Lista 3 (Parte 3)

Sequências monótonas: 1, 4, 5, 7, 8,10,13,15,16,18.

1-6 Use $a_{n+1} - a_n$ para mostrar que a seqüência $\{a_n\}$ dada é estritamente crescente ou estritamente decrescente.

1. $\left\{\dfrac{1}{n}\right\}_{n=1}^{+\infty}$
2. $\left\{1 - \dfrac{1}{n}\right\}_{n=1}^{+\infty}$
3. $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$
4. $\left\{\dfrac{n}{4n-1}\right\}_{n=1}^{+\infty}$
5. $\{n - 2^n\}_{n=1}^{+\infty}$
6. $\{n - n^2\}_{n=1}^{+\infty}$

7-12 Use a_{n+1}/a_n para mostrar que a seqüência $\{a_n\}$ dada é estritamente crescente ou estritamente decrescente.

7. $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$
8. $\left\{\dfrac{2^n}{1+2^n}\right\}_{n=1}^{+\infty}$
9. $\{ne^{-n}\}_{n=1}^{+\infty}$
10. $\left\{\dfrac{10^n}{(2n)!}\right\}_{n=1}^{+\infty}$
11. $\left\{\dfrac{n^n}{n!}\right\}_{n=1}^{+\infty}$
12. $\left\{\dfrac{5^n}{2^{(n^2)}}\right\}_{n=1}^{+\infty}$

13-18 Use diferenciação para mostrar que a seqüência dada é estritamente crescente ou estritamente decrescente.

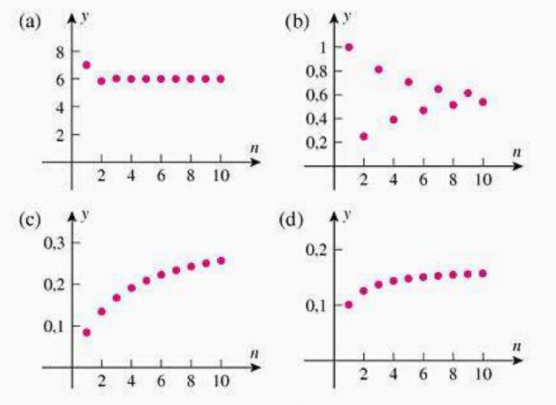
13. $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$
14. $\left\{3 - \dfrac{1}{n}\right\}_{n=1}^{+\infty}$
15. $\left\{\dfrac{1}{n + \ln n}\right\}_{n=1}^{+\infty}$
16. $\{ne^{-2n}\}_{n=1}^{+\infty}$
17. $\left\{\dfrac{\ln(n+2)}{n+2}\right\}_{n=1}^{+\infty}$
18. $\{\text{arc tg } n\}_{n=1}^{+\infty}$

Séries: 3,5,7,9,10,12,15,16.

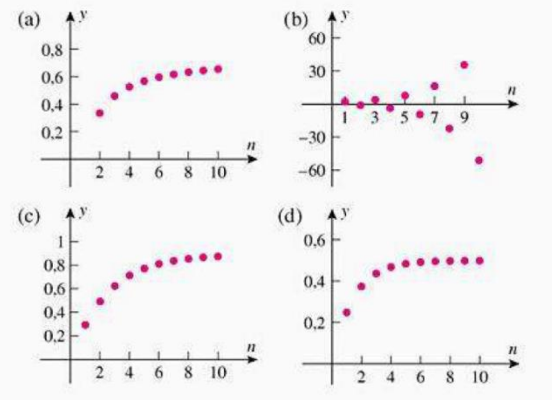
3-14 Determine se a série converge e, se convergir, encontre sua soma.

