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The velocity of rise of single cylindrical air bubbles through liquids contained in vertical tubes

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Abstract—Velocity measurements were made on single cylindrical air bubbles rising in a variety of liquids contained in vertically mounted tubes sealed at the lower end. A general graphical correlation involving $\mathrm{Fr} = u^2/gd$, $\mathrm{Eo} = \rho g d^2/\sigma$ and $Y = g\mu^4/\rho\sigma^3$ is given. It is shown that viscous effects are of minor importance if $\rho^2 g d^3/\mu^2 > 3 \times 10^5$, interfacial effects if $\mathrm{Eo} > 70$ and inertial if $\mathrm{Fr} < 0{\cdot}003$. Evidence suggests that the general correlation is also applicable to limiting velocity data for smaller (non-cylindrical) bubbles.

INTRODUCTION

THE behaviour of a single gas bubble released in a column of liquid contained in a vertical tube depends on the size of the bubble. When the bubble is quite small it remains spherically shaped and rises along a vertical rectilinear path. Larger bubbles become ellipsoidally and irregularly shaped and tend to rise along helical or zig-zag paths. Further increases in the bubble size cause the bubble to be of a spherically capped shape and again to rise along a rectilinear path. With still further increase in size the bubble is constrained by the containing walls into a cylindrical shape.

This paper is mainly devoted to a study of cylindrical air bubbles, which are of concern in the design of the Pohle type of air-lift pump. However, it will also be shown that some data for spherically capped bubbles fits the correlations for cylindrical bubbles.

SURVEY OF PREVIOUS WORK

Although the Pohle type of air-lift pump is useful for a wide range of different fluids, there is little quantitative information on its behaviour, or on such basic phenomena as the effects of fluid properties on the velocity of rise of cylindrical gas bubbles.

The case of a cylindrical gas bubble rising in a tube containing an ideal fluid (i.e. one without viscosity or surface tension) has been treated theoretically by both DUMITRESCU [1] and DAVIES and TAYLOR [2]. Both derived that the dimensionless group $u/\sqrt{gd}$ has a constant value, where u denotes the terminal velocity of rise of the bubble in a vertical tube of diameter d. DUMITRESCU theoretically determined this value as 0·351, and a series of careful experiments with cylindrical air bubbles rising in water in tubes of large diameter resulted in a value of 0·346. DAVIES and TAYLOR estimated a value of 0·328 and a brief experimental check showed this to be substantially correct. Other data for cylindrical air bubbles rising through water have been given by GIBSON [3] (nine tubes of different diameter), BARR [4] (four tubes), LAIRD and CHISHOLM [5] (one tube), GRIFFITH and WALLIS [6] (three tubes), NICKLIN [7] (one tube) and HATTORI [8] (twelve tubes).

HATTORI [8] has also recorded results for air bubbles rising through benzene, glycerine, ethanol, ether, formic acid and liquid paraffin. Articles by FOUST [9], BARR [4] and ABRAMS *et al.* [10] concerning the air-bubble (or Cochius) viscometer also provide limited data.

An excellent article recently published by HARMATHY [11] attempts to give a general correlation for the gravitational movement of a discrete amount of one phase through a continuum of another fluid phase, this continuous medium being either infinite or restricted in extent. As well as considering the movement of gas bubbles through a liquid this article considers the settling of solid particles through a fluid, the movement of drops of one liquid phase through another and the settling of liquid droplets through gases. However, the correlations given are limited to conditions corresponding to Re′ ($= u_\infty d_e \rho/\mu$) > 500, which, it is stated, defines the region where the viscous effects are negligible.

The purpose of the present paper is to extend the data on the rise of cylindrical gas bubbles to cover all liquids, and to define the conditions under which the effects of the individual retarding forces involved become of negligible importance.

DIMENSIONAL ANALYSIS OF PROBLEM

The problem concerns the estimation of the terminal velocity of rise of a single fluid bubble moving under the influence of gravitational, inertial, viscous and interfacial forces, relative to another fluid contained in a vertical cylindrical tube. As a general hydrodynamic solution would be quite complex, even if it were possible, the data will be treated empirically.

Dimensional analysis of the problem leads to a set of dimensionless groups such as

$$\frac{u^2\rho}{gd\Delta\rho}, \quad \frac{\Delta\rho g d^2}{\sigma}, \quad \frac{g\mu^4\Delta\rho}{\rho^2\sigma^3}, \quad \frac{\mu_1}{\mu}, \quad \frac{\text{bubble size}}{d}$$

The bubble size is often given by the equivalent diameter, d_e, for smaller bubbles, however for cylindrical bubbles the length of the bubble, l, is more convenient. Bubble-shape factors are not required as the bubble is fluid and its shape becomes dependent on the groups already given. As the density difference is concerned solely with buoyancy effects it will be associated with the gravitational acceleration, g, and hence the group $\Delta\rho/\rho$ is not required alone.

