Resumo Prob. 2

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- Probabilidade Condicional: $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$
- Formula de Bayes: $E = (E \cap F) \cup (E \cap F^C) \Rightarrow P(E) = P(E|F)P(F) + P(E|F^C)(1 - P(F))$

  Generalização: $P(F_j|E) = \dfrac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$
- Série de Taylor: $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$
- Série Geométrica (Geral): $\displaystyle\sum_{k=k_1}^{k_2} ar^k = a\frac{r^{k_1} - r^{k_2}}{1-r} \quad k_1, k_2 \geq 0, |r| < 1$
- Teorema Binomial: $(x+y)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$
- Esperança, Variância:(Discreto): $\mathbb{E}[X] := \displaystyle\sum_{x \in Dom} x\mathbb{P}(X = x)$ (Continuo): $\mathbb{E}[X] := \displaystyle\int_{-\infty}^{\infty} xf(x)dx$

  $\text{Var}[X] := \mathbb{E}[X^2] - \mathbb{E}^2[X]$ $\mathbb{E}[aX+b] = a\mathbb{E}[X]+b$ $\text{Var}[aX+bY] = a^2\text{Var}[X]+b^2\text{Var}[Y]+2ab\text{Cov}[X,Y]$
- Função Geratriz de Momento: $M_X(t) := \mathbb{E}[e^{tx}] \quad \mathbb{E}[x^n] = \lim_{t\to 0} M_X^{(n)}(t)$
- Função Característica: $\varphi_X(t) := \mathbb{E}[e^{itx}] \quad i^k\mathbb{E}[X^k] = \lim_{t\to 0} \varphi_X^{(k)}(t)$
- Função Gamma: $\Gamma(t) = \displaystyle\int_0^{\infty} x^{t-1}e^{-x}dx \quad \Gamma(n) = (n-1)! \quad \Gamma(t+1) = t \cdot \Gamma(t)$
- Relações entre VA's:$X \sim N(\mu, \sigma^2) \Rightarrow \frac{X-\mu}{\sigma} \sim N(0,1)$

  $X \sim N(\mu_x, \sigma_x^2), Y \sim N(\mu_y, \sigma_y^2) \Rightarrow X \pm Y \sim N(\mu_x \pm \mu_y, \sigma_x^2 + \sigma_y^2)$

  $X_i \sim \exp(\lambda) \Rightarrow \sum_{i=1}^{n} X_i \sim \text{Gamma}(n, \lambda)$

  $X_i \sim \text{Gamma}(\alpha_i, \beta) \Rightarrow \sum_{i=1}^{k} X_i \sim \text{Gamma}(\sum_{i=1}^{k} \alpha_i, \beta)$

  $X_i \sim N(0,1) \Rightarrow \sum_{j=1}^{n} X_j^2 \sim \chi_n^2 \quad \text{Gamma}(\alpha = n/2, \lambda = 1/2) \sim \chi_n^2$

  $Z \sim N(0,1), V \sim \chi_k^2 \Rightarrow \frac{Z}{\sqrt{V/k}} \sim t_k$

  $X_i \sim N(\mu, \sigma^2) \Rightarrow t = \frac{\overline{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$ Onde: $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$

