NONLINEAR TRACKING CONTROL OF A CAR-LIKE MOBILE ROBOT VIA DYNAMIC FEEDBACK LINEARIZATION

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Keywords: Nonlinear tracking control, Mobile robot, Feedback linearization, Constrained-state feedback control

Abstract

Trajectory tracking is an important behaviour for mobile robots. This paper addresses the nonlinear trajectory tracking control problem for a nonholonomic car-like mobile robot with a constrained state. Compared with the existing works on the nonholonomic car-like mobile robots, the state constraint is emphasized in this work. The controllability of the system under the state constraint is checked firstly from the sense of nonlinear geometric control. A nonlinear tracking controller is then achieved by taking advantage of the *dynamic feedback linearization* technique. Moreover, the obtained controller can be exploited to simultaneously solve both the tracking and regulation problems of the car-like mobile robot with the constrained state. The effectiveness of the proposed control law to trajectory tracking control is demonstrated by simulation results.

1 Introduction

The trajectory planning, formation maneuver, and motion control, in particular parking control problems of the mobile robots and ground vehicles have received considerable attention during the last decade [1–13]. One difficulty for the motion planning and control of a car-like mobile robot arises from the so-called nonholonomic constraints imposed by the rolling wheels. Another difficulty comes from the constrained state and saturation inputs, such as bounded steering angle as well as driving velocity.

According to Brockett's theorem, the kinematics model of a nonholonomic mobile robot is open-loop controllable, but not stabilizable by pure smooth, time-invariant feedback law. To stabilize such a system, some time-varying control laws have been developed, e.g., see at [1, 6, 14].

Recently, nonlinear control theory and techniques with applications to the trajectory planning, tracking, and parking control of the mobile robots have attracted a great deal of attention [2, 4, 5, 8, 9, 11, 12]. For instance, a time-varying-based robust control law for the parking problem of a wheeled mobile robot was achieved via the Lyapulov direct method by Tayebi and Rachid [2]. The tracking control problem with saturation constraint for a class of unicycle-modeled mobile robots was solved in [8] by using the backstepping technique and the idea from LaSalle's invariance principle. Jiang and Hijmeijer [4] also proposed a tracking control methodology via time-varying state feedback based on the backstepping technique for a mobile robot. Oriolo et al. [9] reported their works on the motion control problem of wheeled mobile robots (WMRs) in environments without obstacles.

In this contribution, the trajectory tracking control problem is addressed for a car-like mobile robot with a constrained state. A design methodology which fully incorporates the state constraint into the system is presented by using the change of coordinates. For the transformed system, the dynamic feedback linearization technique is used to design the trajectory tracking control law.

The remainder of this paper is organized as follows. Section 2 formulates the control problem. In Section 3, the dynamic feedback linearization is used to design trajectory tracking controller. Section 4 presents the simulation results to validate the effectiveness of the developed control law. Finally, some conclusions are made in Section 5.

2 Statement of the Control Problem

As depicted in Fig. 1, the configuration of the robot denoted by $\boldsymbol{q}{:=}\ (x, y, \theta, \psi)^T \in \mathbb{R}^4$ is 4-D [1, 6]. Where, (x, y) is the Cartesian location of the center of its rear wheels, θ is the heading angle between the body axis and the horizontal axis, and ψ represents the steering angle with respect to the car body. The distance between the location (x, y) and the middle point of the driving wheels is denoted by L. This system has 2 degrees of nonholonomy since the constraints on the system arise by allowing the wheels to roll and spin, but not slip. Thus, the Pfaffian constraints on the mobile robot become [1]:

$$\begin{array}{c} \sin(\theta+\psi)\,\dot{x} - \cos(\theta+\psi)\,\dot{y} - L\,\cos\psi\,\dot{\theta} = 0 \\ \sin\theta\,\dot{x} - \cos\theta\,\dot{y} = 0 \end{array} \tag{1}$$

