University of Liege
Faculty of Applied Sciences

ELEC0041: Modelling and design of electromagnetic systems

Homework 3

Prof. Christophe Geuzaine

RADELET Florian
DUMONT Antoine

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1 Theoretical reminder

A transformer is an electrical device used to change the voltage of a line or a cable into another one (higher or lower). It transmits (in the ideal case) the power from the input to the output. The principle remains the same for all types of transformers observed here, only the connections of the input and output and the geometry of the core vary.
The three types of transformers studied here are presented at Figure 1, 2, 3. These are respectively the single-phase core type transformer, the single-phase shell type transformer and the three-phase transformer. The schematics come from the course "Introduction to electric power and energy systems" from the University of Liege.

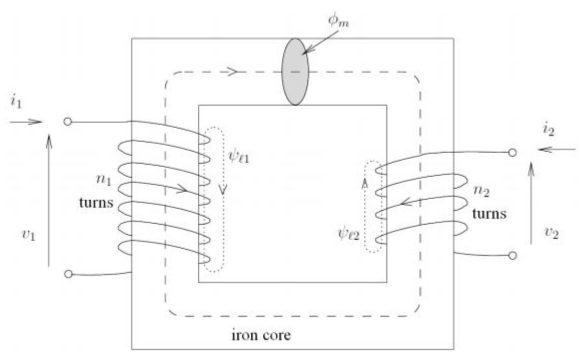


Figure 1
Schematic of a single phase core type transformer

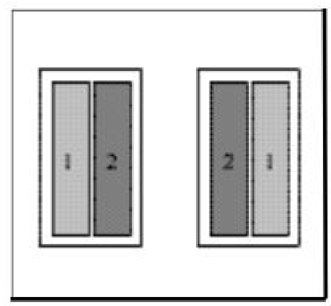


Figure 2
Schematic of a single phase shell type transformer

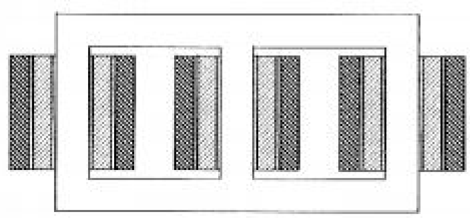


Figure 3
Schematic of a three-phase transformer

The transformers can only work with AC voltages and currents. The coils in the primary are travelled by an alternating current. This generates an alternating (and so varying) magnetic flux ϕ in the core (proportional to the number of turn). The magnetic field is concentrated in the core because its permeability is higher than the one of air ($\mu = 2000\mu_0$ for the iron). This is source of non-ideality as discussed later.
This varying field goes through the coil in the secondary. Thanks to the Faraday law, an electromotive force is created in the secondary. This one is proportional to the magnetic flux ϕ, the frequency and to the number of coil. To recall, the Faraday law is:

$$e = n_2 \frac{d\phi}{dt}$$

In the ideal transformer, the power is preserved between the input and the output, and the primary and secondary are linked by the following equation:

S_N[VA]	50	500	5000
$I_{N,1}$ [A]	0.4167	4.167	41.67
$I_{N,2}$ [A]	4.167	41.67	416.7

Table 1
Nominal currents for different nominal power

$$\frac{V_1}{n_1} = \frac{V_2}{n_2} \text{ and } I_1 n_1 = I_2 n_2$$

But, in reality, the transformer is source of a lot of non-idealities. The first one is the resistance of the wire. This one yields to Joule losses and voltage drops. Then, all the flux does not go into the core (μ_r is finite) and some is lost in the air. This part is called the leakage flux and have to be minimised. In term of losses in the core, these are of two types: the hysteresis and the Eddy currents. The first one is due to the relation between the magnetic flux density **b** and the magnetic field **h** (vector quantities). It is not linear and present an hysteresis cycle. This cannot be taken into account in our case due to the non-linearity of the relation. The second one is due to the fact that the iron is conductive and small currents are created in this one. The solution to that is to use laminated core that increases the electrical resistance and so decreases the Eddy currents.

In our study, the non-linear behaviour of μ with respect to **h** is not taken into account. The proximity effect is also neglected.

The coils of the transformer, in our case, are modelled by equivalent squares. This is a representation to simplify the problem and to avoid representation of each wire of the coil.

2 Single-phase core transformer

Design procedure

Nominal current

The power flowing in the transformer is given by:

$$S_N = \frac{1}{2} U_N I_N$$

with U_N and I_N peak value.
For the primary, the voltage is equal to 240V. The above formula can be used to find the current for a given value of the power. These are assembled in the Table 1. The currents in the secondary are computed by the same procedure except that the voltage is 24V.

Section of the wire

Now that the currents are determined, the section of the wire can be determined. The limit in current density **j** is equal to $1A/mm^2$. This gives the sections and wire diameters given in the

I_N[A]	0.4167	4.167	41.67	416.7
Section $S[mm^2]$	0.4167	4.167	41.67	416.7
Diameter d [mm]	0.7284	2.3034	7.2839	23.0339

Table 2
Section and diameters of the wires for different currents

Table 2 for different currents.
In the design, the coils are replaced by equivalent rectangles. The surface of these are different than the total surface of the real coils due to gap between the wires.

Number of turns

The goal is to derive a link between the number of turn and other known variables. This starts by the Faraday law (given before):

$$e = n\frac{d\phi}{dt}$$

The magnetic flux is created by an AC current. Thus, it is also alternating and is given by:

$$\phi(t) = AB(t) = AB\sin\omega t$$

B is the magnetic flux density in Tesla [T] and A is the area of the core leg. The derivative of the flux is equal to:

$$\frac{d\phi}{dt} = \omega AB\cos\omega t$$

ω is equal to $2\pi f$ with f the frequency of the AC current and voltage.
The combination of the two equations yields to:

$$e = n2\pi fAB\cos\omega t$$

The number of turns per volt can be derived and is equal to:

$$T_V = \frac{1}{2\pi fAB_{max}}$$

Where B_{max} is the maximal flux density and is equal to 1.5. In fact, a core of iron saturates at 1.8-2 T but a security margin is kept.

Note that this formula can be applied to the primary and the secondary. The number of turn is still not known because the area of the core is not fixed at this level. The number of turns obtained with this formula is a theoretical value that has to be checked and modified if it is not realistic.

Geometrical parameters: the coils

The coils are characterised by two parameters, the width and the height. These are also linked with the area. As explained before the coils are replaced by rectangles.

Now, one goal is to determined the height and the width to use. The surface is given by the product of the wire area (the rectangle that contains them) by the number of wire in the two coils. As this is also the product of the width of the rectangle by the height, for one of these two fixed, the second can be found.

