Roteiro 7.

(1) a) $C(s) = R(s)G(s) = \frac{1}{s} \cdot \frac{5}{s+5}$

$C(s) = \frac{A}{s} + \frac{B}{s+5}$

$\frac{1}{\cancel{s}} \frac{5}{s+5} \cancel{s} = \frac{A \cancel{s}}{\cancel{s}} + \frac{B}{s+5} \cdot s \Rightarrow s+0 = \frac{5}{5} \Rightarrow A = 1$

$\frac{1}{s} \cdot \frac{5}{s+5} \cdot (s+5) = \frac{A}{s}(s+5) + \frac{B}{s+5}(s+5) \Rightarrow B = -1$

$C(s) = \frac{1}{s} - \frac{1}{s+5} \qquad c(t) = 1 - e^{-5t}$

b) $C(s) = \frac{1}{s} \frac{20}{s+20}$

$C(s) = \frac{A}{s} + \frac{B}{s+20} \qquad \frac{1}{\cancel{s}} \cdot \frac{20}{s+20} \cancel{s} = \frac{A}{s} s + \frac{B}{s+20} \cdot s \Rightarrow A = 1$

$\frac{1}{s} \frac{20}{s+20} (s+20) = \frac{A}{s}(s+20) + \frac{B}{s+20}(s+20) \Rightarrow B = -1$

$C(s) = \frac{1}{s} - \frac{1}{s+20} \qquad c(t) = 1 - e^{-20t}$

(2) $\frac{(V_1 - V)}{R_1} + \frac{V_1}{R_2} + \frac{1}{L}\int_0^t V_1 dz + C\frac{dV_1}{dt} = 0$

$\frac{(V_1 - V)}{10K} + \frac{V_1}{10K} + \frac{V_1}{200s} + 10\mu s V_1 = 0$

$V_1\left(\frac{2}{10K} + \frac{1}{200s} + 10\mu s\right) = \frac{V(s)}{10K}$

$V(s) = V_o(s)\left(2 + \frac{50}{s} + 0{,}1s\right) = V_o(s)\left(\frac{2s + 50 + 0{,}1s^2}{s}\right)$

$$F(s) = \frac{V_o}{V} = \frac{s}{0{,}1s^2+2s+50} = \frac{10s}{s^2+20s+500}$$

(20) a) $T(s) = \dfrac{16}{s^2+3s+16}$

$\omega_n = \sqrt{16} = 4\ rad/s$

$2\xi\omega_n = 3 \quad \xi = \dfrac{3}{2.4} = \dfrac{3}{8} = 0{,}375$

$T_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{\frac{3}{8}.4} = \dfrac{8}{3} = 2{,}667s$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{4\sqrt{\frac{55}{64}}} = 0{,}847s$

$T_r = \dfrac{1{,}8}{\omega_n} = \dfrac{1{,}8}{4} = 0{,}45s$

$e^{-(\xi\pi/\sqrt{1-\xi^2})}.100 = e^{-\left(\frac{0{,}375\pi}{\sqrt{55/64}}\right)}.100$

b) $T(s) = \dfrac{0{,}04}{s^2+0{,}02s+0{,}04}$

$\omega_n = \sqrt{0{,}04} = 0{,}2\ rad/s$

$\xi = \dfrac{0{,}02}{2.0{,}2} = 0{,}05$

$T_s = \dfrac{4}{0{,}05 . 0{,}2} = 400s$

$T_p = \dfrac{\pi}{0{,}2\sqrt{1-0{,}0025}} = 15{,}73s$

$T_r = \dfrac{1{,}8}{0{,}2} = 9s \qquad \%OS = e^{-\left(\frac{0{,}05\pi}{\sqrt{0{,}9975}}\right)}.100 = 85{,}47\%$

(1)

1\. $\zeta = \dfrac{-\ln(M_p/100)}{\sqrt{\pi^2 + \ln^2(M_p/100)}} = \dfrac{-\ln(0,12)}{\sqrt{\pi^2 + \ln^2(0,12)}} = 0,5593$

$T_s = \dfrac{4}{\zeta \omega_n} \Rightarrow 0,6 = \dfrac{4}{\omega_n \cdot 0,5593} \Rightarrow \omega_n = 11,9197 \text{ rad/s}$

$s_{1,2} = -6,667 \pm 9,881$

b). $\zeta = \dfrac{-\ln(0,1)}{\sqrt{\pi^2 + \ln^2(0,1)}} = 0,1518$

$T_p = \dfrac{\pi}{\omega_n \sqrt{1-\zeta^2}} \Rightarrow \omega_n = \dfrac{\pi}{T_p\sqrt{1-\zeta^2}} = \dfrac{\pi}{5 \cdot 0,988} = 0,6359 \text{ rad/s}$

$s_{1,2} = -0,0965 \pm 0,6285j$

(25) a) $f(t) = cv - kx = ma$

$f(t) = m\ddot{x} + c\dot{x} + kx$

$F(s) = ms^2 X(s) + cs X(s) + kX(s)$

$F(s) = X(s)(5s^2 + 2s + 20)$

$G(s) = \dfrac{X(s)}{F(s)} = \dfrac{1}{5s^2 + 2s + 20} = \dfrac{1/5}{s^2 + \frac{2}{5}s + 4}$

b) $\omega_n = \sqrt{4} = 2 \text{ rad/s}$

$\zeta = \dfrac{2/5}{2 \cdot 2} = \dfrac{1}{10} = 0,1$ $\quad T_p = \dfrac{\pi}{2\sqrt{1-0,01}} = 1,58s$ $\quad M_p = e^{-\left(\frac{\pi \cdot 0,1}{\sqrt{0,99}}\right)} \cdot 100 = 72,92\%$

$T_s = \dfrac{4}{0,1 \cdot 2} = 20s$ $\quad T_r = \dfrac{1,8}{2} = 0,9s$

(26) a) $T - J\ddot{\theta} - D\dot{\theta} - K\theta = 0 \Rightarrow T = J\ddot{\theta} + D\dot{\theta} + K\theta$

$T(s) = Js^2\theta(s) + Ds(\theta(s)) + K\theta(s)$

$T(s) = \theta(s)(2s^2 + 1s + 1)$

$G(s) = \dfrac{\theta(s)}{T(s)} = \dfrac{1}{2s^2 + s + 1}$

b) $\omega_n = \sqrt{1/2} = 0,707$

$\zeta = \dfrac{1/2}{2 \cdot 0,707} = 0,354$

$T_s = \dfrac{4}{\omega_n} = 5,658$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 4,753s$