# Lista V

Cálculo Diferencial e Integral I — 22 de junho de 2024

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## Exercício 1

Calcule as seguintes integrais.

**a** $\int (x+1)\,dx$

**b** $\int \left(3t^2 + \frac{t}{2}\right)\,dt$

**c** $\int (2x^3 - 5x + 7)\,dx$

**d** $\int \left(\frac{1}{x^2} - x^2 - \frac{1}{3}\right)\,dx$

**e** $\int x^{-1/3}\,dx$

**f** $\int (\sqrt{x} + \sqrt[3]{x})\,dx$

**g** $\int \left(8y - \frac{2}{y^{1/4}}\right)\,dy$

**h** $\int 2x(1 - x^{-1/3})\,dx$

**i** $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\,dt$

**j** $\int (-2\cos t)\,dt$

**k** $\int 7\sin\frac{\theta}{3}\,d\theta$

**l** $\int (-3\csc^2 x)\,dx$

**m** $\int \frac{\csc\theta\cot\theta}{2}\,d\theta$

**n** $\int (e^x + 5e^{-x})\,dx$

**o** $\int (e^{-x} + 4^x)\,dx$

**p** $\int (4\sec x\tan x - 2\sec^2 x)\,dx$

**q** $\int (\sin 2x - \csc^2 x)\,dx$

**r** $\int \frac{1+\cos 4t}{2}\,dt$

**s** $\int \left(\frac{1}{x} - \frac{5}{x^2+1}\right)\,dx$

**t** $\int 3x^{\sqrt{3}}\,dx$

**u** $\int (1 + \tan^2\theta)\,d\theta$

**v** $\int \cot^2 x\,dx$

**w** $\int \cos\theta(\tan\theta + \sec\theta)\,d\theta$

## Exercício 2

Diga se cada uma das igualdades a seguir é verdadeira ou falsa. Justifique sua resposta.

**a** $\int (2x+1)^2\,dx = \frac{(2x+1)^3}{3} + C$

**b** $\int 3(2x+1)^2\,dx = (2x+1)^3 + C$

**c** $\int \sqrt{2x+1}\,dx = \sqrt{x^2+x+C}$

**d** $\int \sqrt{2x+1}\,dx = \sqrt{x^2+x} + C$

## Exercício 3

(*)Determine a curva $y = f(x)$ que passa pelo ponto (9,4) e cujo coeficiente angular em cada ponto é $3\sqrt{x}$.

## Exercício 4

Suponha que $f$ e $g$ seja integráveis e que

$$\int_1^2 f(x)\,dx = -4, \quad \int_1^5 f(x)\,dx = 6, \quad \int_1^5 g(x)\,dx = 8.$$

Determine:

**a** $\int_2^2 g(x)\,dx$

**b** $\int_5^1 g(x)\,dx$

**c** $\int_1^2 3f(x)\,dx$

**d** $\int_2^5 f(x)\,dx$

**e** $\int_1^5 [f(x) - g(x)]\,dx$

**f** $\int_1^5 [4f(x) - g(x)]\,dx$

## Exercício 5

Calcule as integrais definidas a seguir.

**a** $\displaystyle\int_{-2}^{0} (2x+5)\,dx$

**b** $\displaystyle\int_{0}^{2} x(x-3)\,dx$

**c** $\displaystyle\int_{0}^{4} \left(3x - \frac{x^3}{4}\right) dx$

**d** $\displaystyle\int_{0}^{1} (x^2 + \sqrt{x})\,dx$

**e** $\displaystyle\int_{0}^{\pi/3} 2\sec^2 x\,dx$

**f** $\displaystyle\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\,d\theta$

**g** $\displaystyle\int_{\pi/2}^{0} \frac{1+\cos 2t}{2}\,dt$

**h** $\displaystyle\int_{0}^{\pi/4} \tan^2 x\,dx$

**i** $\displaystyle\int_{0}^{\pi/8} \sin 2x\,dx$

**j** $\displaystyle\int_{1}^{-1} (r+1)^2\,dr$

**k** $\displaystyle\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du$

**l** $\displaystyle\int_{1}^{\sqrt{2}} \frac{s^2+\sqrt{s}}{s^2}\,ds$

**m** $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin 2x}{2\sin x}\,dx$

**n** $\displaystyle\int_{-4}^{4} |x|\,dx$

**o** $\displaystyle\int_{0}^{\ln 2} e^{3x}\,dx$

**p** $\displaystyle\int_{0}^{1/2} \frac{4}{\sqrt{1-x^2}}\,dx$

**q** $\displaystyle\int_{2}^{4} x^{\pi-1}\,dx$

## Exercício 6

Determine a área *total* da região limitada pela curva $y = f(x)$ e pelo eixo $x$ nos intervalos especificados.

