Lectures on Quantum Mechanics

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Contents

Introduction

Quantum mechanics is certainly the most successful theory of the physical world at small scales. It is also one of the most mysterious.

Some notes here.[1]

[1] *With some additional side-notes*

PART

I

Time Independent Non-degenerate Perturbation Theory

1 Lecture 1

Suppose we have a "known" system with Hamiltonian, eigenfunctions and eigenenergies H_0, Ψ_n and E_n respectively.

$$\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$$

where [2]

$$\langle\hat{\Psi}|\hat{H}|\hat{\Psi}\rangle = E_n, \quad \langle\Psi_n|\Psi_{n'}\rangle = \delta_{nn'}$$

So that $\{\Psi_n\}$ forms a orthonormal basis over the Hilbert space of H_0. We apply a perturbative potential $\tilde{V}$ [3] to H_0, yielding a new Hamiltonian. for the perturbated system,

$$\tilde{H} = \hat{H}_0 + \lambda\tilde{V}, \quad \text{let } \lambda \in [0,1] \tag{1.1}$$

a "bookkeeping parameter" to keep track of the relative signs of from in order of the perturbated potential.

[2] *The student may remember the Dirac delta function from his/her Quantum Mechanics I course.*

[3] $\tilde{V}$ *is very small compared to* H_0

1.1 Method

Express the wave functions and eigenenergies of the perturbated Hamiltonian H, $\tilde{\psi}$, $\tilde{E}_n$ as a power series equation in orders of λ.

$$\tilde{E}_n = \tilde{E}_0^{(0)} + \lambda\tilde{E}_n^{(1)} + \lambda^2\tilde{E}_n^{(2)} + \cdots = \sum_k \lambda^k \tilde{E}_n^{(k)}$$
$$\tilde{\psi}_n = \tilde{\psi}_n^{(0)} + \lambda\tilde{\psi}_n^{(1)} + \lambda^2\tilde{\psi}_n^{(2)} + \cdots = \sum_k \lambda^k \tilde{\psi}_n^{(k)}$$

Now we write the Time independent Schrödinger equation for the perturbated Hamiltonian,

$$\tilde{H}\left|\tilde{\psi}_n\right\rangle = \tilde{E}_n\left|\tilde{\psi}_n\right\rangle \tag{1.2}$$
$$(\hat{H} + \hat{V})\sum_k \lambda^k\left|\psi_n^{(k)}\right\rangle = \sum_k \lambda^k \tilde{E}_n^{(k)} \sum_k \lambda^k\left|\psi_n^{(k)}\right\rangle \tag{1.3}$$
$$\sum_k (\lambda^k \hat{H}_0 + \lambda^{k+1}\hat{V})\left|\hat{\psi}_n^{(k)}\right\rangle = \sum_{k'}\sum_k \lambda^{k'+k}\hat{E}_n^{(k)}\left|\hat{\psi}_n^{(k)}\right\rangle \tag{1.4}$$

$$\sum_k \left(\lambda^k \hat{H}_0 + \lambda^{k+1}\hat{V} - \sum_{k'} \lambda^{k'+k}\hat{E}_n^{(k')}\right)\Big|\psi_n^{(k)}\Big\rangle = 0 \tag{1.5}$$

$$\sum_k \lambda^k \left(\hat{H}_0\Big|\hat{\psi}_n^{(k)}\Big\rangle + \underbrace{(1-\delta_{k0})}_{k\neq 0}\hat{V}\Big|\hat{\psi}^{(k-1)}\Big\rangle - \sum_{k=0}^{k}\hat{E}_n^{(k')}\Big|\hat{\psi}_n^{(k-k')}\Big\rangle\right) = 0 \tag{1.6}$$

