Pergamon

Chemical Engineering Science, Vol. 53, No. 16, pp. 2973–2983, 1998

Printed in Great Britain
0009–2509/98/$–See front matter

PII: S0009–2509(98)00121–3

Coalescence of two gas slugs rising in a co-current flowing liquid in vertical tubes

A. M. F. R. Pinto,* M. N. Coelho Pinheiro† and J. B. L. M. Campos*‡

* Departamento de Engenharia Química, Faculdade de Engenharia da Universidade do Porto, 4099 Porto Codex, Portugal; † Departamento de Engenharia Química, Instituto Superior de Engenharia do Instituto, Politécnico de Coimbra, Apartado 4065, 3030 Coimbra, Portugal

(Received 21 November 1997; received in revised form 14 April 1998; accepted 20 April 1998)

Abstract—An experimental study is presented about the interaction between two slugs rising through a co-current flowing liquid in vertical tubes with 22, 32 and 52 mm internal diameters. For the conditions studied the liquid flow pattern in the wake of the slugs was turbulent and the main liquid flow regime was either laminar or turbulent. When the liquid flow regime is turbulent the minimum distance between slugs above which there is no interaction is about $5D$. For lower distances the slugs coalesce during their rise. When the liquid flow regime is laminar two different situations are observed:

—if the ratio between the average velocity in the full developed film around the slug and the average velocity in the main liquid is greater than 25, coalescence between slugs is observed. The minimum distance between slugs above which there is no interaction is about $10D$;

—if the value of that ratio is lower than 25 and the initial distance between slugs is greater than the wake length of the leading slug, the distance between slugs increases during their rising in the column and slugs coalescence is not observed. This behaviour has never been referred in the literature.

The wake length is found to be $5D$ whatever is the main liquid flow regime. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Two phase flow; slug flow coalescence.

INTRODUCTION

The 'slug flow' is a gas–liquid flow characterized by long gas bubbles almost filling the pipe cross-section, and liquid moving around the bubbles and in bulk between two successive bubbles. The complex slug flow structure can be described, in a simplified way, as a sequence of 'unit cells'; each unit cell consists of a bubble with a liquid film flowing around and a portion of liquid behind the bubble. When the flow is fully developed these unit cells rise in the column in a steady sequence. This steady condition is reached only some distance above the point where the gas is injected and until there, a significant interaction between successive bubbles happens with substantial alterations in bubbles velocity and frequency.

In a recent work Pinto and Campos (1996) related a study about the coalescence of two slugs rising through a stagnant liquid in a vertical column, i.e., in a column with no net liquid flow. The main conclusions were:

(i) there is a minimum distance between slugs, $\ell_{\min}$, above which there is no interaction, this conclusion comes from the work of Moissis and Griffith (1962);

(ii) the minimum distance between two slugs, $\ell_{\min}$, may be divided into two parts:
–one occupied by the wake of the leading bubble (when the nose of the trailing bubble is inside this region, the interaction between slugs is strong and the coalescence is practically instantaneous);
–the other corresponds to the region where the liquid coming from the wake of the leading bubble recovers its initial condition, i.e., the length needed to have again motionless liquid.

The flow pattern in the wake of long bubbles rising through stagnant liquids and almost filling the cross section of large tubes was studied by Campos and Guedes de Carvalho (1988). They concluded that the flow pattern depends on the dimensionless parameter. $N_f = g^{1/2}D^{3/2}/\nu$. Three different patterns were found in the wake over the whole range of this dimensionless parameter:

(i) laminar flow — closed and axisymmetric wake with internal recirculatory flow ($N_f < 500$);
(ii) transitory flow — closed unaxisymmetric wake with internal recirculatory flow ($500 < N_f < 1500$);

‡ Corresponding author.

(iii) turbulent flow — open and perfectly mixed wake ($N_f > 1500$).

The work of Pinto and Campos (1996) showed the dependence of the dimensionless minimum stable liquid length, $\ell_{\min}/D$, on the dimensionless group N_f. Three distinct relations corresponding to the different flow patterns in the wake were identified. According to Pinto and Campos (1996) the length occupied by the wake is approximately 24% of $\ell_{\min}$ whatever is the liquid flow pattern behind the slug.

After the work of Pinto and Campos (1996), exposed briefly above, one question remains: would the conclusions be the same if the slugs rise in a flowing liquid?

Moissis and Griffith (1962) performed an experimental study about the coalescence of slugs rising in flowing liquids and referred as moderate the influence of the liquid flow. Performing a careful analysis of this work some remarks are pertinent: the data obtained have a high scattered level and the regime of the flowing liquid is always turbulent.

Adopting the experimental technique described by Pinto and Campos (1996), based on the signals of differential pressure transducers placed along the column, experiments were performed to investigate the influence of liquid flow on the coalescence of two rising slugs. The experiments cover a large range of liquid Reynolds number (laminar and turbulent liquid flow), but are restricted to turbulent flow pattern in the wake of the bubbles.

THEORY

Velocity of an individual slug in a flowing liquid

This topic has been studied by a great number of investigators, among them Griffith and Wallis (1961), Nicklin et al. (1962), Zuber and Findlay (1965) and Collins et al. (1978), and the main features are well known.

Collins et al. (1978) presented solutions to both laminar and turbulent liquid velocity profiles with the form

$$U = U_C + (gD)^{1/2}\,\phi\left\{\frac{U_C}{(gD)^{1/2}}\right\} \tag{1}$$

where U is the slug velocity, U_C is the liquid velocity at the tube axis and ϕ indicates a functional relationship, the form of which depends upon the shape of the liquid velocity profile.

