Lista 3 (Parte 1)

31-36 Encontre os extremos absolutos da função dada no conjunto fechado e limitado R indicado.

31. $f(x, y) = xy - x - 3y$; R é a região triangular com vértices $(0, 0)$, $(0, 4)$ e $(5, 0)$.

32. $f(x, y) = xy - 2x$; R é a região triangular com vértices $(0, 0)$, $(0, 4)$ e $(4, 0)$

33. $f(x, y) = x^2 - 3y^2 - 2x + 6y$; R é a região quadrada com vértices $(0, 0)$, $(0, 2)$, $(2, 2)$ e $(2, 0)$.

34. $f(x, y) = xe^y - x^2 - e^y$; R é a região retangular com vértices $(0, 0)$, $(0, 1)$ $(2, 1)$ e $(2, 0)$.

5-12 Use multiplicadores de Lagrange para obter os valores máximo e mínimo de f sujeita à restrição dada. Além disso, encontre os pontos nos quais esses valores extremos ocorrem.

5. $f(x, y) = xy$; $4x^2 + 8y^2 = 16$

6. $f(x, y) = x^2 - y^2$; $x^2 + y^2 = 25$

7. $f(x, y) = 4x^3 + y^2$; $2x^2 + y^2 = 1$

8. $f(x, y) = x - 3y - 1$; $x^2 + 3y^2 = 16$

24. Suponha que a temperatura em um ponto (x, y) de uma placa de metal seja $T(x, y) = 4x^2 - 4xy + y^2$. Uma formiga, andando sobre a placa, percorre um círculo de raio 5 centrado na origem. Qual é a maior e a menor temperatura encontrada pela formiga?

RESPOSTAS:

31) $f_x = y - 1 = 0$, $f_y = x - 3 = 0$; critical point (3,1).

Along $y = 0$: $u(x) = -x$; no critical points,

along $x = 0$: $v(y) = -3y$; no critical points,

along $y = -\frac{4}{5}x + 4$: $w(x) = -\frac{4}{5}x^2 + \frac{27}{5}x - 12$; critical point $(27/8, 13/10)$.

(x,y)	$(3,1)$	$(0,0)$	$(5,0)$	$(0,4)$	$(27/8,13/10)$
$f(x,y)$	-3	0	-5	-12	$-231/80$

Absolute maximum value is 0, absolute minimum value is -12.

32) $f_x = y - 2 = 0$, $f_y = x = 0$; critical point (0,2), but (0,2) is not in the interior of R.

Along $y = 0$: $u(x) = -2x$; no critical points,

along $x = 0$: $v(y) = 0$; no critical points,

along $y = 4 - x$: $w(x) = 2x - x^2$; critical point $(1, 3)$.

(x,y)	$(0,0)$	$(0,4)$	$(4,0)$	$(1,3)$
$f(x,y)$	0	0	-8	1

Absolute maximum value is 1, absolute minimum value is -8.

33) $f_x = 2x - 2 = 0$, $f_y = -6y + 6 = 0$; critical point (1,1).

Along $y = 0$: $u_1(x) = x^2 - 2x$; critical point $(1, 0)$,

along $y = 2$: $u_2(x) = x^2 - 2x$; critical point $(1, 2)$

along $x = 0$: $v_1(y) = -3y^2 + 6y$; critical point $(0, 1)$,

along $x = 2$: $v_2(y) = -3y^2 + 6y$; critical point $(2, 1)$

(x,y)	$(1,1)$	$(1,0)$	$(1,2)$	$(0,1)$	$(2,1)$	$(0,0)$	$(0,2)$	$(2,0)$	$(2,2)$
$f(x,y)$	2	-1	-1	3	3	0	0	0	0

Absolute maximum value is 3, absolute minimum value is -1.

34) $f_x = e^y - 2x = 0$, $f_y = xe^y - e^y = e^y(x - 1) = 0$; critical point $(1, \ln 2)$.

Along $y = 0$: $u_1(x) = x - x^2 - 1$; critical point $(1/2, 0)$,

along $y = 1$: $u_2(x) = ex - x^2 - e$; critical point $(e/2, 1)$,

along $x = 0$: $v_1(y) = -e^y$; no critical points,

along $x = 2$: $v_2(y) = e^y - 4$; no critical points (for $0 < y < 1$).