3. $\displaystyle\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1}$
4. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2}$
5. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k-1}\frac{7}{6^{k-1}}$
6. $\displaystyle\sum_{k=1}^{\infty}\left(-\frac{3}{2}\right)^{k+1}$
7. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$
8. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right)$
9. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{9k^2+3k-2}$
10. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k^2-1}$
11. $\displaystyle\sum_{k=3}^{\infty}\frac{1}{k-2}$
12. $\displaystyle\sum_{k=5}^{\infty}\left(\frac{e}{\pi}\right)^{k-1}$
13. $\displaystyle\sum_{k=1}^{\infty}\frac{4^{k+2}}{7^{k-1}}$
14. $\displaystyle\sum_{k=1}^{\infty}5^{3k}7^{1-k}$

15. Associe uma série de um dos Exercícios 3, 5, 7 ou 9 com o gráfico da seqüência de suas somas parciais.



16. Associe uma série de um dos Exercícios 4, 6, 8 ou 10 com o gráfico da seqüência de suas somas parciais.



Testes de convergência: 3, 4, 7(a), 8(a), 9, 10, 12,13,14, 16,17, 20, 21.

3-4 Para cada série p dada, identifique p e determine se a série converge.

3. (a) $\sum_{k=1}^{\infty} \frac{1}{k^3}$ (b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ (c) $\sum_{k=1}^{\infty} k^{-1}$ (d) $\sum_{k=1}^{\infty} k^{-2/3}$

4. (a) $\sum_{k=1}^{\infty} k^{-4/3}$ (b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k}}$ (c) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^5}}$ (d) $\sum_{k=1}^{\infty} \frac{1}{k^{\pi}}$

7-8 Confirme ser aplicável o teste da integral e use-o para determinar se a série converge.

7. (a) $\sum_{k=1}^{\infty} \frac{1}{5k+2}$ (b) $\sum_{k=1}^{\infty} \frac{1}{1+9k^2}$

8. (a) $\sum_{k=1}^{\infty} \frac{k}{1+k^2}$ (b) $\sum_{k=1}^{\infty} \frac{1}{(4+2k)^{3/2}}$

9-24 Determine se a série converge.

9. $\sum_{k=1}^{\infty} \frac{1}{k+6}$
10. $\sum_{k=1}^{\infty} \frac{3}{5k}$
11. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+5}}$
12. $\sum_{k=1}^{\infty} \frac{1}{\sqrt[k]{e}}$
13. $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$
14. $\sum_{k=3}^{\infty} \frac{\ln k}{k}$
15. $\sum_{k=1}^{\infty} \frac{k}{\ln(k+1)}$
16. $\sum_{k=1}^{\infty} k e^{-k^2}$
17. $\sum_{k=1}^{\infty} \left(1+\frac{1}{k}\right)^{-k}$
18. $\sum_{k=1}^{\infty} \frac{k^2+1}{k^2+3}$
19. $\sum_{k=1}^{\infty} \frac{\operatorname{arc\,tg} k}{1+k^2}$
20. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2+1}}$
21. $\sum_{k=1}^{\infty} k^2 \operatorname{sen}^2\left(\frac{1}{k}\right)$
22. $\sum_{k=1}^{\infty} k^2 e^{-k^3}$
23. $\sum_{k=5}^{\infty} 7k^{-1,01}$
24. $\sum_{k=1}^{\infty} \operatorname{sech}^2 k$

Teste da comparação, razão e raiz:

1(a), 2(b), 3(a), 4(a), 11,14,16, 17, 18, 20, 21,22,23,25.

1-2 Faça uma conjectura sobre a convergência ou a divergência da série e confirme-a usando o teste da comparação.