For the case of gas bubbles rising through liquids the density and viscosity of the gas are generally much less than those for the liquid, so that $\Delta\rho/\rho$ may be taken as unity, and also the viscosity ratio μ_1/μ is likely to be unimportant. It will be shown later that for cylindrical bubbles the terminal velocity is practically independent of the length of the bubble. Thus for this case, dimensional analysis leads to a set of three groups, such as

$$\frac{u^2}{gd}, \quad \frac{\rho g d^2}{\sigma}, \quad \frac{g\mu^4}{\rho\sigma^3}$$

Other sets of three independent dimensionless groups could be chosen, e.g. Reynolds, Weber and Froude. However it is desirable to have only one group containing the terminal velocity, u, for this is usually the unknown variable. The group chosen to contain u is the Froude group, u^2/gd, representing the ratio of inertial to gravitational forces. It is also desirable to have one group that does not contain either u or d, i.e. containing only the properties of the fluid. Such a group is $Y = g\mu^4/\rho\sigma^3$, the property group.

When the relative magnitudes of all the retarding forces, namely the viscous, inertial and interfacial forces, are significant then three independent groups are required for correlation. If one or more of these retarding forces is of negligible importance a lesser number of groups is required for correlation, provided suitable groups are chosen. When the inertial forces become of negligible importance the Poiseuille number, Ps $= u\mu/\rho g d^2$, representing the ratio of viscous to gravitational forces, will be used as the group involving u. A group involving surface tension but not viscosity or velocity is $\rho g d^2/\sigma$, termed the Eötvös number, Eo, by HARMATHY [11]. This group is related to the group r/a, used by BARR [4] and others, where a^2 is the "specific cohesion" defined by $a^2 = 2\sigma/\rho g$. It follows that Eo $= 8(r/a)^2$. A group involving viscosity but not surface tension or velocity is $\rho^2 g d^3/\mu^2$. This group will be used for correlation under conditions where the interfacial forces are of minor significance. A tabulation of the groups chosen for correlation under the different conditions is given in Table 2.

EXPERIMENTAL DETAILS

All measurements were made with vertically mounted glass tubes of diameters ranging from 0·500 to 3·87 cm. Tubes, other than precision-bore tubes, were carefully selected for uniformity and circularity. Before each run, each tube was thoroughly cleaned and dried, and then filled with

Table 1. Details of Fluids used

Fluid	*Temp.* (°C)	ρ (g/ml)	μ (cP)	σ (g/sec²)	$Y = \frac{g\mu^4}{\rho\sigma^3}$
Distilled water	26	0·997	0·87	71·5	$1{\cdot}6 \times 10^{-11}$
†0·3% butyric acid soln.	27	0·997	0·88	65·3	$2{\cdot}1 \times 10^{-11}$
0·55% butryic acid soln.	27	0·997	0·89	61·8	$2{\cdot}6 \times 10^{-11}$
0·95% butyric acid soln.	27	0·997	0.90	57·2	$3{\cdot}4 \times 10^{-11}$
1·4% butyric acid soln.	27	0·998	0·91	53·7	$4{\cdot}3 \times 10^{-11}$
2·1% butyric acid soln.	27	0·998	0·92	49·8	$5{\cdot}7 \times 10^{-11}$
3·0% butyric acid soln.	27	0·999	0·97	45·7	$9{\cdot}1 \times 10^{-11}$
40% sucrose soln.	26	1·172	5·65	77·7	$1{\cdot}8 \times 10^{-8}$
58% sucrose soln.	25	1·272	40·5	76·0	$4{\cdot}7 \times 10^{-5}$
ethylene glycol	26	1·113	19·9	47·5	$1{\cdot}3 \times 10^{-5}$
aqueous ethanol	26	0·803	1·385	22·8	$4{\cdot}1 \times 10^{-9}$
Tellus oil‡	25	0·864	52·0	31·0	$1{\cdot}8 \times 10^{-3}$
Voluta oil‡	25	0·902	294	30·8	2·8
glycerol	24	1·260	712	63·1	8·0
90% glycerol soln.	24	1·234	154	64·8	$1{\cdot}6 \times 10^{-2}$
95% glycerol soln.	25	1·246	323	63·9	0·33
sugar syrup§	27	1·42	20,900	77·2	$2{\cdot}9 \times 10^{6}$
diluted sugar syrup	27	1·40	2650	77·0	$7{\cdot}5 \times 10^{2}$
rediluted sugar syrup	26	1·39	1610	76·9	$1{\cdot}05 \times 10^{2}$

† All percentages, by wt.
‡ Hydrocarbon lubricating oils.
§ From cane sugar refining.

the test liquid. Air was contained in a rubber tube attached to the lower end of each glass tube and clamped at two positions with quick opening clamps. A single air bubble, released by snapping open the uppermost clamp, was timed as it passed between two marks, about 60 cm apart, on the tube. All measurements were made from the top of the bubble, and the velocity so determined was corrected for the expansion of the bubble by the method described later (refer effect of bubble length). Each experiment was repeated a sufficient number of times for a reliable average velocity to be obtained. It is expected that the average velocity so determined is within ± 1 per cent of the true value.

