① Gabriel Duarte

a)

$\vec{u}+\vec{v}=(2,6)$; $a\vec{u}=(0,6)$

b) Se, $\vec{u}$ e $\vec{V}$ pertencem a V, então $\vec{v}+\vec{u} \in V$;

Se, $\vec{u}$ pertence a V e α é um escalar qualquer então $\alpha\vec{u} \in V$

c) 1 e 5

d) Axioma 7

$\alpha(\vec{u}+\vec{v}) = \alpha\vec{u}+\alpha\vec{v}$

$3(2+6) = 3(-1,2)+3(3,4)$

$3(2,6) = (-3,6)+(9,12)$

$(6,18)=(6,18)$

Axioma 8

$(a+b)\vec{u} = a\vec{u}+b\vec{u}$

$(a+b)(-1,2) = a(-1,2)+b(-1,2)$

$(-a-b,2a+2b) = (-a,2a)+(-b,2b)$

$(-a-b,2a+2b) = (-a-b,2a+2b)$

Axioma 9

$a(b\vec{u}) = (ab)\vec{u}$

$a(b(-1,2)) = (ab)(-1,2)$

$a(-b,2b) = (-ab,2ab)$

$(-b,2ab) = (-ab,2ab)$

e)

$$1 \cdot \vec{u} = \vec{u}$$

$$1 \cdot (-1, 2) = (0, 2) \neq \vec{u}$$

② Axioma 1 está no final

c)

Axioma 2

$$\underbrace{(1, x_1) + (1, x_2'')}= (1, x_2') + (1, x_1)$$

$$(1, x_1 + y_2'') = (1, x_1 + x_2')$$

Axioma 3:

$$(1, x_1) + [(1, x_2) + (1, x_3)] = [(1, x_1) + (1, x_2)] + (1, x_3)$$

$$(1, x_1) + (1, x_2 + x_3) = (1, x_1 + x_2) + (1, x_3)$$

$$(1, x_1 + x_2 + x_3) = (1, x_1 + x_2 + x_3)$$

Axioma 4:

$$(1, x_1) + 0 = (1, x_1)$$

Axioma 5:

$$(1, x_1) + (-1, -x_1)$$

$$(1-1, x_1 - x_1) = 0$$

Axioma 6:

$$a(1, x_1) = (1, ax_1)$$

Axioma 7:

$$a[(1, x_1) + (1, x_2)] = a(1, x_1) + a(1, x_2)$$

$$a(1, x_1 + x_2) = (1, ax_1) + (1, ax_2)$$

$$(1, ax_1 + ax_2) = (1, ax_1 + ax_2)$$

Axioma 8:

$$(a + b)(1, x_1) = a(1, x_1) + b(1, x_1)$$

$$(1, ax_1 + bx_1) = (1, ax_1) + (1, bx_1)$$

$$(1, ax_1 + bx_1) = (1, ax_1 + bx_1)$$

Axioma 9:

$$a[b \cdot (1, x_1)] = (ab)(1, x_1)$$

$$a(1, bx_1) = (1, abx_1)$$

$$(1, abx_1) = (1, abx_1)$$

Axiomo 10

$1 \cdot (1, x_1) = (1, x_1)$

Axioma 1

$(1, x_1) + (1, x_2)$

$(1, x + x_2)$

3

a) É subespaço

$(a_1, 0, 0) + (a_2, 0, 0) = (a_1 + a_2, 0, 0)$

$\alpha(a_1, 0, 0) = (\alpha a_1, 0, 0)$

b) Não é Subespaço

$(a_1, 1, 1) + (a_2, 1, 1) = (a_1 + a_2, 2, 2)$

$\alpha(a_1, 1, 1) = (\alpha a, \alpha, \alpha)$

c) É subespaço

$(a_1, \overset{a_1+c_1}{\cancel{b_1}}, c_1) + (a_2, \overset{a_2+c_2}{\cancel{b_2}}, c_2) = (a_1 + a_2, a_1 + c_1 + a_2 + c_2, c_1 + c_2)$

$\alpha(a_1, a_1 + c_1, c_1) = (\alpha a_1, \alpha a_1 + \alpha c_1, \alpha c_1)$

d) Não é subespaço

$(a_1, a_1 + c_1 + 1, c_1) + (a_2, a_2 + c_2 + 1, c_2) = (a_1 + a_2, a_1 + c_1 + a_2 + c_2 + 2, c_1 + c_2)$

