# 【核心作业四】泰勒级数的计算

1 用泰勒级数为$e^{2x^2}$在0附近找到一个良好的二次近似，即将其展开为二次泰勒级数。

A. $e^{2x^2} \approx 1 - x - 2x^2$

B. $e^{2x^2} \approx 2x^2$

C. $e^{2x^2} \approx 1 + x + 2x^2$

D. $e^{2x^2} \approx 1 - 2x^2$

E. $e^{2x^2} \approx 1 + 2x^2$

F. $e^{2x^2} \approx x + 2x^2$

2 将$e^{u^2+u}$展开为四次泰勒级数。

A. $e^{u^2+u} \approx 1 + u + \frac{3}{2}u^2 + \frac{7}{6}u^3 + \frac{25}{24}u^4 +$ H.O.T.

B. $e^{u^2+u} \approx 1 - u - \frac{1}{2}u^2 + \frac{2}{3}u^3 + \frac{5}{4}u^4 +$ H.O.T.

C. $e^{u^2+u} \approx 1 - u - \frac{1}{2}u^2 + \frac{5}{6}u^3 + \frac{25}{24}u^4 +$ H.O.T.

D. $e^{u^2+u} \approx 1 + u + \frac{3}{2}u^2 + \frac{4}{3}u^3 + \frac{5}{4}u^4 +$ H.O.T.

E. $e^{u^2+u} \approx 1 + u + \frac{3}{2}u^2 + \frac{7}{6}u^3 + \frac{5}{4}u^4 +$ H.O.T.

F. $e^{u^2+u} \approx 1 - u - \frac{1}{2}u^2 + \frac{2}{3}u^3 + \frac{25}{24}u^4 +$ H.O.T.

3 将$e^{1-\cos x}$展开为四次泰勒级数。

A. $e^{1-\cos x} = 1 + \frac{x^2}{2} + \frac{x^4}{8} +$ H.O.T.

B. $e^{1-\cos x} = 1 + \frac{x^2}{2} - \frac{x^4}{24} +$ H.O.T.

C. $e^{1-\cos x} = 1 + \frac{x^2}{2} + \frac{x^4}{12} +$ H.O.T.

D. $e^{1-\cos x} = 1 - \frac{x^2}{2} + \frac{x^4}{12} +$ H.O.T.

E. $e^{1-\cos x} = 1 - \frac{x^2}{2} + \frac{x^4}{8} +$ H.O.T.

F. $e^{1-\cos x} = 1 - \frac{x^2}{2} - \frac{x^4}{24} +$ H.O.T.

4 计算$\cos(\sin x)$展开为泰勒级数的前三个非零项。

A. $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{x^4}{4} +$ H.O.T.

B. $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{x^4}{6} +$ H.O.T.

C. $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{5x^4}{24} +$ H.O.T.

D. $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{5x^4}{6} + \text{H.O.T.}$

E. $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{5x^4}{4} + \text{H.O.T.}$

F. $\cos(\sin x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \text{H.O.T.}$

5 计算$\frac{\cos(2x)-1}{x^2}$展开为泰勒级数的前三个非零项。

A. $\frac{\cos(2x)-1}{x^2} = -2 + \frac{2x^2}{3} + \frac{x^4}{45} + \text{H.O.T.}$

B. $\frac{\cos(2x)-1}{x^2} = -2 + \frac{2x^2}{3} - \frac{4x^4}{45} + \text{H.O.T.}$

C. $\frac{\cos(2x)-1}{x^2} = -\frac{1}{2} + \frac{x^2}{24} - \frac{x^4}{720} + \text{H.O.T.}$

D. 这个函数在原点附近无法展开为泰勒级数

E. $\frac{\cos(2x)-1}{x^2} = 2 - \frac{x^2}{6} + \frac{x^4}{45} + \text{H.O.T.}$

F. $\frac{\cos(2x)-1}{x^2} = -2 + \frac{x^2}{6} - \frac{2x^4}{45} + \text{H.O.T.}$

6 将函数$\cos x \sin 2x$展开为五次泰勒级数。

提示：别算导数！

A. $\cos x \sin 2x = 2x - \frac{7x^3}{3} + \frac{3x^5}{4} + \text{H.O.T.}$

B. $\cos x \sin 2x = 2x - \frac{7x^3}{3} + \frac{7x^5}{20} + \text{H.O.T.}$

C. $\cos x \sin 2x = 2x - \frac{4x^3}{3} + \frac{3x^5}{4} + \text{H.O.T.}$

D. $\cos x \sin 2x = 2x - \frac{4x^3}{3} + \frac{61x^5}{60} + \text{H.O.T.}$

E. $\cos x \sin 2x = 2x - \frac{4x^3}{3} + \frac{7x^5}{20} + \text{H.O.T.}$

F. $\cos x \sin 2x = 2x - \frac{7x^3}{3} + \frac{61x^5}{60} + \text{H.O.T.}$

7 将函数$x^{-1}e^x sinx$展开为四次泰勒级数。

A. $x^{-1}e^x \sin x = 1 + x + \frac{7x^2}{6} + \frac{x^3}{3} + \frac{2x^4}{15} + \text{H.O.T.}$

B. $x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{3x^4}{40} + \text{H.O.T.}$

C. $x^{-1}e^x \sin x = 1 + x + \frac{7x^2}{6} + \frac{5x^3}{24} - \frac{x^4}{60} + \text{H.O.T.}$

D. $x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{x^4}{24} + \text{H.O.T.}$

E. $x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{x^4}{30} + \text{H.O.T.}$

