Roteiro 3A - 1

1) a) $f(t) = u(t)$

$\mathcal{L}\{f(t)\} = \int_0^{\infty} 1 e^{-st} dt = -\frac{e^{-st}}{s} \Big|_0^{\infty} = \left(-\frac{1}{e^{\infty} s} - \left(\frac{-1}{e^0 s}\right)\right) \qquad F(s) = \frac{1}{s}$

b) $f(t) = t u(t)$

$\mathcal{L}\{f(t)\} = \int_0^{\infty} t e^{-st} dt = -\frac{t e^{-st}}{s} \Big|_0^{\infty} - \left(\frac{-1}{s}\right) \cdot \int_0^{\infty} e^{-st} dt$

$u = t \Rightarrow du = dt \qquad dv = e^{-st} dt \Rightarrow v = \frac{-e^{-st}}{s}$

$F(s) = \frac{1}{s^2}$

c)

$f(t) = \operatorname{sen} \omega t . u(t)$

$\mathcal{L}\{f(t)\} = \int_0^{\infty} \operatorname{sen} \omega t \, e^{-st} dt = -\frac{\operatorname{sen} \omega t \, e^{-st}}{s} \Big|_0^{\infty} - \left(-\frac{\omega}{s}\right) \int_0^{\infty} \cos \omega t \, e^{-st} . dt$

$u = \operatorname{sen} \omega t . \quad du = \omega \cos \omega t \, dt$
$dv = e^{-st} dt . \quad v = \frac{-e^{-st}}{s}$

$u = \cos \omega t \quad du = -\omega \operatorname{sen} \omega t \, dt$
$dv = e^{-st} dt \quad v = -\frac{e^{-st}}{s}$

$\int_0^{\infty} \operatorname{sen} \omega t \, e^{-st} dt = \left[-\frac{\cos \omega t . e^{-st}}{s} \Big|_0^{\infty} - \left(\frac{-\omega}{s}\right)(-1) \int_0^{\infty} \operatorname{sen} \omega t \, e^{-st} dt\right] \frac{\omega}{s}$

$\left(1 + \frac{\omega^2}{s^2}\right) \int_0^{\infty} \operatorname{sen} \omega t \, e^{-st} dt = \left[0 - \left(-\frac{1}{s}\right)\right] \frac{\omega}{s} \qquad F(s) = \frac{\omega}{s^2 + \omega^2}$

2) a) $e^{-at} \operatorname{sen} \omega t . u(t)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-at} . \cos \omega t \, u(t)\} = \mathcal{L}\{\operatorname{sen} \omega (t+a)\} \qquad F(s) = \frac{\omega}{(s+a)^2 + \omega^2}$

b) $f(t) = e^{-at} . \cos \omega t \, u(t)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-at} . \cos \omega t . u(t)\} = \mathcal{L}\{\cos \omega (t+a)\} \qquad F(s) = \frac{s+a}{(s+a)^2 + \omega^2}$

c) $f(t) = e^{at} . \cos \omega t \, u(t)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-at} \cos \omega t \, u(t)\} = \mathcal{L}\{\cos \omega (t-a)\} \qquad F(s) = \frac{s+a}{(s+a)^2 + \omega^2}$

(4) a) $\frac{dx}{dt} + 7x = 5\cos 2t$

$\mathcal{L}\{x(t)\} + 7\mathcal{L}\{x\} = 5\,\mathcal{L}\{\cos 2t\}$

$sX(s) - x(0) + 7X(s) = \frac{5s}{s^2+4}$

$X(s)(s+7) = \frac{5s}{s^2+4}$ $\quad \therefore X(s) = \frac{5s}{(s^2+4)(s+7)}$

$\frac{5s}{(s^2+4)(s+7)} = \frac{A}{s+7} + \frac{Bs+C}{s^2+4}$

$\frac{5}{(s^2+4)(s+7)} (s+7) = A + Bs + C$

$A = \frac{-35}{53}$

$\frac{5s}{(s^2+4)(s+7)} = \frac{A(s^2+4) + (Bs+C)(s+7)}{(s^2+4)(s+7)}$

$As^2 + 4A + Bs^2 + 7Bs + Cs + 7C = 5s$

$(A+B)s^2 + (7B+C)s + (4A+7C) = 0s^2 + 5s + 0$

$A+B=0 \quad B = \frac{35}{53} \quad 7B+C=5 \quad 4A+7C=0$

$C = \frac{-4\cdot 35}{53}\cdot\frac{1}{7} = \frac{20}{53}$

(8) $\frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{5dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt} + \frac{6dx}{dt} + 8x$

$\mathcal{L}\{\dddot{y}(t)\} + 3\mathcal{L}\{\ddot{y}(t)\} + 5\mathcal{L}\{\dot{y}(t)\} + \mathcal{L}\{y(t)\}$