Table 1 gives details of the fluids used. Care was taken to select fluids whose properties were not greatly affected by the passage of the bubble through them. Fluid viscosities were determined by the use of a falling-ball viscometer and surface tensions by a capillary-tube tensiometer. The average diameter of each tube was computed from the volume of distilled water required to fill the tube between the timing marks. Further details of the methods are given by BEARDMORE [12].

DISCUSSION OF RESULTS

Presentation of results

Fig. 1 presents the results for water and dilute aqueous solutions. A linear scale was chosen for the ordinate so that the points corresponding to zero velocity could be shown. Instead of Fr, $\sqrt{\text{Fr}}$ was plotted so that a reasonable spread of data resulted. All results fall along a single curve with the exception of those of GIBSON [3] for larger diameter tubes. It is practically impossible to determine an accurate terminal velocity for the very large diameter tubes, without the use of a movie camera, so that the results of DUMITRESCU [1] and LAIRD and CHISHOLM [5] who used such a technique must be considered as the more reliable data in this region. It can be seen that the results agree well with the correlation of HARMATHY [11].

The fluid behaves as an "ideal fluid" with $u/\sqrt{gd}$ constant at 0·345 for Eo > 70. The

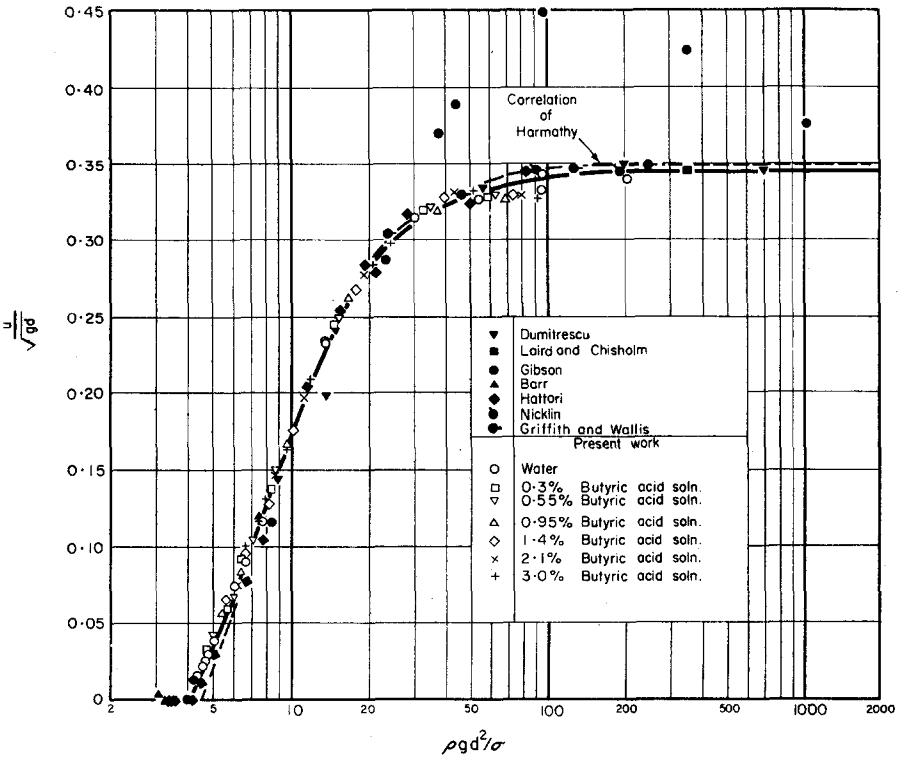

FIG. 1. Results for cylindrical air bubbles rising in water and dilute aqueous solutions contained in vertical tubes.

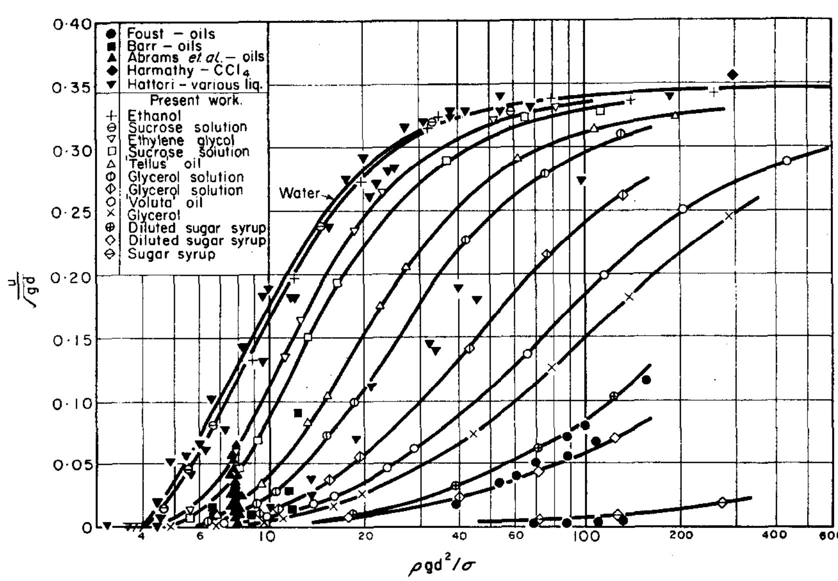

FIG. 2. Results for cylindrical air bubbles rising in vertical tubes (all fluids).

