B+ and Huffman Trees

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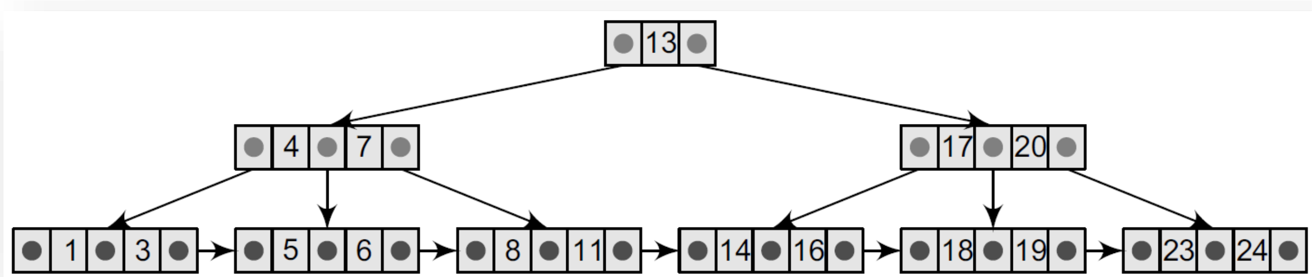
Review

- In a 2-3 tree, proposed by John Hopcroft in 1970, each interior node has either two or three children
  - **2-3 tree is a B-tree of order 3**
  - Nodes with two children are called 2-nodes
    - The 2-nodes have one data value and two children
  - Nodes with three children are called 3-nodes
    - The 3-nodes have two data values and three children
  - All the leaf nodes are at the same level

B+ Trees

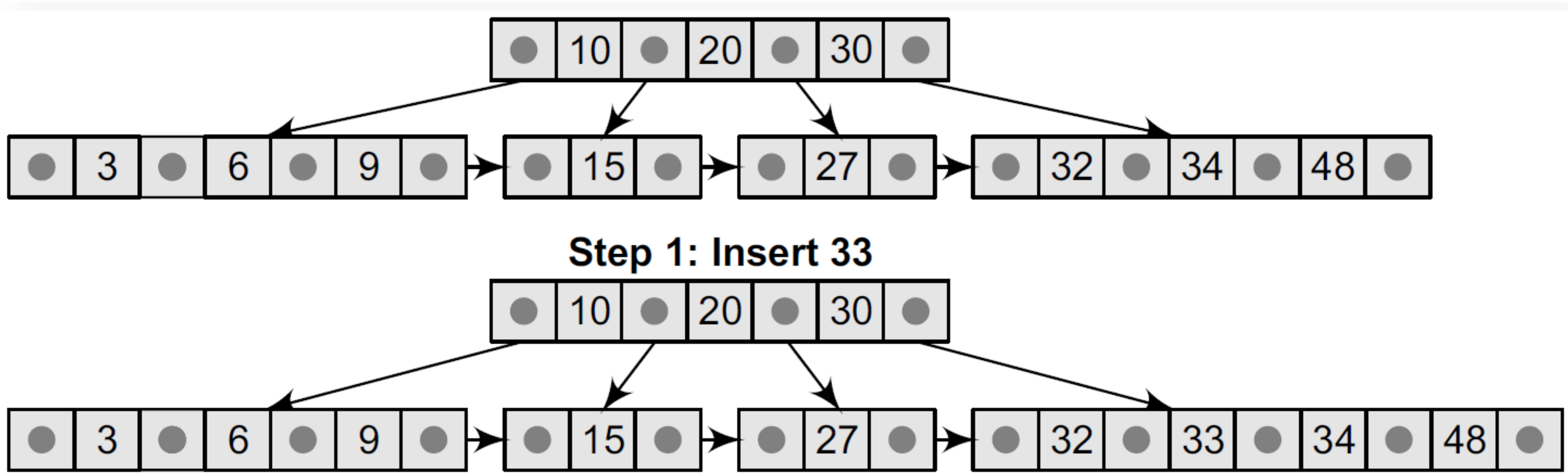
- A B+ tree is a variant of a B tree which stores sorted data in a way that allows for **efficient** insertion, retrieval, and removal of records, each of which is identified by a key
  - A B tree can store both keys and records in its interior nodes
  - A B+ tree stores all the records at the leaf level of the tree and keys are stored in the interior nodes
    - Typically, B+ trees are used to store large amounts of data that cannot be stored in the main memory
    - The leaf nodes of a B+ tree are often linked to one another in a linked list
    - All of the internal nodes are called index nodes or i-nodes
    - All leaf nodes are at the same level



B+ Trees – Insertion.

- For inserting a new element in a B+ tree
  - A new element is simply added in the leaf node if there is space for it
  - If the data node in the tree is full, then that node is split into two nodes

- For a given B+ tree of order 4, please insert 33 in the tree



B+ Trees – Insertion..