$\mathcal{L}\{\dddot{x}(t)\} + 4\mathcal{L}\{\ddot{x}(t)\} + 6\mathcal{L}\{\dot{x}(t)\} + 8\mathcal{L}\{x(t)\}$

$s^3 Y(s) + 3s^2 Y(s) + 5sY(s) + Y(s) = (s^3 + 4s^2 + 6s + 8)X(s)$

$\frac{Y(s)}{X(s)} = \frac{(s^3+4s^2+6s+8)}{(s^3+3s^2+5s+1)}$

(9) a)

$\frac{X(s)}{F(s)} = \frac{7}{s^2+5s+10}$ $\quad (s^2+5s+10)X(s) = 7(F(s))$

$\ddot{x}(t) + 5\dot{x}(t) + 10x = 7f(t)$

$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 10x = 7f$

b) $\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)}$ $\quad (s^2+21s+110)X(s) = 15F(s) \therefore \ddot{x}(t) + 21\dot{x}(t) + 110x = 15f(t)$

$\frac{d^2x}{dt^2} + \frac{21dx}{dt} + 110x = 15f$

c) $\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18}$ $\quad (s^3+11s^2+12s+18)X(s) = (s+3)F(s)$

$\dddot{x}(t) + 11\ddot{x}(t) + 12\dot{x}(t) + 18x(t) = \dot{f}(t) + 3f(t)$

$\frac{d^3x}{dt^3} + \frac{11d^2x}{dt^2} + \frac{12dx}{dt} + 18x = \frac{df}{dt} + 3f$

$$\frac{C(s)}{R(s)} = \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5}$$

$$(s^5+2s^4+4s^3+s^2+4)R(s) = (s^6+7s^5+3s^4+2s^3+s^2+5)C(s)$$

$$\frac{d^5r}{dt^5}+2\frac{d^4r}{dt^4}+4\frac{d^3r}{dt^3}+\frac{d^2r}{dt^2}+4r = \frac{d^6c}{dt^6}+7\frac{d^5c}{dt^5}+3\frac{d^4c}{dt^4}+3\frac{d^4c}{dt^4}+2\frac{d^3c}{dt^3}+\frac{d^2c}{dt^2}+5c$$

(11) $$\frac{C(s)}{R(s)} = \frac{s^4+3s^3+2s^2+s+1}{s^5+4s^4+3s^3+2s^2+3s+2}$$

$$\frac{d^4r}{dt^4}+3\frac{d^3r}{dt^3}+2\frac{d^2r}{dt^2}+\frac{dr}{dt}+r = \frac{d^5c}{dt^5}+4\frac{d^4c}{dt^4}+3\frac{d^3c}{dt^3}+2\frac{d^2c}{dt^2}+3\frac{dc}{dt}+2c$$

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3.2

b) $\mathcal{L}\{f(t)\} = \mathcal{L}\{3+7t+t^2+\delta(t)\} = 3\mathcal{L}\{1\}+7\mathcal{L}\{t\}+\mathcal{L}\{t^2\}+\mathcal{L}\{\delta(t)\}$

$$F(s) = \frac{3}{s}+\frac{7}{s^2}+\frac{2s}{s^3}+1$$

d) $\{f(t)\} = \mathcal{L}\{(t+1)^2\} = \mathcal{L}\{t^2+2t+1\} = \mathcal{L}\{t^2\}+2\mathcal{L}\{t\}+\mathcal{L}\{1\}$

$$F(s) = \frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}$$

f) $f(t) = \operatorname{senh}(t)$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{senh}(t)\} = \frac{1}{s^2-1^2}$$

3.3

a) $f(t) = 3\cos 6t$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3\cos 6t\} = 3\mathcal{L}\{\cos 6t\}$$

$$F(s) = \frac{3s}{s^2+36}$$

b) $f(t) = \operatorname{sen} 2t + 2\cos 2t + e^{-t} \operatorname{sen} 2t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen} 2t + 2\cos 2t + e^{-t}\operatorname{sen} 2t\} = \mathcal{L}\{\operatorname{sen} 2t\} + 2\mathcal{L}\{\cos 2t\} + \mathcal{L}\{e^{-t}\operatorname{sen}$

$F(s) = \dfrac{2}{s^2+2^2} + \dfrac{2s}{s^2+2^2} + \dfrac{2}{(s+1)^2+2^2}$

c) $f(t) = t^2 + e^{-2t} \operatorname{sen} 3t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\} + \mathcal{L}\{e^{-2t}\operatorname{sen} 3t\} = \dfrac{2}{s^3} + \dfrac{3}{(s+2)^2+3^2}$

$F(s) = \dfrac{2}{s^3} + \dfrac{3}{s^2+4s+13}$

3.5

a) $f(t) = \operatorname{sen} t \operatorname{sen} 3t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen} t \operatorname{sen} 3t\} = \mathcal{L}\left\{\frac{1}{2}(\cos(t-3t) - \cos(t+3t))\right\}$