$\alpha(a_1, a_1 + c_1 + 1, c_1) = (\alpha a_1, \alpha a_1 + \alpha c_1 + \alpha, c_1)$

e) É subespaço

$(a_1, b_1, 0) + (a_2, b_2, 0) = (a_1 + a_2, b_1 + b_2, 0)$

$\alpha(a_1, b_1, 0) = (\alpha a_1, \alpha b_1, 0)$

5

a)

$$\left[\begin{array}{ccc|cc} 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$0 + 0 + 0 - (0 + 6 + 0)$

Det $= -6 \rightarrow$ Det $\neq 0$ gera vetor $\mathbb{R}^3$

c)

$$\begin{bmatrix} 3 & 1 & 4 \\ 2 & -3 & 5 \\ 5 & -2 & 9 \\ 1 & 4 & -1 \end{bmatrix} \xrightarrow{L_1 \cdot 1/3} \begin{bmatrix} 1 & 1/3 & 4/3 \\ 2 & -3 & 5 \\ 5 & -2 & 9 \\ 1 & 4 & -1 \end{bmatrix} \xrightarrow[\substack{L_3 - 5L_1 \\ L_4 - L_1}]{L_2 - 2L_1} \begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & -11/3 & 7/3 \\ 0 & -11/3 & 7/3 \\ 0 & 11/3 & -7/3 \end{bmatrix} \xrightarrow{L_2 \cdot -3/10}$$

$$\begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 1 & -21/30 \\ 0 & -11/3 & 7/3 \\ 0 & 11/3 & -7/3 \end{bmatrix} \xrightarrow[\substack{L_3 + \frac{11}{2} L_2 \\ L_4 - \frac{11}{3} L_2}]{} \begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 1 & -21/30 \\ 0 & 0 & 49/10 \\ 0 & 0 & -49/10 \end{bmatrix}$$

Não gera $\mathbb{R}^3$

6

a)

$$\left[\begin{array}{cccc|c} 3 & 1 & 2 & 1 & 0 \\ 8 & 5 & -1 & 4 & 0 \\ 7 & 3 & 2 & 0 & 0 \\ -3 & -1 & 6 & 3 & 0 \end{array}\right] \xrightarrow{L_1 \cdot \frac{1}{3}} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/3 & 0 \\ 8 & 5 & -1 & 4 & 0 \\ 7 & 3 & 2 & 0 & 0 \\ -3 & -1 & 6 & 3 & 0 \end{array}\right]$$

$$\xrightarrow[\substack{L_3 - 7L_1 \\ L_4 + 3L_1}]{L_2 - 8L_1} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/4 & 0 \\ 0 & 7/3 & -19/3 & 4/3 & 0 \\ 0 & -4/3 & -8/3 & -7/3 & 0 \\ 0 & 0 & 8 & 4 & 0 \end{array}\right] \xrightarrow{L_2 \cdot \frac{3}{7}} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -19/7 & 4/7 & 0 \\ 0 & 4/3 & -8/3 & -7/4 & 0 \\ 0 & 0 & 8 & 4 & 0 \end{array}\right]$$

$$\xrightarrow{L_3 - \frac{4}{3}L_2} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -19/7 & 4/7 & 0 \\ 0 & 0 & -49/7 & -11/7 & 0 \\ 0 & 0 & 8 & 4 & 0 \end{array}\right] \xrightarrow{-\frac{7L_3}{49}} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -19/7 & 4/7 & 0 \\ 0 & 0 & 1 & -1/4 & 0 \\ 0 & 0 & 8 & 4 & 0 \end{array}\right]$$

$$\xrightarrow{L_4 - 8L_3} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -19/7 & 4/7 & 0 \\ 0 & 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{array}\right] \xrightarrow{L_4/2} \left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -19/7 & 4/7 & 0 \\ 0 & 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[L_3 - \frac{1}{4}L_4]{\substack{L_1 - \frac{L_4}{3} \\ L_2 - \frac{4}{3}L_4}}$$

$$\left[\begin{array}{cccc|c} 1 & 1/3 & 2/3 & 0 & 0 \\ 0 & 1 & -19/7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[L_3 - \frac{19}{8}L_3]{L_1 - \frac{2}{3}L_3} \left[\begin{array}{cccc|c} 1 & 1/3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{L_1 - \frac{L_2}{3}}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{array}$$