The next thing to determine is the ratio of these two values. This is solved thanks to a trial and error procedure.

Geometrical parameters: the core

The dimensions of the core highly depend of the power. The smaller the nominal power, the smaller the core. The dimensions are free for all the core. These are determined by testing all of them. The criteria are: non-saturation of the core, efficiency of the systems, compactness. The efficiency is defined as the ratio between the output power and the input power.
To find those parameters, a parametric study is made. One parameter is varied while the others are fixed. This is done for the width, the height and the leg width (in the vertical and horizontal one) of the core. The graphics of the efficiency and the maximal magnetic field against the parameter are shown in the Figures 4, 5, 6 and 7. The thickness of the core is supposed equal to the width of the leg in all those cases.

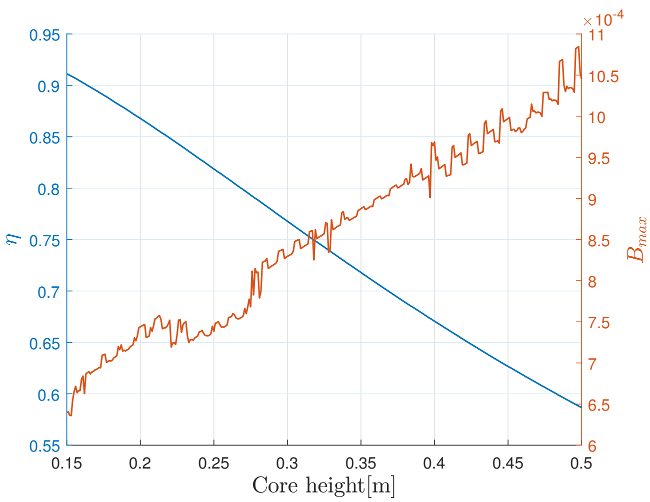


Figure 4
Efficiency as a function of the height of the core with other parameters fixed

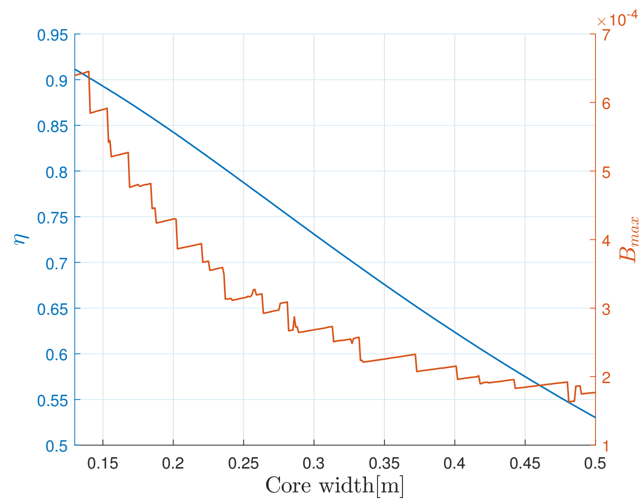


Figure 5
Efficiency as a function of the width of the core with other parameters fixed

The following remarks can be emitted:

- The width of the core must be minimised. In fact, when this one is increased, the reluctance associated with the magnetic circuit is increased and the coupling between the two circuits decreases. Concerning B_{max}, this one increases when the width increases.

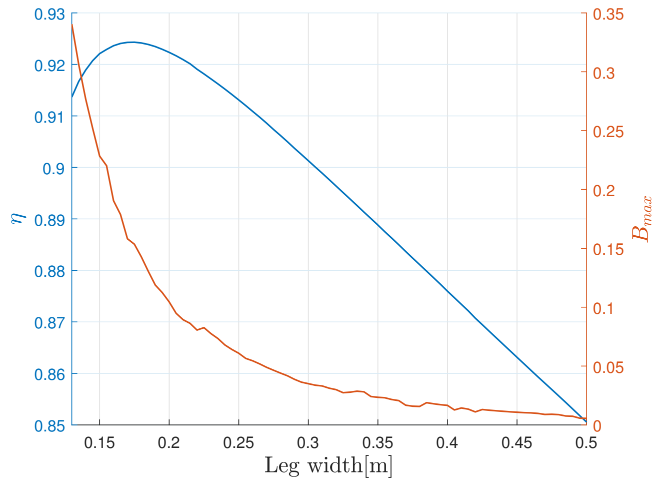


Figure 6
Efficiency as a function of the leg width (horizontal) with other parameters fixed

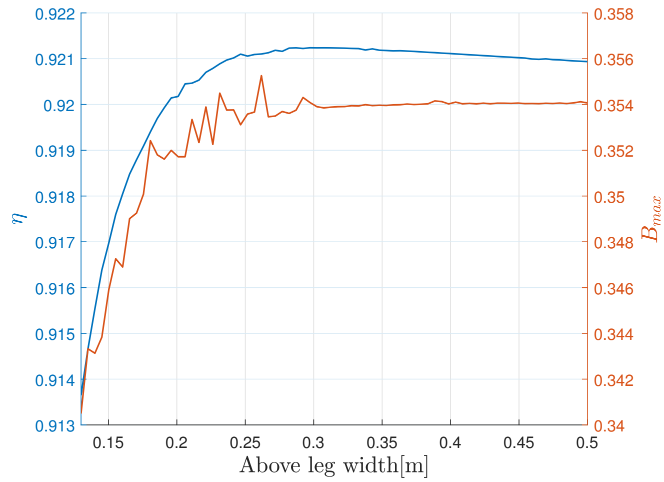


Figure 7
Efficiency as a function of the leg width (vertical) with other parameters fixed

S_N	H_c [m]	H_w[m]	W_c[m]	W_w[m]	H_{coil1}[m]	H_{coil2}[m]	n_1	n_2	η
50	0.135	0.035	0.13	0.02	0.32778	2.3034	600	60	94.61
500	0.175	0.055	0.18	0.06	0.046068	0.5009873	250	25	96.51
5000	0.27	0.12	0.23	0.08	0.109258	0.115169	80	8	97.8

Table 3
Results obtained for the one phase type transformer

- The height has to be reduced as much as possible. This is the same result for the width and for the efficiency. For the B_{max}, it decreases with the height.
- The width of the core leg can be increased to increase the efficiency. At a certain point, it decreases the efficiency or saturates and does not increase anymore (depending on which leg is increased).

The results are provided in the Table 3. H_c, H_w, W_c, W_w, H_{coil1}, H_{coil2}, n_1 and n_2 state respectively for the core height, the window height, the core width, the window width, the coil 1 height, the coil 2 height, the number of turns at the primary and the one at the secondary. η is the efficiency and is equal to the ratio between the output and the input power. The diameter are the one provided in the table 2.

