International Journal of Multiphase Flow 72 (2015) 318–323

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow

The existence and behaviour of large diameter Taylor bubbles ☆

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ARTICLE INFO

Article history:
Received 13 November 2013
Received in revised form 22 April 2014
Accepted 23 April 2014
Available online 9 May 2014

Keywords:
Taylor bubbles
Bubble rise
Oscillations

ABSTRACT

Large tube filling bubbles rising up through quiescent fluid in a vertical tube are commonly known as Taylor bubbles. Their apparent simplicity of form and behaviour has led to them being viewed and modelled as a paradigm for both large bubble dynamics, where there is no continuous gas flow, and slug flow for the case of continuous gas flow. Central to this approach is the question: what diameter tubes support stable Taylor bubbles? In this paper we examine the case of low viscosity Taylor bubbles through experiments and theory and show that they exist in much wider diameter tubes than had previously been reported. In order for the bubbles to be stable a settling period is required to allow the column to sufficiently quiesce. This settling period is compared favourably with the classical stability analysis of Batchelor (1987). We also observe such bubbles rising in an oscillatory manner if the gas input is abruptly curtailed. The oscillations match theoretical predictions well.

1. Introduction

The rise of individual gas bubbles through stagnant fluids represents a classic problem in the multiphase flow literature. The canonical nature of the problem, coupled with its experimental ease, means that Taylor bubbles (large, bullet shaped bubbles rising up through cylindrical tubes) have been studied extensively.

For the inviscid case, much of the literature focuses upon two issues – the bubble's rise rate and stability properties. The former of these has been addressed both analytically and experimentally. Dimensional arguments suggest a relationship of the from

$$U_b = Fr\sqrt{gD}, \tag{1}$$

where U_b is the rise rate, D the tube's diameter, g acceleration due to gravity and Fr is the Froude Number, a constant of proportionality to be determined. Theoretical predictions for Fr exist in the range 0.328–0.369 (Dumitrescu, 1943; Davies and Taylor, 1950; Layzer, 1955; Tung and Parlange, 1976; Viana et al., 2003) with Dumitrescu's 0.351 being regarded as the most accurate (Fabre and Liné, 1992).

The stability of large diameter bubbles is less well understood (Grace et al., 1978; Batchelor, 1987; Kitscha and Kocamustafaogullari, 1989; Krechetnikov, 2009). The most mathematically formal of these approaches is that by Batchelor (1987). In this work, a linear approximation of the evolution of small perturbations to the leading front of a bubble was considered and the effect of the bubble encountering a turbulent eddy was considered. With appropriate approximations, an upper limit of 0.46 m in water was estimated for the diameter of tube capable of maintaining a stable Taylor bubble. Experimentally, probing this stability limit has proven difficult – principly because doing so would require a selection of large diameter columns. Nonetheless, the work which has been done suggests that after a diameter of around 0.10–0.14 m, Taylor bubbles become unstable (Martin, 1976; Kawanishi et al., 1990; Hibiki and Ishii, 2003; Lu and Prosperetti, 2006).

Despite this there is some evidence that larger diameter bubbles do exist. An unpublished conference proceeding Hsu and Simon (1969) suggests the existence of stable bubbles in a 0.30 m tube. Perhaps more significantly, James et al. (2011) report that they find stable bubbles within a 0.24 m tube. They do not, however, provide any further details as to the conditions underwhich they find the bubble nor as to how they measure stability.

In recent years there have been a number of computational papers examining bubble rise. Much of the difficulty in this work comes down to dealing with the interface between the gas and the liquid appropriately. The majority of the literature circumvents explicitly calculating the surface's behaviour by means of methods such as the Volume of Fluid approach (e.g. Taha and Cui, 2006;

☆ The original version of this paper was presented at the 8th International Symposium on Measurement Techniques for Multiphase Flows held in GuangZhou, China, from December 13th to 15th, 2013 under the chairmanship of Professor Xiaoshu Cai. The Symposium was sponsored by the Chinese Society for Measurement Techniques on Multiphase Flow, the Japanese Society for Multiphase Flow and the National Natural Science Foundation of P.R. China.