Converting (1) to a nonlinear control system with the inputs chosen as $u_1 = v\,\cos\psi$ and $u_2 = \dot{\psi}$ yields [1, 6]

$$\begin{array}{ll} \dot{x} = \cos\theta\, u_1, & \dot{y} = \sin\theta\, u_1 \\ \dot{\theta} = \frac{1}{L}\,\tan\psi\, u_1 & \dot{\psi} = u_2 \end{array} \tag{2}$$

where, v is the driving speed, u_1 corresponds to the translational velocity of the rear wheels of the robot and u_2 corresponds to the velocity of the angle of the steering wheels. In this study, the steering angle ψ is constrained by the following relation:

$$|\,\psi\,| \leq M, \quad 0 < M < \frac{\pi}{2} \tag{3}$$

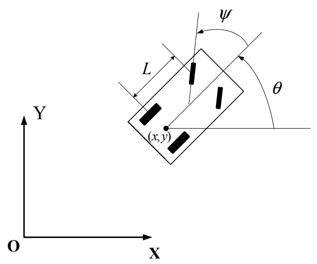

Fig. 1. Kinematic model of a car-like robot

Obviously, (2) is a so-called driftless nonlinear system, i.e.

$$\dot{\boldsymbol{q}} = (g_1, g_2)\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{L}\tan\psi \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2 \quad (4)$$

In what follows, we first discuss the controllability of system (4) under the constraint condition (3). It is noted that the approximate linearization of the system at any point is not controllable. Hence, we only address the controllability in a nonlinear sense. The main result is stated as:

Proposition 1 *The control distribution of the system (4) under the constraint condition (3) still has a constant dimension which equals to the dimension of the states so that the system (4) under condition (3) is locally controllable.*

Proof: According to the rank condition of the controllability, the proof is straightforward. As the system is driftless (the drift term $\boldsymbol{f}$ is $\mathbf{0}$ vector field), the smallest involutive distribution $\boldsymbol{\Delta_C}$ containing span$\{\boldsymbol{f}, \boldsymbol{g_1}, \boldsymbol{g_2}\}$, i.e.,

$$\boldsymbol{\Delta_C} =< \boldsymbol{f}, \boldsymbol{g_1}, \boldsymbol{g_2} \mid \text{span}\{\boldsymbol{f}, \boldsymbol{g_1}, \boldsymbol{g_2}\} > \quad (5)$$

equals to the smallest involutive distribution $\boldsymbol{\Delta_{C_0}}$ containing span$\{\boldsymbol{g_1}, \boldsymbol{g_2}\}$, i.e.,

$$\boldsymbol{\Delta_{C_0}} =< \boldsymbol{f}, \boldsymbol{g_1}, \boldsymbol{g_2} \mid \text{span}\{\boldsymbol{g_1}, \boldsymbol{g_2}\} > \quad (6)$$

Denoting by $[\boldsymbol{g_1}, \boldsymbol{g_2}]$ the Lie bracket of vector field $\boldsymbol{g_1}$ and $\boldsymbol{g_2}$, we begin with span$\{\boldsymbol{g_1}, \boldsymbol{g_2}\}$ and expand this distribution by taking its Lie brackets of its component vector fields until we achieve a distribution of maximum dimension.

$$[\boldsymbol{g_1}, \boldsymbol{g_2}] = (0, 0, -\frac{1}{L}\sec^2\psi, 0)^T \quad (7)$$

and

$$[[\boldsymbol{g_1}, \boldsymbol{g_2}], \boldsymbol{g_1}] = (\frac{1}{L}\sec^2\psi\sin\theta, -\frac{1}{L}\sec^2\psi\cos\theta, 0, 0)^T \quad (8)$$

thus, it is easy to verify that the controllability rank condition holds, i.e.,

$$\dim\Delta_{C_0} = \text{rank}(\boldsymbol{g_1}, \boldsymbol{g_2}, [\boldsymbol{g_1}, \boldsymbol{g_2}], [[\boldsymbol{g_1}, \boldsymbol{g_2}], \boldsymbol{g_1}]) = n = 4 \quad (9)$$

Therefore, the system (4) is locally controllable in a nonlinear sense though there exists a state constraint. □

Although there is a constrained state in the original difficult nonholonomic system, from Proposition 1 one knows that the mobile robot could be moved from one position and orientation to another position and orientation in finite time. In this study we are particularly interested in the following issue:

Problem 1 (Trajectory Tracking) *Given a feasible trajectory $\boldsymbol{q_d}$ which is collision-free. For the mobile robot system (4) under the constraint condition (3), design a trajectory tracking controller $\boldsymbol{u} = (\boldsymbol{u_1}, \boldsymbol{u_2})^{\boldsymbol{T}}$ such that the mobile robot is moved from an initial state $\boldsymbol{q(0)}$ to follow the desired trajectory $\boldsymbol{q_d}$ over the time period $[0, T](T \to \infty)$.*