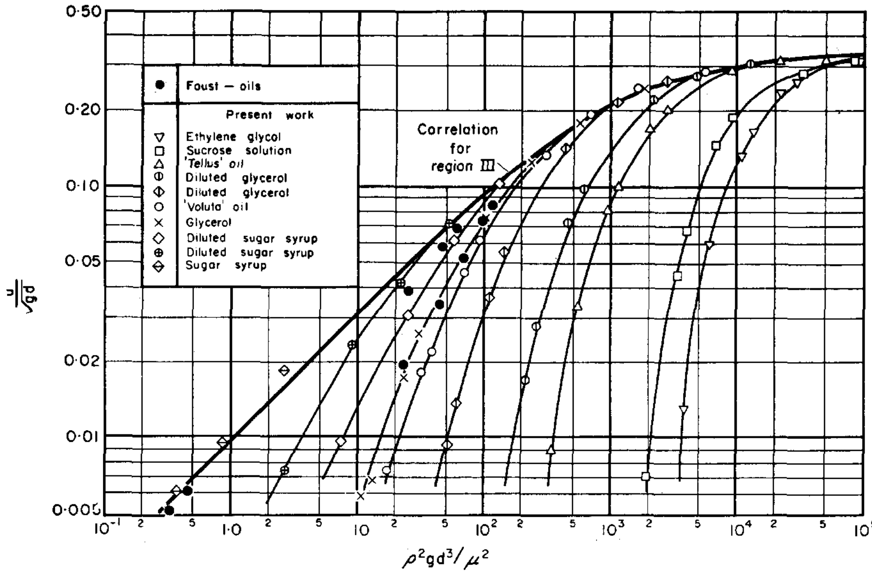

FIG. 3. Plot of data, showing correlation when interfacial forces can be neglected.

criterion for zero bubble velocity has been taken as Eo $\leqslant$ 4, compared with a value of 3·36 given by HATTORI [8], 3·37 by BRETHERTON [13], and a value of 0·58 given by BARR [4]. It is possible that there is no unique critical value of Eo, for it is reasonable to expect the contact angle of the liquid on the tube surface to have an effect on the conditions for zero velocity if wetting of the surface is incomplete. This factor has not been considered in the dimensional analysis. Factors such as the cleanliness or surface roughness of the tube then may affect the experimental determination of a critical value of Eo. If the upper bubble shape can be considered as a hemisphere, and if the contact angle of the liquid on the surface is zero, GIBSON [3] has shown that the critical value of Eo = 4. The present investigators found that a bubble will not rise in distilled water contained in a well cleaned 0·500 cm diameter tube, which is equivalent to Eo = 3·4.

The results for all fluids is shown on Fig. 2. The closeness of fit to the data of other workers is quite good. The separation of the curves for the more viscous liquids can be attributed to the increasing importance of viscous forces on the terminal velocity. Since viscous forces are not involved when the bubble does not move all curves pass through the common point Eo = 4, for zero bubble velocity. As a preliminary estimate, viscous effects are unimportant for $Y < 10^{-8}$. This range includes most of the common low-viscosity liquids.

If interfacial forces do not appreciably affect the velocity, a plot of $u/\sqrt{gd}$ against $\rho^2 g d^3/\mu^2$ (Fig. 3) will give a single curve. It can be seen that this condition is fulfilled over a range of the data. Similarly, if inertial effects are of negligible importance, a plot of Ps against Eo (Fig. 4) should show a single curve. This is fulfilled over a range of data, which actually correspond to small values of u^2/gd. This plot is the type of correlation given by BARR [4] and the agreement of his correlation with the data is quite reasonable, considering the limited amount of data at his disposal. At values of Eo > 70, $\sqrt{\text{Ps}}$ is substantially constant at 0·098 (i.e. Ps = 0·0096), indicating that the retarding forces under these conditions are predominantly viscous.

A general graphical correlation for all regimes is given by Fig. 5, which shows $u/\sqrt{gd}$ plotted against Eo with Y as parameter for the range of variables Eo < 1000, $Y < 10^6$. This figure was obtained by crossplotting the smoothed data of the previous figures. It presents the results of this paper in the most convenient form for future use.