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http://dx.doi.org/10.1016/j.ijmultiphaseflow.2014.04.006
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James et al., 2008). More recently a number of papers have begun explicitly tracking the surface (Kang et al., 2010; Lu and Prosperetti, 2009). Both approaches have led to a much better understanding of the roles played by various parameters and have allowed for greater insight into the flow around and past bubbles. For the problem of calculating the maximum size of bubble observable, however, progress has been harder. The work is hampered by the difficulty of assessing whether the bubble itself is genuinely unstable or whether it is a numerical artifact (Suckale et al., 2010; James et al., 2011).

In parallel to the engineering literature, volcanological papers have also addressed the issues of large bubble dynamics as these bubbles are believed to drive some forms of volcanic activity. Sound measurements are a key diagnostic tool in this setting as they allow measurements to be made from a safe distance. As a source of such sound three possibilities have been suggested: oscillation and bursting at the surface; the breakup and coalescence of bubbles within the volcanic conduit; and oscillations of bubbles within the volcano. Models for all three have been created with the foremost of these being argued to be most important (Vergniolle and Brandeis, 1996; Vergniolle et al., 1996).

In order to probe the apparent contradiction between theoretical and experimental work on maximum Taylor bubble size, in this paper we examine the behaviour and stability of bubbles within two purpose-built tubes with internal diameters 0.121 m and 0.29 m. For the smaller tube we find that Taylor bubbles exist and are stable. For the larger tube we again demonstrate that such bubbles exist and their stability properties are in keeping with the analysis of Batchelor (1987). We also find that if the bubble injection is abruptly curtailed, these bubbles oscillate in length as theorised in the geological literature. We provide a more formal model for these oscillations.

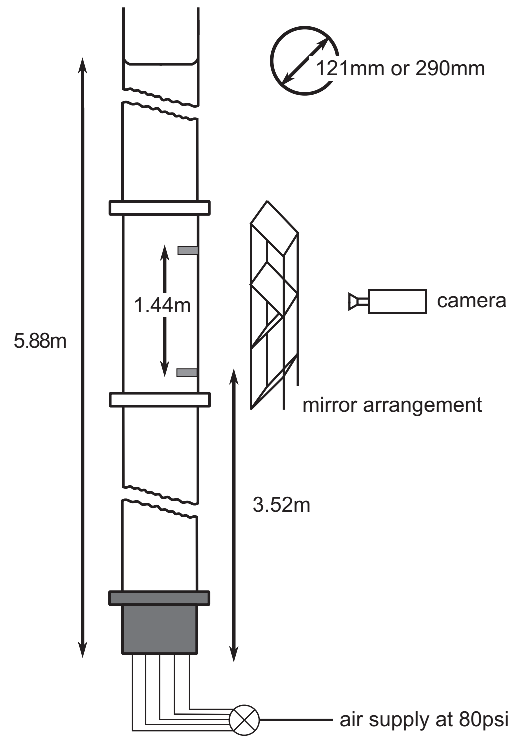

Fig. 1. Diagram of experimental apparatus. Gas is injected at the bottom of the column. Velocity measurements are taken using a system of mirrors in conjunction with a high speed camera.

2. Experiments

The experiments were performed in two vertical, cylindrical acrylic pipes with internal diameters of 0.121 m and 0.290 m, and a height of 9.3 m. Air was fed into the bottom of the larger column by means of 25 separate 5 mm nozzles, controlled in groups of 5 with variable flow rates, and by means of one such group of 5 nozzles in the smaller tube. These were fed by a high pressure (80 psi) mainline supply of compressed air. Bubbles of different lengths were created by injecting air for different time periods (controlled by hand) and at different flow rates (controlled by valve aperture). The tube was partially filled with water to a depth of 5.88 m (note that this level rises as air is injected), with the upper surface being free. Measurements were taken using two cameras, a Phantom V9.1 highspeed camera and a Sanyo Xacti AVC/H.264. The full set up is illustrated in Fig. 1.

