Back to Resources

Disjoint Sets

by Eddie Huang

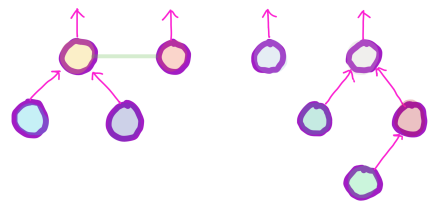
Overview

Disjoint sets allows you to organize elements in sets and be able to query which set an element belongs to in essentially constant time. The data structure to represent disjoint sets is a forest of trees, where each set is represented by a tree and each tree is identified by its root node. Therefore, we call the root node of a tree, the `representative` of its set.

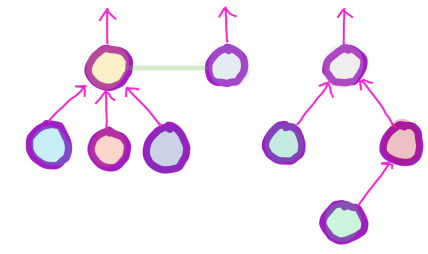
Unionize Sets

There are two types of unions

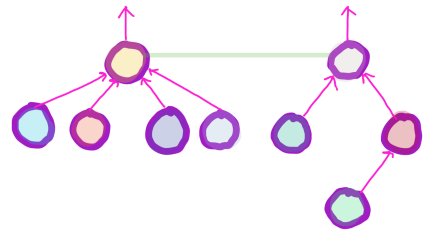
- `union by rank (height)`: minimizes the rank (height) of the disjoint sets by appending the tree of smaller rank to the root of the other
- `union by size`: appends the tree of smaller size to the root of the other



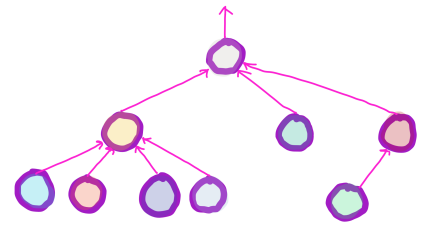
1: Disjoint sets



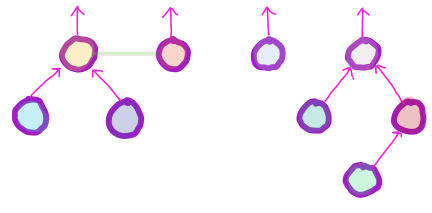
2: Union by Rank



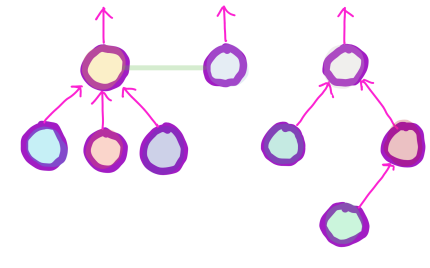
3: Union by Rank



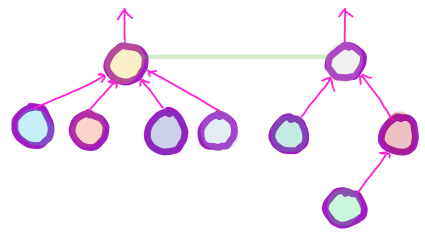
4: Union by Rank



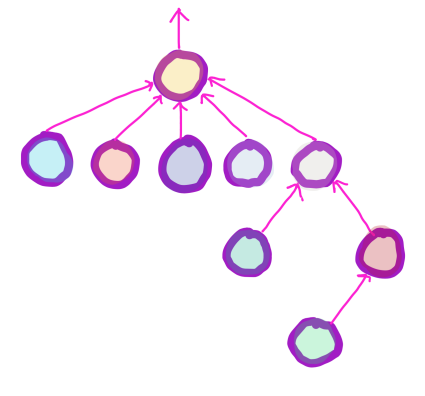
1: Disjoint sets



2: Union by Size



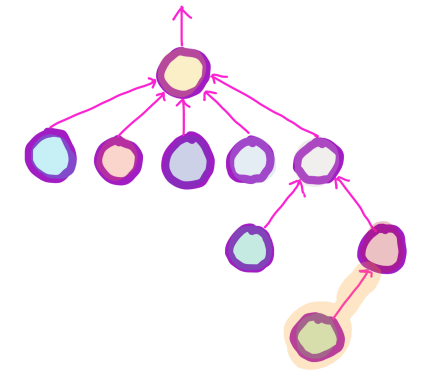
3: Union by Size



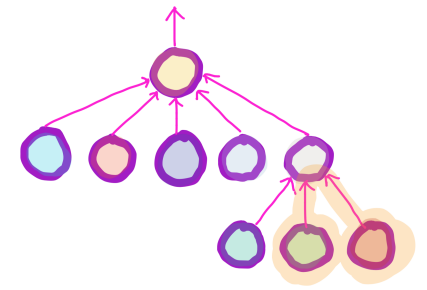
4: Union by Size

Path Compression

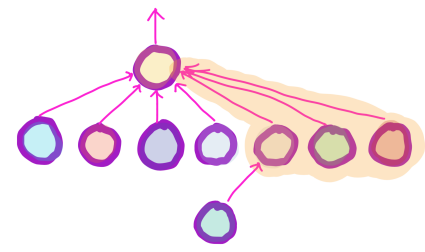
While finding nodes, it is efficient to move all nodes along the path of the query node to the representative node, so that the next we time we query those nodes, the runtime will be faster.



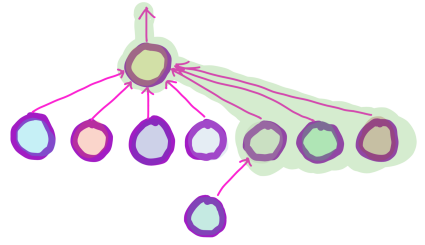
1: Compressing a query node's path



2: Compressing a query node's path



3: Compressing a query node's path



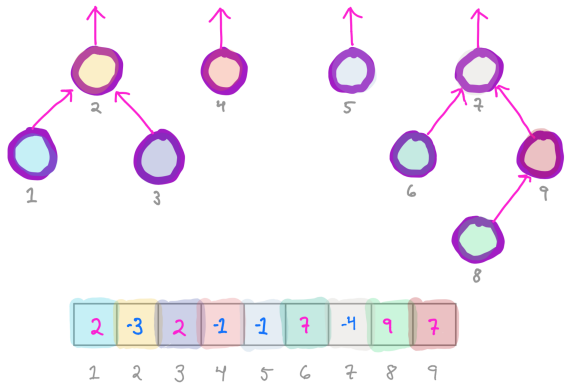
4: Set identified and path compressed

Time Complexity

With path compression and union by rank or size, the run time of the `find` and `union` operations are essentially constant time

Array Implementation

Disjoint sets can be implemented as arrays in which negative numbers indicate that they are representatives and positive numbers are pointers to their parents.



An array implementation of disjoin sets in which the size of the set is stored in the representative