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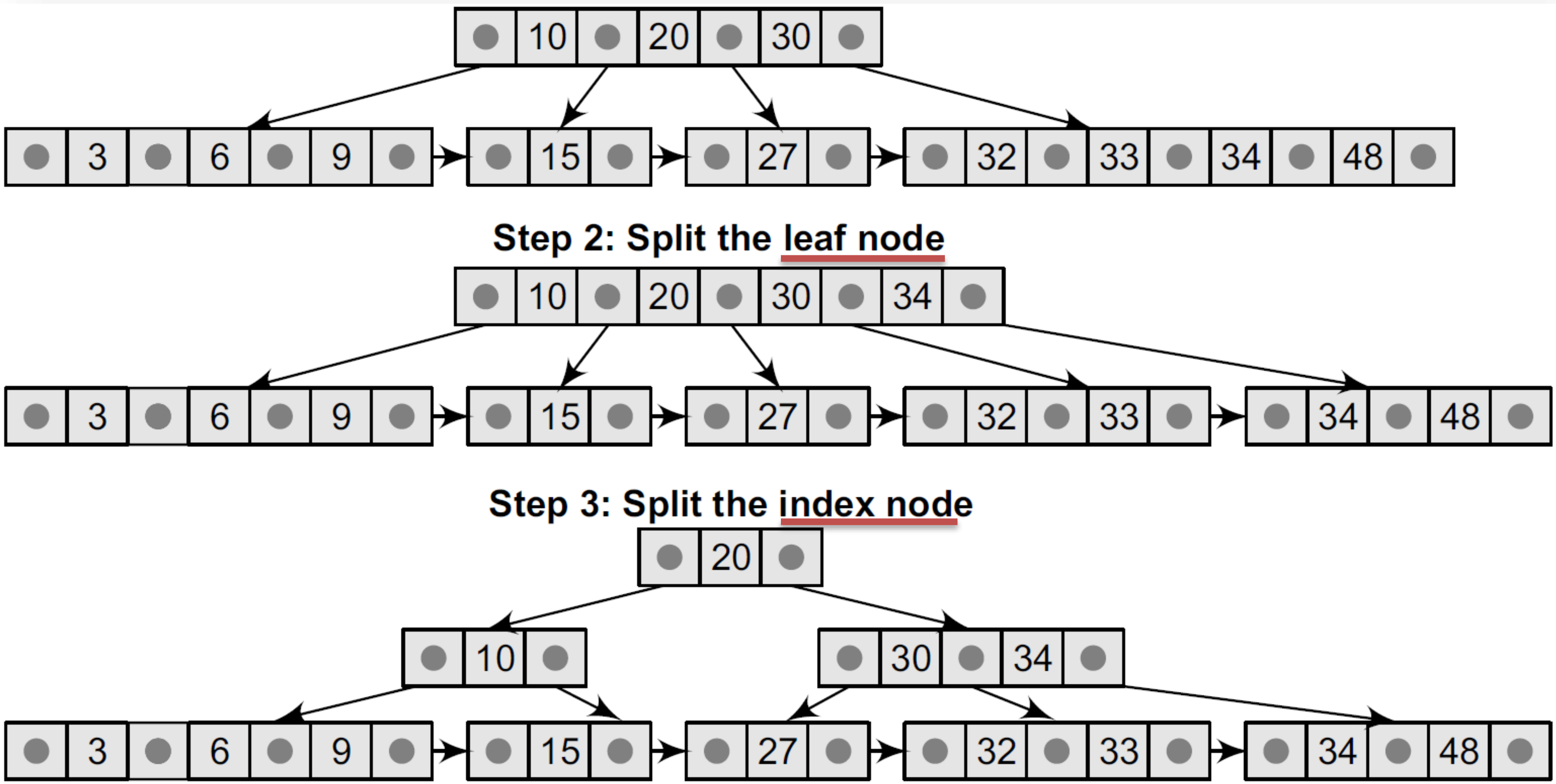
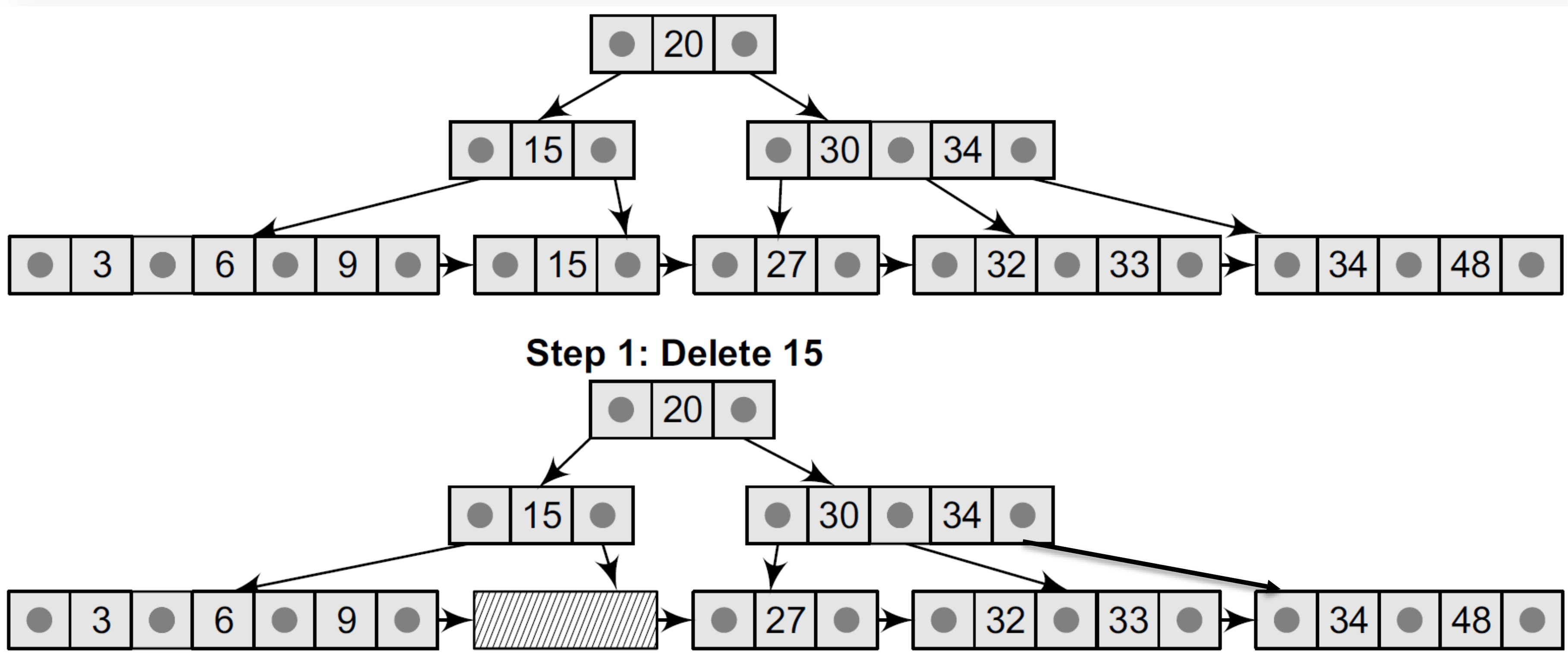


B+ Trees – Deletion.