The method of gas injection meant that in both columns, the large Taylor bubbles of interest were followed by a cloud a small (≈5 mm) bubbles. In the case of the 0.121 m pipe this cloud was relatively short (of the order of 1 m) and disperse. For the larger pipe the cloud was dense and filled up most of the pipe behind the trailing edge. An added effect of this was that for the same length of Taylor bubble, the water level in the larger pipe rose higher after injection than in the narrower pipe.

2.1. Rise rates

Bubble rise rates were measured using the high speed camera and a system of mirrors which allowed us to record two vertically separated points in the pipe 1.44 m apart. The observation window started 3.52 m from the base of the pipe in order to allow for a development region, but sufficiently below the upper liquid surface to avoid any effects from it. In the 0.29 m pipe, the rise of 55 bubbles were recorded, while in the 0.121 m pipe 41 bubbles were examined. In between each run the pipe was left to settle for 120 s as this was found to be sufficient to allow every observed bubble to rise stably. For each bubble the length was found by examining footage of the bubble taken with the Sanyo camera.

2.2. Stability

Within the 0.29 m tube we examined the settling period necessary between runs to ensure that each bubble was stable in an additional series of experimental runs. This was done by observing the behaviour of a bubble as it rose through a 2 m observation window started 3.3 m above the base of the column. For each bubble observed we recorded either that the bubble was stable or unstable depending on its behaviour in this window. The settling time was defined by the time between the last gas from the preceding bubble leaving the observation window until the next bubble entered it. We calculated the probability of bubble stability for a series of increasing settling periods. For each settling period we repeated the experiment to calculate the probability. In total 135 runs were observed with settling times between 50 s and 90 s. A further 20 runs were observed for 120 s. Due to break up it was not possible to measure the length of all the bubbles being tested. Instead, all were injected in a manner that, were it not for break up, would have lead to a 0.45 m bubble had the bubble been stable.

2.3. Oscillations

When the gas was injected into the column, the smoothness of the gas cut-off was observed to alter the bubble's behaviour. For the smoothly rising bubbles described above a slow cut-off was used. If the gas injection was ended abruptly, however, the bubbles oscillated as they rose, undergoing a compressive/decompressive

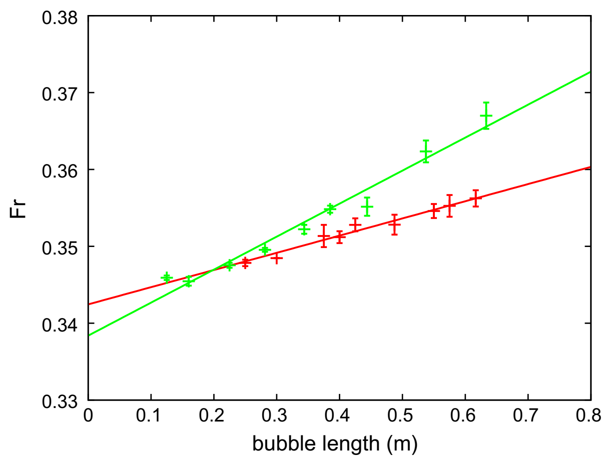

Fig. 2. The non-dimensionalised rise rates (Fr) of bubbles in the 0.29 m (red) and 0.121 m (green) pipe and their dependence on bubble length. The variation in Fr is due to the bubbles expanding as they rise. Consequently extrapolation to a bubble of zero length gives the rise rate of bubble not undergoing expansion – $Fr = 0.342$ in the case of the larger pipe and $Fr = 0.338$ in the case of the narrower one. The different gradients for the two pipes are due to the effect of H on $\dot{L}$ (Eq. (3)). Error bars represent one standard error. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

cycle. This could be clearly observed by the motion of the surface of the liquid rising and falling. The surface's location was tracked using the high speed camera to allow us to measure the oscillations. By compiling the behaviour of sixteen bubble rises we were able to calculate the evolution of the oscillatory frequency as the bubble rose up through the pipe.

3. Results and discussion

3.1. Rise rate

Although Taylor bubbles are qualitatively well defined (large, diameter filling bubbles), they lack more quantitative characterisation. To determine whether our bubbles were fully developed we measured their rise rates to compare with the Froude numbers (Eq. (1)) for inviscid Taylor bubbles reported widely.