Collins et al. (1978) theoretically showed that for a turbulent liquid velocity profile, the slug velocity is predicted by

$$U = U_L\left(\frac{\ln Re_{U_L} + 0.089}{\ln Re_{U_L} - 0.74}\right) + 0.347(gD)^{1/2} \times \phi\left\{\frac{U_L}{(gD)^{1/2}}\,\frac{1.81}{\ln Re_{U_L} - 0.74}\right\} \tag{2}$$

where Re_{U_L} is the Reynolds number of the flowing liquid (with mean velocity U_L). The coefficient of U_L simply expresses the dependence of U_C/U_L on Re_{U_L}. The authors observed an excellent agreement between the results of this equation and those predicted by Nicklin et al. (1962), for turbulent liquid flow

$$U = 1.2U_L + U_s \tag{3}$$

where U_s is the slug rising velocity through a stagnant liquid. The coefficient 1.2 is close to the rate $U_C/U_L = 1.22$ obtained from a one-seventh power-law profile.

For slug velocity in a laminar flowing liquid Collins et al. (1978) arrived to the following theoretical expression:

$$U = 2U_L + 0.347(gD)^{1/2}\,\phi\left\{\frac{2.39\,U_L}{(gD)^{1/2}}\right\}. \tag{4}$$

The authors experimentally confirmed the results predicted by the above equation but they only succeeded when the parabolic velocity profile in the liquid was well established, i.e., when entrance effects on the tube were not felt in the measurement section [the ratio between the entry length and the diameter of the tube is proportional to Re_{U_L} being the constant of proportionality 0.08 according to White (1974) or 0.028 according to Govier and Aziz (1972)].

Comparing, once more, the results of the above equation with those obtained with the equation cited by Nicklin et al. (1962), for laminar liquid flow

$$U = 2U_L + U_s \tag{5}$$

it is found a quite good agreement. The coefficient 2 is close to the rate $U_C/U_L = 2$ obtained from a parabolic profile.

According to White and Beardmore (1962) and Collins et al. (1978) the slug rising velocity through a stagnant liquid, U_s, is a function of g, the acceleration due to gravity, D, the internal diameter of the tube and ρ, μ and σ, respectively, density, viscosity and surface tension of the liquid. From dimensional analysis it follows that the Froude number $Fr = U_s/(gD)^{1/2}$ is an unique function of the dimensionless groups $N_f = g^{1/2}D^{3/2}\rho/\mu$ and $M = g\mu^4/\rho\sigma^3$. For sufficiently large tubes, the slug rising velocity in liquids with low viscosity is independent of liquid properties and so Fr number is independent of N_f and M. According to Collins et al. (1978) this inertia controlled regime is specified by $N_f > 300$ and $N_f{}^{4/3}M^{1/3} > 100$ and White and Beardmore (1962) and Stewart and Davidson (1967) referred in this regime Fr as being equal to 0.35. Therefore,

$$U_s = 0.35(gD)^{1/2}. \tag{6}$$

Taking the above considerations, the individual slug velocity in a flowing liquid can be expressed by

$$U = CU_L + 0.35(gD)^{1/2} \tag{7}$$

where C is a constant that takes the value 1.2 when the liquid flow regime is turbulent and 2 when is laminar.

Flow pattern in the wake of a slug rising in a flowing liquid

According to Campos and Guedes de Carvalho (1988) the flow pattern and the length of the wake of long bubbles rising through stagnant liquids in sufficiently large tubes depend only on the dimensionless parameter N_f. In a moving liquid the flow in the wake is more complex and the determinant parameters are more difficult to identify.

The liquid ahead the nose of the slug, flowing with a lower velocity than the bubble, is picked up and begins to flow around, more precisely between the tube wall and the slug surface. Alongside the slug the mean relative velocity between the liquid and the bubble changes until the boundary layer adjoining the tube wall occupies all the annulus of liquid, i.e. until the liquid film is fully developed (the viscous force on an element of the liquid balances the gravitational force). From there on, the mean relative velocity remains constant and at the rear of the bubble the liquid film 'plunges' into the bubble wake (portion of liquid that rises behind the bubble). The streamlines of the 'plunging' liquid in the wake region determine the flow in the wake:

(i) if the liquid flows around [see Fig. 1(a)], suffering a slowing down process as the open area to the flow increases until it occupies all the cross-sectional area, the wake is said to be closed;
(ii) if turbulent eddies are induced [see Fig. 1(b)] by the collisions between liquid streams at different velocities (magnitude and sense), the wake is said to be open.

The emerging profile (from the wake region) develops in order to restore the main liquid profile (laminar or turbulent).

For the development of this work, it is important to specify the conditions to have a closed or an open

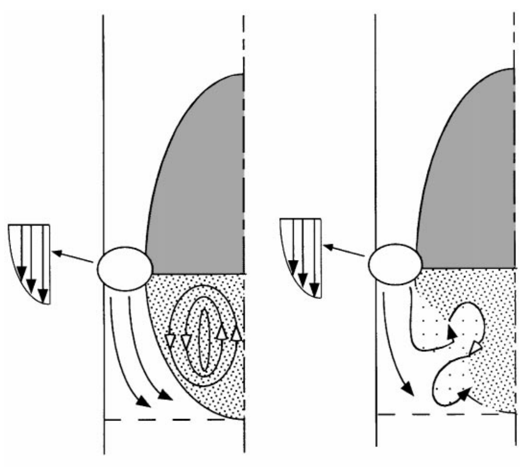

Fig. 1. Schematic representation of the liquid flow at the rear of the slug when its wake is closed (a) and open (b).