- For a B+ tree, deletion is always done from a leaf node
  1. Delete the key and data from the leaves
  2. If the **leaf node underflows**, merge that node with the sibling and **delete** the key in between them
  3. If the **index node underflows**, merge that node with the sibling and **move down** the key in between them

B+ Trees – Deletion..

- For a B+ tree of order 4, please delete node 15 from the tree



B+ Trees – Deletion…

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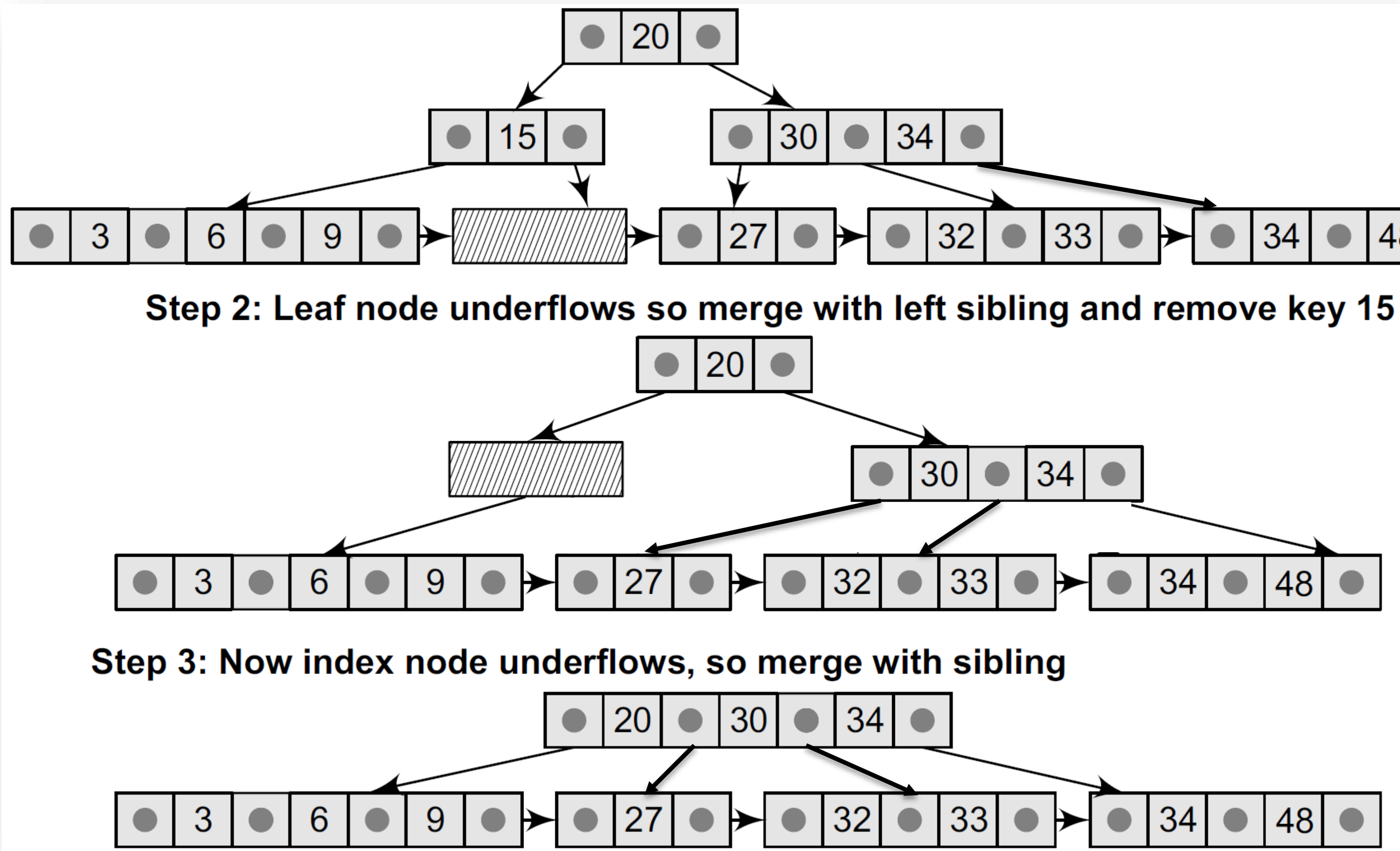