The observed rise rate of any bubble is complicated by the fact that if the upper surface of the liquid is free, then the bubble will expand as it rises. Pressure arguments suggest that in the case of a bubble horizontally constrained (as in our experiment), this expansion will lead to the trailing edge of the bubble rising at a constant rate and the upper surface of the bubble rising at a modified rate given as

$$U = U_b + \dot{L}, \tag{2}$$

where U_b is the rise rate of an unexpanding bubble and L is the bubble's length. Moreover, assuming that the bubble expands as an ideal gas leads to the conclusion that

$$\dot{L} \propto L/H, \tag{3}$$

H being the depth of the bubble below the free surface (White and Beardmore, 1962).

In Fig. 2 we plot the non-dimensionalised rise rates (Froude numbers) of our observed bubbles in the 0.121 m tube and the 0.29 m tube. As expected, the rise rate of the bubbles varies with bubble length in an approximately linear manner. The differing gradients are due to different effective water heads due to the injection method (see Section 2). Extrapolating the data to a theoretical bubble of zero length gives the rise rate of an non-expanding bubble. This corresponds to Froude numbers of 0.338 and 0.342 for the 0.121 m and 0.290 m pipes respectively, consistent with the published experimental values (e.g. Dumitrescu, 1943). We conclude that the bubbles are indeed rising as well developed Taylor bubbles.

3.2. Stability

Taylor bubbles typically break up by large sections being stripped down the side (Fig. 3). Whether this breakup occurs will depend on the quiescence of the fluid the bubble rises into.

For the smaller 0.121 m tube, all the bubbles we created were observed to be stable. Unless the bubbles were released sufficiently close together that the second bubble actually caught the trailing bubbles in the wake of the first bubble, breakup was never observed. This is in keeping with previous experimental work suggesting that bubbles are stable in this diameter of tube (Martin, 1976; Kawanishi et al., 1990).

For the larger diameter (0.29 m) pipe, the bubbles were more susceptible to instability. The experimentally determined probability of stability of these bubbles is plotted in Fig. 4. For a settling period of 120 s, all observed bubbles were stable. We then reduced the settling period to find the time window for which bubbles became unstable. There is no critical value for the settling period, but instead a drop off over a ~50 s window where the bubbles go from being almost certainly stable to certainly unstable. The probabilistic nature of these results come from the chaotic nature of the decaying turbulent eddies which trigger breakup, and the precise time window at which the change happens could depend on column depth. The absolute stability at larger times ($\geqslant$120 s) leads us to confidently conclude that Taylor bubbles are stable within a 0.29 m pipe, as predicted by Batchelor.

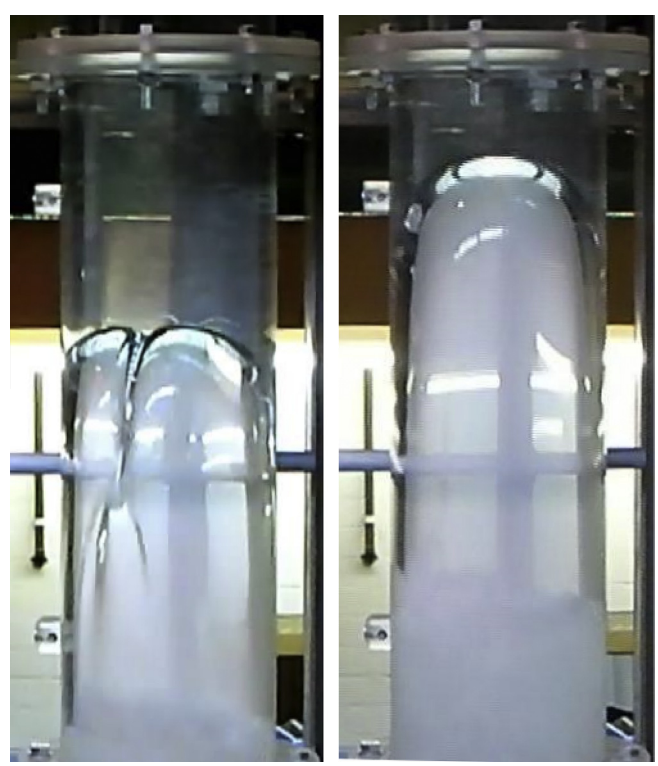

Fig. 3. Two Taylor bubbles rising through the 0.29 m column. The one on the left is undergoing breakup, while the right-hand bubble is stable.