  $U \sim \chi_u^2, V \sim \chi_v^2 \Rightarrow \frac{U/u}{V/v} \sim F(u,v)$

  $X, Y \sim N(0,1) \Rightarrow X/Y \sim \text{Cauchy}(0,1)$
- Desigualdades: Markov: $\forall a > 0 \quad P(|x| \geq a) \leq \frac{\mathbb{E}[X]}{a}$ Chebyshev: $\forall a > 0 \quad P(|x-\mu| \geq a) \leq \frac{Var[X]}{a^2}$
- Tipos de convergência: $\begin{cases} \text{Probabilidade} & \lim_{n\to\infty} \mathbb{P}(|X - a| > \varepsilon) = 0 \Leftrightarrow X \xrightarrow{\text{p}} a \\ \text{Distribuição} & \lim_{n\to\infty} F_W(w) = F_Z(z) \Leftrightarrow W \xrightarrow{\text{d}} F \text{ ou } \lim_{n\to\infty} \varphi_w(t) = \varphi_z(t) \Leftrightarrow W \xrightarrow{\text{d}} F \\ \text{Quase Certamente} & \mathbb{P}\left(\lim_{n\to\infty} X_n = X\right) = 1 \Leftrightarrow X_n \xrightarrow{\text{q.c}} X \end{cases}$
- Lei dos Grandes Números: $\begin{cases} \text{Fraca:} & \overline{X_n} \xrightarrow{\text{p}} \mu \\ \text{Forte:} & \overline{X_n} \xrightarrow{\text{q.c}} \mu \end{cases}$
- Teorema do Limite Central:$X_1, \cdots, X_n$ iid tq. $\mathbb{E}[X] = \mu, Var[X] = \sigma^2 \Rightarrow \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \to N(0,1)$

Distribuição	FDP	Domínio	$\mathbb{E}[x]$	Var[x]	$M_X(t)$	$\varphi_X(t)$
Discretas						
Bernoulli (p)	$\mathbb{P}(X=k)=p^k(1-p)^{1-k}$	$x \in \{0,1\}, p \in [0,1]$	p	$p(1-p)$	$(1-p)+pe^t$	$(1-p)+pe^{it}$
Binomial(n,p)	$\mathbb{P}(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$	$k=0,1,\cdots,n$	np	$np(1-p)$	$(p(e^t-1)+1)^n$	$(p(e^{it}-1)+1)^n$
Poisson(λ)	$\mathbb{P}(X=k)=\frac{e^{-\lambda}\lambda^k}{k!}$	$\lambda>0, k=0,1,\cdots$	λ	λ	$\exp(\lambda(e^t-1))$	$\exp(\lambda(e^{it}-1))$
Geométrica(p)	$\mathbb{P}(X=k)=(1-p)^k p$	$k=0,1,\cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$	$\frac{p}{1-(1-p)e^{it}}$
Continuas						
Uniforme(a,b)	$f(x)=\frac{1}{b-a}$	$x \in [a,b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{t(b-a)}$	$\frac{e^{ibt}-e^{iat}}{it(b-a)}$
Exponencial(λ)	$f(x)=\lambda e^{-\lambda x}$	$\lambda>0, x>0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$	$\frac{\lambda}{\lambda-it}$
Normal(μ,σ^2)	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	$x \in \mathbb{R}$	μ	σ^2	$\exp\left(\mu t+\frac{\sigma^2 t^2}{2}\right)$	$\exp\left(\mu i t-\frac{\sigma^2 t^2}{2}\right)$
Gamma(α,λ)	$f(x)=\dfrac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}$	$\alpha,\lambda>0, x\geq 0$	$\dfrac{\alpha}{\lambda}$	$\dfrac{\alpha}{\lambda^2}$	$\left(\dfrac{\lambda-t}{\lambda}\right)^{-\alpha}$	$\left(\dfrac{\lambda-it}{\lambda}\right)^{-\alpha}$
t de Student (t_v)	$\frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\,\Gamma(\frac{v}{2})}(1+\frac{x^2}{v})^{-\frac{v+1}{2}}$	$x \in \mathbb{R}, v>0$			-	
F(u,v)					-	
Cauchy(α,β)	$f(x)=\dfrac{1}{\pi\beta\left[1+\left[\frac{x-\alpha}{\beta}\right]^2\right]}$	$\beta>0\ x,\alpha \in \mathbb{R}$	-	-	-	
Qui Quadrado (χ^2_v)	$f(x)=\dfrac{1}{2^{v/2}\,\Gamma(v/2)}x^{(v/2)-1}\exp\left(-\dfrac{x}{2}\right)$	$x>0, v>0$	v	$2v$	$(1-2t)^{-v/2}$	$(1-2it)^{-v/2}$
Beta(α,β)	$f(x)=\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$x \in (0,1)$	$\dfrac{\alpha}{\alpha+\beta}$			

Tabela 1: Resumo de Distribuições