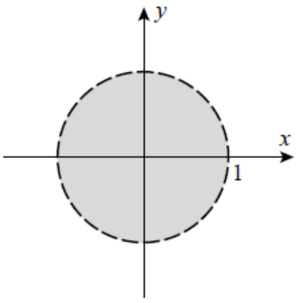
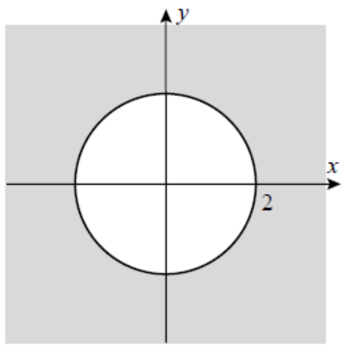
1) (a) $f(2,1) = (2)^2(1) + 1 = 5$ (b) $f(1,2) = (1)^2(2) + 1 = 3$
(c) $f(0,0) = (0)^2(0) + 1 = 1$ (d) $f(1,-3) = (1)^2(-3) + 1 = -2$
(e) $f(3a,a) = (3a)^2(a) + 1 = 9a^3 + 1$ (f) $f(ab,\, a-b) = (ab)^2(a-b) + 1 = a^3b^2 - a^2b^3 + 1$

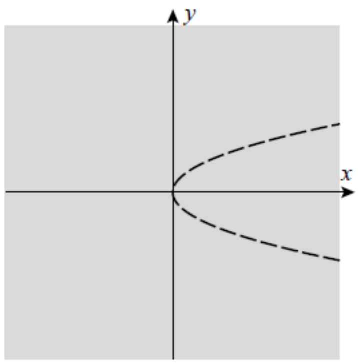
2) (a) $f(x+y, x-y) = (x+y)(x-y) + 3 = x^2 - y^2 + 3$
(b) $f\left(xy, 3x^2y^3\right) = (xy)\left(3x^2y^3\right) + 3 = 3x^3y^4 + 3$

3)
$$F(g(x), h(y)) = F\left(x^3, 3y+1\right) = x^3 e^{x^3(3y+1)}$$

4) (a) 19 (b) -9 (c) 3
(d) $a^6 + 3$ (e) $-t^8 + 3$ (f) $(a+b)(a-b)^2b^3 + 3$

5)


6)


7)


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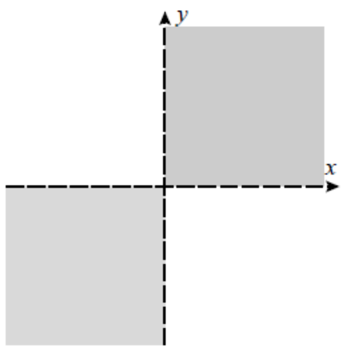
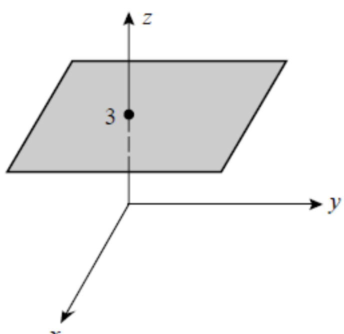
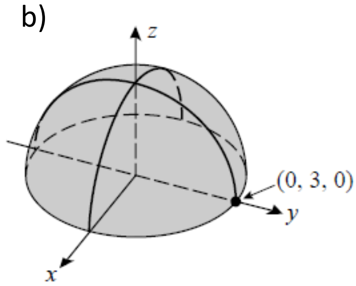
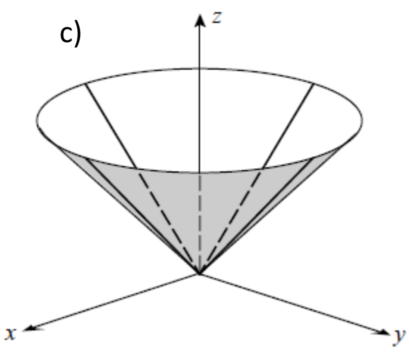


9) All points in 2-space on or between the vertical lines $x = \pm 2$.