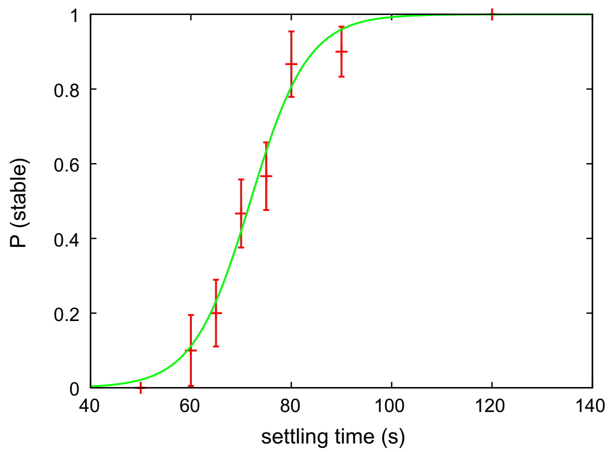

Fig. 4. The probability of a Taylor bubble breaking up within a 2 m observation window in the 0.290 m pipe. The settling time is defined by the delay between the last of the gas from the previous bubble departing the window to the tip of the bubble being considered entering the window. Error bars are found by assuming a binomial distribution to the data's repeated runs and represent one standard deviation. The green line is included to guide the eye. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.3. Oscillatory behaviour

The results discussed so far have all described the progress of bubbles rising smoothly up through the pipe. These were created by gas injection being slowly cut off. If instead the gas injection was terminated suddenly, an entirely different behaviour was observed. Qualitatively, the bubbles no longer rose smoothly up the pipe and instead lurched their way up in fits and spurts. The position of the water surface at the top of the 0.29 m pipe was tracked using the high speed camera, and a typical example is shown in Fig. 5. As the water is essentially incompressible, we interpret the surface behaviour as being the bubble oscillating in length, while undergoing a mean expansion. We could not test this directly by measuring the Taylor bubble's length as its trailing edge is too indistinct and ragged for a precise enough measurement.

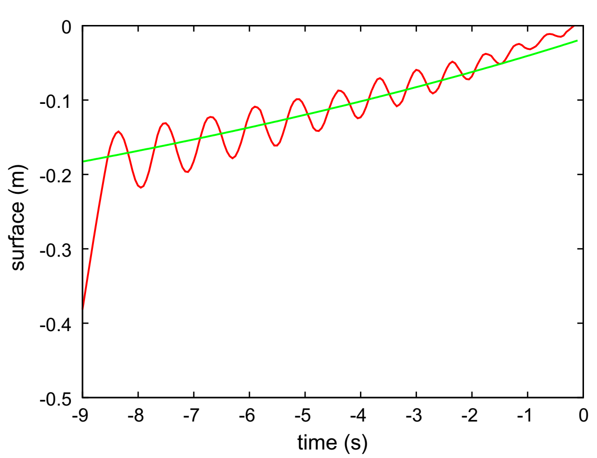

Fig. 5. The evolution of the height of the water's surface from the time the surface enters the shot of the highspeed camera until the bubble bursts it at $t = 0$ and $h = 0$. The red line is taken from the rise of a bubble initially 0.55 m long. The mean rise of the surface is due to the bubble expanding as it decompresses during the rise. This is captured by the green line which has been calculated by assuming the bubble expands as a perfect gas obeying $pV = const$. Time is measured from when the bubble will burst at the surface. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The oscillations can be modelled formally by a model analogous to the Rayleigh–Plesset equation (Plesset, 1949) reduced to one dimension. We consider the case where rise rate is slow compared to the oscillations, and so can neglect gravity. Our theoretical bubble will consist of a cylinder of air of length L that fills the entire cross-section of the pipe. If the water is inviscid and incompressible then the velocity field in the water above the bubble due to the bubble length changing by $L + h(t)$ will simply be given by