1. (a) $\sum_{k=1}^{\infty} \frac{1}{5k^2-k}$ (b) $\sum_{k=1}^{\infty} \frac{3}{k-\frac{1}{4}}$

2. (a) $\sum_{k=2}^{\infty} \frac{k+1}{k^2-k}$ (b) $\sum_{k=1}^{\infty} \frac{2}{k^4+k}$

3. Em cada parte, use o teste da comparação para mostrar que a série converge.
(a) $\sum_{k=1}^{\infty} \frac{1}{3^k+5}$ (b) $\sum_{k=1}^{\infty} \frac{5 \operatorname{sen}^2 k}{k!}$

4. Em cada parte, use o teste da comparação para mostrar que a série diverge.
(a) $\sum_{k=1}^{\infty} \frac{\ln k}{k}$ (b) $\sum_{k=1}^{\infty} \frac{k}{k^{3/2}-\frac{1}{2}}$

11-16 Use o teste da razão para determinar se a série converge. Se o teste for inconclusivo, diga isso.

11. $\sum_{k=1}^{\infty} \frac{3^k}{k!}$
12. $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$
13. $\sum_{k=1}^{\infty} \frac{1}{5k}$
14. $\sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k$
15. $\sum_{k=1}^{\infty} \frac{k!}{k^3}$
16. $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$

17-20 Use o teste da raiz para determinar se a série converge. Se o teste for inconclusivo, diga isso.

17. $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$
18. $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$
19. $\sum_{k=1}^{\infty} \frac{k}{5^k}$
20. $\sum_{k=1}^{\infty} (1-e^{-k})^k$

21-44 Use qualquer método para determinar se a série converge.

21. $\sum_{k=0}^{\infty} \frac{7^k}{k!}$
22. $\sum_{k=1}^{\infty} \frac{1}{2k+1}$
23. $\sum_{k=1}^{\infty} \frac{k^2}{5^k}$
24. $\sum_{k=1}^{\infty} \frac{k!10^k}{3^k}$
25. $\sum_{k=1}^{\infty} k^{50} e^{-k}$
26. $\sum_{k=1}^{\infty} \frac{k^2}{k^3+1}$
27. $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^3+1}$
28. $\sum_{k=1}^{\infty} \frac{4}{2+3^k k}$
29. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$
30. $\sum_{k=1}^{\infty} \frac{2+(-1)^k}{5^k}$
31. $\sum_{k=1}^{\infty} \frac{2+\sqrt{k}}{(k+1)^3-1}$
32. $\sum_{k=1}^{\infty} \frac{4+|\cos k|}{k^3}$
33. $\sum_{k=1}^{\infty} \frac{1}{1+\sqrt{k}}$
34. $\sum_{k=1}^{\infty} \frac{k!}{k^k}$
35. $\sum_{k=1}^{\infty} \frac{\ln k}{e^k}$

Respostas

1. $a_{n+1}-a_n=\dfrac{1}{n+1}-\dfrac{1}{n}=-\dfrac{1}{n(n+1)}<0$ for $n\geq 1$, so strictly decreasing.

4. $a_{n+1}-a_n=\dfrac{n+1}{4n+3}-\dfrac{n}{4n-1}=-\dfrac{1}{(4n-1)(4n+3)}<0$ for $n\geq 1$, so strictly decreasing.

5. $a_{n+1}-a_n=(n+1-2^{n+1})-(n-2^n)=1-2^n<0$ for $n\geq 1$, so strictly decreasing.

7. $\dfrac{a_{n+1}}{a_n}=\dfrac{(n+1)/(2n+3)}{n/(2n+1)}=\dfrac{(n+1)(2n+1)}{n(2n+3)}=\dfrac{2n^2+3n+1}{2n^2+3n}>1$ for $n\geq 1$, so strictly increasing.

8. $\dfrac{a_{n+1}}{a_n}=\dfrac{2^{n+1}}{1+2^{n+1}}\cdot\dfrac{1+2^n}{2^n}=\dfrac{2+2^{n+1}}{1+2^{n+1}}=1+\dfrac{1}{1+2^{n+1}}>1$ for $n\geq 1$, so strictly increasing.

10. $\dfrac{a_{n+1}}{a_n}=\dfrac{10^{n+1}}{(2n+2)!}\cdot\dfrac{(2n)!}{10^n}=\dfrac{10}{(2n+2)(2n+1)}<1$ for $n\geq 1$, so strictly decreasing.