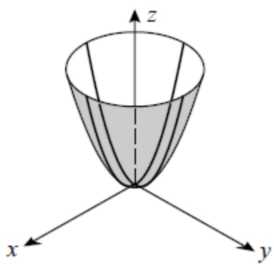
10)

a)


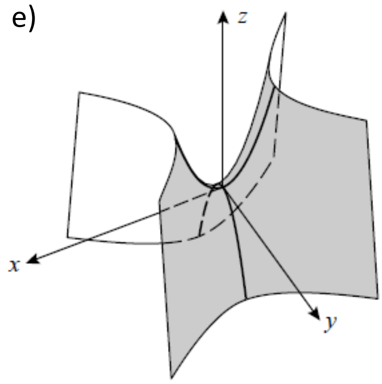
b)


c)


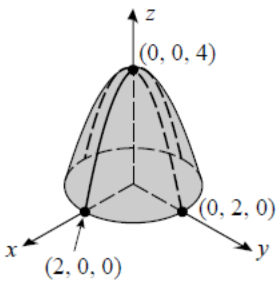
d)



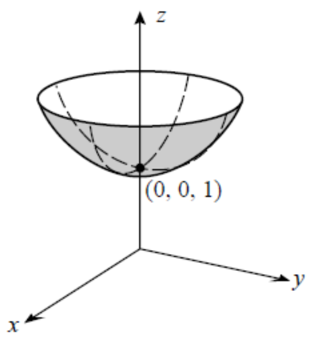
e)



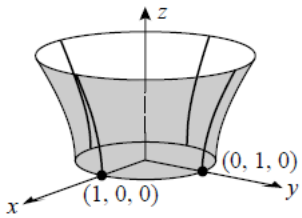
f)



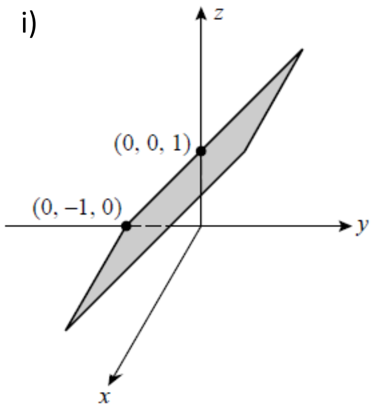
g)



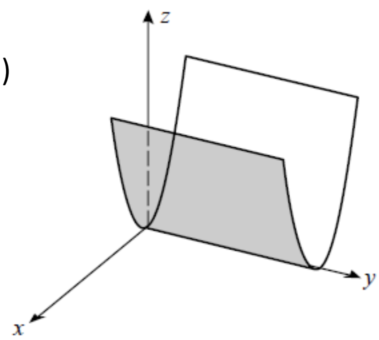
h)



i)

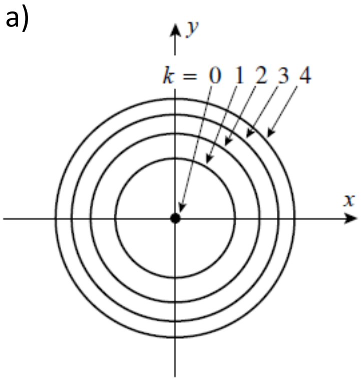


j)

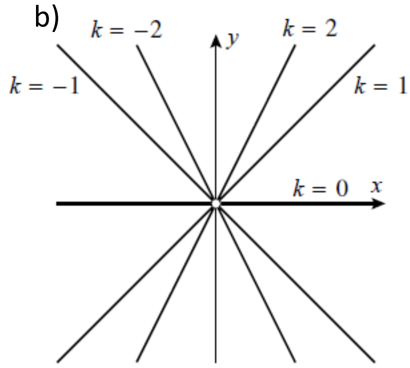


11)

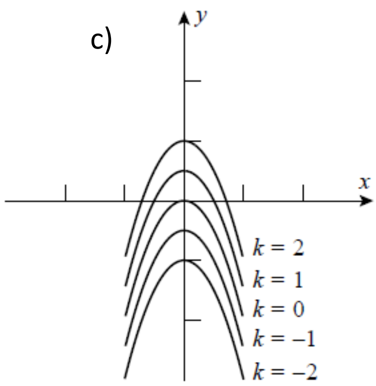
a)



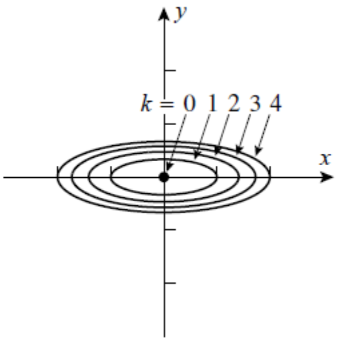
b)