**a** $f(x) = -x^2 - 2x,\ -3 \le x \le 2$

**b** $f(x) = 3x^2 - 3,\ -2 \le x \le 2$

**c** $f(x) = -x^3 - 3x^2 + 2x,\ 0 \le x \le 2$

**d** $f(x) = x^3 - 4x,\ -2 \le x \le 2$

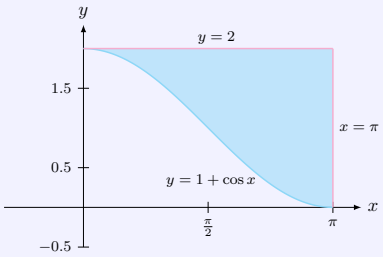
**e** $f(x) = \sqrt[3]{x},\ -1 \le x \le 8$

**f** $f(x) = \sqrt[3]{x} - x,\ -1 \le x \le 8$

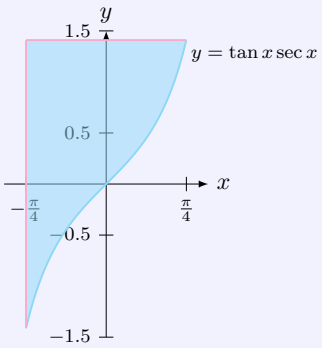
## Exercício 7

(**)Determine o valor das áreas hachuradas a seguir.

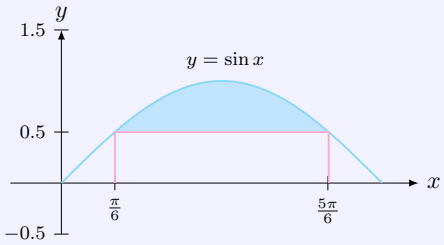
**a**



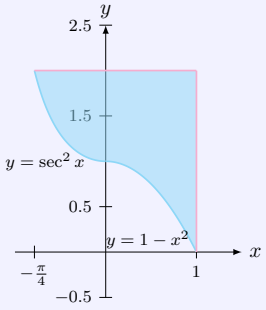

**c**




**b**




**d**



## Exercício 8

Avalie as integrais fazendo-se uma substituição apropriada.

**a** $\int (4x-3)^9\,dx$

**b** $\int x^3\sqrt{5+x^4}\,dx$

**c** $\int \sin 7x\,dx$

**d** $\int \cos\dfrac{x}{3}\,dx$

**e** $\int \sec 4x\tan 4x\,dx$

**f** $\int \sec^2 5x\,dx$

**g** $\int e^{2x}\,dx$

**h** $\int \dfrac{dx}{2x}$

**i** $\int \dfrac{1}{\sqrt{1-4x^2}}\,dx$

**j** $\int \dfrac{1}{1+16x^2}\,dx$

**k** $\int t\sqrt{7t^2+12}\,dt$

**l** $\int \dfrac{x}{\sqrt{4-5x^2}}\,dx$

**m** $\int \dfrac{6}{(1-2x)^3}\,dx$

**n** $\int \dfrac{x^2+1}{\sqrt{x^3+3x}}\,dx$

**o** $\int \dfrac{x^3}{(5x^4+2)^3}\,dx$

**p** $\int \dfrac{\sin(1/x)}{3x^2}\,dx$

**q** $\int e^{\sin x}\cos x\,dx$

**r** $\int x^3e^{x^4}\,dx$

**s** $\int x^2e^{-2x^3}\,dx$

**t** $\int \dfrac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

**u** $\int \dfrac{e^x}{1+e^{2x}}\,dx$

**v** $\int \dfrac{t}{t^4+1}\,dt$

**w** $\int \dfrac{\sin(5/x)}{x^2}\,dx$

**x** $\int \dfrac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$

## Exercício 9

Use a regra da substituição para calcular as integrais a seguir.