3 Trajectory Tracking Control via Dynamic Feedback Linearization

First, we replace the constraint condition (3) by taking advantage of $\psi = M\ \tanh w$, where w is an auxiliary variable. Thus, we can get

$$\dot{\psi} = M\,\text{sech}^2 w\,\mu_2 := u_2, \quad \dot{w} = \mu_2 \quad (10)$$

Substituting the relation $\psi = M\ \tanh w$ and (10) into (2) yields

$$\begin{array}{ll} \dot{x} = \cos\theta\, u_1, & \dot{y} = \sin\theta\, u_1 \\ \dot{\theta} = \frac{1}{L}\tan(M\ \tanh w)\, u_1 := \eta_1(w)\, u_1 & \dot{w} = \mu_2 \end{array} \quad (11)$$

Obviously, (11) is equivalent to (2) and (3). However, the state constraint (3) now is incorporated into system (11), which will make it feasible to deal with the state constraint (3) in our following design methodology.
The *exact feedback linearization problem* in question can be defined as follows [14]:

Definition 1 (Feedback Linearization) *Given a set of vector fields $\boldsymbol{f(x)}, \boldsymbol{g_1(x)}, \cdots, \boldsymbol{g_m(x)}$ and an initial state $\boldsymbol{x^0}$, find (if possible), a neighborhood $\mathcal{U}$ of $\boldsymbol{x^0}$, a pair of feedback functions $\boldsymbol{\alpha(x)}$ and $\boldsymbol{\rho(x)}$ defined on $\mathcal{U}$, a coordinates transformation $\boldsymbol{z} = \boldsymbol{\Phi(x)}$ also defined on $\mathcal{U}$, a matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{B} \in \mathbb{R}^{n\times m}$, such that*

$$\left[\frac{\partial\boldsymbol{\Phi}}{\partial\boldsymbol{x}}(\boldsymbol{f(x)} + \boldsymbol{g(x)}\,\boldsymbol{\alpha(x)})\right]_{\boldsymbol{x}=\boldsymbol{\Phi^{-1}(z)}} = \boldsymbol{Az} \quad (12)$$

$$\left[\frac{\partial\boldsymbol{\Phi}}{\partial\boldsymbol{x}}(\boldsymbol{g(x)}\,\boldsymbol{\rho(x)})\right]_{\boldsymbol{x}=\boldsymbol{\Phi^{-1}(z)}} = \boldsymbol{B} \quad (13)$$

and

$$rank(\boldsymbol{B}\ \boldsymbol{A}\,\boldsymbol{B}\ \cdots\ \boldsymbol{A^{n-1}B}) = \boldsymbol{n} \quad (14)$$

where, $\boldsymbol{\alpha(x)} = (\boldsymbol{L_f^{r_1}} h_1\boldsymbol{(x)}, \cdots, \boldsymbol{L_f^{r_m}} h_m\boldsymbol{(x)})^{\boldsymbol{T}}$

Based on Definition1, we have the following proposition.

Proposition 2 *Define the linearizing output vector of (11) as $\boldsymbol{y} = (y_1, y_2)^T = (x, y)^T$. The nonholonomic kinematic model (11) cannot be transformed into a linear controllable system by means of static state feedback.*

Proof: Differentiating the output $\boldsymbol{y}$ once with respect to time then yields

$$\dot{y}_1 = \cos\theta\, u_1, \qquad \dot{y}_2 = \sin\theta\, u_1 \quad (15)$$

Since the input u_1 already appears in (15), one can obtain the decoupling matrix as the following form:

$$\boldsymbol{\rho}(\boldsymbol{x}) = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{pmatrix} \tag{16}$$

Apparently, rank$(\boldsymbol{\rho}(\boldsymbol{x})) = 1$ everywhere on $\mathcal{U}$, viz., the decoupling matrix $\boldsymbol{\rho}(\boldsymbol{x})$ is *singular*. Hence, a well-defined definition for vector relative degree $\boldsymbol{r}$ does not exist for system (11). Because the relative degree is invariant under static state feedback, one can not transform the original system (11) which does not have a well-defined relative degree into a new system which does have a well-defined relative degree by means of *static* state feedback. □

3.1 Dynamic Feedback Linearization

Proposition 2 implies that we can not find a regular *static* state feedback and a local change of coordinates $\boldsymbol{\xi} = \boldsymbol{\Phi}(\boldsymbol{x})$ to directly transform the original system into an exact linearization form via *static* feedback from the viewpoint of input-output. However, one may resort to the dynamic feedback linearization for exact linearization purposes. For our considered kinematic system (11), a result on dynamic feedback linearization is established as follows.

