THE FLOW OF LIQUIDS IN THIN FILMS

George D. Fulford *

Department of Chemical Engineering,
University of Birmingham, Birmingham, England

I. Introduction

The flow of liquids in thin layers is frequently observed in everyday life; a common example is the flow of rain water on window panes, road surfaces, and roofs.

* *Present address:* E. I. du Pont de Nemours & Company, Inc., Photo Products Department, Research Division, Parlin, New Jersey.

Film flow is a special case of two-phase flow. In the present survey the flow of free liquid sheets (i.e., films bounded by two free surfaces) will not be considered; attention will be confined to film flow along a solid surface of some sort, with only one free surface. The second phase in contact with the free surface of the film may be either a gas or a second (immiscible) liquid, which may be at rest or in motion relative to the solid surface on which the film flows. Film flow is distinguished from other forms of two-phase flow by the presence of large interfaces of basically simple geometrical configuration. Two-phase flows are also often further classified as single-component (e.g., steam-water) or multicomponent flows, but this distinction is not of great importance in the study of the flow behavior of films.

The occurrence and applications of film flow in modern technology are numerous and important. Omitting the field of chemical engineering for the moment, the following applications of thin liquid flows have been found in the recent literature:

(1) Film cooling of rocket motors, turbine blades, reactor tubes, etc. (T17, K18, Y3).

(2) Refrigeration problems (C5).

(3) Conveying of liquids by cocurrent gas streams, e.g., in oil pipelines, boiler tubes, etc. (H17, K23).

(4) Design of channels, camber of roads, drainage works (C8, I3).

(5) Design of dam spillways (B1).

(6) Design of drain pipes and overflow lines (E2, V4).

(7) Study of the transport of radioactive materials by rain water (B12).

(8) Combating soil erosion (I1).

(9) Heat transfer from wavy films of molten materials, e.g., on spacecraft (Z5).

The flow of thin liquid films in channels and columns has also served as the basis of fundamental studies of wave motion (M7), the effects of wall roughness in open-channel flow (R4), the effects of surface-active materials (T9–T12), and the like.

A very early application of liquid film flow in the chemical industry is mentioned in a patent of 1836 (G5): hydrogen chloride gas produced in the Leblanc soda process was absorbed by water films flowing over packings. Much later the film coolers and evaporators used in the German beet sugar industry inspired the earliest detailed theoretical and experimental studies of flow and heat transfer in falling films (C10, N6, N7). Chemical engineering interest in film flow has increased rapidly in recent years.

Typical modern applications of gas/film flows in the process industries are in trickling-type cooling towers, packed and wetted-wall towers for rectification and gas absorption (or desorption), and in various types of coolers and evaporators. Most vapor condensers also make use of film flow, since droplet condensation is comparatively rare. In recent years swept-film equipment has been finding increasing application for solving difficult separation problems (B20, H21).

Extensive treatments of general two-phase flow problems have been given in the monograph by Kutateladze and Styrikovich (K25) and in recent surveys by Dukler and Wicks (D17), and Scott (S4), all of which indicate clearly the important place of film flow in the over-all scheme of two-phase flow phenomena. Film flow is more amenable to detailed study than most other types of two-phase flow, and a detailed knowledge of the phenomena occurring in film flow (with or without an adjacent gas stream) would assist greatly in understanding many of the more complex types of two-phase flow and the mechanisms of heat and mass transfer in such flows. Numerous experimental studies have been made of various two-phase transfer processes, but these have led mainly to empirical correlations of more or less limited applicability.

An attempt is made in the present survey to collect and review the more important results of the studies of liquid film flow scattered throughout the literature. It is hoped that this will avoid future duplication of work and enable investigators to concentrate on the many aspects of film flow which have been insufficiently studied in the past.

Studies of heat and mass transfer to films and film condensation are considered only insofar as the results throw light on the flow behavior; a brief review of such studies has been published elsewhere (F6). In addition, annular gas/liquid flow in horizontal ducts will not be considered here, since this is usually complicated by droplet entrainment.

II. Description of Film Flow

A. Types of Film Flow

As in other flows, various types of film flow can be distinguished. The most important of these are steady flow and uniform flow, in which the properties of the flow are constant with respect to time and with respect to distance in the direction of flow.

Thus, the flow of a smooth film outside the acceleration zone is both steady and uniform. The flow in the smooth entry zone of a wetted-wall column where acceleration occurs is steady but nonuniform, while certain wavy film flows are both unsteady and nonuniform.

B. The Regimes of Film Flow

A dimensional analysis of the problem of film flow (F7) has shown that in general the properties of a film flow *may* depend on the Reynolds, Weber, and Froude numbers of the flow, a dimensionless shear at the free surface of the film, and, for wavy flows, a Strouhal number formed from the frequency of the surface waves, and various geometrical ratios, e.g., the ratios of the wave amplitude and length to the mean film thickness.

The general dependence on the Reynolds, Froude, and Weber numbers appears to be the most useful. In this survey the following definitions of these groups are used:

$$N_{\mathrm{Re}} = \bar{u}\bar{b}\rho/\mu = W/\mu \tag{1}$$

(a Reynolds number four times as large, based on the hydraulic diameter of an infinitely wide film, is frequently used in the literature, but appears to offer no particular advantage):

$$N_{\mathrm{We}} = \bar{u}/(\sigma/\rho\bar{b})^{1/2} \tag{2}$$

$$N_{\mathrm{Fr}} = \bar{u}/(g\bar{b})^{1/2} \tag{3}$$

This definition of N_{Fr} is of particular physical significance in film flow, since here the Froude number denotes the ratio of the mean film velocity to the celerity of a gravity wave in shallow liquid. (Wave celerity is defined as the velocity of a wave relative to the liquid on which the wave propagates.) The Weber number is defined by analogy.