**a** $\displaystyle\int_0^3 \sqrt{y+1}\,dy$

**b** $\displaystyle\int_0^1 r\sqrt{1-r^2}\,dr$

**c** $\displaystyle\int_0^{\pi/4} \tan x\sec^2 x\,dx$

**d** $\displaystyle\int_0^{\pi} 3\cos^2 x\sin x\,dx$

**e** $\displaystyle\int_0^1 t^3(1+t^4)^3\,dt$

**f** $\displaystyle\int_0^{\sqrt{7}} t(t^2+1)^{1/3}\,dt$

**g** $\displaystyle\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr$

**h** $\displaystyle\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv$

**i** $\displaystyle\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx$

**j** $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx$

**k** $\displaystyle\int_{-\pi/2}^0 \left(2+\tan\frac{t}{2}\sec^2\frac{t}{2}\right)\,dt$

**l** $\displaystyle\int_0^{2\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\,dz$

**m** $\displaystyle\int_{-\pi/2}^0 \frac{\sin w}{(3+2\cos w)^2}\,dw$

**n** $\displaystyle\int_0^1 \sqrt{t^5+2t}(5t^4+2)\,dt$

**o** $\displaystyle\int_0^{\pi/6} \cos^{-3}(2\theta)\sin 2\theta\,d\theta$

**p** $\displaystyle\int_0^{\pi} 5(5-4\cos t)^{1/4}\sin t\,dt$

**q** $\displaystyle\int_0^1 (4y-y^2+4y^3+1)^{-2/3}(12y^2-2y+4)\,dy$

**r** $\displaystyle\int_0^1 (y^3+6y^2-12y+9)^{-1/2}(y^2+4y-4)\,dy$

## Exercício 10

Avalie as integrais usando a técnica de integração por partes.

**a** $\int x \sin \frac{x}{2}\, dx$

**b** $\int \theta \cos \pi\theta\, d\theta$

**c** $\int t^2 \cos t\, dt$

**d** $\int x^2 \sin x\, dx$

**e** $\int_1^2 x \ln x\, dx$

**f** $\int_1^e x^3 \ln x\, dx$

**g** $\int \tan^{-1} y\, dy$

**h** $\int \sin^{-1} y\, dy$

**i** $\int x \sec^2 x\, dx$

**j** $\int 4x \sec^2 2x\, dx$

**k** $\int x^3 e^x\, dx$

**l** $\int p^4 e^{-p}\, dp$

**m** $\int (x^2 - 5x)e^x\, dx$

**n** $\int (r^2 + r + 1)e^r\, dr$

**o** $\int x^5 e^x\, dx$

**p** $\int t^2 e^{4t}\, dt$

**q** $\int_0^{\pi/2} \theta^2 \sin 2\theta\, d\theta$

**r** $\int_0^{\pi/2} x^3 \cos 2x\, dx$

**s** $\int_{2/\sqrt{3}}^2 t \sec^{-1} t\, dt$

**t** $\int_0^{1/\sqrt{2}} 2x sin^{-1}(x^2)\, dx$

**u** $\int e^\theta \sin\theta\, d\theta$

**v** $\int e^{-y} \cos y\, dy$

**w** $\int e^{2x} \cos 3x\, dx$

**x** $\int e^{-2x} \sin 2x\, dx$

## Exercício 11

Avalie as integrais usando frações parciais:

**a** $\int \frac{1}{x(x^2+4)^2}\, dx$

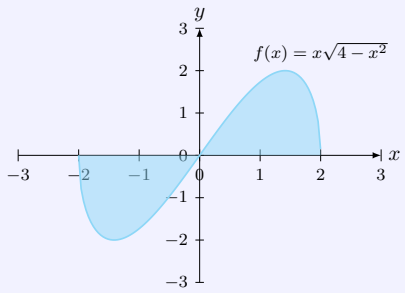
**b** $\int \frac{x^2 + 3x - 1}{x^3 - x}\, dx$

**c** $\int \frac{10}{(x-1)(x^2+9)}\, dx$

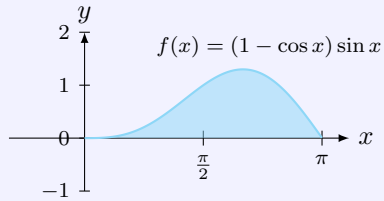
## Exercício 12

(**)Determine o valor das áreas hachuradas.