(x,y)	$(0,0)$	$(0,1)$	$(2,1)$	$(2,0)$	$(1,\ln 2)$	$(1/2,0)$	$(e/2,1)$
$f(x,y)$	-1	$-e$	$e-4$	-3	-1	$-3/4$	$e(e-4)/4 \approx -0.87$

Absolute maximum value is $-3/4$, absolute minimum value is -3.

5) $y = 8x\lambda$, $x = 16y\lambda$; $y/(8x) = x/(16y)$, $x^2 = 2y^2$ so $4(2y^2) + 8y^2 = 16$, $y^2 = 1$, $y = \pm 1$. Test $(\pm\sqrt{2}, -1)$ and $(\pm\sqrt{2}, 1)$. $f(-\sqrt{2}, -1) = f(\sqrt{2}, 1) = \sqrt{2}$, $f(-\sqrt{2}, 1) = f(\sqrt{2}, -1) = -\sqrt{2}$. Maximum $\sqrt{2}$ at $(-\sqrt{2}, -1)$ and $(\sqrt{2}, 1)$, minimum $-\sqrt{2}$ at $(-\sqrt{2}, 1)$ and $(\sqrt{2}, -1)$.

6) $2x = 2x\lambda$, $-1 = 2y\lambda$. If $x \neq 0$ then $\lambda = 1$ and $y = -1/2$ so $x^2 + (-1/2)^2 = 25$, $x^2 = 99/4$, $x = \pm 3\sqrt{11}/2$. If $x = 0$ then $0^2 + y^2 = 25$, $y = \pm 5$. Test $(\pm 3\sqrt{11}/2, -1/2)$ and $(0, \pm 5)$. $f(\pm 3\sqrt{11}/2, -1/2) = 101/4$, $f(0, -5) = 5$, $f(0, 5) = -5$. Maximum $101/4$ at $(\pm 3\sqrt{11}/2, -1/2)$, minimum -5 at $(0,5)$.

7) $12x^2 = 4x\lambda$, $2y = 2y\lambda$. If $y \neq 0$ then $\lambda = 1$ and $12x^2 = 4x$, $12x(x - 1/3) = 0$, $x = 0$ or $x = 1/3$ so from $2x^2 + y^2 = 1$ we find that $y = \pm 1$ when $x = 0$, $y = \pm\sqrt{7}/3$ when $x = 1/3$. If $y = 0$ then $2x^2 + (0)^2 = 1$, $x = \pm 1/\sqrt{2}$. Test $(0, \pm 1)$, $(1/3, \pm\sqrt{7}/3)$, and $(\pm 1/\sqrt{2}, 0)$. $f(0, \pm 1) = 1$, $f(1/3, \pm\sqrt{7}/3) = 25/27$, $f(1/\sqrt{2}, 0) = \sqrt{2}$, $f(-1/\sqrt{2}, 0) = -\sqrt{2}$. Maximum $\sqrt{2}$ at $(1/\sqrt{2}, 0)$, minimum $-\sqrt{2}$ at $(-1/\sqrt{2}, 0)$.

8) $1 = 2x\lambda$, $-3 = 6y\lambda$; $1/(2x) = -1/(2y)$, $y = -x$ so $x^2 + 3(-x)^2 = 16$, $x = \pm 2$. Test $(-2, 2)$ and $(2, -2)$. $f(-2, 2) = -9$, $f(2, -2) = 7$. Maximum 7 at $(2, -2)$, minimum -9 at $(-2, 2)$.

24) $x^2 + y^2 = 25$ is the constraint; $8x - 4y = 2x\lambda$, $-4x + 2y = 2y\lambda$; $(4x - 2y)/x = (-2x + y)/y$, $2x^2 + 3xy - 2y^2 = 0$, $(2x - y)(x + 2y) = 0$, $y = 2x$ or $x = -2y$. If $y = 2x$ then $x^2 + (2x)^2 = 25$, $x = \pm\sqrt{5}$. If $x = -2y$ then $(-2y^2) + y^2 = 25$, $y = \pm\sqrt{5}$. $T(-\sqrt{5}, -2\sqrt{5}) = T(\sqrt{5}, 2\sqrt{5}) = 0$ and $T(2\sqrt{5}, -\sqrt{5}) = T(-2\sqrt{5}, \sqrt{5}) = 125$. The highest temperature is 125 and the lowest is 0.