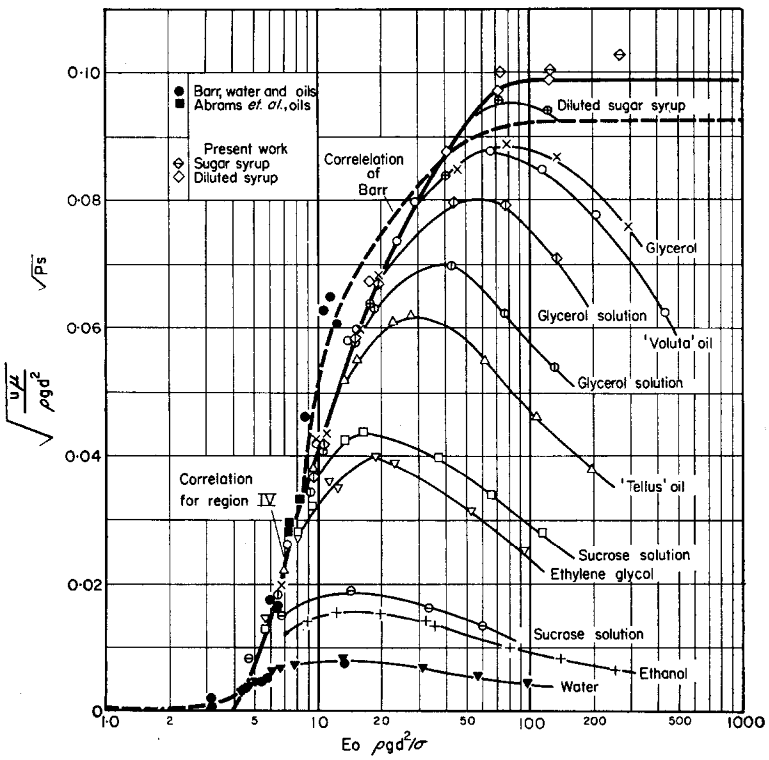

FIG. 4. Plot of data, showing correlation when inertial forces can be neglected.

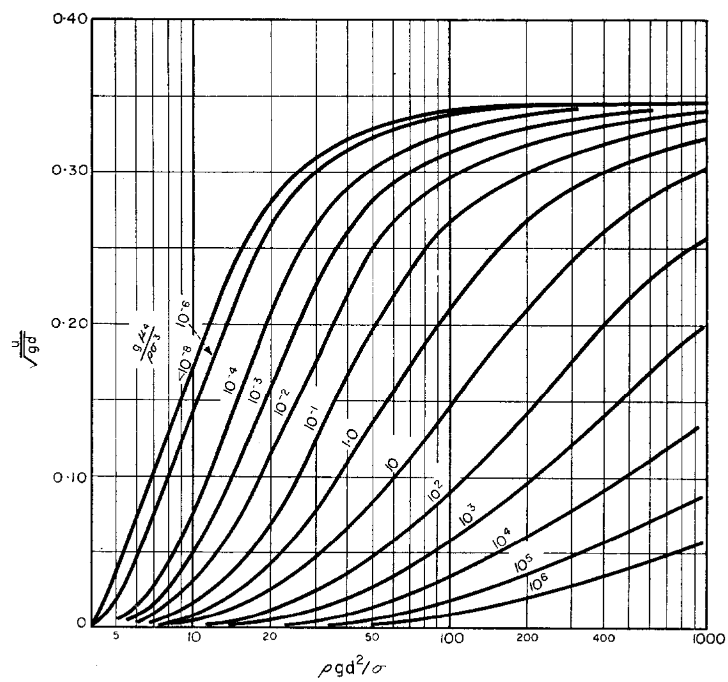

FIG. 5. General correlation for velocity of rise of cylindrical air bubbles in liquids in vertical tubes.

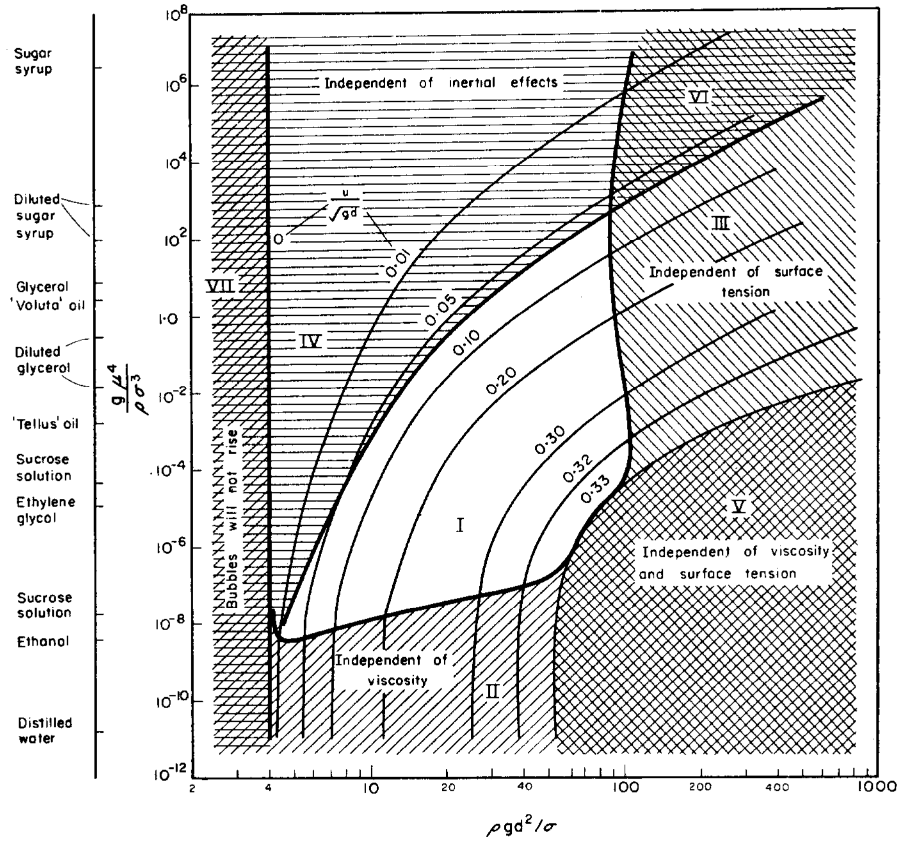

FIG. 6. Crossplot of data, showing regions in which the effect of some variables becomes unimportant.