$$\mathbf{u} = \dot{h}(t)\hat{\boldsymbol{z}}. \tag{4}$$

The momentum component of the Navier–Stokes equations are correspondingly simple,

$$\ddot{h}(t) = -\frac{1}{\rho}\frac{\partial}{\partial z}p(z). \tag{5}$$

If initially the top of the bubble is at height $z = 0$, and the surface of the water is at height $z = H$, then perturbing the bubble in this way will lead to the water surface being perturbed to $z = H + h$. In the manner of the Rayleigh–Plesset equation, we integrate the Navier–Stokes equation along a streamline leading from the top of the bubble to the open surface

$$\int_h^{H+h} \ddot{h}(t)\mathrm{d}z = -\frac{1}{\rho}\int_h^{H+h}\frac{\partial}{\partial z}p(z)\mathrm{d}z, \tag{6}$$

$$\ddot{h}(t)H = -\frac{1}{\rho}[p(H+h) - p(h)]. \tag{7}$$

We note that $p(H + h) = p_{atm}$ and that $p(h)$ is equal to pressure within the bubble. Progress is made by assuming that the bubble expands polytropically (with polytropic exponent γ),

$$pV^{\gamma} = k. \tag{8}$$

In the case of small changes of volume,

$$p(V+\Delta V) = \frac{k}{(V+\Delta V)^{\gamma}}, \tag{9}$$

$$= \frac{k}{V^{\gamma}} - \gamma k\frac{\Delta V}{V^{\gamma+1}} + O(\Delta V^2), \tag{10}$$

$$= p(V)\left[1 - \gamma\frac{\Delta V}{V}\right] + O(\Delta V^2), \tag{11}$$

$$\Rightarrow p(L+h) = p(L)\left[1 - \gamma\frac{h}{L}\right] + O(h^2). \tag{12}$$

Substituting this back into the evolution equation for h (7), along with the assumption that in equilibrium the bubble's internal pressure is simply hydrostatic, leads to the equation

$$\ddot{h}(t) + \frac{\gamma}{L}\left[g + \frac{p_{atm}}{\rho H}\right]h = 0, \tag{13}$$

the equation for a simple harmonic oscillator with frequency

$$\omega^2 = \frac{\gamma}{L}\left[g + \frac{p_{atm}}{\rho H}\right]. \tag{14}$$

Indeed, under the assumptions of simple harmonic oscillations, Vergniolle et al. (1996) derived a similar frequency. This approach only models the frequency of the oscillations and not their amplitude as it is a linear model.

In Fig. 6 we compare the predicted frequency of oscillation of two lengths of bubble within the 0.29 m pipe to observations. The frequencies are taken from a series of ten runs for the shorter bubble and six for the longer one. As the bubble rises the frequency increases as the mass of water on top of the bubble is reduced. While the bubble is deep within the tube the model predicts the behaviour well, however as the bubble nears the surface and the relative volumes of fluid above the bubble and going around the

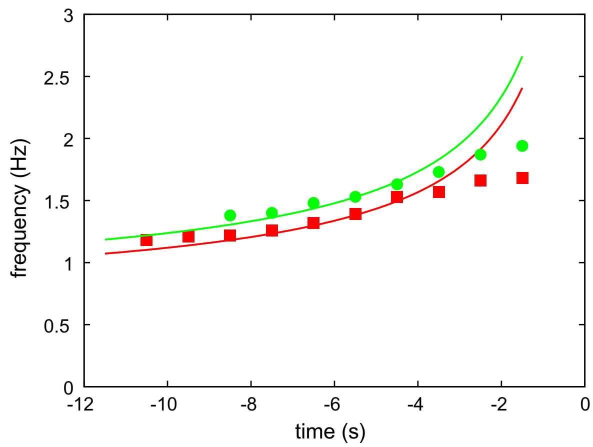

Fig. 6. The evolving frequency of oscillating bubbles as they rise up the pipe for two different mean lengths of bubble (red is 0.55 m, green is 0.45 m). The points represent experimental data taken from ten runs in the case of the longer bubble and six in the case of the shorter bubble. The lines come from the theoretical model given by Eq. (14), where the polytropic exponent has been taken to be 1. Time is measure from the bubble bursting at the surface. The model fits well until the bubble approaches the surface. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

bubble's nose become comparable, the model begins to diverge from observations.