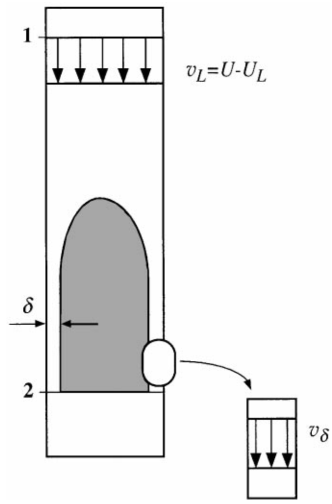

Fig. 2. Gas slug rising in a flowing liquid as seen by an observer moving with the slug.

wake. Figure 2 shows the liquid motion when the slug rises in the column as seen by an observer moving with the slug. The mean relative (downward) velocity of the liquid flowing undisturbed ahead the nose of the slug is

$$V_L = U - U_L = (CU_L + U_s) - U_L = (C-1)U_L + U_s. \tag{8}$$

The Reynolds number of the flowing liquid based on the mean relative velocity V_L, $Re_{V_L}(=V_L D/\nu)$ is for inertial controlled regime

$$Re_{V_L} = \frac{D[(C-1)U_L + U_s]}{\nu} = \frac{D[(C-1)U_L]}{\nu} + 0.35N_f. \tag{9}$$

If the wake flow pattern is determined only by Re_{V_L}, by comparison with the study of Campos and Guedes de Carvalho (1988) for no net liquid flow ($U_L = 0$), it can be concluded that:

(i) the wake is closed and axisymmetric with internal recirculatory flow when

$$Re_{V_L} < 0.35 \times 500 < 175 \tag{10}$$

(ii) the wake is closed and unaxisymmetric with internal recirculatory flow when

$$0.35 \times 500 < Re_{V_L} < 0.35 \times 1500 \rightarrow 175 < Re_{V_L} < 525 \tag{11}$$

(iii) the wake is open and perfectly mixed when

$$Re_{V_L} > 0.35 \times 1500 > 525. \tag{12}$$

These limits for the different flow patterns in the wake have not been confirmed. They were established supposing that they are determined only by $Re_{V_L}$and so they must be used with care.

The present work was carried out with slugs rising in the inertial control regime ($N_f > 300$) and with turbulent flow in the wake ($Re_{V_L} \gg 525$). During the experiments, the turbulence in the wake was helpfully visualized by the randomly movement of small bubbles rising inside the wake.

The minimum stable length between two slugs in a flowing liquid column

As was described by Pinto and Campos (1996) the minimum stable length between two slugs rising through a stagnant liquid can be clearly divided into two zones:

(i) one corresponds to the length of the wake of the leading bubble;
(ii) the other includes the distance required to restore the liquid profile between bubbles.

The magnitude of the second zone should depend on the Reynolds number of the flowing liquid, Re_{U_L} and on the shape of the velocity profile of the liquid emerging from the wake of the leading bubble. This shape should strongly depend on the ratio between the mean downward velocity of the jet-like flow surrounding the bottom section of the bubbles (u_δ) and the upward main liquid velocity (U_L).

Reynolds number of the flowing liquid

The classical study of Osborne Reynolds about liquid steady flows in tubes showed that: the flow regime is laminar and the velocity profile is parabolic when $Re_{U_L} < 2100$, the flow regime is turbulent and the velocity profile is approximately flat when $Re_{U_L} > 4000$ and somewhere between 2100 and 4000 the flow shows some disturbances and it is said to be in the transition regime.

Classical studies related also the length of tube needed to have a fully developed parabolic profile in the liquid when the flow starts with a flat profile (entrance effect on tubes); the ratio between this length and the tube diameter was shown to be proportional to Re_{U_L} as referred before.

Another conclusion of these studies was that the length needed to have a fully developed turbulent profile is much shorter than that to have a fully developed laminar profile.

Velocity profile in the liquid emerging from the turbulent wake of the leading bubble

The sketch of Fig. 3 illustrates two distinct velocity profiles in the liquid emerging from the wake of a slug rising through a flowing liquid in laminar regime.

When the velocity of the liquid emerging from the wake is uniform [Fig. 3(a)] or almost uniform, the liquid velocity anywhere along the reattachment profile region is always lower than $2U_L$ (the maximum liquid velocity when the velocity profile is parabolic). This means that, when two slugs are rising in a column, if the leading bubble has in front of its nose a developed undisturbed parabolic profile and the trailing bubble travels in the reattachment profile region, the velocity of the trailing bubble is always lower than the velocity of the leading one. As a result, the distance between slugs increases during their rise in the column until the nose of the second one is outside the reattachment profile zone.

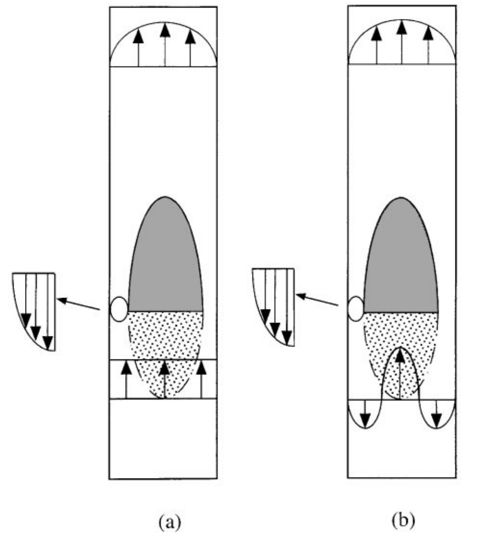

Fig. 3. Velocity profiles of the liquid emerging from the wake of a slug rising through a flowing liquid in laminar regime: (a) uniform velocity and (b) velocity at the tube axis greater than 2 U_L.

Figure 3(b) sketches a liquid emerging profile with a region where the velocity is greater than $2U_L$, for instance, at the center of the tube. In this situation, it is probable that each cross section of the reattachment region has some liquid flowing up with a velocity greater than $2U_L$. Then, a trailing bubble travelling in the reattachment profile zone has always in front of the nose an upward velocity greater than the velocity in front of the nose of the leading one ($2U_L$ for parabolic profile). As a result, the distance between bubbles decreases until they coalesce. This situation was visualized in the experiments by the radial movement of the nose of the trailing bubble (travelling from the center to the wall of the column) trying to have always in its front the fastest upward liquid velocity before coalescence.