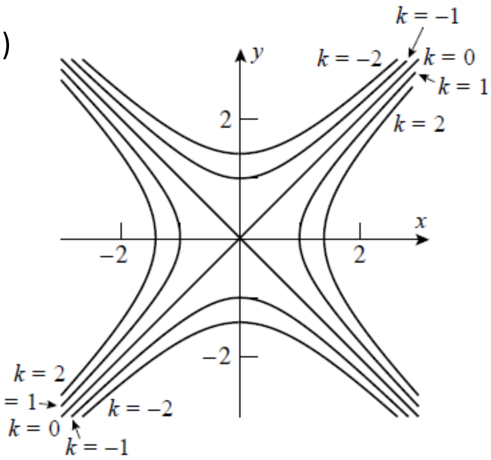
c)



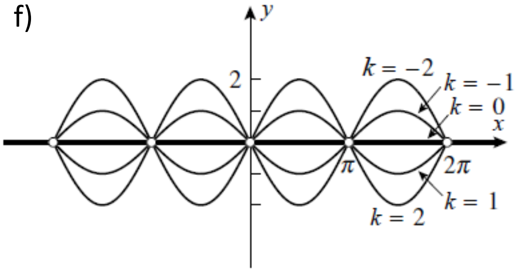
d)



e)



f)



12)

(a) $f(x,y) = 1 - x^2 - y^2$, because $f = c$ is a circle of radius $\sqrt{1-c}$ (provided $c \le 1$), and the radii in (a) decrease as c increases.

(b) $f(x,y) = \sqrt{x^2+y^2}$ because $f = c$ is a circle of radius c, and the radii increase uniformly. (a) is the contour plot of $f(x,y) = 1 - x^2 - y^2$, because $f = c$ is a circle of radius $\sqrt{1-c}$ (provided $c \le 1$), and the radii in (a) decrease as c increases.

(c) $f(x,y) = x^2 + y^2$ because $f = c$ is a circle of radius $\sqrt{c}$ and the radii in the plot grow like the square root function.

13) a) 35 b) -8 c) 0

14)

a) Along $x = 0$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2} = \lim_{y\to 0} \frac{3}{2y^2}$ does not exist.

b) Along $x = 0:$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y}{2x^2+y^2} = \lim_{y\to 0} \frac{1}{y}$ does not exist.

c) Along $y = 0:$ $\displaystyle\lim_{x\to 0} \frac{x}{x^2} = \lim_{x\to 0} \frac{1}{x}$ does not exist, so the original limit does not exist.

d) Along $y = 0:$ $\displaystyle\lim_{x\to 0} \frac{1}{x^2}$ does not exist, so the original limit does not exist.

15)

a) Let $z = x^2 + y^2$, then $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin\left(x^2+y^2\right)}{x^2+y^2} = \lim_{z\to 0^+} \frac{\sin z}{z} = 1.$

b) Let $z = x^2 + y^2$, then $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos\left(x^2+y^2\right)}{x^2+y^2} = \lim_{z\to 0^+} \frac{1-\cos z}{z} = \lim_{z\to 0^+} \frac{\sin z}{1} = 0.$

c) Let $z = x^2 + y^2$, then $\displaystyle\lim_{(x,y)\to(0,0)} e^{-1/\left(x^2+y^2\right)} = \lim_{z\to 0^+} e^{-1/z} = 0.$

d) With $z = x^2 + y^2$, $\displaystyle\lim_{z\to 0} \frac{1}{\sqrt{z}} e^{-1/\sqrt{z}}$; let $w = \dfrac{1}{\sqrt{z}}$, $\displaystyle\lim_{w\to+\infty} \frac{w}{e^w} = 0.$

16)

Exemplo 7 Determine $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$.