Limits of correlations

The various regimes are indicated on Fig. 6 which shows the contour lines for $\sqrt{\text{Fr}}$. These were drawn by determining from Figs. 2, 3 and 4 the conditions under which the various retarding forces may be neglected (without causing an error in velocity of greater than ± 5 per cent). Approximately, surface tension effects may be neglected if Eo > 70, inertial effects if $\sqrt{\text{Fr}} < 0{\cdot}05$ and viscosity effects if $\rho^2 g d^3/\mu^2 > 3 \times 10^5$. Table 2 summarizes these results. The condition given by HARMATHY [11] for the neglecting of viscous forces is Re′ > 500. Using the correlations for spherical particles in turbulent movement in an infinite medium, this becomes $\rho^2 g d_e^3/\mu^2 > 8 \times 10^5$. Although this is of the same order as that already given, if $d \simeq d_e$, it is an unsatisfactory criterion for cylindrical bubbles, because a change in the length of the bubble, which does not affect the velocity, or presumably the forces acting on it, affects d_e. In a similar way, the general correlation given by HARMATHY for the prediction of bubble velocity is possibly needlessly complicated for application to cylindrical bubbles by the introduction of u_∞ and d_e. These quantities apply to a spherical bubble of the same volume.

As the forces affecting the bubble shape and velocity differ in the different regimes, the region in which an investigation is conducted should be stated, and in the absence of further data it is expected that the results apply only to that region. The work of LAIRD and CHISHOLM [5] applies to bubbles rising unaffected by viscous or interfacial forces i.e. region V. The data of FOUST [9] were

Table 2. Correlating Groups used and Limits of Correlations

Region on Fig. 6	*Velocity independent of effects of*	*Correlating groups chosen*	*Correlation*	*Limits of correlation (approx.)*
I	none	Fr, Eo, Y	see Fig. 5	General correlation experimental limits $3 < \text{Eo} < 400$, $10^{-12} < Y < 10^{8}$
II	viscous forces	Fr, Eo	see Fig. 1	$\rho^2 g d^3/\mu^2 > 3 \times 10^5$
III	interfacial forces	Fr, $\rho^2 g d^3/\mu^2$	see Fig. 3	$\text{Eo} > 70$
IV	inertial forces	Ps, Eo	see Fig. 4	$\sqrt{\text{Fr}} < 0{\cdot}05$
V	viscous and interfacial forces	Fr	$u/\sqrt{gd} = 0{\cdot}345$	$\text{Eo} > 70$, $\rho^2 g d^3/\mu^2 > 3 \times 10^5$
VI	inertial and interfacial	Ps	$\mu u/\rho g d^2 = 0{\cdot}0096$	$\sqrt{\text{Fr}} < 0{\cdot}05$, $\text{Eo} > 70$
VII	inertial and viscous forces	no bubble movement	$u = \text{O}$	$\text{Eo} < 4$

mainly in region VI, while that of BARR [4] in region IV. Most other workers worked with fluids corresponding to region II.

Data for non-cylindrical bubbles

When the velocity of a single bubble rising in a liquid in a given tube is plotted against the bubble size, it is found that above a certain bubble size the velocity becomes independent of bubble size [14–18.] This is termed the "limiting velocity" and appears to correspond to the bubbles becoming of a spherically capped shape. This limiting velocity depends on the containing tube size, and fits the correlation for cylindrical bubbles with reasonable accuracy (Fig. 7). It will be noted that most of the experimental work is limited to region V of Fig. 6.

For given liquid properties it appears that when the bubble is of such a size that it is constrained by the tube to a spherically capped shape, the velocity depends only on the curvature of the cap and hence on the tube diameter. It is interesting to note that the theoretical derivations of both DUMITRESCU [1] and DAVIES and TAYLOR [2] considered only the upper cap-shaped interface of the cylindrical bubble.