For the 50 VA transformer, as shown in the Figure 8, increasing the number of turns does not increase the efficiency that much after 600 turns. This is the reason why it is fixed to this value in the case of the 50 VA transformer. The efficiency can be increased by increasing the section of the core. However, it leads to a very big core.
Concerning the second transformer, the number of turns can also be increased in order to obtain a higher efficiency but this would have lead to bigger coils without affecting that much the efficiency. In this case, a higher number of turns yields to a higher width or height of the core (the diameter of the wire is bigger). The two effects counteract each other and the efficiency does not increase a

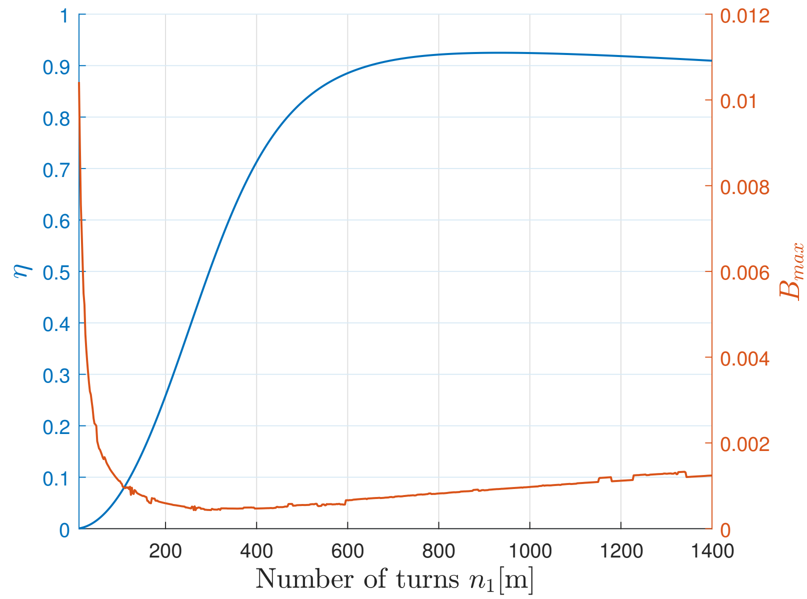


Figure 8
Efficiency as a function of the number of turn at the primary (other parameters fixed)

lot.

For the last transformer, the same parameters than for the second one can be increased to increase the efficiency but this would lead to very big geometries. The efficiency is equal to more or less 96% which is sufficient.

All the plots have been made for the 50 VA transformer but the results are similar for the other power. They have not been made with the best geometry found in the end but these are just there to show the behaviour of the system subjected to specific modification.

The Figure 9 shows the magnetic field in the core. One can see that the core does not saturate (the value does not exceed 2T), exception made of the corner effect.

Parameters of the equivalent schematics

The equivalent schematic has been derived during the course *ELEC 0431: Electromagnetic energy conversion*. It is recalled at Figure 10.
The resistance $R = R_1 + R_2'$ models the Joule losses in the coil wires (at the primary and secondary). The impedance $X = X_{l1} + X_{l2}'$ represents the leakage flux (flux that does not go in the core). The inductance X_m is equal to a resistance R_{H+F} put in parallel with an impedance X_μ. The first one represents the losses by Eddy currents and by hysteresis in the core. The second one is there to model the fact that the permeability of the core is finite. This decreases the reluctance and then the flux in the core. In practice the inductance X_m is bigger than the other elements and the current in this branch is very small. The number 1 and 2 relate the variable with the elements of the primary and the secondary (respectively).

The equivalent parameters of the circuit are determined thanks to two tests: a no-load test (open-circuit) and short-circuit test. The two are applied using the resistance R_{out}. It is set to a very

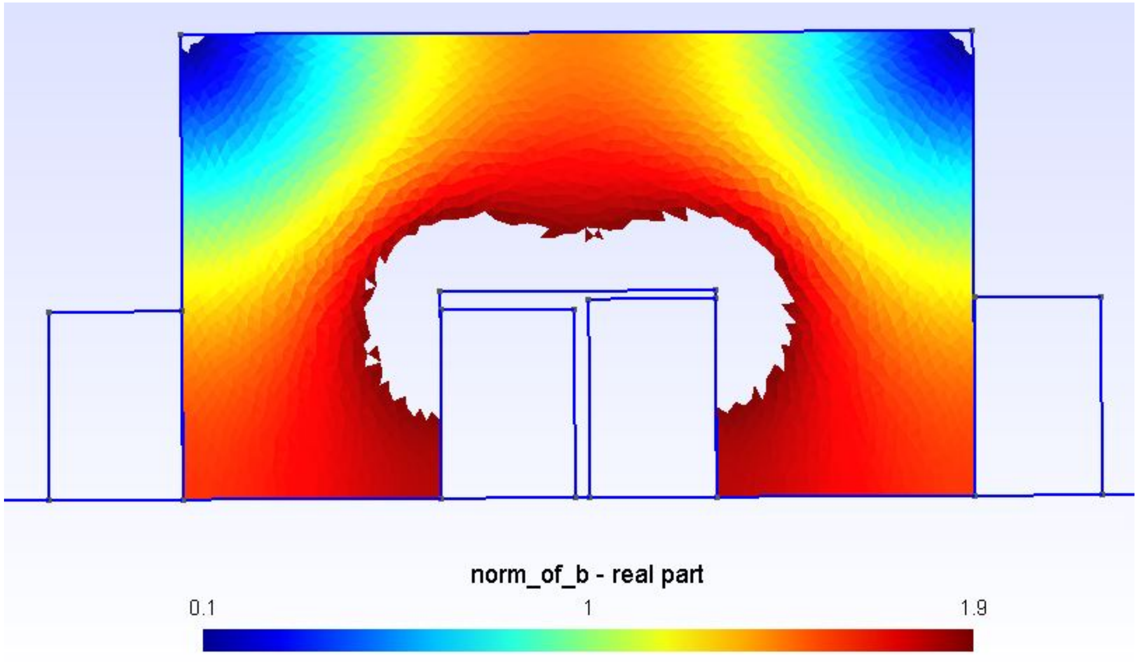


Figure 9
Magnetic field for the 5000 VA core transformer

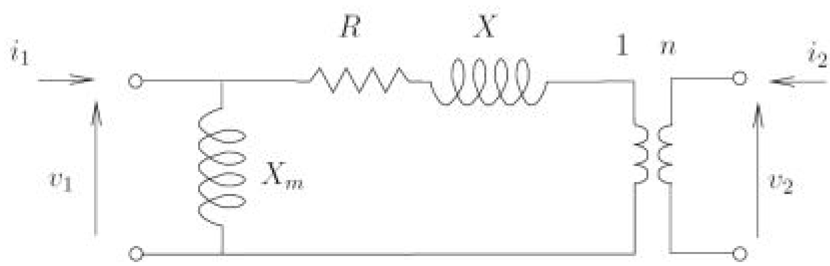


Figure 10
Equivalent simplified schematic of a single phase transformer

high value for the first one and to a very small one for the second one. Then, the active and reactive power are computed knowing the phase and the value of the current and voltage. Those tests make us able to determine the sum of the terms of the primary and secondary but not the elements separately.