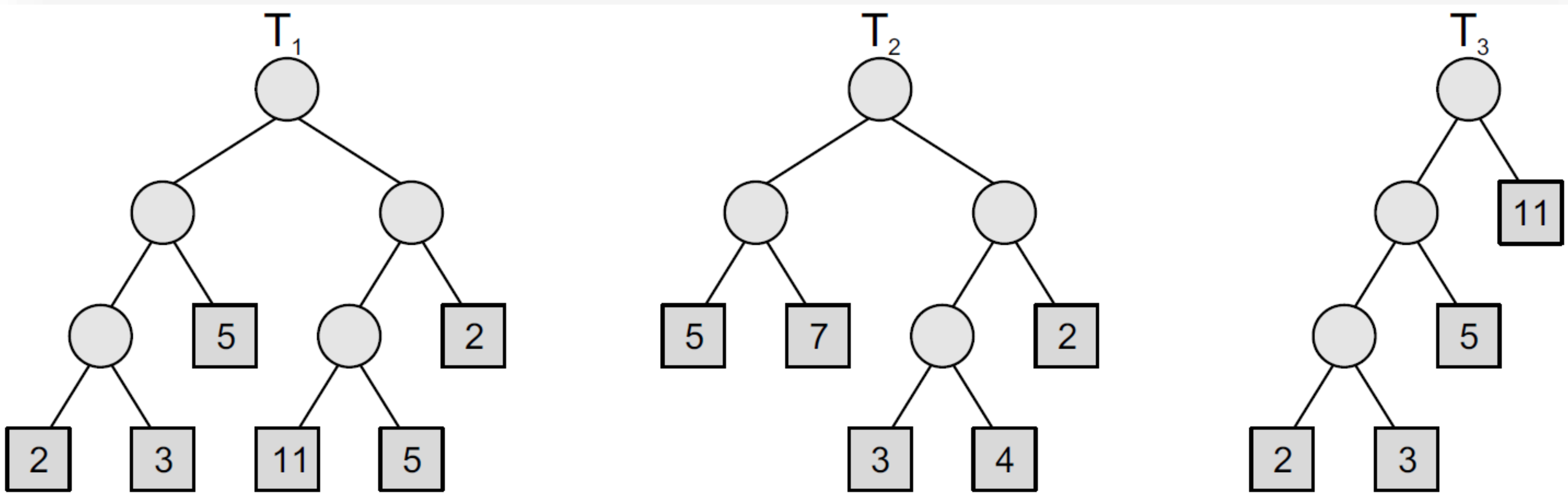
Huffman Trees

- Huffman coding is an entropy encoding algorithm developed by David A. Huffman in 1952 that is widely used as a lossless data compression technique

- The key idea behind Huffman algorithm is that it encodes the most common characters using shorter strings of bits than those used for less common source characters
  - The internal node is used to link to its child nodes
  - The external node contains the actual character and weight

Weighted External Path Length

- The weighted external path length
  - For T_1
    - $2 \times 3 + 3 \times 3 + 5 \times 2 + 11 \times 3 + 5 \times 3 + 2 \times 2 = 77$
  - For T_2
    - $5 \times 2 + 7 \times 2 + 3 \times 3 + 4 \times 3 + 2 \times 2 = 49$
  - For T_3
    - $2 \times 3 + 3 \times 3 + 5 \times 2 + 11 \times 1 = 36$

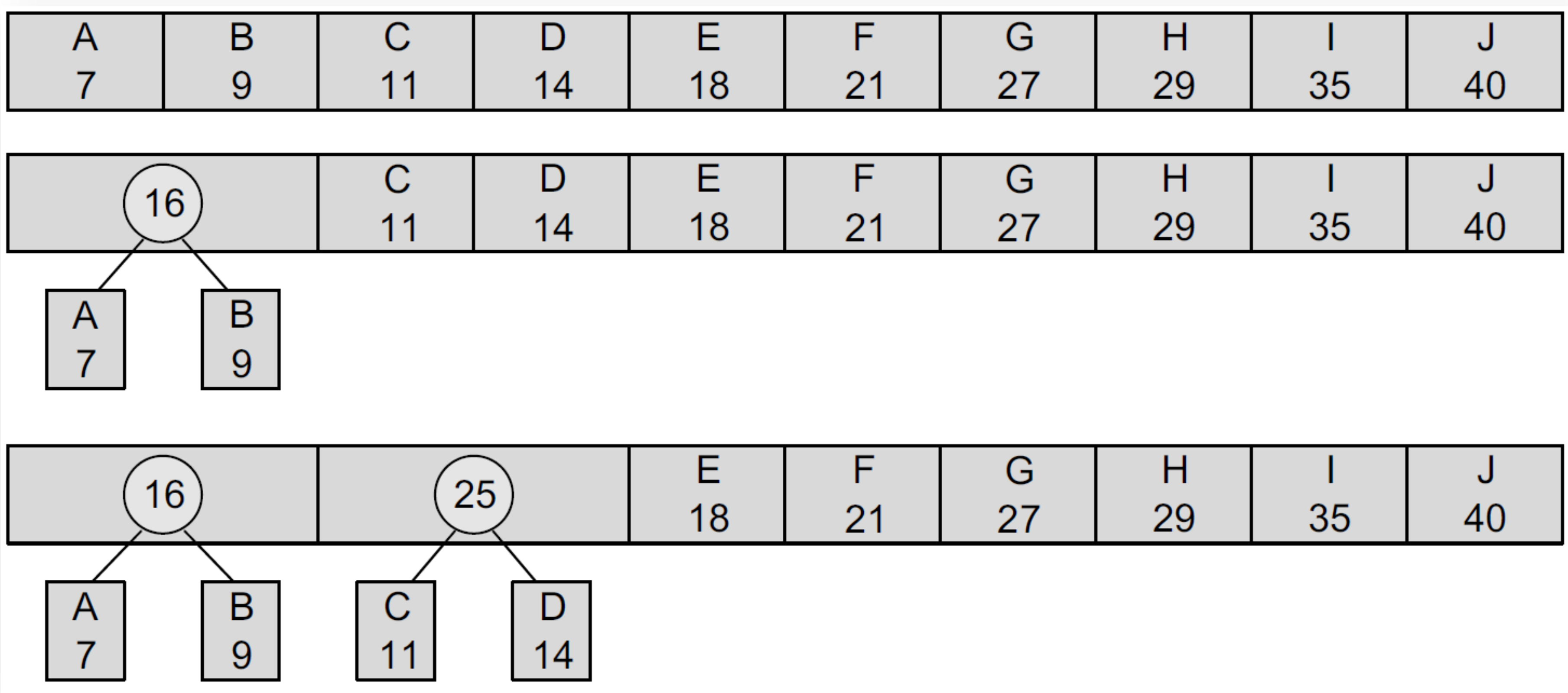


Creating a Huffman Tree

- Given n nodes and their weights, the Huffman algorithm is used to find a tree with a **minimum** weighted path length
  - Creating a new node whose children are the two nodes with the smallest weight
  - The weight of the new node is the sum of the two children
  - Repeat the process until the tree has only one node

Example.