HARMATHY [11] has also noted that the limiting velocity of smaller bubbles fits the correlations for cylindrical bubbles, and has suggested that these correlations can be used provided,

$$d_e/d > 1 - 0{\cdot}175\sqrt{(\text{Eo} - 4{\cdot}5)}, \qquad \text{for } 4{\cdot}5 < \text{Eo} < 20$$

and $$d_e/d > 0{\cdot}31 \qquad \text{for Eo} \geqslant 20.$$

PEEBLES and GARBER [16] correlated their limiting velocity results for air bubbles in various fluids by $u = 1{\cdot}18\ (\sigma g/\rho)^{0{\cdot}25}$ which, by introducing d, becomes

$$\sqrt{\text{Fr}} = 1{\cdot}18\ \text{Eo}^{-0{\cdot}25}$$

This curve is shown on Fig. 7. While the values for their narrow range of data lie close to those for cylindrical bubbles, the correlating curves cross at a very considerable angle. PEEBLES and GARBER suggest that their correlation is applicable provided

$$d_e/d \geqslant 19{\cdot}4/\text{Eo} \qquad \text{for Eo} > 90.$$

Other methods of measurement

All measurements used for the correlations were made by timing a bubble moving in an otherwise stationary liquid phase contained in a tube. Velocity measurements have also been made by determining the downflow rate of liquid necessary to maintain the bubble stationary in the tube [4]. Although this corresponds to a different liquid flow pattern, measurements with an air bubble in water give results which lie close to, but a little below the data obtained in the normal way (Fig. 7). DAVIES

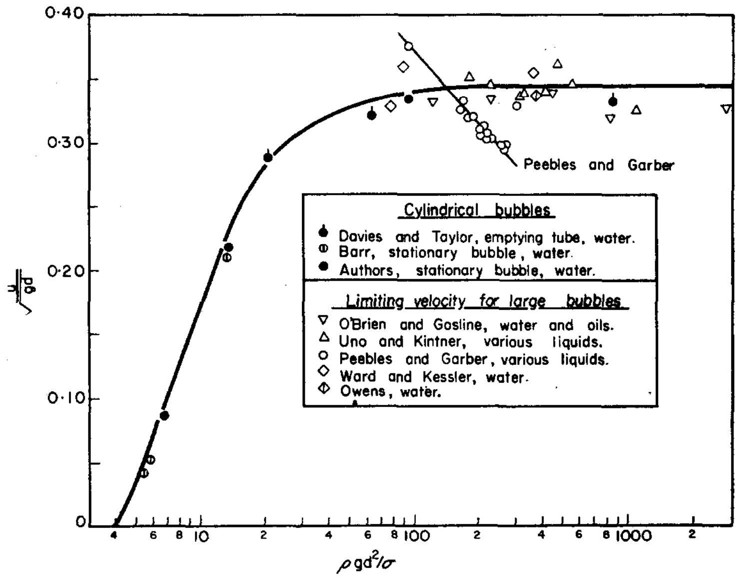

FIG. 7. Comparison of results for low viscosity fluids, showing data obtained by other methods of measurement, and from limiting velocities of large bubbles.

and TAYLOR [2] obtained data by allowing liquid to empty from vertical tubes sealed at the upper end.

Effect of other variables

Diameter of tube. The velocity varies quite markedly with tube diameter, especially for the smaller tubes ($4 < Eo < 20$) with low viscosity liquids. As the velocity thus gives a sensitive test for changes in tube diameter, this was used as a means of selecting tubes of uniform diameter. The average velocities for different sections of a tube were determined, and a suitable portion of the tube, if such were found, was selected for subsequent measurements. That such variations are caused by dimensional variations and not by acceleration or depth effects, is shown by repeating the measurements with the tube inverted.

Length of bubble. It has been widely recorded that the length of the bubble has but little effect on the terminal velocity. However, LAIRD and CHISHOLM [5], using air in water in a 5·08 cm tube, found a significant increase in velocity with increasing bubble length. They measured the usual velocity, u', determined from the movement of the cap of the bubble relative to the tube. Measurement of u' for smaller tubes shows a significant, though smaller, variation.

The velocity, u', is the sum of the velocity of the cap relative to the undisturbed liquid above the bubble, u, and a bulk movement of the cap and liquid due to the expansion of the bubble. To remove the effects of bubble expansion, NICKLIN [7] used a tube sealed at both ends, and found that the velocity of a cylindrical air bubble was independent of bubble length. This suggests that the variation of velocity with bubble length is due solely to the expansion of the bubble. That this is so, is found when the data is corrected for bubble expansion by the simply derived expression.

$$u = u' \, (1 - \rho g l / p_t g_c).$$

Thus, u, the velocity of the cap of the bubble referred to the undisturbed liquid (e.g. liquid surface) is independent of bubble length. The data in this paper were corrected for the expansion of the gas bubble, but for the majority of cases the correction was negligibly small.

Test results showed that the head of liquid over the bubble also does not affect the velocity of rise.

Acceleration to ultimate velocity. Bubble velocities measured over several different sections of a pre-

cision bore tube were identical. The distance necessary for the bubble to reach terminal velocity appeared to be much less than two tube diameters.
Angle of inclination of tube. Tests were made on bubbles rising in water contained in a given tube inclined at different angles to the vertical (Fig. 8). The velocity (measured along the inclined tube) increases quite markedly with the angle of inclination. All subsequent measurements were made in tubes carefully aligned to the vertical.