13. $f(x)=x/(2x+1)$, $f'(x)=1/(2x+1)^2>0$ for $x\geq 1$, so strictly increasing.

15. $f(x)=1/(x+\ln x)$, $f'(x)=-\dfrac{1+1/x}{(x+\ln x)^2}<0$ for $x\geq 1$, so strictly decreasing.

16. $f(x)=xe^{-2x}$, $f'(x)=(1-2x)e^{-2x}<0$ for $x\geq 1$, so strictly decreasing.

18. $f(x)=\tan^{-1}x$, $f'(x)=1/(1+x^2)>0$ for $x\geq 1$, so strictly increasing.

3. geometric, $a=1$, $r=-3/4$, sum $=\dfrac{1}{1-(-3/4)}=4/7$

5. geometric, $a=7$, $r=-1/6$, sum $=\dfrac{7}{1+1/6}=6$

7. $s_n=\displaystyle\sum_{k=1}^{n}\left(\frac{1}{k+2}-\frac{1}{k+3}\right)=\frac{1}{3}-\frac{1}{n+3}$, $\displaystyle\lim_{n\to+\infty}s_n=1/3$

9. $s_n=\displaystyle\sum_{k=1}^{n}\left(\frac{1/3}{3k-1}-\frac{1/3}{3k+2}\right)=\frac{1}{6}-\frac{1/3}{3n+2}$, $\displaystyle\lim_{n\to+\infty}s_n=1/6$

10. $s_n=\displaystyle\sum_{k=2}^{n+1}\left[\frac{1/2}{k-1}-\frac{1/2}{k+1}\right]=\frac{1}{2}\left[\sum_{k=2}^{n+1}\frac{1}{k-1}-\sum_{k=2}^{n+1}\frac{1}{k+1}\right]$

$\displaystyle=\frac{1}{2}\left[\sum_{k=2}^{n+1}\frac{1}{k-1}-\sum_{k=4}^{n+3}\frac{1}{k-1}\right]=\frac{1}{2}\left[1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right];\ \lim_{n\to+\infty}s_n=\frac{3}{4}$

12. geometric, $a=(e/\pi)^4$, $r=e/\pi<1$, sum $=\dfrac{(e/\pi)^4}{1-e/\pi}=\dfrac{e^4}{\pi^3(\pi-e)}$

15 e 16: não tenho a resposta. Plotar alguns valores para ver como fica o gráfico.

3. **(a)** p=3, converges

4. **(a)** p=4/3, converges

7. **(a)** $\displaystyle\int_1^{+\infty}\frac{1}{5x+2}=\lim_{\ell\to+\infty}\frac{1}{5}\ln(5x+2)\Big]_1^{\ell}=+\infty$, the series diverges by the Integral Test.

8. **(a)** $\displaystyle\int_1^{+\infty}\frac{x}{1+x^2}dx=\lim_{\ell\to+\infty}\frac{1}{2}\ln(1+x^2)\Big]_1^{\ell}=+\infty$, the series diverges by the Integral Test.

9. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k+6}=\sum_{k=7}^{\infty}\frac{1}{k}$, diverges because the harmonic series diverges.

10. $\displaystyle\sum_{k=1}^{\infty}\frac{3}{5k}=\sum_{k=1}^{\infty}\frac{3}{5}\left(\frac{1}{k}\right)$, diverges because the harmonic series diverges.

12. $\displaystyle\lim_{k\to+\infty}\frac{1}{e^{1/k}}=1$, the series diverges because $\displaystyle\lim_{k\to+\infty}u_k=1\neq 0$.

13. $\displaystyle\int_1^{+\infty}(2x-1)^{-1/3}dx=\lim_{\ell\to+\infty}\frac{3}{4}(2x-1)^{2/3}\Big]_1^{\ell}=+\infty$, the series diverges by the Integral Test.