- Create a Huffman tree with the following sorted nodes



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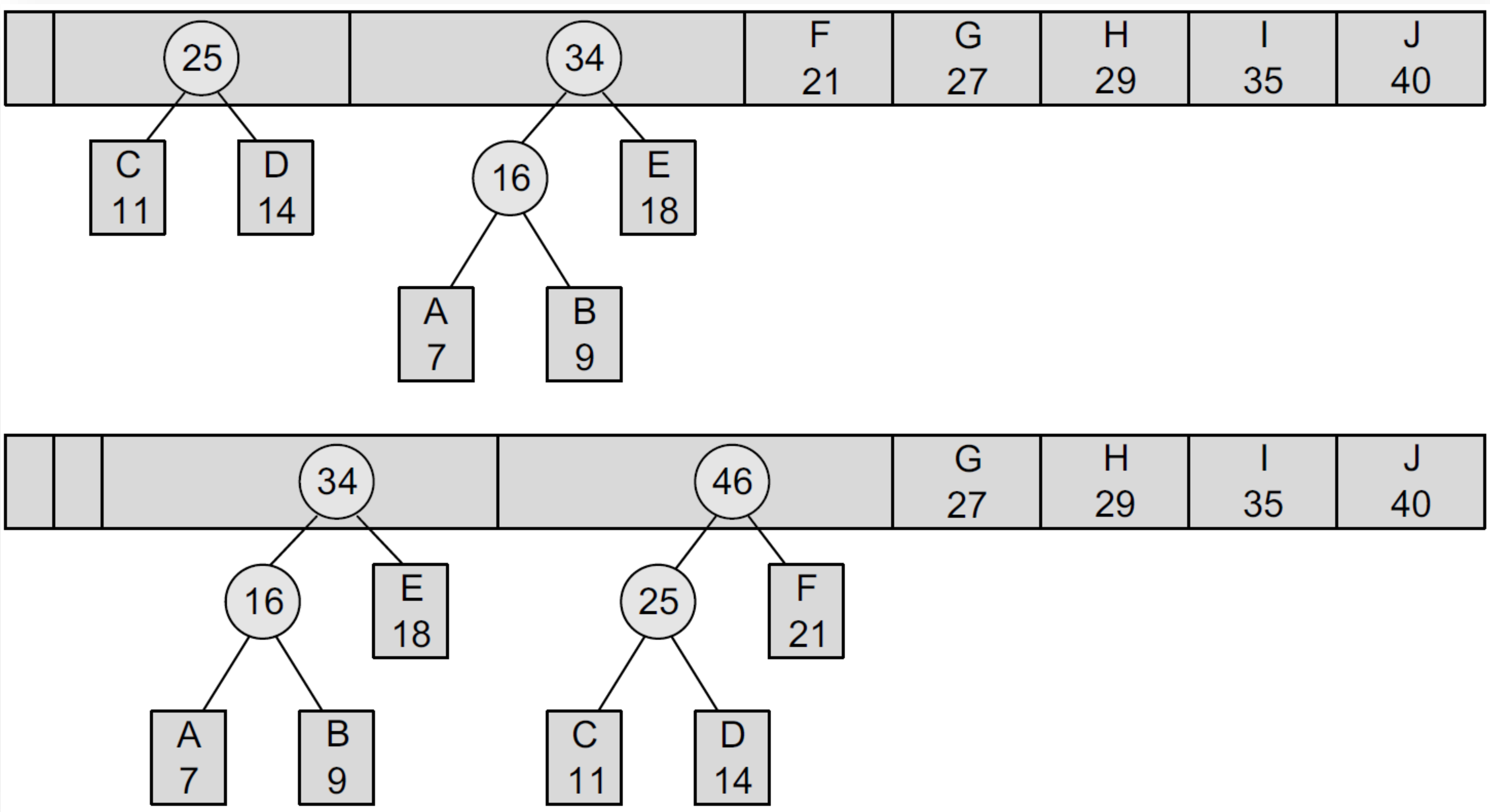

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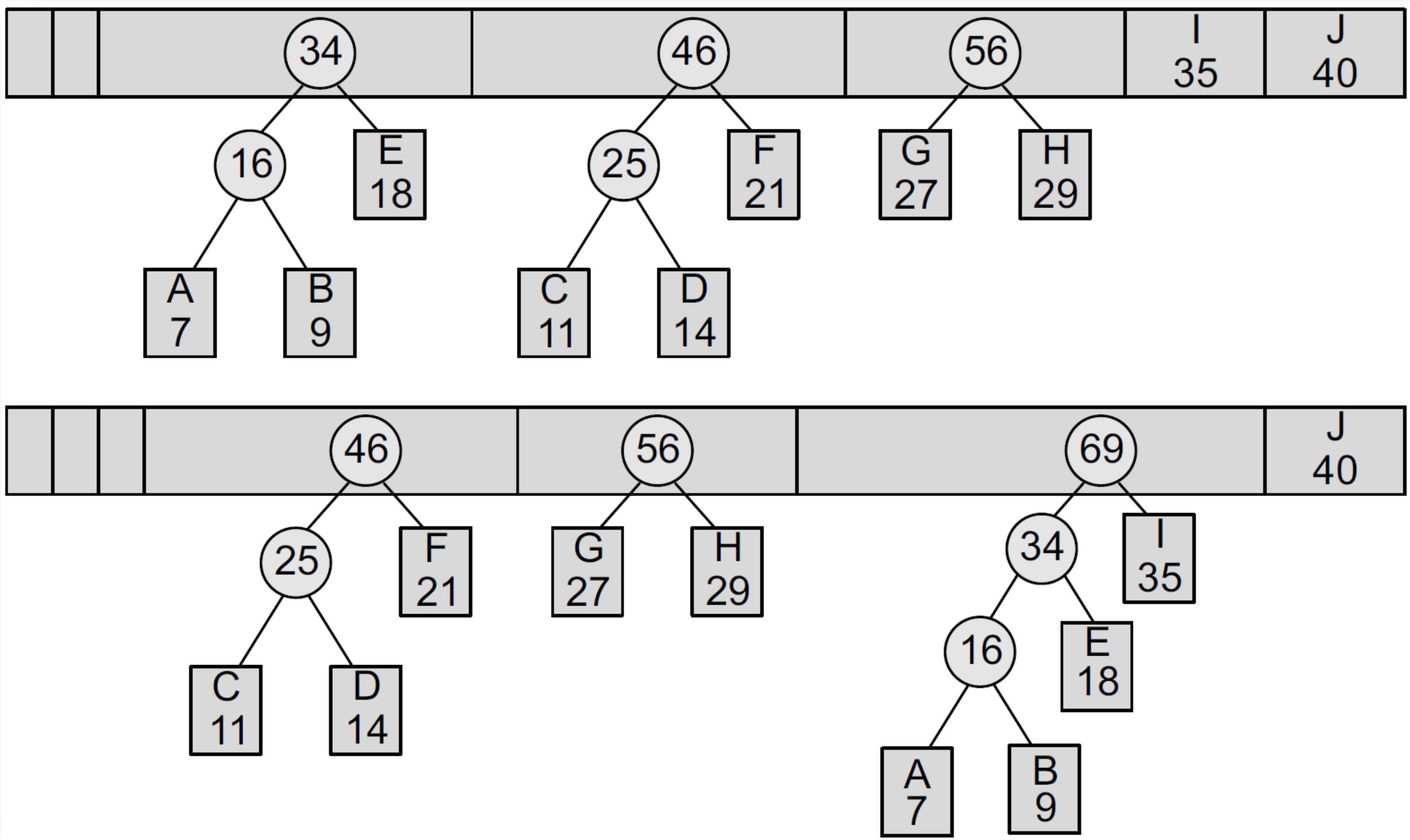