Since 1.6 is true for any value of λ, it must hold for each power of λ separately:

$$\begin{aligned}
\mathcal{O}(1)_{k=0} &= \hat{H}\Big|\hat{\psi}_n^{(0)}\Big\rangle + (1-\delta_{k0})\hat{V}\Big|\hat{\psi}_n^{(-1)}\Big\rangle \\
&= \sum_{k'=0}^{0}\hat{E}_n^{(k')}\Big|\hat{\psi}_n^{(0-k')}\Big\rangle \\
\implies \hat{H}_0\Big|\hat{\psi}_n^{(0)}\Big\rangle &= \hat{E}_n^{(0)}\Big|\hat{\psi}_n^{(0)}\Big\rangle \\
\Big\langle\psi_n\Big|\underbrace{H_0}_{\text{hermitian}}\Big|\hat{\psi}_n^{(0)}\Big\rangle &= \hat{E}_n^{(0)}\Big\langle\psi_n\Big|\hat{\psi}_n^{(0)}\Big\rangle
\end{aligned}$$

$$\Big\langle\psi_{n'}\Big|\hat{H}_0^\dagger = E_{n'}\Big\langle\psi_{n'}\Big|$$

$$(E_{n'} - \hat{E}_n^{(0)})\Big\langle\psi_{n'}\Big|\hat{\psi}_n^{(0)}\Big\rangle = 0$$

So,

$$\hat{E}_n^{(0)} = E_n \ \text{ or } \ \Big\langle\psi_n\Big|\hat{\psi}_n^{(0)}\Big\rangle = 0 \tag{1.7}$$

If the system $\hat{H}_0$ is nondegenerate:

$$\implies \Big|\hat{\psi}_n^{(0)}\Big\rangle = \Big|\psi_n\Big\rangle$$

If the system is degenerate, then:

$$\hat{H}_0\Big|\psi_{n'}\Big\rangle = E_n\Big|\psi_{n'}\Big\rangle \ \text{ for } n' \in \{n, \cdots, n+k\}$$

Then:

$$\Big|\hat{\psi}_n^{(0)}\Big\rangle = \sum_{k'=0}^{k} C_{n+k'}\Big|\psi_{n+k'}\Big\rangle$$

$$\hat{E}_n = E_n\mathcal{O}(\lambda)$$

$$\Big|\hat{\psi}_n\Big\rangle = |\psi_n\Big\rangle + \mathcal{O}(\lambda)$$

$$\mathcal{O}(\lambda) : \hat{H}_0\Big|\hat{\psi}_n^{(1)}\Big\rangle + \hat{V}\Big|\hat{\psi}_n^{(0)}\Big\rangle = E_n^{(0)}\Big|\hat{\psi}_n^{(1)}\Big\rangle + \hat{E}_n^{(1)}\Big|\hat{\psi}_n^{(0)}\Big\rangle$$

multiply by $\Big\langle\psi_{n'}\Big|$:

$$\Big\langle\psi_{n'}\Big|\hat{H}_0\Big|\hat{\psi}_n^{(1)}\Big\rangle + \Big\langle\psi_{n'}\Big|\hat{V}\Big|\hat{\psi}_n^{(0)}\Big\rangle = E_n^{(0)}\Big\langle\psi_{n'}\Big|\hat{\psi}_n^{(1)}\Big\rangle + \hat{E}_n^{(1)}\Big\langle\psi_{n'}\Big|\hat{\psi}_n^{(0)}\Big\rangle$$

$$\Big\langle\psi_{n'}\Big|\hat{H}_0 = E_{n'}\Big\langle\psi_{n'}\Big|$$

$$\left|\hat{\psi}_n^{(0)}\right\rangle = \left|\psi_n\right\rangle \quad \text{(non-degenerate)}, \quad \hat{E}_n^{(0)} = E_n$$

$$(E_{n'} - E_n)\left\langle \psi_{n'} \middle| \hat{\psi}_n^{(1)} \right\rangle + \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle = \hat{E}_n^{(1)} \delta_{n'n} \tag{1.8}$$

if $\mathbf{n' = n}$:

$$\cancelto{0}{(E_{n'} - E_n)}\left\langle \psi_{n'} \middle| \hat{\psi}_n^{(1)} \right\rangle + \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle = \hat{E}_n^{(1)} \delta_{n'n}$$