Proposition 3 *The nonholonomic kinematic model (11) can be dynamic feedback linearizable by using a dynamic compensator such as*

$$\dot{u}_1 = p_1, \qquad \dot{p}_1 = \mu_1 \tag{17}$$

where μ_1 is a new control input.

Proof: According to the definition of exact feedback linearization, the proof is straightforward. Observing (15), one finds that the first derivatives of $y_i, i = 1,2$ are only affected by the input u_1 while the other control inputs do not appear. Thus, in order to achieve a well-defined relative degree, we can expect to render $\dot{y}_i, i = 1,2$ independent of u_1 by means of adding some integrators in the first control input channel as done in (17).
Differentiating (15) further with respect to time yields

$$\begin{aligned} \ddot{y}_1 &= \cos\theta\,\dot{u}_1 - \sin\theta\,\dot{\theta}\,u_1 = \cos\theta\,p_1 - \sin\theta\,\eta_1(w)\,u_1^2 \\ \ddot{y}_2 &= \cos\theta\,\dot{\theta}\,u_1 + \sin\theta\,\dot{u}_1 = \cos\theta\,\eta_1(w)\,u_1^2 + \sin\theta\,p_1 \end{aligned} \tag{18}$$

Since neither control input μ_1 nor μ_2 appears in (18), one can continue to differentiate it and get

$$\begin{aligned} y_1^{(3)} &= -\cos\theta\,\eta_1^2(w)\,u_1^3 - 3\,\sin\theta\,\eta_1(w)\,u_1\,p_1 \\ &\quad - \tfrac{\partial\eta_1}{\partial w}\sin\theta\,u_1^2\,\mu_2 + \cos\theta\,\mu_1 \\ y_2^{(3)} &= -\sin\theta\,\eta_1^2(w)\,u_1^3 + 3\,\cos\theta\,\eta_1(w)\,u_1\,p_1 \\ &\quad + \tfrac{\partial\eta_1}{\partial w}\cos\theta\,u_1^2\,\mu_2 + \sin\theta\,\mu_1 \end{aligned} \tag{19}$$

in which $\frac{\partial\eta_1}{\partial w} = M\,\sec^2(M\,\tanh w)\,\mathrm{sech}(w)/L$.
From (19), one finds that both μ_1 and μ_2 already appear in $\boldsymbol{y}^{(3)}$. Hence, the original system (11) plus the dynamic compensator (17) will have a well-defined relative degree $\boldsymbol{r} = (3,3)$ whenever $u_1 \neq 0$ and its sum ($r = r_1 + r_2 = 6 = n$) exactly equals to the dimension of the augmented system, which fully satisfies Lemma 5.2.1 in [14]. Therefore, the (dynamic) exact linearization problem in question can be solvable.

Moreover, the set of functions

$$\begin{aligned} z_1 &= y_1 = x \\ z_2 &= \dot{y}_1 = \dot{x} = \cos\theta\,u_1 \\ z_3 &= \ddot{y}_1 = \ddot{x} = -\sin\theta\,\eta_1(w)\,u_1^2 + \cos\theta\,p_1 \\ z_4 &= y_2 = y \\ z_5 &= \dot{y}_2 = \dot{y} = \sin\theta\,u_1 \\ z_6 &= \ddot{y}_2 = \ddot{y} = \cos\theta\,\eta_1(w)\,u_1^2 + \sin\theta\,p_1 \end{aligned} \tag{20}$$

completely defines a local coordinates transformation. In the new coordinates, the extended system is fully feedback linearized and described by the two chains of integrators and no extra equations are involved, i.e.,

$$\begin{aligned} \dot{z}_1 &= z_2, & \dot{z}_2 &= z_3, & \dot{z}_3 &= v_1 \\ \dot{z}_4 &= z_5, & \dot{z}_5 &= z_6, & \dot{z}_6 &= v_2 \end{aligned} \tag{21}$$

with $v_1 = y_1^{(3)} = x^{(3)}, v_2 = y_1^{(3)} = y^{(3)}$.
Denoting $\boldsymbol{x}= (x, y, \theta, w, u_1, p_1)^T$ and $\boldsymbol{u}= (\mu_1, \mu_2)^T$ then follows

