~~D7..~~

Roteiro 9 A

Atv 2

17-a) $\frac{V_1(s)}{R_1} = \frac{V_0(s)}{L s} + \frac{V_0(s)}{R_2} \Rightarrow \frac{V_1(s)}{1} = \frac{1}{s} V_0(s) + \frac{V_0(s)}{2}$

$V_i(s) = V_0(s) \cdot \left(\frac{1}{s} + \frac{1}{2}\right) = V_0(s)\left(\frac{2+s}{2s}\right) \quad \frac{V_0(s)}{V_1(s)} = \frac{2s}{s+2}$

17-b) $\frac{V_1 - V_i}{1} + \frac{V_A}{1} + \frac{1}{1}\int_0^t (V_1 - V_0)\, dt = 0$

$V_1(s) - V_i(s) + V_1(s) + \frac{1}{s}(V_1(s) - V_0(s)) = 0$

$V_1\left(2 + \frac{1}{s}\right) - \frac{V_0}{s} = V_i \Rightarrow V_i = V_1\left(\frac{2s+1}{s}\right) - \frac{V_0}{s}$

$\frac{1}{1}\int_0^t (V_0 - V_1)\, dt + 2\frac{dV_0}{dt} = 0$

$\frac{1}{s}(V_0(s) - V_1(s)) + 2s V_0(s) = 0$

$V_0\left(\frac{1}{s} + 2s\right) = \frac{V_1}{s} \Rightarrow V_0(2s^2+1) = V_1 \quad V_i = (V_0(2s^2+1))\left(\frac{2s+1}{s}\right) - \frac{V_0}{s}$

$V_i = V_0\left(\frac{4s^3+2s^2+2s+1-1}{s}\right) = \frac{V_0(s)}{V_i(s)} = \frac{s}{4s^3+2s^2+2s}$

18-a) $\frac{1}{2}\int_0^t (V_1 - V)\, d\tau + \frac{V_1}{2} + \frac{V_1 - V_L}{2} = 0$

$\frac{1}{2} \cdot \frac{1}{s}(V_1(s) - V(s)) + \frac{V_1(s)}{2} + \frac{V_1(s) - V_L(s)}{2} = 0$

$V_1\left(\frac{1}{2s} + \frac{1}{2} + \frac{1}{2}\right) = \frac{V_L}{2} - \frac{V}{2s}$

$\frac{V_L - V_1}{2} + \frac{1}{2}\int_0^t V_L(\tau)\, d\tau = 0$

$\frac{V_L(s) - V_1(s)}{2} + \frac{1}{2s} V_L(s) = 0$

$V_L\left(\frac{1}{2} + \frac{1}{2s}\right) = \frac{V_1}{2} \quad V_L\left(\frac{s+1}{s}\right) = V_1$

$\frac{V}{2s} = \left(V_L\left(\frac{s+1}{s}\right)\right) \cdot \left(\frac{1+2s}{2s}\right) - \frac{V_L}{2} = V_L\left(\frac{2+2s^3+2s+1}{2s^2} - \frac{1}{2}\right)$

$\frac{V}{2s} = V_L\left(\frac{s^2+3s+1}{2s^2}\right) \Rightarrow \frac{V_L(s)}{V(s)} = \frac{s}{s^2+3s+1}$

b) $\frac{V_1 - V}{2+\frac{2}{n}} + \frac{V_1}{2+\frac{2}{n}} + \frac{V_1 - V_\ell}{2} = 0$

$V_1\left(\frac{2}{2+\frac{2}{n}} + \frac{1}{2}\right) - \frac{V_\ell}{2} = \frac{V}{2+\frac{2}{n}} \Rightarrow V_1\left(\frac{2n}{n+1} + 1\right) - V_\ell = V\left(\frac{n}{n+1}\right)$

$V\left(\frac{n}{n+1}\right) = V_1\left(\frac{3n+1}{n+1}\right) - V_\ell$

$\frac{V_\ell - V_1}{2} + \frac{1}{2n} V_\ell = 0 \Rightarrow V_\ell\left(\frac{1}{2} + \frac{1}{2n}\right) = \frac{V_1}{2}$ $\quad V_1 = V_\ell\left(\frac{n+1}{n}\right)$

$V\left(\frac{n}{n+1}\right) = V_\ell\left(\frac{n+1}{n}\right)\left(\frac{3n+1}{n+1}\right) - V_\ell$

$V\left(\frac{n}{n+1}\right) = V_\ell\left(\frac{3n+1}{n} - \frac{n}{n}\right) \Rightarrow V\left(\frac{n}{n+1}\right) = V_\ell\left(\frac{2n+1}{n}\right)$

