We have the dynamical system:

With:

Note that the system is driftless. Now, let’s see the controllability of the system. For that, we take Lie Bracket to the direction vectors , and we must be sure that they are linear independent of each other. In other words:

It can be prove by induction that the next terms are . We need now:

The pair is controllable in linear sense, because the matrix is full rank. This means, that the columns are linearly independent and automatically our last desired condition is hold (operating enough brackets until we get ). Note also that we are relying on the fact that the diagonal matrix has all its components different. Otherwise, the matrix would not be full rank.

Note also that this controllability is in the sense of the variables. Remember that and are rotations of an angle of the coordinates of to the final point .

In the MATLAB program we define a constant angular velocity . All robots will be spinning at that rate times (Recall , ). So we have the new system:

This system has the classic form of the linear systems in modern control. So we can apply a simple LQR controller here. Simulation and graphics are showed in MATLAB script. There is also an additional commented code where we make the angular velocity variable piecewise according to the total distance from each robot to their final points.

Simple Proof of requirement:

Suppose we have two robots with . Recalling their dynamics

Recalling controllability condition:

But not controllability doesn’t imply directly we can drive to 0. In fact, this is possible but only in a subspace of all the state space. In that case we must compute the Reachable set of the system, the values for which it is possible to have .

But in general, it is not always possible to drive to 0. Since it is the sum of squares of distances to the final points from the actual points, we will not be able to reach our desired position from any initial position, just only a subset of all possible points.