Não são lineanmente dependentes

c)

$$\left[\begin{array}{cccc|c} 0 & -2 & 0 & 0 & 0 \\ 3 & 0 & -4 & -8 & 0 \\ -3 & 0 & -2 & 4 & 0 \\ 6 & -6 & -2 & -4 & 0 \end{array}\right] \xrightarrow[L_1 \leftrightarrow L_2]{L_2/3} \left[\begin{array}{cccc|c} 1 & 0 & -4/3 & -8/3 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ -3 & 0 & -2 & 4 & 0 \\ 6 & -6 & -2 & -4 & 0 \end{array}\right] \xrightarrow[L_4 - 6L_1]{L_3 + 3L_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -4/3 & -8/3 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -6 & -4 & 0 \\ 0 & -6 & 6 & 12 & 0 \end{array}\right] \xrightarrow[-L_3/6]{-\frac{L_2}{2}} \left[\begin{array}{cccc|c} 1 & 0 & -4/3 & -8/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2/3 & 0 \\ 0 & -6 & 6 & 12 & 0 \end{array}\right] \xrightarrow[\substack{6L_3 \\ L_4 -}]{L_4 + 6L_2}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -4/3 & -8/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 8 & 0 \end{array}\right] \xrightarrow[L_3 - \frac{2}{3}L_4]{\frac{L_4}{8}} \left[\begin{array}{cccc|c} 1 & 0 & -4/3 & -8/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[L_1 + \frac{8}{3}L_4]{L_1 + \frac{4}{3}L_3}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$
$x_4 = 0$

Não são Lineanmente dependentes

8

a)

$$\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 0 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \end{array}\right]$$

$6 + 0 + 0 - (0 + 0 + 0)$

$Det = 6 \rightarrow Det \neq 0$

São Lineanmente independentes

c)

$$\left[\begin{array}{ccc|cc} 2 & -3 & 1 & 2 & -3 \\ 4 & 1 & 1 & 4 & 1 \\ 0 & -7 & 1 & 0 & -7 \end{array}\right]$$

$2 + 0 - 28 - (0 - 14 - 12$

$-26 + 26$

$Det = 0$

Não são Lineanmente independentes

9

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 1 & 0 \\ 6 & -1 & -8 & 0 & 0 \\ 3 & -1 & -12 & -1 & 0 \\ -6 & 0 & -4 & 2 & 0 \end{array}\right] \xrightarrow{L_1/3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 6 & -1 & -8 & 0 & 0 \\ 3 & -1 & -12 & -1 & 0 \\ -6 & 0 & -4 & 2 & 0 \end{array}\right] \xrightarrow[L_4+6L_1]{L_2-6L_1,\ L_3-3L_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & -1 & -8 & -2 & 0 \\ 0 & -1 & -12 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 \end{array}\right]$$

$$\xrightarrow[L_3+L_2]{-L_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 8 & 2 & 0 \\ 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & -4 & 4 & 0 \end{array}\right] \xrightarrow[L_4+L_3]{-L_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 8 & 2 & 0 \\ 0 & 0 & 4 & -2 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{array}\right] \xrightarrow{L_4/2,\ L_3+2L_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 8 & 2 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[L_2-8L_3]{L_3/4,\ L_2-2L_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{L_1-\frac{L_4}{3}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

$x_4 = 0$

CamScanner

(10)

b)

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ -4 & 8 & 0 & 1 \end{array}\right] \xrightarrow[-\ell_2/4]{\ell_1/2} \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 1 & -2 & 0 & -1/4 \end{array}\right] \xrightarrow{\ell_2-\ell_1} \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 0 & -7/2 & -1/2 & -1/4 \end{array}\right]$$

$$\xrightarrow{\ell_2\cdot -\frac{2}{7}} \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 0 & 1 & \frac{1}{7} & \frac{1}{14} \end{array}\right] \xrightarrow{\ell_1-\frac{3}{2}\ell_2} \left[\begin{array}{cc|cc} 1 & 0 & 2/7 & -3/28 \\ 0 & 1 & 1/7 & 1/14 \end{array}\right]$$

$$\begin{bmatrix} 2/7 & -3/28 \\ 1/7 & 1/14 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5/28 \\ 3/14 \end{bmatrix} \qquad w = \left(\frac{5}{28}, \frac{3}{14}\right)_S$$

(11)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\ell_3/3]{\ell_2/2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right] \xrightarrow{\ell_2-\frac{3}{2}\ell_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right] \xrightarrow[\ell_1-3\ell_3]{\ell_1-2\ell_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right]$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 1/3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$