$$\begin{aligned} \implies \hat{E}_n^{(1)} &= \left\langle \psi_n \middle| \hat{V} \middle| \hat{\psi}_n^{(1)} \right\rangle \\ \hat{E}_n &= E_n + \left\langle \psi_{n'} \middle| \hat{V} \middle| \hat{\psi}_n^{(1)} \right\rangle + \mathcal{O}(\lambda^2) \\ \left|\hat{\psi}_n\right\rangle &= \left|\psi_n\right\rangle + \mathcal{O}(\lambda) \end{aligned} \tag{1.9}$$

if $\mathbf{n' \neq n}$:

$$(E_{n'} - E_n)\left\langle \psi_{n'} \middle| \hat{\psi}_n^{(1)} \right\rangle = -\left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle + \hat{E}_n^{(1)} \cancelto{0}{\delta_{n'n}}$$

$$\left\langle \psi_{n'} \middle| \hat{\psi}_n^{(1)} \right\rangle = \frac{\left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_{n'}} \tag{1.10}$$

Since $\{\psi_n\}$ forms an orthonormal basis,

$$\implies \sum_{n'} \left|\psi_{n'}\right\rangle \left\langle \psi_{n'}\right| = \mathbb{1} \ \text{ Identity matrix}$$

$$\begin{aligned} \left|\hat{\psi}_n^{(1)}\right\rangle = \mathbb{1}\left|\hat{\psi}_n^{(1)}\right\rangle &= \sum_{n'} \left|\psi_{n'}\right\rangle \left\langle \psi_{n'} | \hat{\psi}_n^{(1)} \right\rangle \\ &= \sum_{n'=n} \frac{\left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_{n'}} \left|\psi_{n'}\right\rangle \end{aligned} \tag{1.11}$$

Since $\left|\hat{\psi}_n^{(0)}\right\rangle = \left|\psi_n\right\rangle$, then $\left|\hat{\psi}_n^{(1)}\right\rangle$ cannot depend on $\left|\psi_n\right\rangle$.

2 Lecture 2

In the previous lecture we defined the perturbed hamiltonian as: [4] 1.1 as:

$$\tilde{H} = \hat{H}_0 + \lambda \hat{V}$$

[4] $\tilde{H}$: *perturbed hamiltonian,* $\hat{H}_0$: *unperturbed hamiltonian,* λ: *bookkeeping parameter,* $\hat{V}$: *perturbing potential.*

Definition 2.1. since $\hat{V}$ needs to be smaller than $\hat{H}_0$, λ keeps track of this, i.e., dependence on $\hat{V}$.
Writing the Schrödinger equation:

$$(\hat{H}_o + \lambda \hat{V})\hat{\psi}_n = \tilde{E}_n \tilde{\psi}_n$$

$\tilde{E}_n$ and $\tilde{\psi}_n$ are eigenenergies and wavefunctions for the perturbated hamiltonian.
Expand $\tilde{E}_n$ and $\tilde{\psi}_n$ in power of λ:

$$\tilde{E}_n = \hat{E}_n^{(0)} + \lambda \tilde{E}_n^{(1)} + \lambda^2 \tilde{E}_n^{(2)} + \mathcal{O}(\lambda^3)$$
$$\tilde{\psi}_n = \hat{\psi}_n^{(0)} + \lambda \tilde{\psi}_n^{(1)} + \lambda^2 \tilde{\psi}_n^{(2)} + \mathcal{O}(\lambda^3)$$

Substitute for $\tilde{E}_0$ and $\tilde{\psi}_n$ into the (SE) and collect terms of like powers of λ.

$$\mathcal{O}(1) : \hat{H}_0 \left|\tilde{\psi}_n^{(0)}\right\rangle = \tilde{E}_0^{(0)} \left|\psi_n^{(0)}\right\rangle$$

multiply by $\left\langle \psi_{n'} \right|$:

$$\left\langle \psi_{n'} \middle| \hat{H}_o \middle| \tilde{\psi}_n^{(0)} \right\rangle = \tilde{E}_o^{(0)} \left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle$$

$\hat{H}_o = H_o^\dagger$ since $\hat{H}_o$ is hermitian.

