We have the dynamical system:

With:

Note that the system is driftless. Now, let’s see the controllability of the system. For that, we take Lie Bracket to the direction vectors , and we must be sure that they are linear independent of each other. In other words:

It can be prove by induction that the next terms are . We need now:

The pair is controllable in linear sense, because the matrix is full rank. This means, that the columns are linearly independent and automatically our last desired condition is hold (operating enough brackets until we get ). Note also that we are relying on the fact that the diagonal matrix has all its components different. Otherwise, the matrix would not be full rank.

Note also that this controllability is in the sense of the variables. Remember that and are rotations of an angle of the coordinates of to the final point .

In the MATLAB program we define a constant angular velocity . All robots will be spinning at that rate times (Recall , ). So we have the new system:

This system has the classic form of the linear systems in modern control. So we can apply a simple LQR controller here. Simulation and graphics are showed in MATLAB script. There is also an additional commented code where we make the angular velocity variable piecewise according to the total distance from each robot to their final points.