This model explains the nature of the oscillations recorded, but not their origin. The oscillations come about because of the manner in which the bubbles are created. When the gas jets at the base of the column are turned on, the air being forced into the bottom of the tube causes the entire column of liquid to rise. When the gas is turned off, the liquid comes to a stop. If the gas flow is stopped abruptly, the sudden deceleration of the mass of liquid above the bubble applies an inertial force to the bubble. This leads to the compressive/decompressive cycle described above. If the gas jets are turned off more slowly this effect is ameliorated as in the experiments of Sections 3.1 and 3.2.

4. Stability model

The analysis of bubble stability is a non-trivial problem. In the absence of a strong stabilising force (such as surface tension), bubbles exist in a permanent state of flux. Any interface between a large bubble and the body of fluid it is rising through can be expected to be linearly unstable due to well-known Rayleigh–Taylor type effects. The density difference between the two fluids is such that any disturbance to the surface will grow exponentially. If the bubble and liquid were stationary, this would be enough to ensure breakup. However, the liquid's motion past the bubble complicates the problem by advecting disturbances on the surface off the top of the bubble. If the disturbance is washed off the bubble before it grows large enough to break the bubble up, the bubble will appear stable. This is in essence a competition of timescales – one dictating the growth rate of disturbances and one the rate at which they are removed.

Batchelor (1987) analysed this competition in the case of inviscid fluids using linear analysis. The problem, he noted, is further complicated by the straining nature of the liquid's flow around the bubble. This inevitably leads to the distortion of any disturbance to the interface. Nonetheless, he arrived at a single equation to describe the evolution of the amplitude of a disturbance to such a bubble

$$\frac{d^2A}{dt^2} + 3k\frac{dA}{dt} + A\left[2k^2 - gn\left(1 - \frac{n^2}{n_c^2}\right)\right] = 0. \tag{15}$$

The disturbance has amplitude A, while it can be taken to have either Fourier-Fourier or Fourier–Bessel form. The mode has wavenumber n, which evolves in time from an initial value n_0 given by the size of disturbance encountered, simply as

$$n = n_0 \exp\{-kt\}. \tag{16}$$

Finally, R is the radius of curvature of the tip of the bubble (taken to be 1/3rd of the tube's diameter), $k = \sqrt{g/R}$ and n_c is the critical wavenumber (that which gives maximum growth rate) for the problem of Rayleigh–Taylor instability in the instance of a flat interface. In the case of water and air it corresponds to a wavelength of 1.71 cm.

Although this equation describes the evolution of a disturbance, in order to assess whether a disturbance will break up a bubble a criterium is required. Batchelor suggested that this should depend on the wavenumber of the disturbance as if the wavelength ($\lambda = 2\pi/n$) is "appreciably larger than R, the disturbed interface is obtained approximately by displacing the undisturbed interface as a whole." He consequently argues in favour of the criterium that once the disturbance has been stretched to a wavelength *equal* to R breakup will occur if $A > 0.082R$. The coefficient being that required to generate a positive curvature of $2/R$. Though this was his preferred criterium he also provided for the choice of $A > 0.041R$, corresponding to there being a point of precisely zero curvature.

