B2I: Ball and Ball 2Dimension

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Forces u_1 is applied in the vertical direction, u_2 is applied in horizontal direction, both forces are applied to the ball 1

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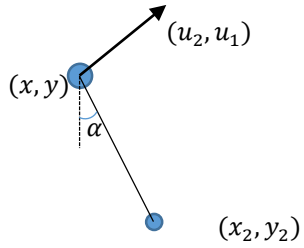

Mode 1:

$$\ddot{x} = \frac{m_2 \cos\alpha \sin\alpha\, u_1 + \left(2m_1 + m_2(1 + \cos^2(\alpha))\right) u_2 + (m_2 + 2m_1) m_2 l \sin(\alpha)\, \dot{\alpha}^2}{(m_1 + m_2)(2m_1 + m_2)}$$

$$\ddot{y} = -g + \frac{m_2 \cos\alpha \sin\alpha\, u_2 + \left(2m_1 + m_2(1 + \sin^2(\alpha))\right) u_1 - (m_2 + 2m_1) m_2 l \cos(\alpha)\, \dot{\alpha}^2}{(m_1 + m_2)(2m_1 + m_2)}$$

$$\ddot{\alpha} = -\frac{u_1 \sin(\alpha) + u_2 \cos(\alpha)}{l(2m + m_2)}$$

Mode 2:

$$\ddot{x} = \frac{1}{m_1} u_2$$

$$\ddot{y} = \frac{1}{m_1}(-m_1 g + u_1)$$

$$\ddot{x}_2 = 0$$

$$\ddot{y}_2 = -g$$

Guard:

$$h_{12} = \frac{m_2}{m_1 + m_2}(m_1 l \dot{\alpha}^2 + u_1 \cos(\alpha) - u_2 \sin(\alpha)) < 0 \rightarrow Change$$

$$x_2 = x + l \sin\alpha$$
$$y_2 = y - l \cos\alpha$$

$$\alpha = \frac{\pi}{2} + \text{atan2}(y_2 - y, x_2 - x)$$

$$h_{22} = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - l \geq 0 \ \wedge \frac{dh_{22}}{dt} > v_{trig} \rightarrow Change$$

$$e = -\frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}}$$

$$m_2 v_{2f} + m_1 v_{1f} = m_2 v_{2i} + m_1 v_{1i}$$

$$v_{2f} - v_{1f} = -e(v_{2i} - v_{1i})$$

$$v2i = \dot{x}_2 \sin(\alpha) - \dot{y}_2 \cos(\alpha)$$
$$v1i = \dot{x} \sin(\alpha) - \dot{y} \cos(\alpha)$$
$$v2f = \frac{(m2 * v2i + m * v1i) - m * e * (v2i - v1i)}{m + m2}$$
$$v1f = v2f + e * (v2i - v1i)$$

$$v2ort = \dot{x}_2 \cos(\alpha) + \dot{y}_2 \sin(\alpha)$$
$$v1ort = \dot{x} \cos(\alpha) + \dot{y} \sin(\alpha)$$

$$\dot{x}_2 = v2ort \cos(\alpha) + v2f \sin(\alpha)$$
$$\dot{y}_2 = v2ort \sin(\alpha) - v2f \cos(\alpha)$$
$$\dot{x} = v1ort \cos(\alpha) + v1f \sin(\alpha)$$
$$\dot{y} = v1ort \sin(\alpha) - v1f \cos(\alpha)$$

$$h_{21} = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - l \geq 0 \ \wedge v_{trig} > \frac{dh_{21}}{dt} > 0 \rightarrow Change$$
$$\dot{\alpha} = \frac{1}{l}\left((\dot{x}_2 - \dot{x}) * \cos\alpha + (\dot{y}_2 - \dot{y}) * \sin\alpha\right)$$