$$\left\langle \psi_{n'} \middle| \hat{H}_o^\dagger = E_{n'} \left\langle \psi_{n'} \right| \text{ (SE) for } \hat{H}_o$$

$$\implies E_{n'} \left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle = \hat{E}_o^{(0)} \left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle$$
$$E_{n'}^{(0)} = E_{n'} \quad \text{or} \quad \left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle = 0$$

$\mathbb{1} = \sum_{n'} \left|\psi_{n'}\right\rangle \left\langle \psi_{n'} \right|$

$\tilde{\psi}_n^{(0)} = \sum_{n'} \left\langle \psi_{n'} \middle| \psi_n^{(1)} \right\rangle$

If E_n is k-level degenerate, then:

$$\left|\tilde{\psi}_n^{(0)}\right\rangle = \sum_{n'=n}^{n+k-1} \left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle \left|\psi_{n'}\right\rangle$$

Consider $\hat{H}_o$ to be nondegenerate:

$$\Longrightarrow E_n^{(0)} = E_n, \quad \left|\left\langle \psi_n \middle| \tilde{\psi}_n^{(0)} \right\rangle\right|^2 = 0$$

$$\sum_{n'=n}^{n+k-1} \left|\left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle\right|^2 = 1$$

Since $\left\langle \psi_{n'} \middle| \psi_n^{(0)} \right\rangle = 0$ for $n' \neq n$:

$$\Longrightarrow \left| \tilde{\psi}_n^{(0)} \right\rangle = \left| \psi_{n'} \right\rangle$$

The n^{th} eigenenergy of the perturbated system (hamiltonian) $\tilde{E}_n$ is the n^{th} eigenenergy of the unperturbed system to Zeroth order in the perturbation, i.e $\mathcal{O}(\lambda^0) = \mathcal{O}(1),\ \lambda = 0$.

$\Longleftrightarrow$ If we turn the perturbation off we get the eigenenergy of the unperturbed system.

$\because$ The n^{th} wavefunction for the perturbed hamiltonian $\tilde{\Psi}_n$ is a linear combination of the unperturbed hamiltonian's degenerate wavefunctions with energy E_n to Zeroth order in the perturbation, i.e $\mathcal{O}(\lambda^0) = \mathcal{O}(1)$ or $\lambda = 0$

$$\Longrightarrow \left| \hat{\psi}_n^{(0)} \right\rangle = \sum_{n'=n}^{n+k-1} \left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(0)} \right\rangle \left| \psi_{n'} \right\rangle$$

,where

$$\sum_{n'=n}^{n+k-1} \left|\left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(0)} \right\rangle\right|^2 = 1$$

Even if we "turn off" the perturbation, it can still left the degeneracy of $\{\left|\psi_n\right\rangle, \cdots, \left|\psi_{n+k-1}\right\rangle\}$ for k-fold degenerate systems, unless the unperturbed wavefunctions are eigenstates of the perturbation.

Assuming $\hat{H}_o$ is non-degenerate

$$\Longrightarrow \left|\left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(0)} \right\rangle\right|^2 = 1$$

wlog $\psi_n = \tilde{\psi}_n^{(0)}$ (Choice of 1 for phase factor).

$$\tilde{E}_n = E_n + \mathcal{O}(\lambda)$$
$$\tilde{\psi}_n = \psi_n + \mathcal{O}(\lambda)$$

Gather together terms of order 1 in λ in the (SE):

$$\mathcal{O}(\lambda) : \lambda \left(\hat{H}_o \middle| \tilde{\psi}_n^{(1)} \right\rangle + \hat{V} \left| \tilde{\psi}_n^{(0)} \right\rangle = \lambda \left(E_n \left| \tilde{\psi}_n^{(1)} \right\rangle + E_n^{(1)} \left| \psi_n \right\rangle \right)$$

Set $\lambda = 1$,

$$\hat{H}_o \left| \tilde{\psi}_n^{(1)} \right\rangle + \hat{V} \left| \tilde{\psi}_n^{(0)} \right\rangle = E_n \left| \tilde{\psi}_n^{(1)} \right\rangle + E_n^{(1)} \left| \psi_n \right\rangle$$

This is a recursive equation relating the 1st order corrections in $\tilde{E}_n$ and $\tilde{\psi}_n$ to the Zeroth ordeŕcorrections, in the unperturbed eigenenergy and wavefunction, E_n and ψ_n respectively $\because$ the $\mathcal{O}^{th}$ relates the kth order corrections to all previous orders.

