Lista 3 (Parte 2)

Exercícios 1 a 30.

EXERCÍCIOS 10.1 Recurso Gráfico **C** CAS

1. Em cada parte, encontre a fórmula para o termo geral da seqüência, começando com $n = 1$.

(a) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ (b) $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$

(c) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$ (d) $\frac{1}{\sqrt{\pi}}, \frac{4}{\sqrt[3]{\pi}}, \frac{9}{\sqrt[4]{\pi}}, \frac{16}{\sqrt[5]{\pi}}, \ldots$

2. Em cada parte, encontre duas fórmulas para o termo geral da seqüência, uma começando com $n = 1$ e a outra com $n = 0$.

(a) $1, -r, r^2, -r^3, \ldots$ (b) $r, -r^2, r^3, -r^4, \ldots$

3. (a) Escreva os quatro primeiros termos da seqüência $\{1+(-1)^n\}$, começando com $n = 0$.

(b) Escreva os quatro primeiros termos da seqüência $\{\cos n\pi\}$, começando com $n = 0$.

(c) Use os resultados nas partes (a) e (b) para expressar o termo geral da seqüência 4, 0, 4, 0,... de duas formas diferentes, começando com $n = 0$.

4. Em cada parte, encontre a fórmula para o termo geral usando fatoriais e começando com $n = 1$.

(a) $1\cdot 2, 1\cdot 2\cdot 3\cdot 4, 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6, 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8, \ldots$

(b) $1, 1\cdot 2\cdot 3, 1\cdot 2\cdot 3\cdot 4\cdot 5, 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7, \ldots$

5-22 Escreva os cinco primeiros termos da seqüência, determine se ela converge e, se isso acontecer, encontre o limite.

5. $\left\{\frac{n}{n+2}\right\}_{n=1}^{+\infty}$ **6.** $\left\{\frac{n^2}{2n+1}\right\}_{n=1}^{+\infty}$ **7.** $\{2\}_{n=1}^{+\infty}$

8. $\left\{\ln\left(\frac{1}{n}\right)\right\}_{n=1}^{+\infty}$ **9.** $\left\{\frac{\ln n}{n}\right\}_{n=1}^{+\infty}$ **10.** $\left\{n \operatorname{sen}\frac{\pi}{n}\right\}_{n=1}^{+\infty}$

11. $\{1+(-1)^n\}_{n=1}^{+\infty}$ **12.** $\left\{\frac{(-1)^{n+1}}{n^2}\right\}_{n=1}^{+\infty}$

13. $\left\{(-1)^n \frac{2n^3}{n^3+1}\right\}_{n=1}^{+\infty}$ **14.** $\left\{\frac{n}{2^n}\right\}_{n=1}^{+\infty}$

15. $\left\{\frac{(n+1)(n+2)}{2n^2}\right\}_{n=1}^{+\infty}$ **16.** $\left\{\frac{\pi^n}{4^n}\right\}_{n=1}^{+\infty}$

17. $\left\{\cos\frac{3}{n}\right\}_{n=1}^{+\infty}$ **18.** $\left\{\cos\frac{\pi n}{2}\right\}_{n=1}^{+\infty}$

19. $\{n^2 e^{-n}\}_{n=1}^{+\infty}$ **20.** $\{\sqrt{n^2+3n}-n\}_{n=1}^{+\infty}$

21. $\left\{\left(\frac{n+3}{n+1}\right)^n\right\}_{n=1}^{+\infty}$ **22.** $\left\{\left(1-\frac{2}{n}\right)^n\right\}_{n=1}^{+\infty}$

23-30 Encontre o termo geral da seqüência, começando com $n = 1$, determine se a seqüência converge e, se isso acontecer, encontre o limite.

23. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$ **24.** $0, \frac{1}{2^2}, \frac{2}{3^2}, \frac{3}{4^2}, \ldots$

25. $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}, \ldots$ **26.** $-1, 2, -3, 4, -5, \ldots$

27. $\left(1-\frac{1}{2}\right), \left(\frac{1}{3}-\frac{1}{2}\right), \left(\frac{1}{3}-\frac{1}{4}\right), \left(\frac{1}{5}-\frac{1}{4}\right), \ldots$

28. $3, \frac{3}{2}, \frac{3}{2^2}, \frac{3}{2^3}, \ldots$

29. $(\sqrt{2}-\sqrt{3}), (\sqrt{3}-\sqrt{4}), (\sqrt{4}-\sqrt{5}), \ldots$

30. $\frac{1}{3^5}, -\frac{1}{3^6}, \frac{1}{3^7}, -\frac{1}{3^8}, \ldots$

(d) Determine se as seqüências nas partes (a), (b), (c) convergem. Em caso afirmativo, encontre o limite.