The physical conditions to observe one or other profile should depend as already referred, on the ratio between the mean downward velocity of the jet-like flow surrounding the bottom section of the bubbles and the upward liquid velocity; as low is this ratio flatter should be the velocity profile shape. In Appendix A it is demonstrated that this ratio depends on the dimensionless parameters Re_{V_L}, U/U_L and N_f.

BASIS OF THE EXPERIMENTAL METHOD AND EXPERIMENTAL SET-UP

The basis of the experimental method is carefully described by Pinto and Campos (1996). Succinctly, it is based on the fast response of differential pressure

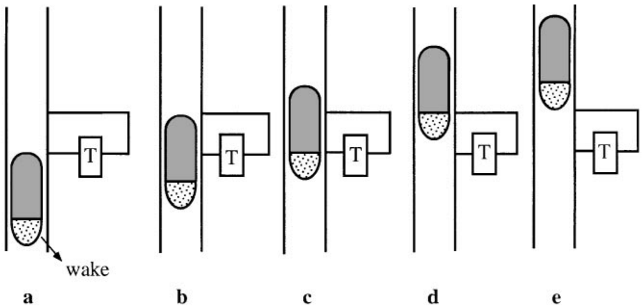

Fig. 4. Schematic representation of a slug rising in the section of the column where the pressure transducer taps are connected.

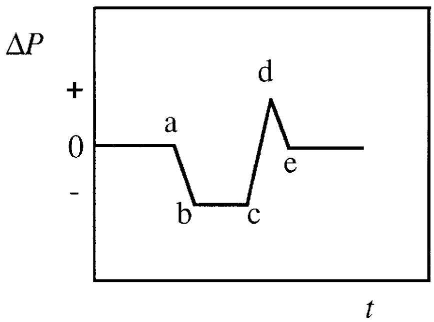

Fig. 5. Pressure signal from transducer during the rise of a slug bubble; a, b, c, d and e refer to Fig. 4.

transducers used to monitor the movement of the rising gas slugs through the flowing liquid in the column. Five particular instants in the rise of a single slug are depicted in Fig. 4 and the corresponding output from the transducer (converted to pressure) is shown in Fig. 5 [for more details see Pinto and Campos (1996)].

Values of bubble velocity and distance between bubbles are obtained from the time lag between the signals of two pressure transducers connected to pairs of taps at different locations along the tube (see Fig. 6).

The experimental set-up used in this study is sketched in Fig. 6. The coalescence experiments were performed in three acrylic columns with about 5.0 m height and 22, 32 and 52 mm internal diameters. The flow rate in the test column was maintained by means of a peristaltic pump; in order to have a continuous flow rate of liquid in the test column a damping chamber was placed between the pump outlet and the column. To measure the liquid flow rate the plastic hose dipped in the liquid inside the tank was emerged and the volume collected in a graduated container was timed. Triplicates of the measurements were performed to ensure reproducibility. Experiments were carried out with water and with aqueous glycerol solutions.

Near the bottom of the test column there was a mechanism for the slugs injection, sketched also in Fig. 6. The slugs were released by manipulating the ball valves. Initially, with valves 1 and 4 open and valve 2 and 3 closed, the spaces between valves were filled with air. The two gas slugs were released into the test column by closing valves 1 and 4 and opening valve 2 and, with some delay, valve 3. The volume of gas injected in the test column were enough to generate slugs with stabilized liquid films (see Campos and Guedes de Carvalho, 1988).

Two differential pressure transducers (T_1 and T_2) connected to the test column at more than 2.5 m above the point of slugs injection read continuously the pressure change as the slugs rose. The length between pressure taps could be chosen as 65, 70 or 80 mm bearing in mind the desired error (always less than 5%) for the velocity and the distance between slugs [for more details see Pinto and Campos (1996)]. The signals from the transducers were acquired at a frequency of 250 or 300 Hz by a computer with an analog-digital board and recorded for later processing, after conversion into pressure data.

During the experiments the liquid temperature was continuously measured by means of thermocouples placed in two points: near the transducers taps and between the slugs injection section and the damping chamber. For high liquid flow rates in the test column the temperature difference registered was insignificant. When low liquid flow rates were used the greater temperature difference registered was about 0.5°C.

The liquid viscosity at the different values of temperature registered during the experiments was measured with a rotating Brookfield viscometer.

For each column used, experiments with single slugs rising through the flowing liquid for a large range of Re_{U_L} (from laminar to turbulent regime) were performed in order to verify the validity of eq. (7). Experiments with single slugs rising through the quiescent liquid with a ball valve either closed or opened at the top of the test column, were also performed. The aim of these experiments was to obtain an estimate of the increase in the slug velocity due to the slug expansion during its rise. This estimate was used to correct all the velocity data shown in Fig. 7.

