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Resolução P1 - Álgebra Linear

1) a) $2x_1 - 4x_2 + x_3 = 6$
$-4x_1 + 3x_3 = -1$
$x_2 - x_3 = 3$

b) $\begin{bmatrix} 2 & -4 & 1 & 6 \\ -4 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{bmatrix} \xrightarrow{1/2 L_1} \begin{bmatrix} 1 & -2 & 1/2 & 3 \\ -4 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{bmatrix} L_2 \leftrightarrow L_3$

$\begin{bmatrix} 1 & -2 & 1/2 & 3 \\ 0 & 1 & -1 & 3 \\ -4 & 0 & 3 & -1 \end{bmatrix} \xrightarrow{4L_1 + L_3} \begin{bmatrix} 1 & -2 & 1/2 & 3 \\ 0 & 1 & -1 & 3 \\ 0 & -8 & 5 & 11 \end{bmatrix} \xrightarrow{8L_2 + L_3}$

$\begin{bmatrix} 1 & -2 & 1/2 & 3 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -3 & 35 \end{bmatrix} \xrightarrow{-1/3 L_3} \begin{bmatrix} 1 & -2 & 1/2 & 3 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -35/3 \end{bmatrix} \xrightarrow{2L_2 + L_1}$

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$\begin{bmatrix} 1 & 0 & -3/2 & 9 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -35/3 \end{bmatrix} \xrightarrow{L_3 + L_2} \begin{bmatrix} 1 & 0 & -3/2 & 9 \\ 0 & 1 & 0 & -26/3 \\ 0 & 0 & 1 & -35/3 \end{bmatrix} \quad 3/2 L_3 + L_1$

$\begin{bmatrix} 1 & 0 & 0 & -17/2 \\ 0 & 1 & 0 & -26/3 \\ 0 & 0 & 1 & -35/3 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow$ $x_1 = -17/2$
$x_2 = -26/3$
$x_3 = -35/3$

c) $A\vec{x} = \vec{b} \longrightarrow \begin{bmatrix} 2 & -4 & 1 \\ -4 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$

$\left|\begin{matrix} 2 & -4 & 1 \\ -4 & 0 & 3 \\ 0 & 1 & -1 \end{matrix}\right| \begin{matrix} 2 & -4 \\ -4 & 0 \\ 0 & 1 \end{matrix}$ $\det(A) = 6$
sendo $\neq 0$

d) como $\det(A) \neq 0$, então sim, é invertível

1) b)

$$\left[\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ -4 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_1/2} \left[\begin{array}{ccc|ccc} 1 & -2 & 1/2 & 1/2 & 0 & 0 \\ -4 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{L_2 \leftrightarrow L_3} \left[\begin{array}{ccc|ccc} 1 & -2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ -4 & 0 & 3 & 0 & 1 & 0 \end{array}\right] \xrightarrow{L_1+L_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & -8 & 5 & 2 & 1 & 0 \end{array}\right] \xrightarrow{8L_2+L_3} \left[\begin{array}{ccc|ccc} 1 & -2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & -3 & 2 & 1 & 8 \end{array}\right] \xrightarrow{1/3 L_3} \left[\begin{array}{ccc|ccc} 1 & -2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2/3 & -1/3 & -8/3 \end{array}\right] \xrightarrow{2L_2+L_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3/2 & 1/2 & 0 & 2 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2/3 & -1/3 & -8/3 \end{array}\right] \xrightarrow{L_3+L_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3/2 & 1/2 & 0 & 2 \\ 0 & 1 & 0 & -2/3 & -1/3 & -5/3 \\ 0 & 0 & 1 & -2/3 & -1/3 & -8/3 \end{array}\right] \xrightarrow{3/2 L_3+L_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & -1/2 & -2 \\ 0 & 1 & 0 & -2/3 & -1/3 & -5/3 \\ 0 & 0 & 1 & -2/3 & -1/3 & -8/3 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} -1/2 & -1/2 & -2 \\ -2/3 & -1/3 & -5/3 \\ -2/3 & -1/3 & -8/3 \end{bmatrix}$$

2) $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, $E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

$E^T = \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$ $\quad 3D = \begin{bmatrix} 3 & 15 & 6 \\ -3 & 0 & 3 \\ 9 & 6 & 12 \end{bmatrix}$