Short-circuit test

In this case, the voltage in the secondary is null. The magnetic losses are very small and the two powers are equal to:

$$P_{sc} = 0.5(R_1 + R_2')I_{cc}^2 \text{ and } Q_{sc} = 0.5(X_1 + X_2')I_{cc}^2$$

The active and reactive powers are computed using the formulae $P = \frac{1}{2}UI\cos\phi = Re(S)$ and $Q = \frac{1}{2}UI\sin\phi = Im(S)$ with ϕ the angle between the current and the voltage and S the nominal power in VA. From this, the sum of the resistances and of the reactances can be obtained. These are given in the Table 4.

S_N	P_{sc} [W]	Q_{sc} [VAr]	$R_1 + R_2'$ [Ω]	$X_{l1} + X_{l2}'$ [Ω]	P_v [W]	Q_v [VAr]	R_{H+F} [Ω]	X_μ [Ω]
50	1163	-430	21.8149	8.0213	0.0935	-15.7	24.7641	67.3487
500	8961	-13420	0.9908	1.4841	0.2406	-118.1	3.2138	2.1456
5000	43171	-171570	0.0397	0.1579	0.9177	-1153.2	0.6671	0.1679

Table 4
Parameters of the equivalent circuit for the one phase type transformer

Open-circuit test

Now, the current in the secondary is null. Then, the current in the primary is very small and the losses from all the elements except X_m are neglected. This gives:

$$P_v = \frac{U_1^2}{2R_{H+F}} \text{ and } Q_v = \frac{U_1^2}{2X_\mu}$$

The active and reactive powers are computed using the same principles as before. The results are assembled in the Table 4.

As can be seen in the table 4, the sources of non-idealities decrease as the power increases. This is the reason why the efficiency of the transformer increases with the power. For the first transformer, the resistance of the wire seems very big. This is due to the fact that the wires are very thin and the number of turn is quite long. Then, the total resistance increases. As said before the magnetising inductance X_μ is big compared to other value.

External characteristic of the transformer

The external characteristic is the graph showing the evolution of the secondary voltage again the secondary current. This changes in current is induced by a change in the output resistance. The ones of this transformer (respectively for 50 VA, 500 VA and 5000 VA) are given in the Figures 11, 12 and 13 (blue curves).
One can see that the external characteristic of all the transformer is of the type of the one seen during the course *ELEC0431*. In our case, the load induces a phase of 0 rad (its a resistance).

3 Single-phase shell type transformer

Design procedure

The design procedure is exactly the same than for the first type. The currents, the wire sections and diameters are the same. The design of the coils also follows the same procedure. A remark can be emitted concerning the external legs of the transformer. The width of these is equal to half the width of the middle one because they both contains a half of the flux in the central one.

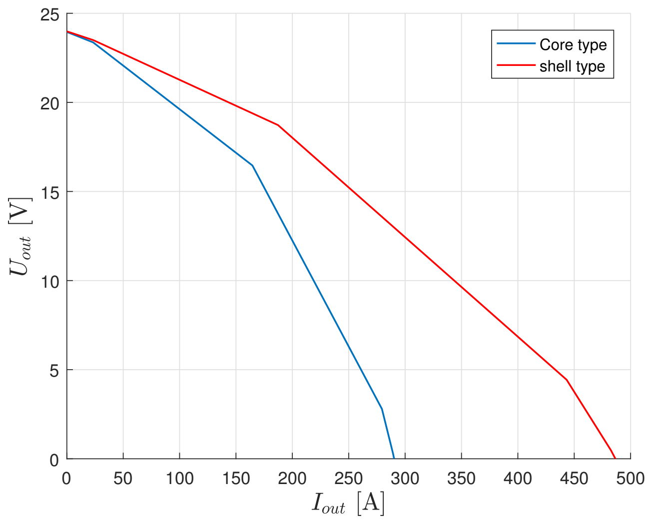


Figure 11
External characteristic of the 50 VA transformers

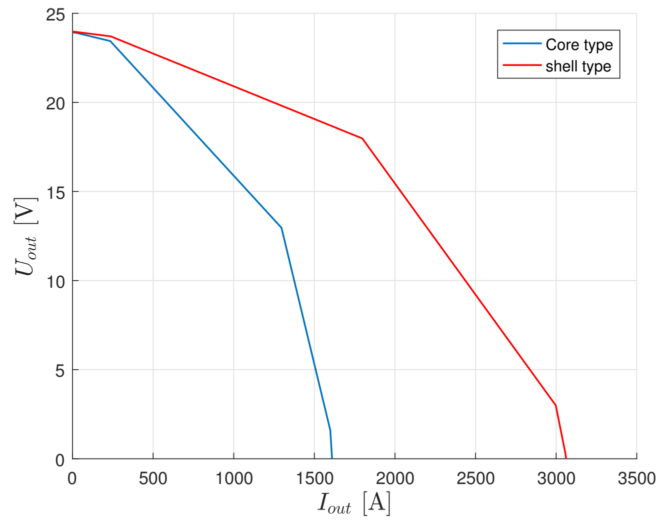


Figure 12
External characteristic of the 500 VA transformers

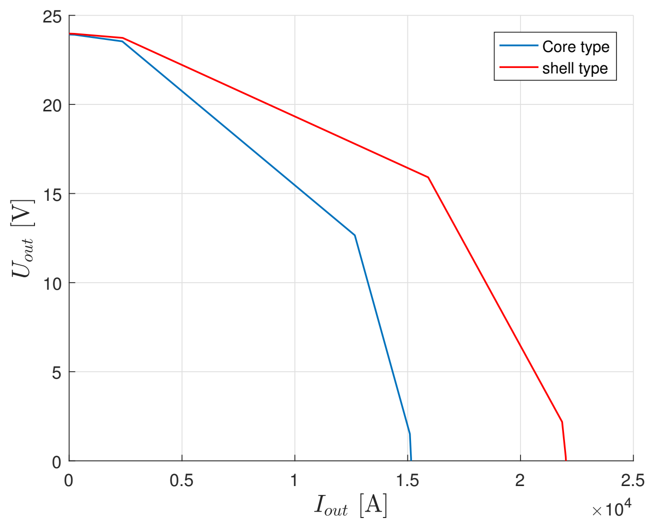


Figure 13
External characteristic of the 5000 VA transformers

This gives the dimensions assemble in the table 5. In terms of dimensions, the core type has the advantage to have a smaller width. However, its height is bigger for all the case than the shell type height.
After test, the best efficiency is obtained by putting the primary inside. The secondary is then the coil that circles the primary.