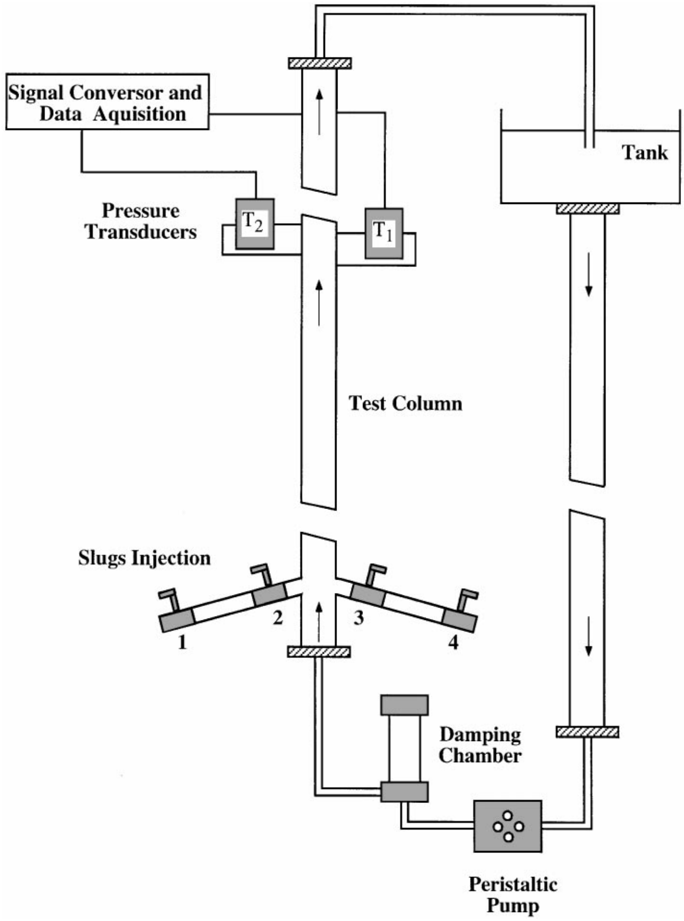

Fig. 6. Experimental set-up.

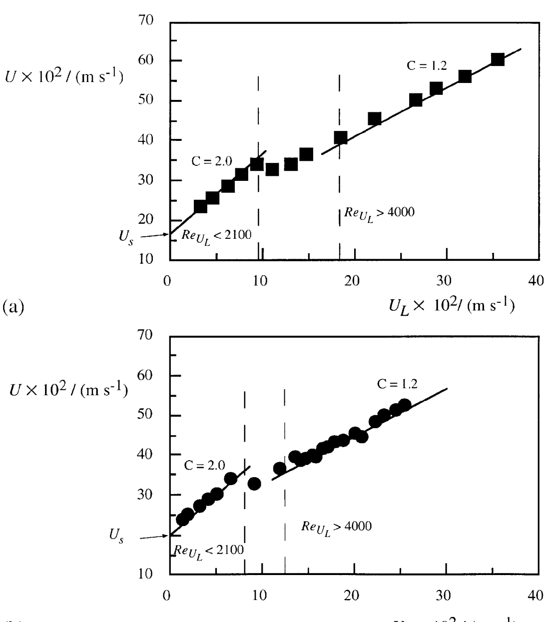

Fig. 7. Velocity of an isolated slug rising through flowing water in a vertical tube with 22 mm (a) and 32 mm (b) internal diameter for a large range of Re_{U_L} (from laminar to turbulent flow).

RESULTS AND DISCUSSION

Velocity of an individual slug in a flowing liquid

The plot of Fig. 7(a) and (b) shows the values of the velocity of a slug, U, rising isolated in a co-current flowing liquid with velocity U_L, in 22 or 32 mm columns of internal diameter, respectively.

According to eq. (7) the correlation between U and U_L is linear. The slope (C) of the straight lines will be about 2.0 for laminar flowing liquid regime and 1.2 for turbulent regime and the intercept value (U_s) will be equal to the velocity of an individual slug rising in the tube with no net liquid flow, $U_L = 0$. The full lines in the plots of Fig. 7 are obtained from eq. (7) (valid for a slug rising in the inertial controlled regime) with $C = 2.0$ and 1.2 which are a good fit to experimental data for $Re_{U_L} < 2100$ and $Re_{U_L} > 4000$, respectively.

The regime in the flowing liquid was laminar for all the experiments performed in the 52 mm internal diameter column. The value of $C = 2.0$ was obtained for the tested experimental conditions.

Although the agreement between predictions and experimental data is auspicious, some doubts remain on the value of the liquid Reynolds number for which the transition from laminar to turbulent regime begins. When the experiments were performed with water or very dilute glycerol solutions, the transition started at the classical value of $Re_{U_L} = 2100$; however, when liquids with moderate viscosity were used the transition seemed to start at lower values of Re_{U_L}. This abnormal behaviour is under investigation at the present.

Velocity of a disturbed slug rising through a turbulent flowing liquid

For each set of column, liquid tested and liquid flowrate, experiments were performed to obtain the individual velocities of slugs rising in pairs, for a large range of distances of separation, ℓ. The ratio between the velocity of the trailing slug, U_2, and the velocity of the leading one (rising undisturbed), U_1, is represented in Fig. 8 vs the dimensionless distance ℓ/D. The experimental conditions were chosen in order to have a turbulent flow in the wake ($Re_{V_L} > 525$) and a turbulent flow in the liquid between slugs ($Re_{U_L} > 4000$). The data shown cover a wide range of Re_{U_L}($6000 < Re_{U_L} < 9000$) and some features become apparent from the plot:

(i) the minimum distance above which there is no interaction between slugs for co-current upward flow is about $5D$ and is insensitive to Re_{U_L};

(ii) there is a sharp variation of U_2/U_1 for $\ell < 5D$, which means that almost all the minimum length is occupied by the wake of the leading slug. The length needed for the turbulent profile to be recovered is short and negligible when compared with the wake length;

(iii) the minimum distance, $\ell_{\min}$, obtained with no net liquid flow (full symbols in the plot) is higher than that for turbulent liquid flowing upwards. Although, in both situations, the rate U_2/U_1 has identical values for $\ell < 5D$, in the former the value 1.0 is reached only for $\ell = 12D$. In the range $5D < \ell < 12D$ this ratio has a moderate variation which suggests that the distance needed to have again motionless liquid is much higher than the value of $\ell_{\min}$ needed to restore the fully developed turbulent profile in the liquid;

(iv) as a consequence of the above considerations, it can be concluded that the length of a turbulent wake of a slug rising through a turbulent liquid flow is about $5D$ whatever are the gas–liquid flow conditions.