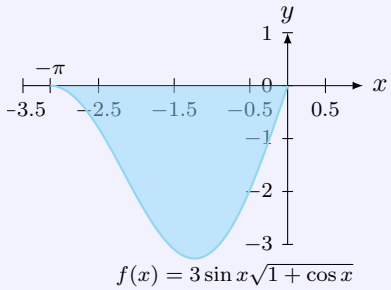
**a**



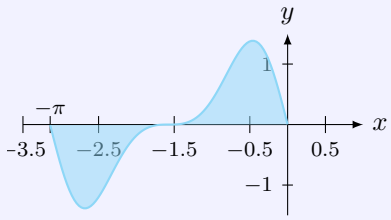

**b**



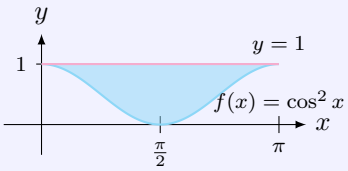

**c**



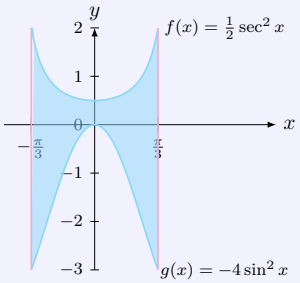

**d**



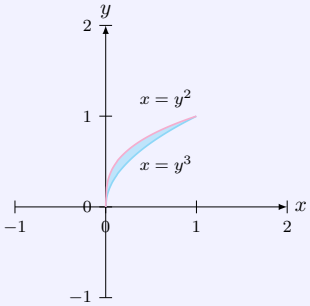

**e**



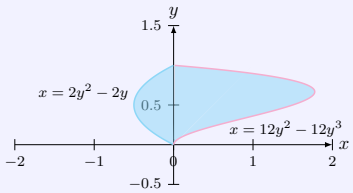

**f**




**g**




**h**




## Exercício 13

Calcule as integrais impróprias abaixo ou diga quando são divergentes.

**a** $\displaystyle\int_2^\infty e^{-5x}\,dx$

**b** $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\,dx$

**c** $\displaystyle\int_0^1 \frac{1}{x}\,dx$

**d** $\displaystyle\int_{-2}^3 \frac{1}{x^4}\,dx$

**e** $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$

**f** $\displaystyle\int_0^1 x\ln x\,dx$

**GABARITO**

## Resp. Exerc. 1

**a** $\frac{x^2}{2} + x + C$

**b** $t^3 + \frac{t^2}{4} + C$

**c** $\frac{x^4}{2} - 5\frac{x^2}{2} + 7x + C$

**d** $-\frac{1}{x} - \frac{x^3}{3} - \frac{x}{3} + C$

**e** $\frac{3}{2}x^{2/3} + C$

**f** $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

**g** $4y^2 - \frac{8}{3}y^{3/4} + C$

**h** $x^2 - \frac{6}{5}x^{5/3} + C$

**i** $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

**j** $-2\sin t + C$

**k** $-21\cos\frac{\theta}{3} + C$

**l** $3\cot x + C$

**m** $-\frac{1}{2}\csc\theta + C$

**n** $e^x - 5e^{-x} + C$

**o** $-e^{-x} + \frac{4^x}{\ln 4} + C$

**p** $4\sec x - 2\tan x + C$

**q** $-\frac{1}{2}\cos 2x + \cot x + C$

**r** $\frac{t}{2} + \frac{\sin 4t}{8} + C$

**s** $\ln|x| - 5\tan^{-1}x + C$

**t** $\frac{3x^{\sqrt{3}+1}}{\sqrt{3}+1} + C$

**u** $\tan\theta + C$

**v** $-\cot x - x + C$

**w** $-\cos\theta + \theta + C$

## Resp. Exerc. 2

**a** Inválida. Derive o lado direito para ver.

**b** Inválida. Derive o lado direito para ver.

**c** Inválida. Derive o lado direito para ver.

**d** Inválida. Derive o lado direito para ver.

## Resp. Exerc. 3

$y = 2x^{3/2} - 50$.