2.1 Two Unknowns

Use the fact that:

$$\left\langle \psi_{n'} \middle| \psi_n \right\rangle = \delta_{n'n} \quad \text{and} \quad \hat{H}_o \left| \psi_{n'} \right\rangle = E_{n'} \left| \psi_{n'} \right\rangle$$

To obtain equations for $\tilde{E}_n^{(1)}$ and $\tilde{\psi}_n^{(1)}$.

multiply by $\left\langle \psi_{n'} \right|$:

$$\left\langle \psi_{n'} \middle| \hat{H}_o^\dagger \middle| \tilde{\psi}_n^{(1)} \right\rangle + \left\langle \psi_{n'} \middle| \hat{V} \middle| \tilde{\psi}_n^{(0)} \right\rangle = E_n \left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(1)} \right\rangle + E_n^{(1)} \underbrace{\left\langle \psi_{n'} \middle| \psi_n \right\rangle}_{\delta_{n'n}}$$

$$(E_{n'} - E_n) \left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(1)} \right\rangle + \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle = \tilde{E}_n^{(1)} \delta_{n'n} \tag{2.1}$$

$\left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(1)} \right\rangle$: Coefficient of $\left| \psi_{n'} \right\rangle$.

$\tilde{E}_n^{(1)}$: First order correction to the eigenenergy.

If $n' = n$, (2.2.1) yields to: $\tilde{E}_n^{(1)}$

$$\boxed{\tilde{E}_n^{(1)} = \left\langle \psi_n \middle| \hat{V} \middle| \psi_n \right\rangle}$$

is the expectation value of the perturbation in the n^{th} eigenstate of the unperturbed hamiltonian.

If $n' \neq n$, (2.2.1) yields to: coefficients of $\left| \tilde{\psi}_n^{(1)} \right\rangle$ and $\lambda = 1$.

$$\left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(1)} \right\rangle = \frac{\left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_{n'}}$$

$$\left|\tilde{\psi}_n^{(1)}\right\rangle = \sum_{n'\neq n} \frac{\left\langle \psi_{n'}\middle|\hat{V}\middle|\psi_n\right\rangle}{E_n - E_{n'}}\left|\psi_{n'}\right\rangle$$

$$\tilde{E}_n = E_n + \left\langle \psi_n\middle|\hat{V}\middle|\psi_n\right\rangle + \mathcal{O}(\lambda^2)$$

$$\left|\tilde{\psi}_n\right\rangle = \left|\psi_n\right\rangle + \sum_{n'\neq n} \frac{\left\langle \psi_{n'}\middle|\hat{V}\middle|\psi_n\right\rangle}{E_n - E_{n'}}\left|\psi_{n'}\right\rangle + \mathcal{O}(\lambda^2)$$

The first order correction to the eigenenergy is the energy of the perturbation of you stay in the n^{th} eigenstate of the unperturbed system. So V has to be "weak" so that you can remain in $\left|\psi_n\right\rangle$ under its action.

Note: if $\hat{V}$ (the perturbation) is an odd function,

$$\implies \hat{E}_n^{(1)} = \int d^3\vec{r}\Big|\underbrace{\psi(\vec{r})}_{\text{even}}\Big|^2 \overbrace{\hat{V}(\vec{r},t)}^{\text{odd}} = 0$$

The first order corrections to wavefunctions are proportional to the matrix elements:

$$\left\langle \psi_{n'}\middle|\hat{V}\middle|\psi_n\right\rangle,$$

i.e., that under the action of the perturbation you can transition from n^{th} to n'^{th} eigenstate of the unperturbed hamiltonian and inversely proportional to the difference (energy) between the eigenenergies of the n^{th} and n'^{th} eigenstates of the perturbed hamiltonian.