These results are far better than the one of the core type studied before. This means that this type of transformer is better. The higher efficiency can be used to decrease the number of turns or the width for example. In this table are presented the results for geometries similar to the one of before (in term of number of turns, ...).

The final magnetic field is shown in the Figure 14. This is the transformer of 5000 VA. This shows that the magnetic field is smaller than 1.75 T (corner effect neglected) and the core does

S_N	H [m]	$window$[m]	$width$[m]	H_{coil1}[m]	H_{coil2}[m]	n_1	n_2	η
50	0.088	0.0365	0.05	0.0182	0.0184	600	60	98.17%
500	0.128	0.074	0.05	0.0368544	0.0364195	250	25	98.24%
5000	0.23	0.123	0.08	0.0728	0.0691	80	8	99.06%

Table 5
Results obtained for the shell type transformer

not saturate.

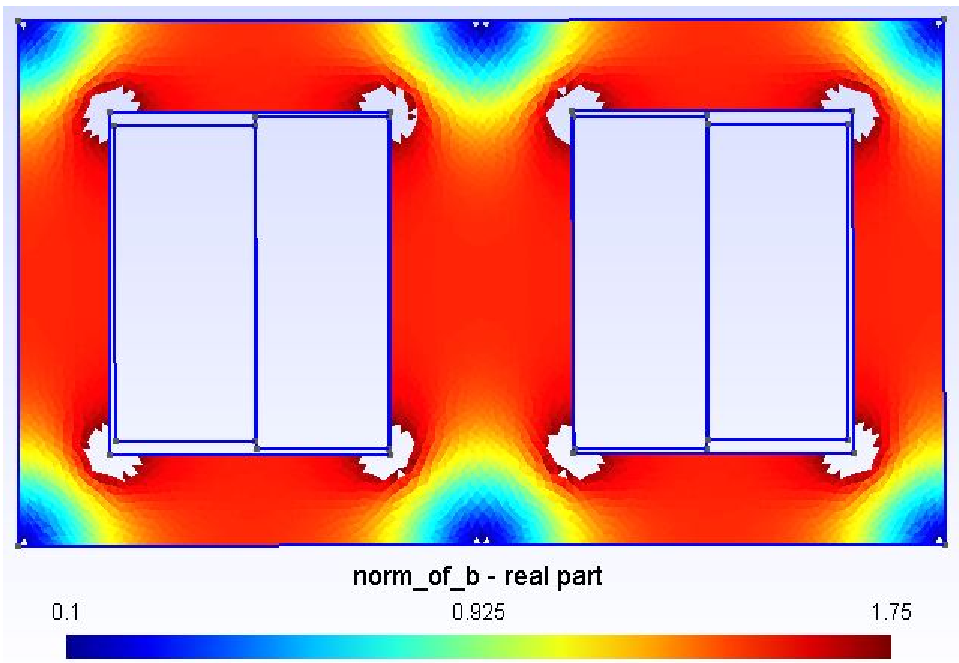


Figure 14
Magnetic field for the 5000 VA shell transformer

Parameter of the equivalent schematics

The equivalent schematic is the same as before (Figure 10). The determination of the parameter is done thanks to the same procedure. The results are given in the Table 6.
The same conclusions than the previous case can be derived. The equivalent parameters decrease with the power. If the equivalent parameters of the two circuits are compared, it can be remark that the equivalent parameters are smaller in the case of the shell type transformer. This is link to the fact that its efficiency is better because these elements represent the losses in the system.

External characteristic of the transformer

The curves can be found in the Figures 11, 12 and 13 (red curves).

S_N	P_{sc} [W]	Q_{sc} [VAr]	$R_1 + R_2'$ [Ω]	$X_{l1} + X_{l2}'$ [Ω]	P_v [W]	Q_v [VAr]	R_{H+F} [Ω]	X_μ [Ω]
50	922.8960	-3366.4	2.1815	7.9571	0.0093	-15.6924	31.2061	8.5552
500	1291.4	-19332	0.0991	1.4831	0.0241	-118.0938	22.3015	1.4897
5000	4577.1	-182330	0.004	0.1579	0.0918	-1153.2	6.2921	0.158

Table 6
Parameters of the equivalent circuit for the shell type transformer (single phase)

The same conclusions than for the previous case can be obtain. One can see that the current obtained for low level of voltage is higher than the one in the previous case. This is due to the fact the transformer is better than the previous one (less losses).

4 Further studies

In this section, the analysis of varying some parameters is done. All the other paramaters are kept constant (in particular the geometry).

Effect of the winding electric resistivity

The study is performed for the shell type 50 VA transformer but can easily be done in the same way for all cases. The plot of the efficiency when the conductivity of the wire varies is given in the Figure 15. Note that the x-axis is of type logarithmic. The conductivity σ is the inverse of the resistivity.

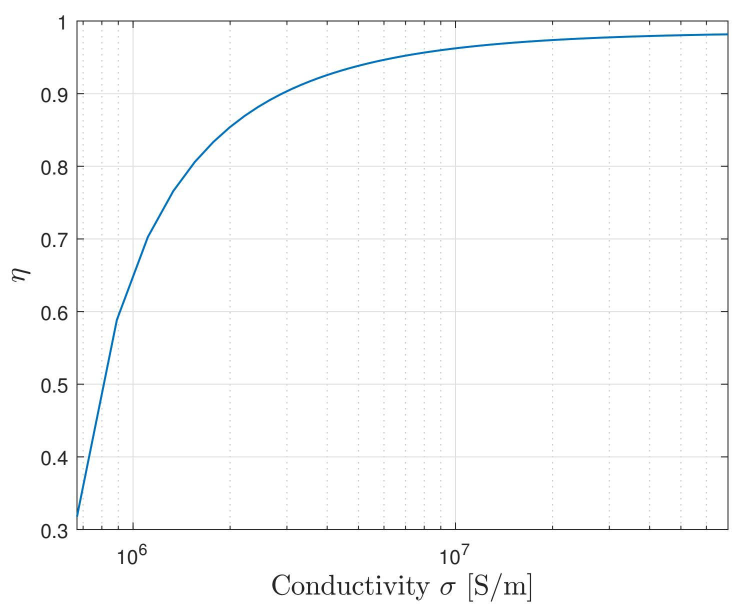


Figure 15
Efficiency when the wire conductivity varies

The test has been performed for conductivity in the range $[6.7 * 10^5 : 7 * 10^7]$. This corresponds to the conductivity of some metals that could be used. The first one is the *nichrome*, an alloy made of nickel and chrome that is usually used in the heating resistance. The last value is the one of the silver.