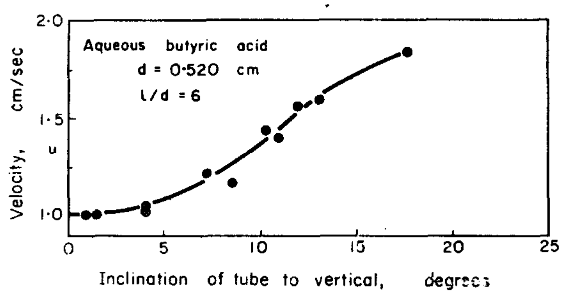

FIG. 8. Effect of tube inclination on bubble velocity.

Cleanliness of tubes. A bubble rose with the same velocity in water in a tube (0·586 cm diameter) when the tube surface was thoroughly clean as when it was obviously filmed and not fully wetted by water. Provided the test fluid fully covers the tube surface as the bubble passes, differences in velocity should not be expected. However at lower velocities dirty tubes probably would cause the fluid to channel past the gas bubble.
Nature of Bubble fluid. This paper has dealt only with air bubbles. It is reasonable to expect the results to apply to bubbles of other gases. HARMATHY [11] has observed that for conditions when viscous forces may be neglected the results apply to liquid bubbles, provided due allowance is made for the density difference term $\Delta\rho$. The range of fluids tested however was quite limited.

CONCLUSIONS

(1) A general graphical correlation (Fig. 5) is given for the prediction of the rate of rise of cylindrical gas bubbles in liquids in vertical tubes.

(2) In a clean tube bubbles will not rise unless $Eo > 4$.

(3) The velocity is unaffected by viscosity if $\rho^2 g d^3/\mu^2 > 3 \times 10^5$, surface tension if $Eo > 70$ and inertial effects if $\sqrt{Fr} < 0{\cdot}05$.

(4) Limiting velocities of non-cylindrical bubbles appear to be also correlated by Fig. 5.

(5) Provided the velocity is measured relative to the undisturbed liquid above the bubble, it appears that the velocity is unaffected by the length of bubble or head of liquid over the bubble.

(6) The inclination of the tube and the tube uniformity have a marked effect on velocity.

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NOTATION

a = $\sqrt{2\sigma/\rho g}$
d = Diameter of tube
d_e = Equivalent diameter of bubble, i.e. the diameter of a sphere with same volume as the bubble
g = Gravitational acceleration
g_c = Conversion factor
l = Length of bubble
p_i = Absolute pressure inside bubble
r = Radius of tube = $d/2$
u = Terminal velocity of bubble, relative to undisturbed liquid
u' = Terminal velocity of bubble, relative to fixed tube
u_∞ = Terminal velocity of spherical bubble of same volume as bubble, in an infinite extent of fluid
μ = Viscosity of fluid of continuous phase
μ_1 = Viscosity of bubble fluid
ρ = Density of fluid of continuous phase
$\Delta\rho$ = Difference in density between continuous fluid and bubble
σ = Interfacial tension between bubble and fluid
Eo = Eötvös number = $\rho g d^2/\sigma$
Fr = Froude number = u^2/gd
Ps = Poiseuille group = $\mu u/\rho g d^2$
Re = Reynolds group = $ud\rho/\mu$
Re′ = Reynolds group as used by HARMATHY [11] = $u_\infty d_e \rho/\mu$
Y = Property group = $g\mu^4/\rho\sigma^3$

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Resumé—Mesure de vitesse de bulles d'air de forme cylindrique dans divers liquides contenus dans dans des tubes verticaux scellés à leur extrêmité inférieure. Les autres établissent une relation comprenant:

$$\mathrm{Fr} = u^2/gd,\ \mathrm{Eo} = \rho g d^2/\sigma \text{ et } \mathrm{Y} = g\mu/8/\rho\sigma^3.$$

L'influence de la viscosité est peu importante si $\rho^2 g d^3/\mu^2 > 3 \times 10^5$, celle de l'interface également si $\mathrm{Eo} > 70$ ainsi que celle de l'inertie si $\mathrm{Fr} < 0{,}003$. Il est évident que la relation générale s'étend aux vitesses limites pour les petits bulles (non cylindriques).

Zusammenfassung—Die Geschwindigkeiten einzelner zylindrischer Luftblasen, die in verschiedenen Flüssigkeiten aufstiegen, wurden gemessen; die Flüssigkeiten befanden sich in senkrecht stehenden, unten verschlossenen Rohren. Es wird eine allgemeine graphische Beziehung angegeben, die die Kennzahlen $\mathrm{Fr} = u^2/g{\cdot}d$, $\mathrm{Eo} = \rho{\cdot}g{\cdot}d^2/\sigma$ und $Y = g{\cdot}\mu^4/\rho{\cdot}\sigma^3$ enthält. Es wird gezeigt, dass der Viskositätseinfluss für $\rho^2{\cdot}g{\cdot}d^3/\mu^2 > 3{\cdot}10^5$, dass Grenzflächeneffekte für $\mathrm{Eo} > 70$ und dass der Einfluss der Trägheit für $\mathrm{Fr} < 0{,}003$ nur von geringer Bedeutung sind. Allem Anschein nach ist die allgemeine Beziehung auch für die Grenzgeschwindigkeit kleinerer (nicht zylindrischer) Blasen andwendbar.