34. Para quais valores positivos de b a seqüência $b, 0, b^2, 0, b^3, 0, b^4, \ldots$ converge? Justifique sua resposta.

C **35.** (a) Use evidência numérica para fazer uma conjectura sobre o limite da seqüência $\{\sqrt[n]{n^3}\}_{n=2}^{+\infty}$.

(b) Use um CAS para confirmar a sua conjectura.

C **36.** (a) Use evidência numérica para fazer uma conjectura sobre o limite da seqüência $\{\sqrt[n]{3^n+n^3}\}_{n=2}^{+\infty}$.

(b) Use um CAS pra confirmar sua conjectura.

37. Supondo que a seqüência dada pela Fórmula (2) desta seção convirja, use o método do Exemplo 10 para mostrar que o limite dessa seqüência é $\sqrt{a}$.

38. Considere a seqüência

$$a_1 = \sqrt{6}$$
$$a_2 = \sqrt{6+\sqrt{6}}$$
$$a_3 = \sqrt{6+\sqrt{6+\sqrt{6}}}$$
$$a_4 = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$$
$$\vdots$$

(a) Encontre uma fórmula de recursão para a_{n+1}.

(b) Supondo que a seqüência convirja, use o método do Exem-

RESPOSTAS

1. **(a)** $\dfrac{1}{3^{n-1}}$ **(b)** $\dfrac{(-1)^{n-1}}{3^{n-1}}$ **(c)** $\dfrac{2n-1}{2n}$ **(d)** $\dfrac{n^2}{\pi^{1/(n+1)}}$

2. **(a)** $(-r)^{n-1}$; $(-r)^n$ **(b)** $(-1)^{n+1}r^n$; $(-1)^n r^{n+1}$

3. **(a)** $2, 0, 2, 0$ **(b)** $1, -1, 1, -1$ **(c)** $2(1+(-1)^n)$; $2 + 2\cos n\pi$

4. **(a)** $(2n)!$ **(b)** $(2n-1)!$

5. $1/3,\ 2/4,\ 3/5,\ 4/6,\ 5/7, \ldots$; $\displaystyle\lim_{n\to+\infty} \frac{n}{n+2} = 1$, converges

6. $1/3,\ 4/5,\ 9/7,\ 16/9,\ 25/11, \ldots$; $\displaystyle\lim_{n\to+\infty} \frac{n^2}{2n+1} = +\infty$, diverges

7. $2, 2, 2, 2, 2, \ldots$; $\displaystyle\lim_{n\to+\infty} 2 = 2$, converges

8. $\ln 1,\ \ln\frac{1}{2},\ \ln\frac{1}{3},\ \ln\frac{1}{4},\ \ln\frac{1}{5}, \ldots$; $\displaystyle\lim_{n\to+\infty} \ln(1/n) = -\infty$, diverges

9. $\dfrac{\ln 1}{1}, \dfrac{\ln 2}{2}, \dfrac{\ln 3}{3}, \dfrac{\ln 4}{4}, \dfrac{\ln 5}{5}, \ldots$;
$\displaystyle\lim_{n\to+\infty} \frac{\ln n}{n} = \lim_{n\to+\infty} \frac{1}{n} = 0 \left(\text{apply L'Hôpital's Rule to } \frac{\ln x}{x}\right)$, converges

10. $\sin\pi,\ 2\sin(\pi/2),\ 3\sin(\pi/3),\ 4\sin(\pi/4),\ 5\sin(\pi/5), \ldots$;
$\displaystyle\lim_{n\to+\infty} n\sin(\pi/n) = \lim_{n\to+\infty} \frac{\sin(\pi/n)}{1/n} = \lim_{n\to+\infty} \frac{(-\pi/n^2)\cos(\pi/n)}{-1/n^2} = \pi$, converges

11. $0, 2, 0, 2, 0, \ldots$; diverges

12. $1,\ -1/4,\ 1/9,\ -1/16,\ 1/25, \ldots$; $\displaystyle\lim_{n\to+\infty} \frac{(-1)^{n+1}}{n^2} = 0$, converges

13. $-1,\ 16/9,\ -54/28,\ 128/65,\ -250/126, \ldots$; diverges because odd-numbered terms approach -2, even-numbered terms approach 2.