This shows that the efficiency drops a lot with the increase of the resistivity. Indeed, when the resistivity of the wire increases, the resistance of the coils in the primary and secondary increases (by Pouillet's law). Then, the Joule losses (RI^2) increases and the power at the output of the transformer decreases. At a certain level, the increase becomes less significative (for an increase of σ).

One can also observe the drop in the voltage in the secondary when the resistivity increases. This is due to the parameter R of the equivalent circuit. It increases when the resistivity increases and the voltage at the primary of the ideal transformer (of the equivalent circuit) decreases. This causes the voltage in the secondary to decrease.

To conclude, one should use the material with the higher conductivity (or the lower resistivity) as possible. The copper is ideal because it has a very good conductivity and is cheaper. Silver is way too expensive to be used.

Effect of the core magnetic permeability

The study is performed in the same way than for the resistivity just above. The efficiency when the parameter varies is shown in the Figure 16.

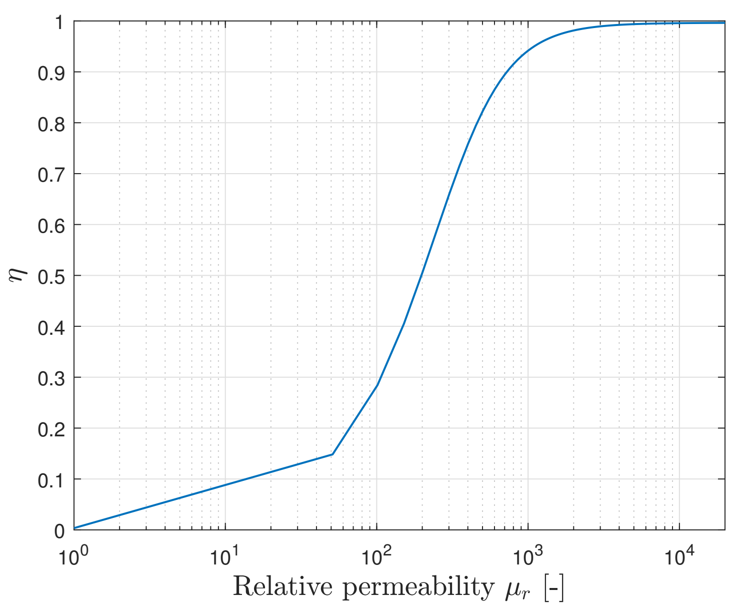


Figure 16
Efficiency when the relative permeability of the core changes

This relative permeability has been tested for value in the range [1 : 20000]. The first one corresponds to the case where there is no core, the second one is the relative permeability of *mu-metal*.

This one is very expensive and should not be used in practice. The relative permeability of the iron is equal to 2000. It is smaller than the one of the *mu-metal* but it is still pretty good. Moreover, the iron is cheap. The x-axis of the graph is in logarithmic scale for more clarity.

When μ_r approaches 2000, it can be seen that the increase in the efficiency of the transformer does not increase that much anymore. Material with a relative permeability lower than 750 should not be considered (efficiency lower than 90%).
One can see that the efficiency drops when the parameter decreases. It reaches nearly 0 in the case where there is no core and nearly 1 in the case of the mu-metal. In fact, when it decreases, it increases the reluctance of the core ($=l/S\mu$). This one is linked to the magnetic flux by:

$$\mathcal{F} = \mathcal{R}\Phi$$

The magnetic flux decreases and so the link between the two sides of the transformer decreases. This lead to a lower efficiency.

Moreover, an ideal core is considered to have a magnetic permeability infinite and so has a perfect coupling between both sides. The *mu-metal* gives an efficiency nearly one due to his relative permeability that tends to infinity.

As a conclusion, one can say that the use of iron is a pretty good choice. Better material could have been used but these are far more expensive.

Effect of an air gap

An air gap is sometimes added in order to not saturate the core. As the permeability of the air is very small compared to the one of the iron, the Reluctance of the magnetic circuit is approximately equal to the one of the air gap. However, this decreases the flux and thus the magnetic flux. Then, the efficiency of the transformer drops with the length of the air gap.

Effect of a non laminated core

The core is made of iron. This one is a conductor and so eddy currents can be induced in it. To avoid them, the core is laminated and so the resistance in the core is increased (and the eddy current decreased). To model the fact that the core is not laminated, the conductivity σ of this one has to be set to a non zero value. The Table 7 shows the efficiency without lamination for the shell type transformer. This shows that the efficiency drops too much. This type of transformer cannot be used in practice. It also shows the Joule losses in the core. These seems very big but the test is performed with the resistance needed to have the nominal current (in the ideal case) and the power transiting are very big.
The fact to add lamination in the core can be added by divided the conductivity of the core by a corrective factor. The more the lamination of the core, the more the division. Obviously, this yields to a better efficiency (when it increases).
The Figure 17 shows the value of the magnetic field in the core when the core is non laminated. This shows that the distribution is very modified.