$$\boldsymbol{\alpha}(\boldsymbol{x}) + \boldsymbol{\rho}(\boldsymbol{x})\,\boldsymbol{u} = \boldsymbol{v} \tag{22}$$

where

$$\boldsymbol{\alpha}(\boldsymbol{x}) = \begin{pmatrix} -\cos\theta\,\eta_1^2(w)\,u_1^3 - 3\,\sin\theta\,\eta_1(w)\,u_1\,p_1 \\ -\sin\theta\,\eta_1^2(w)\,u_1^3 + 3\,\cos\theta\,\eta_1(w)\,u_1\,p_1 \end{pmatrix} \tag{23}$$

and

$$\boldsymbol{\rho}(\boldsymbol{x}) = \begin{pmatrix} \cos\theta & -\frac{\partial\eta_1}{\partial w}\sin\theta\,u_1^2 \\ \sin\theta & \frac{\partial\eta_1}{\partial w}\cos\theta\,u_1^2 \end{pmatrix} \tag{24}$$

Clearly,

$$\det\boldsymbol{\rho}(\boldsymbol{x}) = \frac{\partial\eta_1}{\partial w}\sin\theta\,u_1^2$$

□

Remark 1 *Observing the dynamic feedback linearized system (21), it is obviously advantageous to the controller design because of its linear and controllable structure of system. Hence, the well-established linear control theory, such as linear quadratic regulator (LQR), decoupling design techniques, etc., can be implemented to system analysis and controller synthesis. On the other hand, the decoupling matrix $\boldsymbol{\rho}(\boldsymbol{x})$ has a potential singularity at $u_1 = 0$. Therefore, this difficulty must be seriously taken into account when designing control laws for the car-like mobile robot.*

3.2 Controller Design for Trajectory Tracking

Assume that the desired, feasible trajectory $(x_d(t), y_d(t))^T$ is given for the position coordinates in the form of specified time functions. The output tracking error is denoted by $(x_e, y_e)^T = (x - x_d, y - y_d)^T$.
Our control purpose is to design an appropriate state-feedback controller such that the car-like mobile robot is forced to asymptotically track the desired trajectory from some initial tracking error $(x_e(0), y_e(0))^T$, and both x_e and y_e converge to zero as $t \to +\infty$.
The following proposition establishes a dynamic feedback control solution to the trajectory tracking problem for the car-like mobile robot system (11).

Proposition 4 *Consider the nonholonomic kinematic model (11), the dynamic compensator (17), and the dynamic linearized system (21). Let $x_d(t)$ and $y_d(t)$ be a set of desired trajectories for the position coordinates x and y which is collision-free in its working space. Moreover, we have two Hurwitz polynomials as follows*

$$\begin{aligned} H_1(s) &= s^3 + \lambda_2\, s^2 + \lambda_1\, s + \lambda_0 \\ H_2(s) &= s^3 + \gamma_2\, s^2 + \gamma_1\, s + \gamma_0 \end{aligned} \tag{25}$$

Constructing a control law in the new coordinates (20) as the following form

$$\begin{aligned} v_1 &= x_d^{(3)} - \lambda_2\,(z_3 - \ddot{x}_d) - \lambda_1\,(z_2 - \dot{x}_d) - \lambda_0\,(z_1 - x_d) \\ v_2 &= y_d^{(3)} - \gamma_2\,(z_6 - \ddot{y}_d) - \gamma_1\,(z_5 - \dot{y}_d) - \gamma_0\,(z_4 - y_d) \end{aligned} \tag{26}$$

Thus, if the constant real coefficients $\{\lambda_2, \lambda_1, \lambda_0\}$ and $\{\gamma_2, \gamma_1, \gamma_0\}$ are chosen appropriately such that

1. *The change of coordinates (20) is invertible;*
2. *The decoupling matrix $\boldsymbol{\rho}(\boldsymbol{x})$is nonsingular.*

then, the following controller

$$\boldsymbol{u} = \boldsymbol{\rho}^{-1}(\boldsymbol{x})\,(\boldsymbol{v} - \boldsymbol{\alpha}(\boldsymbol{x})) \tag{27}$$

will render the robot to asymptotically track the desired trajectory from some initial tracking error $(x_e(0), y_e(0))^T$, and both x_e and y_e converge to zero as $t \to +\infty$. Where v is written as in (26).