Example....

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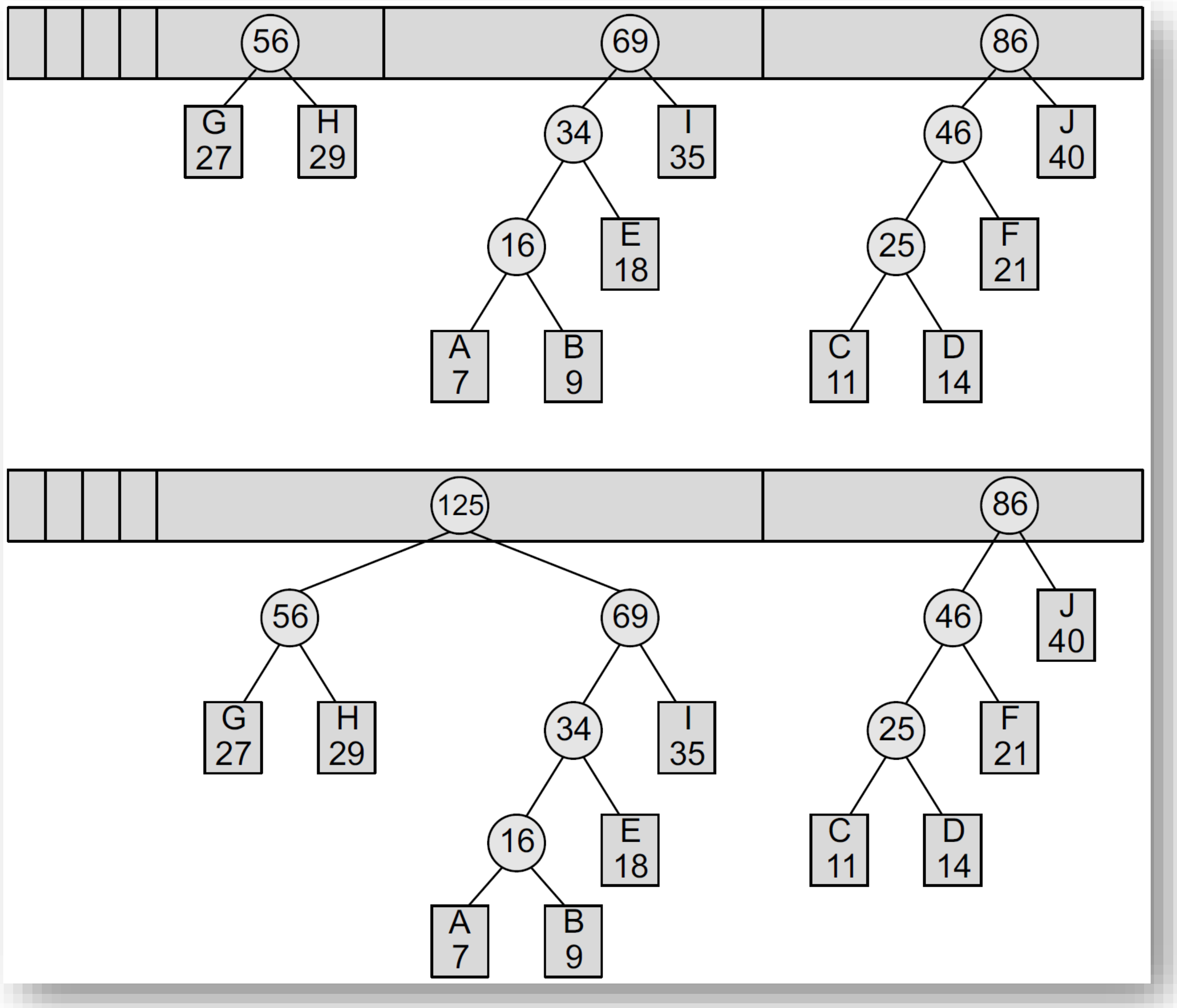

Example.....