S_N	η without lamination	Joule losses in the core [W]
50	11.6837	957.44
500	10.2091	10710
5000	5.5586	105237

Table 7
Table of the efficiency with and without lamination of the core (shell type)

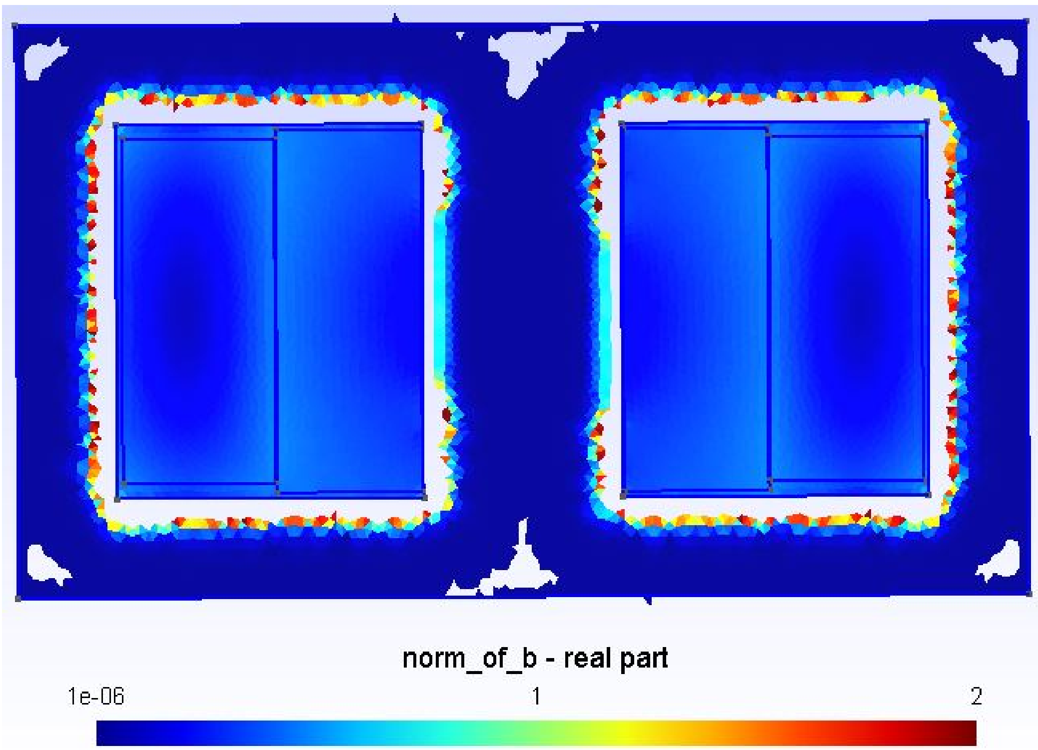


Figure 17
Magnetic field in the core when this one is not laminated

Effect of a tank/shield

In this section, the effect of adding a shield around the transformer is analysed. The shield is added in the geometry and the efficiency is analysed. The magnetic field is shown in the Figure 18.

The shield is there to limit the influence of the magnetic field outside the transformer. In our case, the field is relatively small so the shield is not useful. In this case, the power is not so big and the leakage is pretty well confined in the core. In our case, the efficiency is not affected by the shield. These conclusions are the same all the studied transformer.

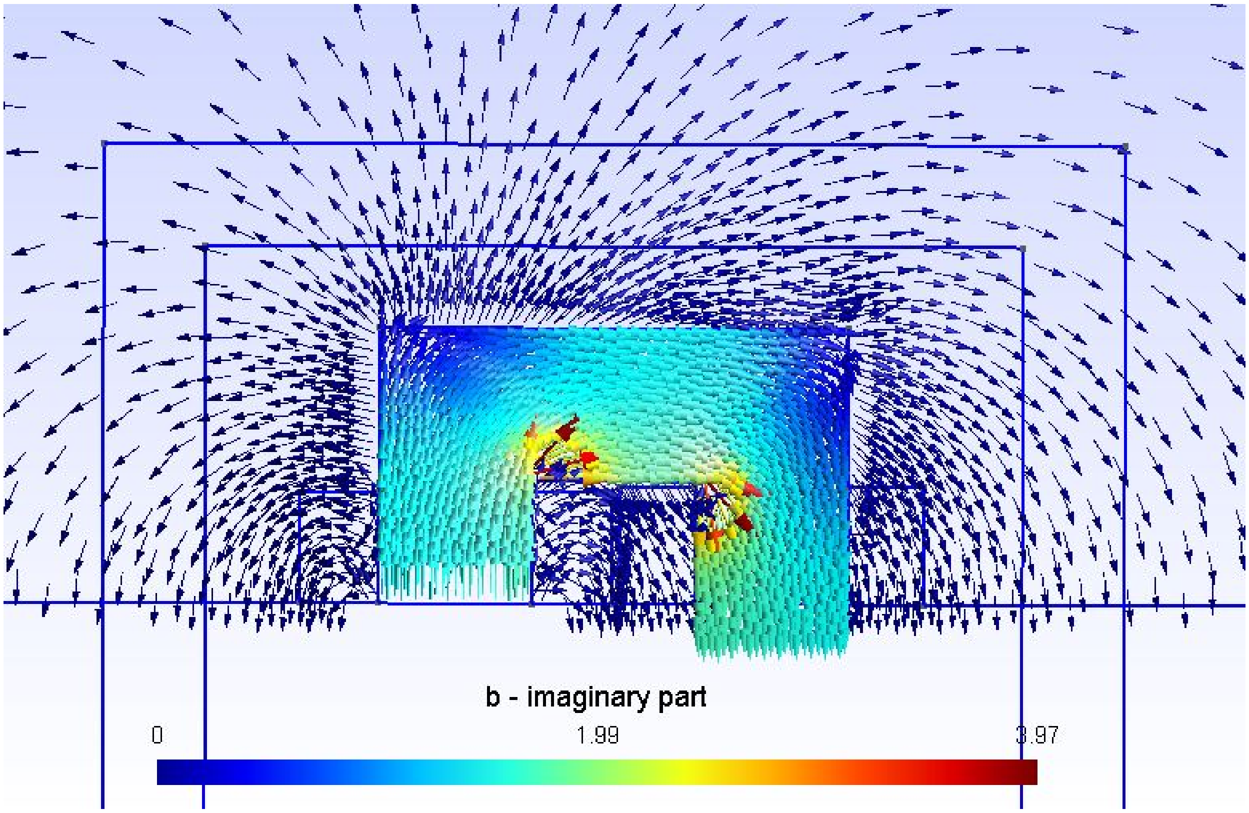


Figure 18
Test of the shield for the 5000 VA phase core transformer

Effect of a higher operating frequency

The analysis is practised on the shell type 50 VA transformer but the results are similar for all the other types. The Figure 19 shows the evolution of the efficiency when the frequency increases.

