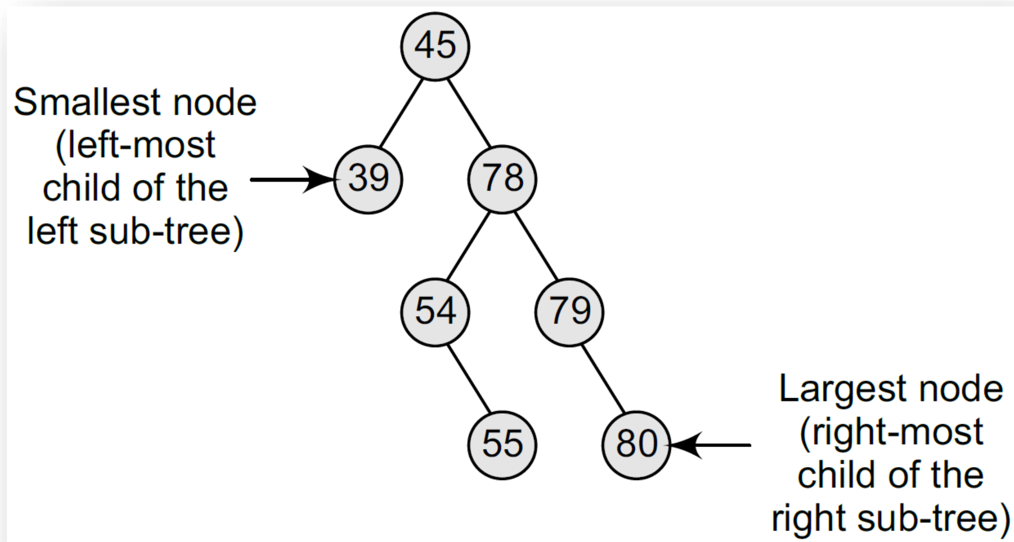
- Mirror image of a binary search tree is obtained by interchanging the left sub-tree with the right sub-tree at every node of the tree

```
MirrorImage(TREE)

Step 1: IF TREE != NULL
            MirrorImage(TREE -> LEFT)
            MirrorImage(TREE -> RIGHT)
            SET TEMP = TREE -> LEFT
            SET TREE -> LEFT = TREE -> RIGHT
            SET TREE -> RIGHT = TEMP
        [END OF IF]
Step 2: END
```



Finding the Smallest/Largest Node

- The very basic property of the binary search tree states that the smaller value will occur in the left sub-tree
  - If the left sub-tree is NULL, then the value of the root node will be smallest



- To find the node with the largest value, we find the value of the rightmost node of the right subtree
  - If the right sub-tree is empty, then the root node will be the largest value in the tree

Questions?

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