Proof: Since $(x_e, y_e)^T = (x - x_d, y - y_d)^T$, we can construct a stable tracking error dynamics by taking advantage of *Hurwitz* polynomials $H_1(s)$ and $H_2(s)$ as follows

$$\begin{aligned} (x^{(3)} - x_d^{(3)}) + \lambda_2(\ddot{x} - \ddot{x}_d) + \lambda_1(\dot{x} - \dot{x}_d) &= \lambda_0(x_d - x) \\ (y^{(3)} - y_d^{(3)}) + \gamma_2\,(\ddot{y} - \ddot{y}_d) + \gamma_1\,(\dot{x} - \dot{y}_d) &= \gamma_0\,(y_d - y) \end{aligned} \tag{28}$$

Using $v_1 = x^{(3)}, v_2 = y^{(3)}$, and the change of coordinates (20) yields (26).
Thus, if the decoupling matrix $\boldsymbol{\rho}(\boldsymbol{x})$ is nonsingular and the inverse transformation of (20) exists, then one finds that the following control law

$$\boldsymbol{u} = \boldsymbol{\rho}^{-1}(\boldsymbol{x})\,(\boldsymbol{v} - \boldsymbol{\alpha}(\boldsymbol{x}))$$

is realizable and the trajectory tracking control problem can be solvable. □

Remark 2 *The obtained trajectory controller (27) can be used to parking purpose. In this case, the desired parking path which is collision-free and feasible for the controller (27) is required to supply ahead of time or it can be generated in real time. The detailed example for parking control with trajectory tracking method will be illustrated in the simulation section of this paper.*

4 Simulation Results

First, we illustrate the point-to-point tracking control for parking. Next, simulation is carried out for circular trajectory tracking control. For all the numerical examples demonstrated later, the parameters L and M are chosen to be 1 and $\pi/3$, respectively.

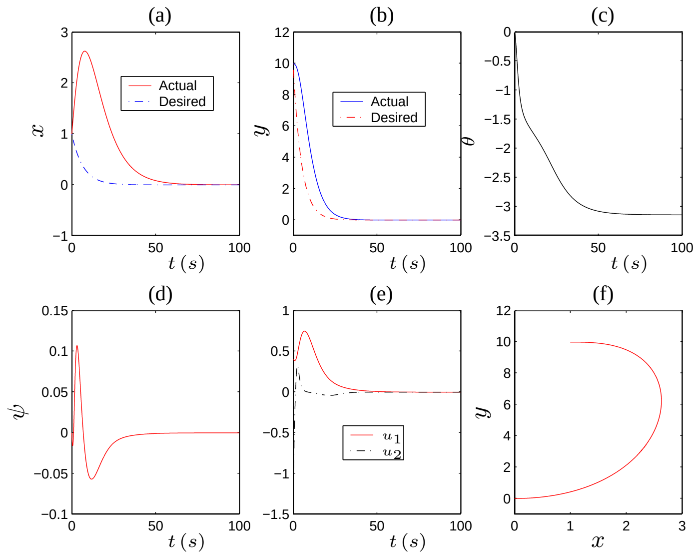

Fig. 2. Time histories of variables and inputs in the point-to-point tracking control for parking

4.1 Point-to-Point Tracking Control for Parking

In this simulation, the mobile robot is required to move from an initial *point* $\boldsymbol{x}(\mathbf{0})$ to another *point* $\boldsymbol{x}(\infty)$ following a desired trajectory $\boldsymbol{x_d}(\boldsymbol{t})$ as $t \to \infty$. For the parking problem via tracking control, we take the desired path in $x - y$ plane as the following form

$$\begin{aligned} x_d(t) &= A_1\, e^{-r_1\, t} + x(\infty) \\ y_d(t) &= A_2\, e^{-r_2\, t} + y(\infty) \end{aligned} \tag{29}$$

where $r_1, r_2 > 0, A_1 = x(0) - x(\infty)$, and $A_2 = y(0) - y(\infty)$. We pick the following choice of design parameters, initial state, and final condition

$$\begin{aligned} &\lambda_2 = 0.6, \lambda_1 = 0.11, \lambda_0 = 0.006 \\ &\gamma_2 = 0.9, \gamma_1 = 0.26, \gamma_0 = 0.024 \\ &r_1 = 0.15, r_2 = 0.20, x(0) = 1.0, y(0) = 10.0 \\ &\theta(0) = 0, \psi(0) = 0.0, u_1(0) = 0.4, p_1(0) = 0.0 \\ &x(\infty) = 0.0, y(\infty) = 0.0, \theta(\infty) = -\pi \\ &\psi(\infty) = 0.0, u_1(\infty) = 0.0, p_1(\infty) = 0.0 \end{aligned} \tag{30}$$