$E^T + 3D = \begin{bmatrix} 9 & 14 & 10 \\ -2 & 1 & 4 \\ 12 & 8 & 15 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$

$E^T + 3D = \begin{bmatrix} 23 & 38 \\ 3 & 6 \\ 43 & 31 \end{bmatrix}$

3) $C(DC)^{-1}C = D^{-1}C$

a) $C(C^{-1}D^{-1})C = D^{-1}C$

$\underbrace{\cancel{CC^{-1}}}_{I}D^{-1}C = D^{-1}C$

$D^{-1}C = D^{-1}C$

3) b) $(C^{-1}+D^{-1})^{-1} = C(C+D)^{-1}D$

$C+D = C(C^{-1}+D^{-1})D$

$C+D = \underbrace{\cancel{CC^{-1}}}_{I}D + C\underbrace{\cancel{D^{-1}D}}_{I}$

$C+D = D+C$

3) c) $(C+DD^T)^{-1}D = C^{-1}D(I+D^TC^{-1}D)^{-1}$

$C^{-1}D + (DD^T)^{-1}D = C^{-1}DI^{-1} + C^{-1}D(D^TC^{-1}D)^{-1}$

$C^{-1}D + D^{-1}(D^T)^{-1}D = C^{-1}DI + C^{-1}D(D^T)^{-1}(C^{-1})^{-1}D^{-1}$

$C^{-1}D + I(D^T)^{-1} = C^{-1}D + C^{-1}(D^T)^{-1}C$

$C^{-1}D + (D^T)^{-1} = C^{-1}D + (D^T)^{-1}$

4) a) $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$

$\det(A) = 1.7 - 4.2$

$\det(A) = 7 - 8$

$\det(A) = -1$

$\det(A^T) = 1.7 - 2.4$

$\det(A^T) = 7 - 8$

$\det(A^T) = -1$

b) $\begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & 1 \\ 8 & 1 & 5 \end{bmatrix}$

$A^T = \left[\begin{array}{ccc|cc} 3 & 2 & 8 & 3 & 2 \\ 4 & -7 & 1 & 4 & -7 \\ 1 & -1 & 5 & 1 & -1 \end{array}\right]$

$\left[\begin{array}{ccc|cc} 3 & 4 & 1 & 3 & 4 \\ 2 & -7 & -1 & 2 & -7 \\ 8 & 1 & 5 & 8 & 1 \end{array}\right]$

$-105 + 2 + (-32) - (-56 + (-3) + 40)$

-116

$-105 + (-32) + 2 - (-56 + (-3) + 40)$

-116

Assim $\det(A) = \det(A^T)$

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5)

$$\begin{vmatrix} 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 \end{vmatrix} \begin{array}{l} 1^\circ \text{ coluna} \\ 2^\circ \text{ coluna} \\ 3^\circ \\ 4^\circ \\ 5^\circ \end{array}$$

$$\det = \cancel{0\cdot(-1)^{1+2}\cdot\begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{vmatrix}} + \cancel{0\cdot(-1)^{2+2}\begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{vmatrix}} + \cancel{0\cdot(-1)^{3+2}\begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{vmatrix}} + 1\cdot(-1)^{4+2}\begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{vmatrix}$$

$$0 \qquad 0 \qquad 0$$

$$+ \cancel{0\cdot(-1)^{5+2}\begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}} \quad 0$$

$$\det = 1\cdot(-1)^6 \begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{vmatrix} \begin{array}{l} 1^\circ\ 0 \\ 2^\circ \to 0 \\ 3^\circ \to 0 \\ 4^\circ\ 0 \end{array} + 4\cdot(-1)^{2+3}\begin{vmatrix} 0 & 0 & -3 \\ 0 & 4 & 0 \\ 5 & 0 & 0 \end{vmatrix}$$

$$\det \begin{vmatrix} 0 & 0 & -3 \\ 0 & 4 & 0 \\ 5 & 0 & 0 \end{vmatrix} \begin{matrix} 0 & 0 \\ 0 & 4 \\ 5 & 0 \end{matrix}$$

$$\det = 60$$

$$\det = 4\cdot(-1)^5\cdot 60$$

$$\boxed{\det = 240}$$