$\frac{V_L(n)}{V(n)} = \frac{n^2}{(2n+1)(n+1)} \Rightarrow \frac{V_L(n)}{V(n)} = \frac{n^2}{2n^2+3n+1}$

20) a) $\frac{1}{2}\int_0^t (V_1 - V_i)\,d\tau + \frac{V_1}{1} + \frac{1}{3}\int_0^t (V_1 - V_0)\,d\tau = 0$

$\frac{1}{2n}(V_1 - V_i) + V_1 + \frac{1}{3n} V_0 = 0 \Rightarrow V_1\left(\frac{1}{2n} + 1\right) + \frac{V_0}{3n} = \frac{V_1}{2n}$

$V_1\left(\frac{2n+1}{2n}\right) + \frac{V_0}{3n} = \frac{V_i}{3n} \Rightarrow \frac{V_i}{2} = V_i\left(\frac{2n+1}{2}\right) + \frac{V_0}{3}$

$\frac{1}{3}\int_0^t (V_0 - V_1)\,d\tau + \frac{1}{2}\frac{dV_0}{dt} = 0$

$\frac{1}{3}\cdot\frac{1}{2}\cdot(-V_0) + \frac{n}{2}(V_1 - V_0) = 0 \Rightarrow \frac{V_0}{3n} + \frac{nV_0}{2} = \frac{nV_1}{2}$

$V_0\left(\frac{1}{3n} + \frac{n}{2}\right) = V_1\left(\frac{n}{2}\right) \Rightarrow V_0\left(\frac{2+3n^2}{6n}\right)\cdot\left(\frac{2}{n}\right) = V_1$

$V_1 = V_0\left(\frac{3n^2+2}{3n^2}\right)$

$\frac{V_i}{2} = V_0\left(\frac{3n^2+2}{3n^2}\right)\left(\frac{2n+1}{2}\right) + \frac{V_0}{3}$ $\quad \frac{V_i}{2} = V_0\left(\left(\frac{6n^3+3n^2+4n+2}{6n^2}\right) + \frac{1}{3}\right) = V_0\left(\frac{6n^3+5n^2+4n+2}{6n^2}\right)$

$\frac{V_0(n)}{V_i(n)} = \frac{3n^2}{6n^3+5n^2+4n+2}$

Roteiro 7

R:

20) b)

$$\frac{1}{1}\int_0^1 (v_1 - v_i)\, d\tau + \frac{1}{1}\int_0^1 v_1\, d\tau + 1\frac{dv_1}{dt} + \frac{v_1 - v_0}{1} = 0$$

$$\frac{1}{s}(v_1 - v_i) + \frac{1}{s}v_1 + sv_1 + (v_1 - v_0) = 0$$

$$V_1\left(\frac{1}{s} + \frac{1}{s} + s + 1\right) - V_0 = \frac{V_i}{s} \Rightarrow V_1\left(\frac{2 + s^2 + s}{s}\right) - V_0 = \frac{V_i}{s}$$

$$\frac{(v_0 - v_1)}{1} + \frac{dv_0}{dt} + \frac{1}{1}\int_0^1 (v_0 - v_i)\, d\tau = 0$$

$$v_0 - V_1 + sV_0 + \frac{1}{s}(V_0 - V_i) = 0 \Rightarrow V_0\left(1 + s + \frac{1}{s}\right) - \frac{V_i}{s} = V_1$$

$$\left\{ \left[V_0\left(\frac{s^2+s+1}{s}\right) - \frac{V_i}{s} \right] \right\} \cdot \left(\frac{s^2+s+2}{s}\right) - V_0 = \frac{V_i}{s}$$

$$V_0\left\{\left(\frac{s^2+s+1}{s}\right)\left(\frac{s^2+s+2}{s}\right) - 1\right\} = V_i\left\{\frac{1}{s} + \frac{s^2+s+2}{s^2}\right\}$$

$$V_0\left(\frac{s^4 + 2s^3 + 4s^2 + 3s + 2}{s^2} - \frac{s^2}{s^2}\right) = V_i\left(\frac{s + s^2 + s + 2}{s^2}\right)$$

$$V_0(s^4 + 2s^3 + 3s^2 + 3s + 2) = V_i(s^2 + 2s + 2)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{s^2 + 2s + 2}{s^4 + 2s^3 + 3s^2 + 3s + 2}$$