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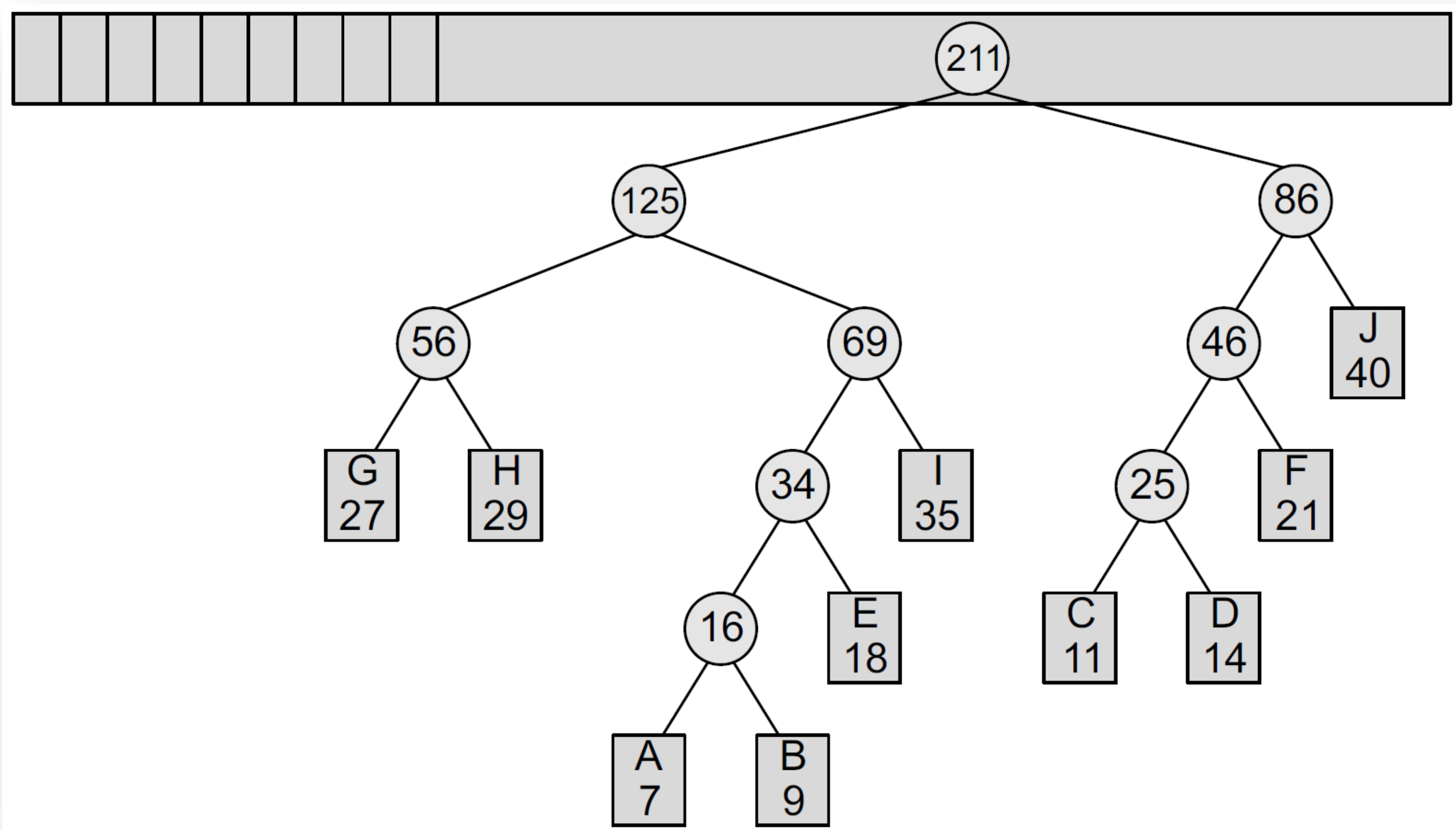
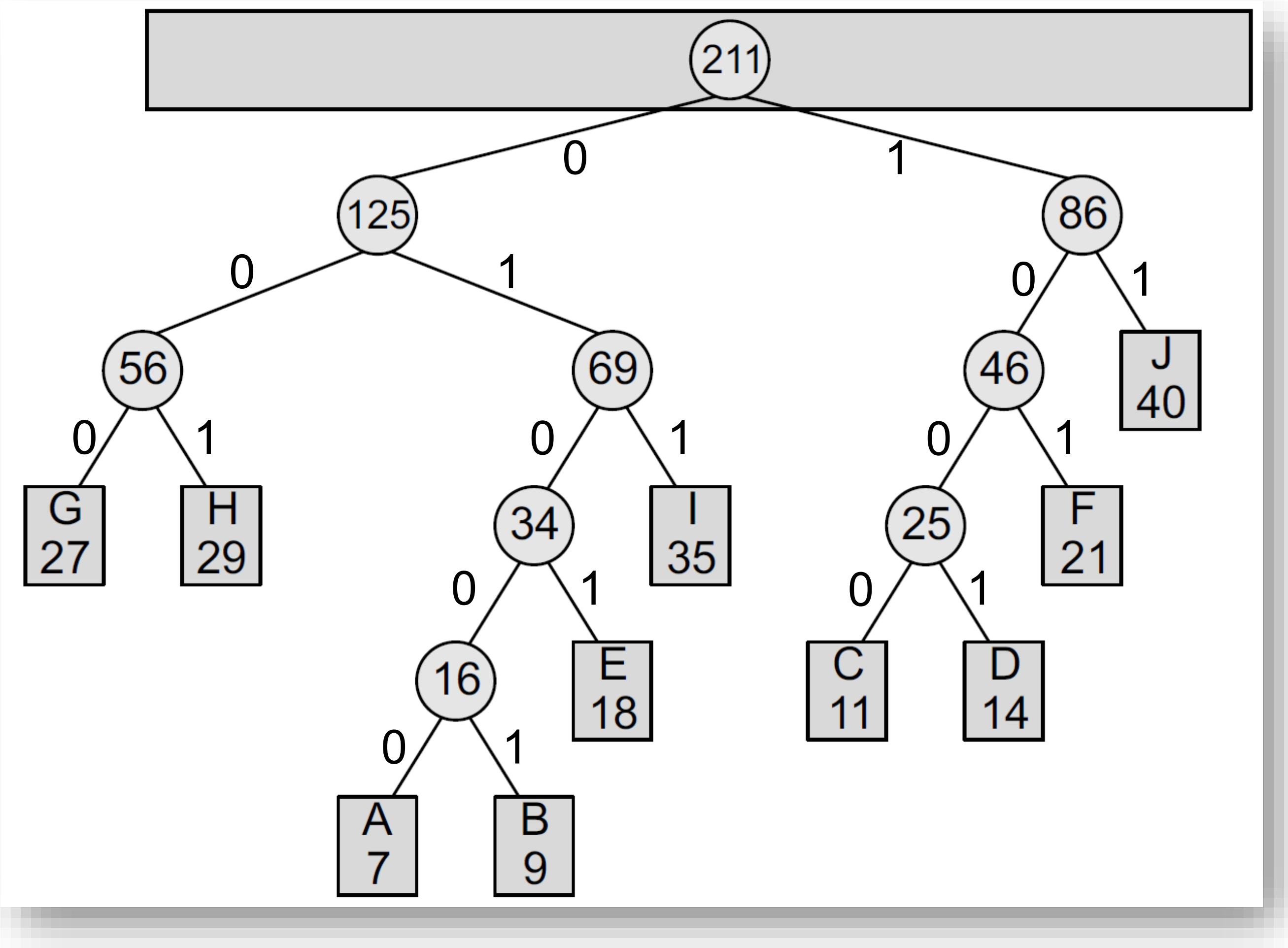