14. $\dfrac{\ln x}{x}$ is decreasing for $x\geq e$, and $\displaystyle\int_3^{+\infty}\frac{\ln x}{x}=\lim_{\ell\to+\infty}\frac{1}{2}(\ln x)^2\Big]_3^{\ell}=+\infty$,
so the series diverges by the Integral Test.

16. $\displaystyle\int_1^{+\infty}xe^{-x^2}dx=\lim_{\ell\to+\infty}-\frac{1}{2}e^{-x^2}\Big]_1^{\ell}=e^{-1}/2$, the series converges by the Integral Test.

17. $\displaystyle\lim_{k\to+\infty}(1+1/k)^{-k}=1/e\neq 0$, the series diverges.

20. $\displaystyle\int_1^{+\infty}\frac{1}{\sqrt{x^2+1}}dx=\lim_{\ell\to+\infty}\sinh^{-1}x\Big]_1^{\ell}=+\infty$, the series diverges by the Integral Test.

21. $\displaystyle\lim_{k\to+\infty}k^2\sin^2(1/k)=1\neq 0$, the series diverges.

1. **(a)** $\dfrac{1}{5k^2-k}\leq\dfrac{1}{5k^2-k^2}=\dfrac{1}{4k^2}$, $\displaystyle\sum_{k=1}^{\infty}\frac{1}{4k^2}$ converges, so the original series also converges.

2 **(b)** $\dfrac{2}{k^4+k}<\dfrac{2}{k^4}$, $\displaystyle\sum_{k=1}^{\infty}\frac{2}{k^4}$ converges, so the original series also converges.

3. **(a)** $\dfrac{1}{3^k+5}<\dfrac{1}{3^k}$, $\displaystyle\sum_{k=1}^{\infty}\frac{1}{3^k}$ converges, so the original series also converges.

4. **(a)** $\dfrac{\ln k}{k}>\dfrac{1}{k}$ for $k\geq 3$, $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k}$ diverges, so the original series also diverges.

11. $\rho=\displaystyle\lim_{k\to+\infty}\frac{3^{k+1}/(k+1)!}{3^k/k!}=\lim_{k\to+\infty}\frac{3}{k+1}=0<1$, the series converges.

14. $\rho=\displaystyle\lim_{k\to+\infty}\frac{(k+1)(1/2)^{k+1}}{k(1/2)^k}=\lim_{k\to+\infty}\frac{k+1}{2k}=1/2<1$, the series converges.

16. $\rho=\displaystyle\lim_{k\to+\infty}\frac{(k+1)/[(k+1)^2+1]}{k/(k^2+1)}=\lim_{k\to+\infty}\frac{(k+1)(k^2+1)}{k(k^2+2k+2)}=1$, the result is inconclusive.

17. $\rho=\displaystyle\lim_{k\to+\infty}\frac{3k+2}{2k-1}=3/2>1$, the series diverges.

18. $\rho=\displaystyle\lim_{k\to+\infty}k/100=+\infty$, the series diverges.

20. $\rho=\displaystyle\lim_{k\to+\infty}(1-e^{-k})=1$, the result is inconclusive.

21. Ratio Test, $\rho=\displaystyle\lim_{k\to+\infty}7/(k+1)=0$, converges.

22. Limit Comparison Test, compare with the divergent series $\displaystyle\sum_{k=1}1/k$, $\rho=\displaystyle\lim_{k\to+\infty}\frac{k}{2k+1}=1/2$, which is finite and positive, therefore the original series diverges.

23. Ratio Test, $\rho=\displaystyle\lim_{k\to+\infty}\frac{(k+1)^2}{5k^2}=1/5<1$, converges.

25. Ratio Test, $\rho=\displaystyle\lim_{k\to+\infty}e^{-1}(k+1)^{50}/k^{50}=e^{-1}<1$, converges.