Velocity of a disturbed slug rising through a laminar flowing liquid

A plot of the values of U_2/U_1 versus ℓ/D for experimental conditions corresponding to turbulent flow in the wake ($Re_{V_L} > 525$) and laminar flow in the liquid between slugs ($Re_{U_L} < 2100$), is shown in Fig. 9. The data cover a wide range of Re_{U_L}($575 < Re_{U_L} < 1800$) and a curious behaviour can be observed: in some experiments (full symbols in the plot) the ratio U_2/U_1 is lower than 1.0 whatever ℓ is in the range $5D < \ell < \approx 25D$. This means that in some experimental conditions the distance between two slugs increases and by consequence they do not coalesce, situation never previously referred to in the literature, as far we are aware.

By contrast, in other experiments (open symbols in the plot) the ratio U_2/U_1 is substantially higher than 1.0 for $\ell < 5D$ and decreases gradually to 1.0 in the range $5D < \ell < 10D$.

The justification of these different behaviours can be found in the above section where the influence of the shape of the emerging liquid profile (from the

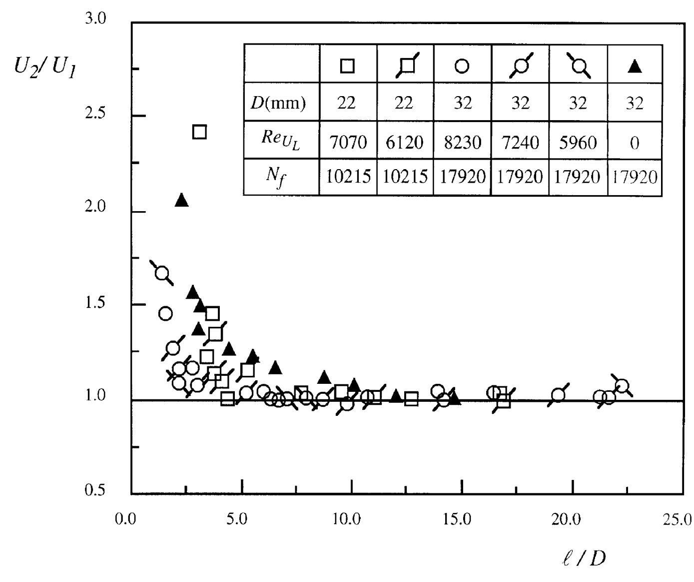

Fig. 8. Variation of U_2/U_1 with ℓ/D for experiments with turbulent regime in the flowing liquid. The full symbols refer to experiments with stationary liquid. The open symbols are for co-current upward turbulent flow. (The minimum distance above which there is no interaction between slugs is about $5D$ and is insensitive to Re_{U_L}.)

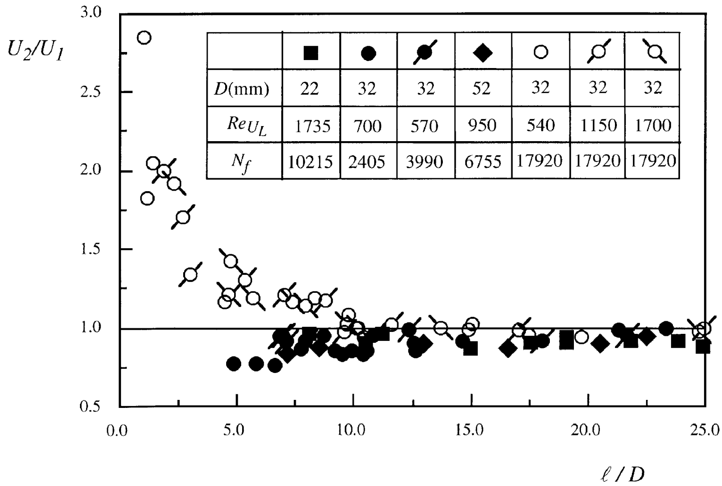

Fig. 9. U_2/U_1 vs ℓ/D for experiments with laminar regime in the flowing liquid. Open symbols are for experiments with coalescence of the slugs. The full symbols refer to experiments with no coalescence of slugs, corresponding to the unexpected values of U_2/U_1 less than 1.

Table 1. Values of u_δ/U_L calculated from eqs (A5) and (A7) at the experimental conditions used

N_f	D (mm)	Re_{V_L}	U/U_L	u_δ/U_L	
2405	32	1555	3.3	7.1	NC
6670	32	3805	3.0	10.2	NC
6645	32	3620	3.1	11.0	NC
6605	32	3755	3.5	12.6	NC
6510	32	3535	3.7	14.3	NC
10215	22	4840	3.3	15.0	NC
3990	32	2015	4.5	15.8	NC
7360	32	3690	4.0	16.8	NC
10215	22	4785	3.7	18.1	NC
10215	22	5355	4.0	19.1	NC
6755	52	3315	4.5	19.5	NC
7305	32	3550	4.5	20.1	NC
7195	32	3600	4.6	20.5	NC
6755	52	3210	4.8	21.5	NC
10215	22	4580	4.3	22.1	NC
7305	32	3390	5.3	25.2	C
6755	52	3060	5.4	25.5	C
6755	52	2935	6.1	30.4	C
4470	32	1920	7.2	31.2	C
7220	32	3105	6.4	32.9	C
17920	32	8160	5.3	36.2	C
6755	52	2830	7.1	36.6	C
7010	32	2945	7.3	38.5	C
7195	32	3020	8.9	48.6	C
6755	52	2705	8.9	48.8	C
17920	32	7585	6.8	50.0	C
4470	32	1745	13.3	65.7	C
6755	52	2585	12.8	74.3	C
6985	32	2750	13.2	77.2	C
17920	32	6945	11.3	92.9	C
6755	52	2495	19.7	119.6	C
6755	52	2470	24.1	148.4	C
6755	52	2430	38.8	244.3	C

Note: The symbol (C) identifies experiments where a decrease of the distance between two slugs during their rise in the column was observed with the consequent coalescence and symbol (NC) corresponds to experiments where an increase of this distance was observed.

wake) on the velocity of the trailing slug was discussed. However a question remains: what determines this behaviour?