Since u_1 converges to zero as $t \to \infty$, from (24) the decoupling matrix $\boldsymbol{\rho}(\boldsymbol{x})$ will be singular. However, from (29) one finds that the parking position is also reached once the singularity occurs. Hence, there is no need for control action on the system when the decoupling matrix $\boldsymbol{\rho}(\boldsymbol{x})$ is singular.
Fig. 2 illustrates the time histories of variables and control inputs. Note that x and y can track the desired trajectory x_d and y_d, respectively. The tracking errors quickly converge to zero as time increases.

4.2 Circular Trajectory Tracking Control

For the circular trajectory tracking, the desired trajectory is defined by

$$x_d(t) = R\,\cos(\omega\, t), \quad y_d(t) = R\,\sin(\omega\, t) \tag{31}$$

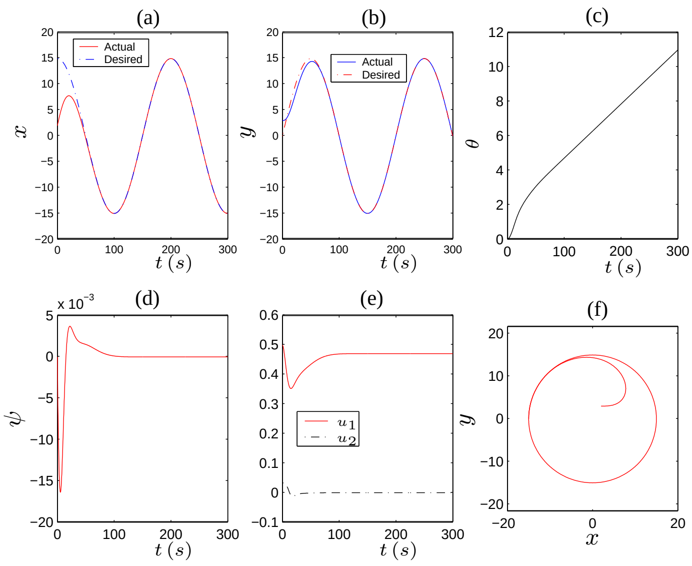

Fig. 3. Time histories of variables and inputs in the circular trajectory tracking control

where R and ω are appropriate positive constants.
The circular trajectory tracking for the robot was simulated with the following design parameters and initial conditions

$$\begin{aligned} &\lambda_2 = \gamma_2 = 0.3, \lambda_1 = \gamma_1 = 0.03, \lambda_0 = \gamma_0 = 0.001 \\ &R = 15, \omega = 0.01\,\pi, x(0) = 2.0, y(0) = 3.0 \\ &\theta(0) = 0, \psi(0) = 0.0, u_1(0) = 0.5, p_1(0) = 0.0 \end{aligned} \tag{32}$$

The time histories of variables and control inputs during the trajectory tracking control are plotted in Fig. 3.

5 Conclusion

The trajectory tracking control issue for a nonholonomic car-like mobile robot with a constrained state has been addressed in this paper. Compared with the existing works regarding the nonholonomic car-like mobile robots, the state constraint has been emphasized in this work. It has been shown that the dynamic feedback linearization is an efficient design tool for the trajectory tracking problem. Although there exists a constrained state in the car-like nonholonomic mobile robot system, the well-known dynamic feedback linearization technique has been used to deal with such a bounded state constraint by using a change of coordinate and input transformation. The corresponding results have been established in this paper. Furthermore, the established controller can be used to simultaneously solve both the tracking and regulation problems of the car-like mobile robot with the constrained state.
The effectiveness of the controller obtained for the trajectory tracking control was demonstrated by simulation results.

Acknowledgments

This research was performed in part under the support of the Japan Society for the Promotion of Science (**JSPS**). The authors also thank the Engineering and Physical Sciences Research Council (**EPSRC**) for its continuous funding under grant GR/S45508/01 (2003-2005).

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