14. $1/2,\ 2/4,\ 3/8,\ 4/16,\ 5/32, \ldots$; $\displaystyle\lim_{n\to+\infty} \frac{n}{2^n} = \lim_{n\to+\infty} \frac{1}{2^n \ln 2} = 0$, converges

15. $6/2,\ 12/8,\ 20/18,\ 30/32,\ 42/50, \ldots$; $\displaystyle\lim_{n\to+\infty} \frac{1}{2}(1+1/n)(1+2/n) = 1/2$, converges

16. $\pi/4,\ \pi^2/4^2,\ \pi^3/4^3,\ \pi^4/4^4,\ \pi^5/4^5, \ldots$; $\displaystyle\lim_{n\to+\infty} (\pi/4)^n = 0$, converges

17. $\cos(3),\ \cos(3/2),\ \cos(1),\ \cos(3/4),\ \cos(3/5), \ldots$; $\displaystyle\lim_{n\to+\infty} \cos(3/n) = 1$, converges

18. $0,\ -1,\ 0,\ 1,\ 0, \ldots$; diverges

19. $e^{-1},\ 4e^{-2},\ 9e^{-3},\ 16e^{-4},\ 25e^{-5}, \ldots$; $\displaystyle\lim_{x\to+\infty} x^2 e^{-x} = \lim_{x\to+\infty} \frac{x^2}{e^x} = 0$, so $\displaystyle\lim_{n\to+\infty} n^2 e^{-n} = 0$, converges

20. $1,\ \sqrt{10}-2,\ \sqrt{18}-3,\ \sqrt{28}-4,\ \sqrt{40}-5, \ldots$;
$\displaystyle\lim_{n\to+\infty} (\sqrt{n^2+3n} - n) = \lim_{n\to+\infty} \frac{3n}{\sqrt{n^2+3n}+n} = \lim_{n\to+\infty} \frac{3}{\sqrt{1+3/n}+1} = \frac{3}{2}$, converges

21. $2,\ (5/3)^2,\ (6/4)^3,\ (7/5)^4,\ (8/6)^5, \ldots$; let $y = \left[\dfrac{x+3}{x+1}\right]^x$, converges because

$$\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln\dfrac{x+3}{x+1}}{1/x} = \lim_{x\to+\infty} \frac{2x^2}{(x+1)(x+3)} = 2, \text{ so } \lim_{n\to+\infty} \left[\frac{n+3}{n+1}\right]^n = e^2$$

22. $-1,\ 0,\ (1/3)^3,\ (2/4)^4,\ (3/5)^5, \ldots$; let $y = (1-2/x)^x$, converges because

23. $\left\{\dfrac{2n-1}{2n}\right\}_{n=1}^{+\infty}$; $\lim\limits_{n\to+\infty}\dfrac{2n-1}{2n}=1$, converges

24. $\left\{\dfrac{n-1}{n^2}\right\}_{n=1}^{+\infty}$; $\lim\limits_{n\to+\infty}\dfrac{n-1}{n^2}=0$, converges

25. $\left\{\dfrac{1}{3^n}\right\}_{n=1}^{+\infty}$; $\lim\limits_{n\to+\infty}\dfrac{1}{3^n}=0$, converges

26. $\{(-1)^n n\}_{n=1}^{+\infty}$; diverges because odd-numbered terms tend toward $-\infty$, even-numbered terms tend toward $+\infty$.

27. $\left\{\dfrac{1}{n}-\dfrac{1}{n+1}\right\}_{n=1}^{+\infty}$; $\lim\limits_{n\to+\infty}\left(\dfrac{1}{n}-\dfrac{1}{n+1}\right)=0$, converges

28. $\{3/2^{n-1}\}_{n=1}^{+\infty}$; $\lim\limits_{n\to+\infty}3/2^{n-1}=0$, converges

29. $\{\sqrt{n+1}-\sqrt{n+2}\}_{n=1}^{+\infty}$; converges because

$$\lim_{n\to+\infty}(\sqrt{n+1}-\sqrt{n+2})=\lim_{n\to+\infty}\frac{(n+1)-(n+2)}{\sqrt{n+1}+\sqrt{n+2}}=\lim_{n\to+\infty}\frac{-1}{\sqrt{n+1}+\sqrt{n+2}}=0$$

30. $\{(-1)^{n+1}/3^{n+4}\}_{n=1}^{+\infty}$; $\lim\limits_{n\to+\infty}(-1)^{n+1}/3^{n+4}=0$, converges