F. $x^{-1}e^x \sin x = 1 + x + \frac{7x^2}{6} + \frac{x^3}{24} + \frac{3x^4}{40} + \text{H.O.T.}$

8 计算$\frac{e^{2x}\sinh x}{2x}$展开为泰勒级数的前三个非零项。

A. $\frac{e^{2x}\sinh x}{2x}=\frac{1}{2}+\frac{x}{2}+\frac{11x^2}{12}+$ H.O.T.

B. $\frac{e^{2x}\sinh x}{2x}=\frac{1}{2}+\frac{x}{2}+\frac{x^2}{12}+$ H.O.T.

C. $\frac{e^{2x}\sinh x}{2x}=\frac{1}{2}+\frac{x}{2}+\frac{13x^2}{12}+$ H.O.T.

D. $\frac{e^{2x}\sinh x}{2x}=\frac{1}{2}+x+\frac{11x^2}{12}+$ H.O.T.

E. $\frac{e^{2x}\sinh x}{2x}=\frac{1}{2}+x+\frac{x^2}{12}+$ H.O.T.

F. $\frac{e^{2x}\sinh x}{2x}=\frac{1}{2}+x+\frac{13x^2}{12}+$ H.O.T.

# 【核心作业四】答案与解析

1. E

令$f(x)=e^{2x^2}$，则有$f(0)=1$，$f'(x)=4xe^{2x^2}$，$f'(0)=0$，$f''(x)=4e^{2x^2}+16x^2e^{2x^2}$，$f''(0)=4$

所以$f(x)\approx f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2=1+2x^2$

2. A

令$f(x)=e^{u^2+u}$,

$f'(x)=(2u+1)e^{u^2+u}$,

$f''(x)=2e^{u^2+u}+(2u+1)^2e^{u^2+u}$,

$f'''(x)=6(2u+1)e^{u^2+u}+(2u+1)^3e^{u^2+u}$,

$f^{(4)}=12e^{u^2+u}+12(2u+1)^2e^{u^2+u}+(2u+1)^4e^{u^2+u}$

所以$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+$ H.O.T.

$=1+x+\frac{3}{2}x^2+\frac{7}{6}x^3+\frac{25}{24}x^4$

3. A

令$f(x)=e^{u^2+u}$,

$f'(x)=(2u+1)e^{u^2+u}$,

$f''(x)=2e^{u^2+u}+(2u+1)^2e^{u^2+u}$,

$f'''(x)=6(2u+1)e^{u^2+u}+(2u+1)^3e^{u^2+u}$,

$f^{(4)}=12e^{u^2+u}+12(2u+1)^2e^{u^2+u}+(2u+1)^4e^{u^2+u}$

所以$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+$ H.O.T.

$=1+x+\frac{3}{2}x^2+\frac{7}{6}x^3+\frac{25}{24}x^4$

4. C

令$f(x)=\cos(\sin x)$,

$f'(x)=-\sin(\sin x)\cos x$,

$f''(x)=-\cos(\sin x)\cos^2 x+\sin(\sin x)\sin x$,

$f'''(x)=\sin(\sin x)(\cos^3 x+\cos x)+(3\sin x\cos x)\cos(\sin x)$,

$f^{(4)}=\cos(\sin x)(\cos^4 x+7\cos^2 x-3)+\sin(\sin x)(-3\cos^2 x\sin x-\sin x)$

所以$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+$ H.O.T.

$=1-\frac{x^2}{2}+\frac{5}{24}x^4$

5. B

因为 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} +$ H.O.T.

于是 $\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} +$ H.O.T.

$\cos 2x - 1 = -\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} +$ H.O.T.

$\frac{\cos 2x - 1}{x^2} = -\frac{2^2}{2!} + \frac{2^4}{4!}x^2 - \frac{2^6}{6!}x^4 +$ H.O.T.

$\frac{\cos 2x - 1}{x^2} \approx -2 + \frac{2}{3}x^2 - \frac{4}{45}x^4$

6. F

因为 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} +$ H.O.T.，$\sin(2x) = (2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} +$ H.O.T.

则$\cos x \sin 2x = (1 - \frac{x^2}{2!} + \frac{x^4}{4!})((2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!}) +$ H.O.T.

$= 2x - \frac{7x^3}{3} + \frac{61x^5}{60} +$ H.O.T.

7. E

由$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} +$ H.O.T.

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} +$ H.O.T.

得$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!}$ H.O.T.

所以$x^{-1}e^x \sin x = 1 + x + \frac{x^2}{3} - \frac{x^4}{30} +$ H.O.T.

8. F

由$\sinh x = \frac{e^x - e^{-x}}{2}$，得$\frac{e^{2x}\sinh x}{2x} = \frac{e^{3x} - e^x}{4x}$，

又$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} +$ H.O.T.，$e^{3x} = 1 + \frac{3x}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} +$ H.O.T.

则$e^{3x} - e^x = 2x + 4x^2 + \frac{13}{3}x^3$

所以$\frac{e^{2x}\sinh x}{2x} = \frac{1}{2} + x + \frac{13x^2}{12} +$ H.O.T.