In each cross-section of the wake there is liquid flowing down, coming from the film, and liquid flowing up in the wake of the slug, being the mean velocity, by continuity, U_L. This difference in velocities (in sense and magnitude) is attenuated along the wake. From the plot of Fig. 9 it can be concluded that the wake length is always about $5D$ (see region where the ratio U_2/U_1 changes sharply), which means that the length where occurs this attenuation is independent of the flow conditions. Although occurring in the same length, the attenuation must be stronger (i.e. the emerging liquid profile must be closer to flat) when the absolute plunging liquid film velocity, u_δ, is low and when the upward main liquid velocity, U_L, is high.

If the above analysis is correct, as the ratio u_δ/U_L is increased the more probable must be the coalescence of two slugs. This is confirmed in Table 1, where the calculated values of u_δ/U_L are shown for all the experimental conditions studied (the data are arranged by ascending order of u_δ/U_L). For each experimental condition, it is referred if it was observed a decrease of the distance between two slugs during their rise in the column with the consequent coalescence (C) or if it was observed an increase of this distance (NC). As it can be observed there is a change of the behavior when the value of u_δ/U_L is around 25. The values of u_δ are calculated from eqs (A5) and (A7), with $u_\delta = v_\delta - U$, where v_δ is the average liquid velocity in the film relative to the slug. Also shown in Table 1 are the values of the dimensionless parameters that according to eq. (A10) determine the ratio u_δ/U_L.

Some other information is apparent from the plot in Fig. 9 for experiments where there was coalescence between slugs:

(i) the minimum length to have coalescence is about $10D$ whatever is Re_{U_L};

(ii) for $\ell < 5D$ the ratio U_2/U_1 decreases sharply with ℓ/D which means that the trailing slug travels inside the wake of the leading one. This wake length (about $5D$) seems to be independent of Re_{U_L};

(iii) for $5D < \ell < 10D$ the trailing slug travels in the reattachment laminar profile zone. Unexpectedly the length of this zone seems to be also independent of Re_{U_L}.

Using the data shown in Fig. 9 for experiments where the distance between slugs increases during their rise, it can be concluded that:

(i) the wake length is equal or lower than $5D$; there are no data for distances below $5D$ as a consequence of the trailing bubble slowing down; at 2.5 m above the slugs injection section it is impossible to obtain a slug inside the wake of another;

(ii) the length of the reattachment profile zone is high and difficult to quantify; for $\ell = 25D$ some values of U_2/U_1 are still slightly below 1.0, however this length seems to be independent of Re_{U_L}.

SUMMARY AND CONCLUSIONS

An experimental study about the interaction between two slugs rising through a co-current upward flowing liquid is reported. The liquid pattern in the wake of the slugs was turbulent and the main liquid flow regime was either turbulent or laminar.

When the liquid flow regime is turbulent (i) the minimum distance above which there is no interaction between slugs for co-current upward flow is about $5D$ and is insensitive to Re_{U_L}; (ii) there is a sharp variation of U_2/U_1 for $\ell < 5D$, which means that almost all the minimum length is occupied by the wake of the leading slug whatever are the gas–liquid flow conditions.

When the liquid flow regime is laminar two different situations are observed:

(i) if the ratio between the average velocity in the full developed film around the slug and the average velocity in the main liquid is greater than 25, coalescence between slugs is observed. The minimum distance between slugs above which there is no interaction is about $10D$;

(ii) if the value of that ratio is lower than 25 and the initial distance between slugs is greater than the wake length of the leading slug, the distance between slugs increases during their rising in the column and slugs coalescence is not observed. This behaviour has never been referred in the literature.

For laminar regime in the flowing liquid the wake length was found to be also $5D$.

NOTATION

A	cross-sectional surface area, m^2
C	constant
D	internal column diameter, m
g	acceleration due to gravity, m s^{-2}
ℓ	distance between slugs, m
ℓ_{min}	minimum stable liquid length, m
u	liquid velocity in the film, relative to the tube wall, m s^{-1}
u_δ	liquid average velocity in the film, relative to the tube wall, m s^{-1}
U	slug velocity, m s^{-1}
U_1	velocity of the leading slug, m s^{-1}
U_2	velocity of the trailing slug, m s^{-1}
U_C	liquid velocity at the tube axis, m s^{-1}
U_L	average velocity of upward liquid flow in the tube, m s^{-1}
U_s	velocity of an individual slug in stagnant liquid, m s^{-1}
v	liquid velocity in the film, relative to the slug, m s^{-1}
v_δ	liquid average velocity in the film, relative to the slug, m s^{-1}
V_L	average velocity of upward liquid flow in the tube, relative to the slug, m s^{-1}
r	radial distance, m
t	time, s

Dimensionless groups

Fr Froude number $\left(=\dfrac{U_s}{(gD)^{1/2}}\right)$

M dimensionless group $\left(=\dfrac{g\mu^4}{\rho\sigma^3}\right)$

N_f dimensionless group $\left(=\dfrac{g^{1/2}D^{3/2}\rho}{\mu}\right)$

Re_{U_L} Reynolds number of the flowing liquid $\left(=\dfrac{DU_L}{v}\right)$

Re_{V_L} Reynolds number of the flowing liquid $\left(=\dfrac{DV_L}{v}\right)$

ξ dimensionless film thickness $\left(=\dfrac{2\delta}{D}\right)$

Greek letters

ΔP pressure difference between the taps of the column where the transducer is connected, Pa

δ equilibrium film thickness, m

v kinematic viscosity, $m^2\ s^{-1}$

μ liquid viscosity, Pa s

ρ liquid density, $kg\ m^{-3}$

σ surface tension of the liquid, $N\ m^{-1}$

ϕ functional relationship

REFERENCES

Brown, R. A. S. (1965) The mechanics of large gas bubbles in tubes. I. Bubble velocities in stagnant liquids. *Can. J. Chem. Engng* **43**, 217–223.