Note:

$$\left|\left\langle \psi_{n'}\middle|\hat{V}\middle|\psi_n\right\rangle\right| \ll \left|E_n - E_{n'}\right|$$

Otherwise our expression for the wavefunctions of the perturbed system won't converge.

$$\left|\left\langle \psi_{n'}\middle|\hat{V}\middle|\psi_n\right\rangle\right| \ll \left|E_n - E_{n'}\right|$$

for our application of perturbation theory to apply.

$$\implies \lambda \gg 1$$

$$\mathcal{O}(\lambda^2) : \hat{H}_o\left|\tilde{\psi}_n^{(2)}\right\rangle + \hat{V}\left|\tilde{\psi}_n^{(1)}\right\rangle = E_n\left|\tilde{\psi}_n^{(2)}\right\rangle + \tilde{E}_n^{(1)}\left|\tilde{\psi}_n^{(1)}\right\rangle + \tilde{E}_n^{(2)}\left|\psi_n\right\rangle$$

Multiply by $\left\langle \psi_{n'}\right|$:

$$\left\langle \psi_{n'}\middle|\hat{H}_o\middle|\tilde{\psi}_n^{(2)}\right\rangle + \left\langle \psi_{n'}\middle|\hat{V}\middle|\tilde{\psi}_n^{(1)}\right\rangle = E_n\left\langle \psi_{n'}\middle|\tilde{\psi}_n^{(2)}\right\rangle + \tilde{E}_n^{(1)}\left\langle \psi_{n'}\middle|\tilde{\psi}_n^{(1)}\right\rangle + \tilde{E}_n^{(2)}\left\langle \psi_{n'}\middle|\psi_n\right\rangle$$

$$(E_{n'} - E_n)\left\langle \psi_{n'} \middle| \hat{\psi}_n^{(2)} \right\rangle + \left\langle \psi_{n'} \middle| \hat{V} \sum_{k \neq n} \frac{\left\langle \psi_k \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_k} \middle| \psi_k \right\rangle$$
$$= \left\langle \psi_n \middle| \hat{V} \middle| \psi_n \right\rangle \left\langle \psi_{n'} \middle| \sum_{n=k} \frac{\left\langle \psi_k \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_k} \middle| \psi_k \right\rangle + \tilde{E}_n^{(2)} \delta_{n'n}$$

$$(E_{n'} - E_n)\left\langle \psi_{n'} \middle| \hat{\psi}_n^{(2)} \right\rangle + \sum_{k=n} \frac{\left\langle \psi_k \middle| \hat{V} \middle| \psi_n \right\rangle \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_k \right\rangle}{E_n - E_k}$$
$$= \frac{\left\langle \psi_n \middle| \hat{V} \middle| \psi_n \right\rangle \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_{n'}} \left(1 - \delta_{n'n} \right) + \tilde{E}_n^{(2)} \delta_{n'n}$$

If $n' = n$:

$$\boxed{\tilde{E}_n^{(2)} = \sum_{k \neq n} \frac{\left| \left\langle \psi_k \middle| \hat{V} \middle| \psi_n \right\rangle \right|^2}{E_n - E_k}}$$

If $n' \neq n$:

$$\left\langle \psi_{n'} \middle| \tilde{\psi}_n^{(2)} \right\rangle = \sum_{k \neq n} \frac{\left\langle \psi_k \middle| \hat{V} \middle| \psi_n \right\rangle \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_k \right\rangle}{(E_n - E_k)(E_n - E_{n'})} + \frac{\left\langle \psi_n \middle| \hat{V} \middle| \psi_n \right\rangle \left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_{n'}}$$

$$\left| \tilde{\psi}_n^{(2)} \right\rangle = \sum_{n' \neq n} \left| \psi_{n'} \right\rangle \left[\frac{\left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_k \right\rangle \left\langle \psi_k \middle| \hat{V} \middle| \psi_n \right\rangle}{(E_n - E_K)(E_n - E_{n'})} + \frac{\left\langle \psi_{n'} \middle| \hat{V} \middle| \psi_n \right\rangle \left\langle \psi_n \middle| \hat{V} \middle| \psi_n \right\rangle}{E_n - E_{n'}} \right]$$

3 Lecture 3