$F(s) = \dfrac{s}{2s^2+8} - \dfrac{s}{2s^2+32}$

b) $f(t) = \operatorname{sen}^2 t + 3\cos^2 t$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen}^2 t + 3\cos^2 t\} = \mathcal{L}\{\frac{1}{2}(1-\cos 2x) + \frac{3}{2}(1+\cos 2x\}$

$F(s) = \dfrac{1}{2}\mathcal{L}\{4 + 2\cos 2x\} = 2\mathcal{L}\{1\} + \mathcal{L}\{\cos 2x\} = \dfrac{2}{s} + \dfrac{s}{s^2+2^2}$

$F(s) = \dfrac{2}{s} + \dfrac{s}{s^2+4}$

c) $f(t) = \dfrac{\operatorname{sen} t}{t}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{\dfrac{\operatorname{sen} t}{t}\right\} = \operatorname{arctg} \dfrac{1}{s}$

$F(s) = \operatorname{arctg} \dfrac{1}{s}$

a) $F(s) = \frac{2}{s(s+2)}$ $\quad \frac{2}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$

$\frac{2}{\cancel{s}(s+2)} \cdot \cancel{s} = \frac{A}{\cancel{s}} \cdot \cancel{s} + \frac{B}{s+2} \cdot s$ $\quad A = \frac{2}{2} = 1$

$\frac{2}{s(s+2)} (s+2) = \frac{A}{s} (s+2) + \frac{B}{s+2} (s+2)$

$B = \frac{2}{-2} = -1$ $\quad F(s) = \frac{1}{s} = \frac{1}{s+2}$

$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{\frac{1}{s}\} - \mathcal{L}^{-1}\{\frac{1}{s+2}\} = 1 - e^{-2t}$

e) $F(s) = \frac{1}{s^2+4}$ $\quad \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2} \mathcal{L}^{-1}\{\frac{2}{s^2+4}\}$

$F(s) \frac{1}{2} \cdot \frac{2}{s^2+4}$ $\quad f(t) = \frac{1}{2} \operatorname{sen} 2t$

j) $F(s) \frac{e^{-s}}{s^2}$

$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{e^{-s} \cdot \frac{1}{s^2} = u(t-1) \cdot \mathcal{L}^{-1}\{\frac{1}{s^2}\}$

$f(t) = u(t-1) \cdot (t-1)$

3.9

a) $\ddot{y}(t) + \dot{y}(t) + 3y(t) = 0$; $y(0) = 1$; $\dot{y}(0) : 2$

$\mathcal{L}\{\ddot{y}(t)\} + \mathcal{L}\{\dot{y}(t)\} + \mathcal{L}\{y(t)\} = \mathcal{L}\{0\}$

$(s^2 Y(s) - sy(0) - \dot{y}(0)) + (sY(s) - y(0)) + 3Y(s) = 0$

$s^2 Y(s) - s - 2 + sY(s) - 1 + 3Y(s) = 0$

$(s^2+s+3) Y(s) = s+3)$ $\quad Y(s) = \frac{s+3}{s^2+s+3}$

f) $\ddot{y}(t) + y(t) = t \quad y(0) = 1 \quad \dot{y}(0) = -1$

$\mathcal{L}\{\ddot{y}(t)\} + \mathcal{L}\{y(t)\} = \mathcal{L}\{t\}$

$s^2 Y(s) - s y(0) - \dot{y}(0) + Y(s) = \dfrac{1}{s^2}$

$(s^2+1) Y(s) - s + 1 = \dfrac{1}{s^2} \Rightarrow Y(s) = \dfrac{1}{s^2(s^2+1)} + \dfrac{s}{(s^2+1)} - \dfrac{1}{s^2+1}$

$\dfrac{1}{s^2(s^2+1)} = \dfrac{As+B}{s^2} + \dfrac{Cs+D}{s^2+1} \qquad As+B(s^2+1) + (Cs+D)s^2 = 1$

$As^3 + As + Bs^2 + B + Cs^3 + Ds^2 = 1$

$(A+C)s^3 + (B+D)s^2 + As + B = 0s^3 + 0s^2 + 0s + 1$

$A+C = 0 \qquad C = 0$

$B+D = 0 \qquad D = -1$

$A = 0$

$B = 1$

$Y(s) = \dfrac{0.s+1}{s^2} + \dfrac{0s+(-1)}{s^2+1} + \dfrac{s}{s^2+1} - \dfrac{1}{s^2+1}$

$Y(s) = \dfrac{1}{s^2} - \dfrac{1}{s^2+1} + \dfrac{s}{s^2+1} - \dfrac{1}{s^2+1} - \dfrac{1}{s^2} - \dfrac{2}{s^2+1} + \dfrac{s}{s^2+1}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right\} - 2\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\dfrac{s}{s^2+1}\right\}$

$y(t) = t - 2\,\text{sen}\,t + \cos t$