Example……

- Create a Huffman tree with the following sorted nodes

Data	Weight	Code
A	7	01000
B	9	01001
C	11	1000
D	14	1001
E	18	0101
F	21	101
G	27	000
H	29	001
I	35	011
J	40	11



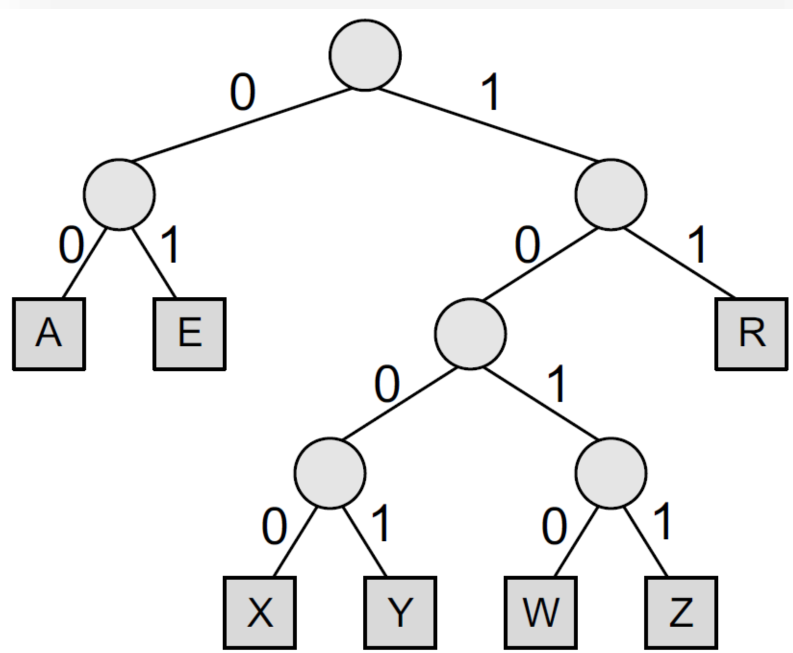
Example…….

- Create a Huffman tree with the following sorted nodes

Data	Frequency	Code	Total Length	Code	Total Length
A	7	01000	35	0000	28
B	9	01001	45	0001	36
C	11	1000	44	0010	44
D	14	1001	56	0011	56
E	18	0101	72	0100	72
F	21	101	63	0101	84
G	27	000	81	0110	108
H	29	001	87	0111	116
I	35	011	105	1000	140
J	40	11	80	1001	160
			668		**844**

Data Coding

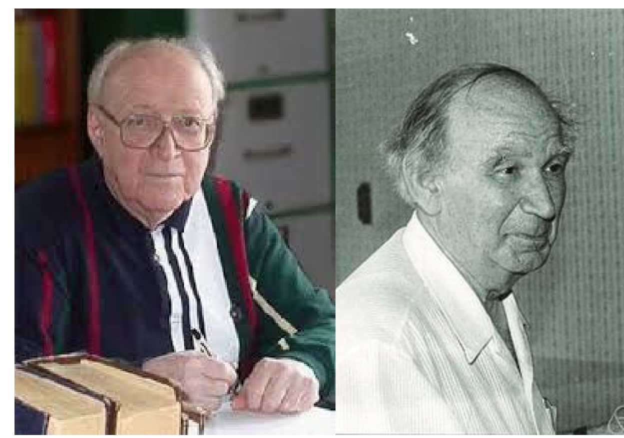
- For a Huffman tree, every left branch is coded with 0 and every right branch is coded with 1
  - For a character sequence: AAERZ
    - By Huffman Coding Scheme: 000001111011
    - By Original Coding Scheme: 000000001010110



Character	Code	Original Coding
A	00	000
E	01	001
R	11	010
W	1010	011
X	1000	100
Y	1001	101
Z	1011	110

Summary

- We introduce a lots of variants of binary search tree, which is the fundamental
  - Huffman Tree is created in 1952
    - Data compression
  - Binary Search Tree, 1960
  - AVL Tree, 1962
    - Proposed by Georgy **A**delson-**V**elsky & Evgenii **L**andis
  - 2-3 Tree, 1970
  - Red-Black Tree, 1972
    - std::map in C++
      - ✓ The re-balance process is faster than AVL tree
  - B Tree, 1972
    - A B-tree of order 3 is a 2-3 tree
  - B+ Tree, 1973
    - NTFS uses B+ trees for directory and security-related metadata indexing
    - MySQL indexing
  - Splay Tree, 1985
    - For memory management algorithms



Schedule

- Midterm exam will be held at 11/7 (Mon.) 10:20~12:10

Questions?

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