Solução Sejam (r, θ) as coordenadas polares do ponto (x, y) com $r \geq 0$. Então, temos

$$x = r\cos\theta, \quad y = r \operatorname{sen}\theta, \quad r^2 = x^2 + y^2$$

Além disso, uma vez que $r \geq 0$, temos $r = \sqrt{x^2+y^2}$, de modo que $r \to 0^+$ se, e somente se, $(x, y) \to (0, 0)$. Assim, podemos reescrever o limite dado como

$$\begin{aligned}\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) &= \lim_{r\to 0^+} r^2 \ln r^2 \\ &= \lim_{r\to 0^+} \frac{2\ln r}{1/r^2} && \boxed{\text{Isso converte o limite para uma forma indeterminada do tipo } \infty/\infty.} \\ &= \lim_{r\to 0^+} \frac{2/r}{-2/r^3} && \boxed{\text{Regra de L'Hopital.}} \\ &= \lim_{r\to 0^+} (-r^2) = 0 \end{aligned}$$

17) a) $\displaystyle\lim_{r\to 0} r\ln r^2 = \lim_{r\to 0}(2\ln r)/(1/r) = \lim_{r\to 0}(2/r)/(-1/r^2) = \lim_{r\to 0}(-2r) = 0.$

b) $\displaystyle\frac{x^2y^2}{\sqrt{x^2+y^2}} = \frac{(r^2\cos^2\theta)(r^2\sin^2\theta)}{r} = r^3\cos^2\theta\sin^2\theta$, so $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{x^2+y^2}} = 0.$

18) (a) $2e^{2x}\sin y$ (b) $e^{2x}\cos y$ (c) $2\sin y$ (d) 0 (e) $\cos y$ (f) e^{2x} (g) 0 (h) 4

19) (a) $\dfrac{\partial z}{\partial x} = \dfrac{3}{2\sqrt{3x+2y}}$; slope $= \dfrac{3}{8}$. (b) $\dfrac{\partial z}{\partial y} = \dfrac{1}{\sqrt{3x+2y}}$; slope $= \dfrac{1}{4}$.

20) (a) $\dfrac{\partial z}{\partial x} = e^{-y}$; slope $= 1$. (b) $\dfrac{\partial z}{\partial y} = -xe^{-y} + 5$; slope $= 2$.

21)

(a) $\dfrac{\partial z}{\partial x} = -4\cos(y^2 - 4x)$; rate of change $= -4\cos 7$.

(b) $\dfrac{\partial z}{\partial y} = 2y\cos(y^2 - 4x)$; rate of change $= 2\cos 7$.

22)

a) $\partial z/\partial x = 8xy^3 e^{x^2y^3}$, $\partial z/\partial y = 12x^2y^2 e^{x^2y^3}$.

b) $\partial z/\partial x = -5x^4y^4 \sin(x^5y^4)$, $\partial z/\partial y = -4x^5y^3 \sin(x^5y^4)$.

c) $\partial z/\partial x = x^3/(y^{3/5}+x) + 3x^2 \ln(1+xy^{-3/5})$, $\partial z/\partial y = -(3/5)x^4/(y^{8/5}+xy)$.

d) $\partial z/\partial x = ye^{xy}\sin(4y^2)$, $\partial z/\partial y = 8ye^{xy}\cos(4y^2) + xe^{xy}\sin(4y^2)$.

23)

a) $\frac{3}{2}\left(x^2+y^2+z^2\right)^{1/2}\left(2x+2z\frac{\partial z}{\partial x}\right) = 0$, $\partial z/\partial x = -x/z$; similarly, $\partial z/\partial y = -y/z$.

b) $\frac{4x-3z^2(\partial z/\partial x)}{2x^2+y-z^3} = 1$, $\frac{\partial z}{\partial x} = \frac{4x-2x^2-y+z^3}{3z^2}$; $\frac{1-3z^2(\partial z/\partial y)}{2x^2+y-z^3} = 0$, $\frac{\partial z}{\partial y} = \frac{1}{3z^2}$.

24)

a) $f_x = 8x - 8y^4$, $f_y = -32xy^3 + 35y^4$, $f_{xy} = f_{yx} = -32y^3$.

b) $f_x = x/\sqrt{x^2+y^2}$, $f_y = y/\sqrt{x^2+y^2}$, $f_{xy} = f_{yx} = -xy(x^2+y^2)^{-3/2}$.

c) $f_x = e^x \cos y$, $f_y = -e^x \sin y$, $f_{xy} = f_{yx} = -e^x \sin y$.

d) $f_x = 4/(4x-5y)$, $f_y = -5/(4x-5y)$, $f_{xy} = f_{yx} = 20/(4x-5y)^2$.

25) **(a)** f_{xyy} **(b)** f_{xxxx} **(c)** f_{xxyy} **(d)** f_{yyyxx}