## Resp. Exerc. 4

**a** 0 **b** $-8$ **c** $-12$ **d** 10 **e** $-2$ **f** 16

## Resp. Exerc. 5

**a** 6

**b** $-10/3$

**c** 8

**d** 1

**e** $2\sqrt{3}$

**f** 0

**g** $-\pi/4$

**h** $1 - \frac{\pi}{4}$

**i** $\frac{2-\sqrt{2}}{4}$

**j** $-8/3$

**k** $-3/4$

**l** $\sqrt{2} - \sqrt[4]{8} + 1$

**m** $-1$

**n** 16

**o** 7/3

**p** $2\pi/3$

**q** $\frac{1}{\pi}(4^\pi - 2^\pi)$

## Resp. Exerc. 6

**a** 28/3 **b** 12 **c** 1/2 **d** 8 **e** 51/4 **f** 83/4

## Resp. Exerc. 7

**a** $\pi$ **b** $\sqrt{3} - \frac{\pi}{3}$ **c** $\frac{\pi\sqrt{2}}{2}$ **d** $\frac{1}{3} + \frac{\pi}{2}$

## Resp. Exerc. 8

**a** $\frac{1}{40}(4x-3)^9+C$

**b** $\frac{1}{6}(5+x^4)^{3/2}+C$

**c** $-\frac{1}{7}\cos 7x+C$

**d** $3\sin(x/3)+C$

**e** $\frac{1}{4}\sec 4x+C$

**f** $\frac{1}{5}\tan 5x+C$

**g** $\frac{1}{2}e^{2x}+C$

**h** $\frac{1}{2}\ln|2x|+C$

**i** $\frac{1}{2}\sin^{-1}(2x)+C$

**j** $\frac{1}{4}\tan^{-1}(4x)+C$

**k** $\frac{1}{21}(7t^2+12)^{3/2}+C$

**l** $-\frac{1}{5}\sqrt{4-5x^2}+C$

**m** $\frac{3}{2}\frac{1}{(1-2x)^3}+C$

**n** $\frac{2}{3}\sqrt{x^3+3x}+C$

**o** $-\frac{1}{40(5x^4+2)^2}+C$

**p** $\frac{1}{3}\cos(1/x)+C$

**q** $e^{\sin x}+C$

**r** $\frac{1}{4}e^{x^4}+C$

**s** $-\frac{1}{6}e^{-2x^3}+C$

**t** $\ln|e^x-e^{-x}|+C$

**u** $\tan^{-1}(e^x)+C$

**v** $\frac{1}{2}\tan^{-1}(t^2)+C$

**w** $\frac{1}{5}\cos(5/x)+C$

**x** $2\tan\sqrt{x}+C$

## Resp. Exerc. 9

**a** 14/3

**b** 1/3

**c** 1/2

**d** 2

**e** 15/16

**f** 45/8

**g** 0

**h** 10/3

**i** 4

**j** $\frac{\sqrt{10}-3}{2}$

**k** 1/6

**l** 3

**m** 0

**n** $-1/15$

**o** $2\sqrt{3}$

**p** 3/4

**q** $3^{5/2}-1$

**r** 3

**s** $-2/3$

## Resp. Exerc. 10

**a** $-2x\cos\frac{x}{2}+4\sin\frac{x}{2}+C$

**b** $\frac{\theta}{\pi}\sin\pi\theta+\frac{1}{\pi^2}\cos\pi\theta+C$

**c** $t^2\sin t+2t\cos t-2\sin t+C$

**d** $-x^2\cos x+2x\sin x+2\cos x+C$

**e** $\ln 4-\frac{3}{4}$

**f** $\frac{3e^4+1}{16}$

**g** $y\tan^{-1}y-\frac{1}{2}\ln(1+y^2)+C$

**h** $y\sin^{-1}y+\sqrt{1-y^2}+C$

**i** $x\tan x+\ln|\cos x|+C$

**j** $2x\tan 2x-\ln|\sec 2x|+C$

**k** $(x^3-3x^2+6x-6)e^x+C$

**l** $-(p^4+4p^3+12p^2+24p+24)e^{-p}+C$

**m** $(x^2-7x+7)e^x+C$

**n** $(r^2-r+2)e^r+C$

**o** $(x^5-5x^4+20x^3-60x^2+!20x-120)e^x+C$

**p** $\left(\frac{t^2}{4}-\frac{t}{8}+\frac{1}{32}\right)e^{4t}+C$

**q** $\frac{\pi^2-4}{8}$

**r** $\frac{3(4-\pi^2)}{16}$

**s** $\frac{5\pi-3\sqrt{3}}{9}$

**t** $\frac{\pi+6\sqrt{3}-12}{12}$

**u** $\frac{1}{2}(e^\theta\sin\theta-e^\theta\cos\theta)+C$

**v** $\frac{1}{2}(e^{-y}\sin y-e^{-y}\cos y)+C$

**w** $\frac{e^{2x}}{13}(3\sin 3x+2\cos 3x)+C$

**x** $-\frac{e^{-2x}}{4}(\sin 2x+\cos 2x)+C$

## Resp. Exerc. 11

**a** $\frac{1}{8}\ln\left(x^2\right)-\frac{1}{8}\ln\left(x^2+4\right)+k$

**b** $\ln(|x|)-\frac{3}{2}\ln(|x+1|)+\frac{3}{2}\ln(|x-1|)+k$

**c** $-\frac{1}{3}\arctan\left(\frac{x}{3}\right)+\ln(|x-1|)-\frac{1}{2}\ln\left(x^2+9\right)+k$

**Resp. Exerc. 12**

**a** $16/3$ **c** $2^{5/2}$ **e** $\pi/2$ **g** $1/12$

**b** $2$ **d** $2$ **f** $4\pi/3$ **h** $4/3$

**Resp. Exerc. 13**

**a** $\frac{1}{5}e^{-10}$ **c** A integral é divergente. **e** A integral é divergente.

**b** $0$ **d** A integral é divergente. **f** $-\frac{1}{4}$