If we assume that any such disturbance was created by the bubble encountering a turbulent eddy of size $2\pi/n_0$ and velocity dA/dt, then it is possible to calculate the minimum velocity of such an eddy required to trigger breakup. This is shown in Fig. 7. The stability curves were calculated by solving (15) numerically and finding the critical values by iterative refinement. In our stability experiment, the source of such turbulent eddies would be the wake of the preceding bubble. These eddies decay with time. The precise way that they do so is a complex and much debated problem beyond the scope of this paper. Despite this, we do make a simple observation. The trailing bubbly wake of the Taylor bubbles we observed in the 0.29 m pipe rose at a rate of ~0.25 m/s. Comparing this velocity to Fig. 7 shows that indeed this is sufficient to break up the following bubble. Decay laws such as that suggested by Kolmogorov (Kolmogorov, 1941) suggest that eddy velocities decay as a power of time, for example $u^2(t) \sim t^{-10/7}$. Applying this crudely

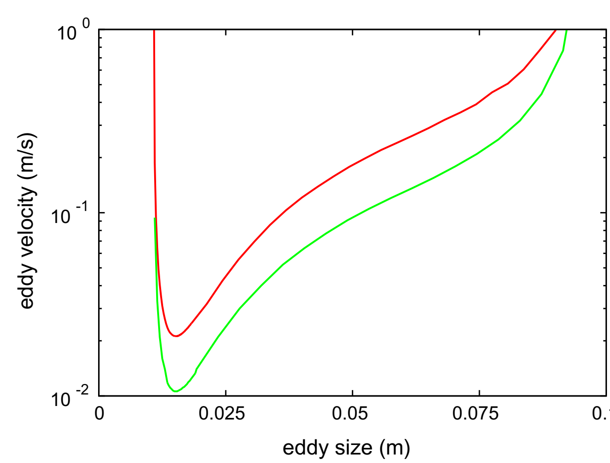

Fig. 7. The minimum amplitude of a turbulent eddy as a function of eddy size predicted by Batchelor that will lead to breakup of a Taylor bubble rising through water in a 0.29 m tube. The two lines correspond to too different stability criterion; $A_{\lambda=R} = 0.082R$ (red) and $A_{\lambda=R} = 0.041R$ (green). Both criterion give a minimum critical amplitude for disturbances of initial length scale 0.0152 m but with amplitudes of 0.0211 m/s for the former and 0.0106 m/s for the latter. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

suggests that the typical eddy velocities will decay enough to allow bubbles to remain unbroken after a delay of ~30–80 s (depending on the breakup criterium chosen). Although not an exact match to the observed settling time required (Fig. 4), it is approximately in line with it. This suggests that Batchelor's approach contains all the important physics required to capture the key dynamics.

5. Conclusions

In this paper we observed stable Taylor bubbles in larger diameter pipes than had previously been reported, and have shown that their rise can be either smooth or oscillatory in nature. In the smooth case, the rise rates of these bubbles follow the same dimensional relationship as for much narrower tubes. The stability of these large diameter bubbles is in keeping with much of the theoretical literature (Grace et al., 1978; Batchelor, 1987; Kitscha and Kocamustafaogullari, 1989; Krechetnikov, 2009) and specifically the modelling of breakup in terms of the bubble encountering turbulent eddies left behind in the pipe appears reasonable. It seems unlikely that these results mean that it should be possible to observe slug flow in the case of continuous gas flux. The settling times found here to allow for stable bubble passage correspond to a typical separation distance in the 0.29 m pipe of ~30 m, a large enough distance as to be unfeasible in any realisable experimental and most industrial set ups.

In order to probe the maximum diameter of pipe which can support Taylor bubbles, ideally a series of increasingly wide tubes would be experimented upon. This seems a formidable task, and so perhaps computer simulations offer the best chance of a solution. Full numerical simulations of this problem seem currently well beyond the bounds of computer power, suggesting CFD algorithms are the best option. For these to produce reliable results, the precise models used – in particular those describing the turbulent wake – must be carefully chosen.

The oscillations observed here are well described in terms of a simple harmonic oscillator, the equation for which we have found by a method analogous to the derivation of the Rayleigh–Plesset equation. They are expected, however, to be important to fields where acoustic signals are used to analyse the flow – such as in industrial flow diagnostics or in volcanic monitoring. In the latter of these there is a difference in terms of scale and of course viscosity, however much of the fundamental physics remains the same. The oscillations may also have implications for the stability of these bubbles. We detected no influence, but it may be that if the amplitude or frequency of the oscillations increases that this changes.

Acknowledgements

The authors would like to thank J.D. Perrott who helped with running the experiments. CCTP and SA were supported by NERC Grant NE/G016593/1.

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