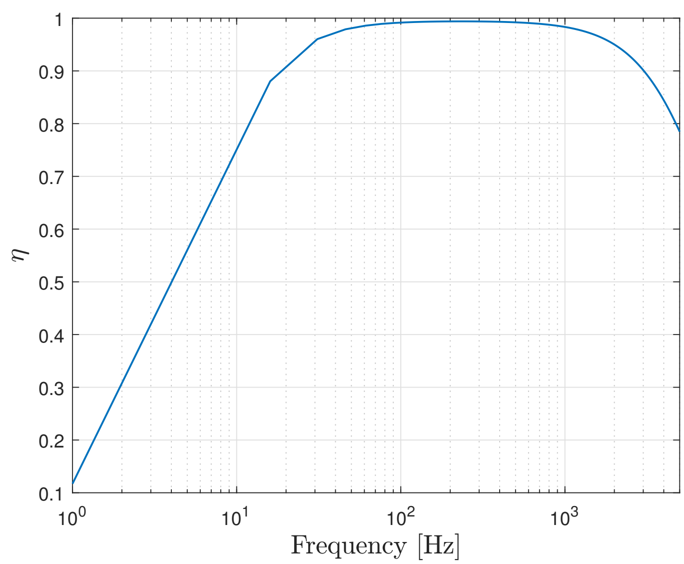


Figure 19
Efficiency in function of the frequency

Note that the x-axis is of type logarithmic.

At first, the efficiency increases with the frequency. This is due to the fact that the electromotive force is proportional to the derivative of the voltage. This one increases when the frequency increases. After 100 Hz, it remains constant and pretty high but it decreases at around 1000 Hz. This is due to the fact that the transformer behaves more like a capacitor at this frequency.

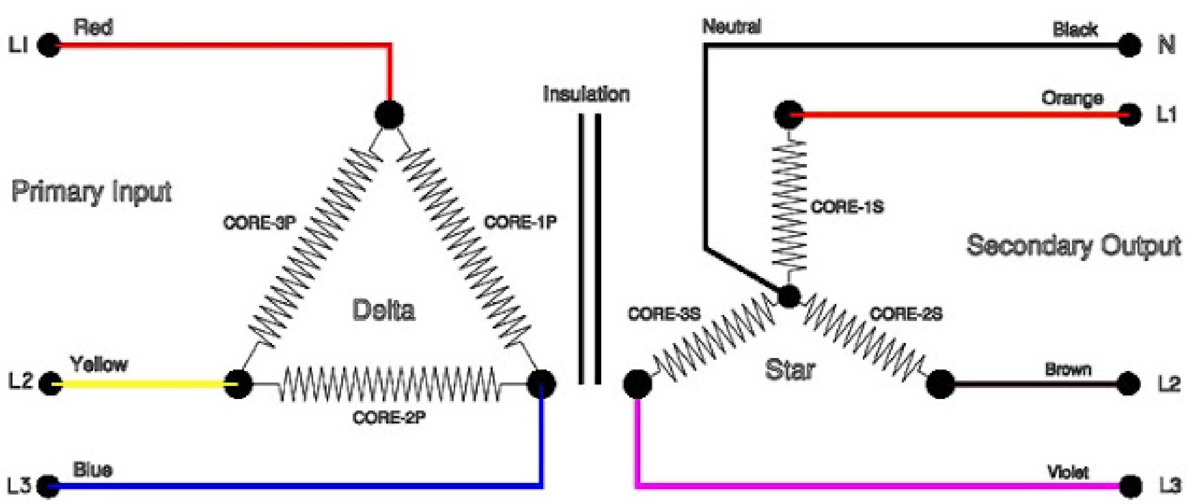
In this case, the Eddy currents are not considered. These induce losses proportional to the square of the frequency. If there were taken into account, the efficiency would certainly not increasing that much. Moreover, the skin depth in the wires decreases with the increase of the frequency. This yields to a higher resistance in the wire. In our model, the skin depth is not taken into account.

To conclude, in our model the efficiency first increases with the frequency. In reality, the losses due to Eddy current and Joule losses increase with the frequency and the efficiency would be different.

5 Three-phase transformer

The shell type transformer can be used to transform a 3 phase voltage. By making a coil around the different legs for the different phases, the transformer is able to transform the tension from the 3 primaries to the 3 secondaries.

There are mostly two different ways to make the connections: in star or in delta. L1, L2 and L3 are the different input/output voltage and have different phases.



There is a shift of phase between the primary and the secondary. This comes from the transformer which gives a floating secondary voltage. However we find the phase shift of 120° between the different phases.

The simulations show the magnetic flux stays in the core and flow between the different legs.
The same conclusion can be deduced than for the 1 phase type for the dimensions, the space between the leg should be reduced, the height and the width reduced and the width of the leg

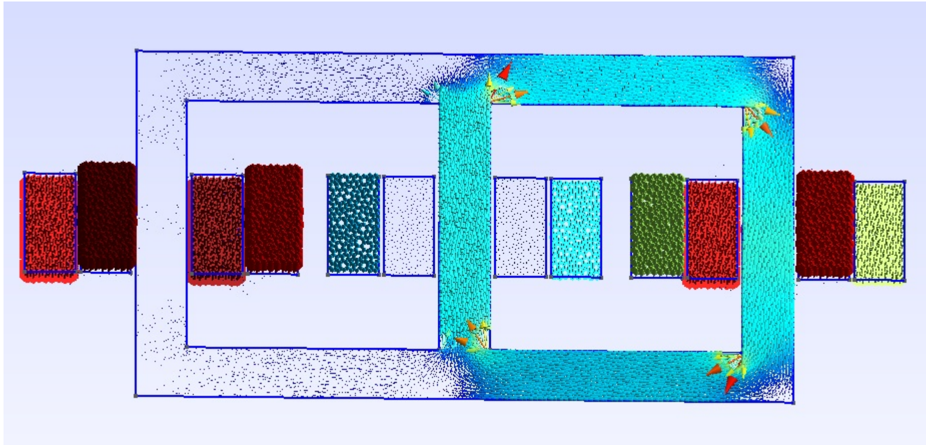


Figure 20
imaginary current and magnetic field

increased. The coils are set in the way presented in the diagram before because this minimise the leakage flux.

The behaviour of the flux is more complex than in the case of the 1 phase transformer. This one sis the sum of the 3 flux induced by the 3 dephased currents.

Note that there exists also shell three phase transformer. These are constitute by adding two legs at the external side.

6 Conclusion

Two main types of transformer have been studied here: the core type and the shell type. This has been found that the second one is better than the first and that it should be used in practice.

The influence of different parameter has also been studied. At the end, it has been obtained that the efficiency increases with the conductivity of the wire and the relative permeability of the wire. The effect of a shield is not observable that much here as the power are small but this can become useful if the power increases. A study of the lamination of the core has also been made. It gives that, without lamination, the losses in the core are far too big and the efficiency is becomes very bad. The frequency also influence the behaviour and the efficiency. The effect of a possible air gap has also been analysed and it comes out that this is used to avoid saturation of the core. The main drawback is that it decreases the efficiency of the transformer.

At the end, a little analysis of the behaviour of a three phase transformer has been performed.