Campos, J, B. L. M. and Guedes de Carvalho, J. R. F. (1988) An experimental study of the wake of gas slugs rising in liquids. *J. Fluid Mech.* **196**, 27–37.

Collins, R., De Moraes, F. F., Davidson, J.F. and Harrison, D. (1978) The motion of large gas bubble rising through liquid flowing in a tube. *J. Fluid Mech.* **89**, 497–514.

Govier, G. W. and Aziz, K. (1972) *The Flow of Complex Mixtures in Pipes*, pp. 388–414, Van Nostrand, New York, U.S.A.

Griffith, P. and Wallis, G. B. (1961) Two-phase slug flow. *J. Heat Transfer* **83**, 307–320.

Moissis, R. and Griffith, P. (1962) Entrance effects in a two-phase slug flow. *J. Heat Transfer* **84**, 29–39.

Nicklin, D. J., Wilkes, J. O. and Davidson, J. F. (1962) Two-phase flow in vertical tubes. *Trans. Inst. Chem. Engrs* **40**, 61–68.

Pinto, A. M. F. R. and Campos, J. B. L. M. (1996) Coalescence of two gas slugs rising in a vertical column of liquid. *Chem. Engng Sci.* **51**, 45–54.

Stewart, P. S. B. and Davidson, J. F. (1967) *Powder Technol*, **1**, 61–80.

White, F. M. (1974) *Viscous Fluid Flow*, pp. 388, McGraw-Hill, New York.

White, E. T. and Beardmore, R. H. (1962) The velocity of rise of single cylindrical air bubbles through liquids contained in vertical tubes. *Chem. Engng Sci.* **17**, 351–361.

Zuber, N. and Findlay, J. A. (1965) Average volumetric concentration in two-phase flow systems. *J. Heat Transfer* **87**, 453–468.

APPENDIX A: METHOD FOR THE ESTIMATION OF δ AND u_δ/U_L

Several investigators studied the liquid flow around the slug when rising in stagnant liquids and estimated the thickness of the fully developed film (δ). In the present study, where the slug is rising in a flowing liquid new equations must be developed in order to estimate δ.

For a fully developed liquid film flowing in laminar regime, the velocity profile using a frame moving with the slug as reference (v) is (see e.g. Brown, 1965)

$$v(r) = u(r) + U = \frac{gD^2}{4v} \times \left[\frac{1-(2r/D)^2}{4} - \frac{[D-2\delta/D]^2}{2}\ln\frac{D}{2r}\right] + U \quad \text{(A1)}$$

where $u(r)$ is the liquid velocity (relative to the tube wall) at a distance r from the axis of the tube. The mean relative velocity of the liquid in the annular region $v_\delta(=\int_A v(r)\,\mathrm{d}A/A)$ is then

$$v_\delta = \frac{gD^2}{4v}\left(\frac{1}{1-(1-\xi)^2}\right) \times \left[\frac{2}{3}\xi^3(1-\xi) + \frac{1}{10}\xi^5 + \frac{1}{60}\xi^6\right] + U \quad \text{(A2)}$$

where ξ is the dimensionless film thickness, $2\delta/D$.

As δ is small when compared with $D/2$ one should expect that the plane wall solution

$$v_\delta = \frac{gD^2}{12v}\,\xi^2 + U \quad \text{(A3)}$$

gives a good estimate of v_δ.

The velocity v_δ be related with the velocity V_L by performing a material balance to the liquid between cross-sections 1 and 2 of Fig. 2 resulting in

$$\frac{\pi D^2}{4}V_L = \pi\left[\frac{D^2}{4} - \frac{(D-2\delta)^2}{4}\right]v_\delta. \quad \text{(A4)}$$

The same balance for the plane wall case gives

$$\frac{\pi D^2}{4}V_L = \pi D\delta\, v_\delta \rightarrow v_\delta = \frac{1}{2\xi}V_L. \quad \text{(A5)}$$

Combining eqs (A2) and (A4) with $V_L = U - U_L$ leads to

$$\frac{\pi}{4}D^2(U-U_L) = \pi\left[\frac{D^2}{4} - \frac{(D-2\delta)^2}{4}\right]\left\{\frac{gD^2}{4v}\left(\frac{1}{1-(1-\xi)^2}\right) \times \left[\frac{2}{3}\xi^3(1-\xi) + \frac{1}{10}\xi^5 + \frac{1}{60}\xi^6\right] + U\right\} \quad \text{(A6)}$$

that after rearrangement gives

$$\xi = \left\{ \left(\frac{4v}{gD^2} [(1-\xi)^2 U - U_L] - \frac{1}{10}\xi^5 - \frac{1}{60}\xi^6 \right) \frac{3}{2(1-\xi)} \right\}^{1/3}. \quad \text{(A7)}$$

An approximate equation for the calculation of ξ is obtained from the plane wall solution [eqs (A3) and (A5)]:

$$\xi = \left[\frac{6v}{gD^2} (U - U_L) \right]^{1/3}. \quad \text{(A8)}$$

The right-hand side of this equation can be re-arranged introducing the dimensionless numbers N_f and Re_{V_L}

$$\xi = \left(\frac{6Re_{V_L}}{N_f{}^2} \right)^{1/3}. \quad \text{(A9)}$$

Eliminating ξ from eqs (A5) and (A9) and after re-arrangment, the following relation between u_δ/U_L and the dimensionless numbers N_f, Re_{V_L} and U/U_L is obtained

$$\frac{u_\delta}{U_L} = \frac{1}{2} \left(\frac{U}{U_L} - 1 \right) \left(\frac{6Re_{V_L}}{N_f{}^2} \right)^{-1/3} - \frac{U}{U_L}. \quad \text{(A10)}$$