26) $f(t) = 4\dot{x}_2(t) + 2x_2(t) + 4\dot{x}_1(t) = m_2\ddot{x}_2(t) = 0$

$$F(s) - 4sX_2(s) + 2X_2(s) + 4sX_1(t) = 0$$

$$F(s) = X_2(4s - 2) - X_1(4s)$$

$$-4\dot{x}_1(t) + 4\dot{x}_2(t) - 8\ddot{x}_1(t) - 2\dot{x}_1(t) = 0$$

$$-6sX_1(s) + 4sX_2(s) - 8s^2X_1(s) = 0$$

$$X_2(4s) = X_1(8s^2 + 6s) \Rightarrow X_1 = X_2\left(\frac{2}{4s+3}\right)$$

$$F(s) = X_2(4s-2) = X_2\left(\frac{2}{4s+3}\right)\cdot(4s) = X_2\left(\frac{4s(4s+2) - 2(4s+3) \cdot 8s}{4s+3}\right)$$

$$F(s) = X_2\left(\frac{16s^2 - 4s - 6}{4s+3}\right) \Rightarrow \frac{X_2(s)}{F(s)} = \frac{4s+3}{16s^2 - 4s - 6}$$

Roteiro 7

27) $-9x_1(t)-6\dot{x}_1-\ddot{x}_1(t)+5x_2(t)+3\dot{x}_2(t)=0$

$-9X_1(s)-6sX_1(s)-s^2X_1(s)+5X_2(s)+3sX_2(s)=0$

$X_2(3s+5)=X_1(s^2+6s+9)$

$X_2=X_1\left(\frac{s^2+6s+9}{3s+5}\right)$

$-5x_2(t)+5x_1(t)-5\dot{x}_2(t)+3\dot{x}_1(t)-2\ddot{x}_2(t)+f(t)=0$

$-5X_2(s)+5X_1(s)-5sX_2(s)+3sX_1(s)-2s^2X_2(s)+F(s)=0$

$X_1(3s+5)-X_2(2s^2+5s+5)+F(s)=0$

$F(s)=X_2(2s^2+5s+5)-X_1(3s+5)$

$F(s)=X_1\frac{(s^2+6s+9)}{3s+5}(2s^2+5s+5)-X_1(3s+5)$

$F(s)=X_1\left(\frac{(2s^4+17s^3+53s^2+75s+45)-(9s^2+30s+25)}{3s+5}\right)$

$F(s)=X_1\left(\frac{2s^4+17s^3+44s^2+45s+20}{3s+5}\right)$

28) $-6x_1(t)-2\dot{x}_1(t)+2\dot{x}_2(t)-4\ddot{x}_1(t)=0$

$-6X_1(s)-2sX_1(s)+2sX_2(s)-4s^2X_1(s)=0$

$X_2(2s)=X_1(4s^2+2s+6)\Rightarrow X_1=X_2\left(\frac{2s}{4s^2+2s+6}\right)$

$-4\dot{x}_2(t)-4\ddot{x}_2(t)+2\dot{x}_1(t)-6x_2(t)+6x_1(t)+f(t)=0$

$-4sX_2(s)-4s^2X_2(s)+2sX_1(s)-6X_2(s)+6X_3(s)+F(s)=0$

$X_2(4s^2+4s+6)=X_1(2s)-X_3(6)=F(s)$

$-6x_3(t)-2\dot{x}_3(t)-4\ddot{x}_3(t)+6x_2(t)=0$

$-6X_3(s)-2sX_3(s)-4s^2X_3(s)+6X_2(s)=0$

$F(s)=X_2\left(\frac{16s^4+24s^3+52s^2+36s}{6}\right)+\left(\frac{X_2(s)}{F(s)}\right)=\frac{6}{16s^4+24s^3+52s^2+36s}$

Rot

(34) $-1\ddot{\theta}_1(t) + T(t) - 1.\dot{\theta}_1(t) - 1.\theta_1(t) + \theta_2(t) + 1.\dot{\theta}_2(t) = 0$

$-s^2\theta_1(s) + T(s) - \theta_1(s) - s(\theta_1(s)) + \theta_2(s) + s.\theta_2(s) = 0$

$T(s) = \theta_1(s^2+s+1) - \theta_2(s+1)$

$\theta_1(s) + 1.\theta(s) - s\theta_2(s) - \theta_2(s) - \theta_2(s) - s^2.\theta_2(s) = 0$

$\theta_1(s+1) = \theta_2(s^2+s+2)$

$\theta_2 = \theta_1\left(\frac{s+1}{s^2+s+2}\right)$

$T(s) = \theta_1(s^2+s+1) - \theta_1\left(\frac{s+1}{s^2+s+2}\right).(s+1)$

$T(s) = \theta_1\left(\frac{(s^2+s+1)(s^2+s+2)-(s+1)^2}{s^2+s+2}\right) = \theta_1\left(\frac{s^4+2s^3+3s^2+s+1}{s^2+s+2}\right)$

$\frac{\theta_1(s)}{T(s)} = \frac{s^2+s+2}{s^4+2s^3+3s^2+s+1}$

8

s^4+2s^3+1