It is well known in fluid flow studies that below a certain critical value of the Reynolds number the flow will be mainly laminar in nature, while above this value, turbulence plays an increasingly important part. The same is true of film flow, though it must be remembered that in thin films a large part of the total film thickness continues to be occupied by the relatively nonturbulent "laminar sublayer," even at large flow rates ($N_{\mathrm{Re}} \gg N_{\mathrm{Re}_{\mathrm{crit}}}$). Hence, the transition from laminar to turbulent flow cannot be expected to be so sharply marked as in the case of pipe flow (D12). Nevertheless, it is of value to subdivide film flow into laminar and turbulent regimes depending on whether ($N_{\mathrm{Re}} \lessgtr N_{\mathrm{Re}_{\mathrm{crit}}}$).

However, the flow regime of a film cannot be defined uniquely as laminar or turbulent, as in the case of pipe flow, due to the presence of the free surface. Depending on the values of N_{Fr} and N_{We}, the free surface may be smooth, or covered with gravity waves or capillary or mixed capillary-gravity waves of various types. Thus, under suitable conditions, it is possible to have smooth laminar flow, wavy laminar or turbulent flow, where the wavy flows may be subdivided into gravity or capillary types. It is to be noted particularly that the presence of waves is no indication that the flow as a whole is turbulent.

It can be seen that a large number of film-flow regimes exists. In the past it has been customary to describe the boundaries between the various regimes in terms of the Reynolds number only. This has given rise to a large number of "critical Reynolds numbers" for the first formation of waves, the appearance of capillary waves, the onset of turbulence, etc. (cf. B14). Recently it has been shown that it is possible to describe the various film-flow regimes in terms of N_{Re}, N_{Fr}, and N_{We}, and a rational nomenclature has been proposed (F7). The most important flow regimes discussed in this survey are the smooth laminar, the wavy laminar, and the (wavy) turbulent regimes. For instance, in the case of water films flowing on steep or vertical walls, smooth laminar flow occurs only at very low flow rates, gravity-type surface disturbances predominate at moderate N_{Re}, while capillary effects become important mainly at larger flow rates. (For water films, the Froude number is larger than the Weber number at a given flow rate except at very small slopes of the wall.)

It is clear that the flow regime is a complicated but predictable function of the physical properties of the liquid, the flow rate, and the slope of the channel. It has been shown that, for water films, gravity waves first appear in the region $N_{\mathrm{Fr}} = 1$–2, capillary surface effects become important in the neighborhood of $N_{\mathrm{We}} = 1$, and the laminar-turbulent transition occurs in the zone $N_{\mathrm{Re}} = 250$–500 (F7).

III. Theoretical Treatments of Film Flow

A. Introduction

From the brief discussion above it is apparent that the flow of viscous liquids in the form of thin films is usually accompanied by various phenomena, such as waves at the free surface. These waves greatly complicate any attempt to give a general theoretical treatment of the film flow problem; Keulegan (K14) considers that certain types of wavy motion are the most complex phenomena that exist in fluid motion. However, by making various simplifying assumptions it is possible to derive a number of relationships which are of great utility, since they describe the limits to which the flow behavior should tend as the assumptions are approached in practice.

In the present section the general equations are set up, and the main results of the various treatments of smooth laminar flow, wavy flow, turbulent flow, and flow of films with an adjoining gas stream will be reviewed briefly.

B. Laminar Flow

1. *General Equations*

The most general equations for the laminar flow of a viscous incompressible fluid of constant physical properties are the Navier-Stokes equations. In terms of the rectangular coordinates x, y, z, these may be written:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} = -\frac{\partial \Omega}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\,\nabla^2 u \tag{4}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} = -\frac{\partial \Omega}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\,\nabla^2 v \tag{5}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} = -\frac{\partial \Omega}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\,\nabla^2 w \tag{6}$$

where u, v, w are the velocities in the x, y, z directions, t is the time, ρ and ν are the density and kinematic viscosity, p is the pressure, Ω is the force potential of the field in which flow occurs, and ∇^2 is the Laplacian operator.

In addition, the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

must be satisfied.

Since only gravity fields will be considered here, the negative derivatives of Ω are merely equal to the components of g in the respective directions.

For flow on flat plates, the following coordinate convention will be used, except where otherwise noted: the x-axis is directed along the plate surface in the direction of the greatest slope, the z-axis is directed across the plate, and the y-axis is taken perpendicular to the plate.

The forms assumed by the general equations for various simplified cases together with the boundary conditions and solutions will now be discussed, starting with the simplest possible case.

2. *Smooth, Laminar, Two-Dimensional Film Flow*

If the flow is steady, uniform, and two-dimensional (i.e., flow of a smooth film on an infinitely wide plate outside the acceleration zone), Eqs. (4) to (6) reduce directly to the very simple case originally obtained by Hopf (H18) and Nusselt (N6):

$$\frac{d^2u}{dy^2} + \frac{g}{\nu}\sin\theta = 0 \tag{8}$$

$$\frac{dp}{dy} = \rho g \cos \theta \tag{9}$$

$$\frac{dp}{dz} = 0 \tag{10}$$

where θ is the slope of the wall. The continuity equation is satisfied automatically. With the boundary conditions

$$u = 0 \qquad \text{at} \quad y = 0 \text{ (no slip at wall)}$$

$$\frac{du}{dy} = 0 \qquad \text{at} \quad y = b \text{ (no drag at interface)}$$

the velocity distribution is given by the semiparabolic equation:

$$u = \frac{g}{\nu} (\sin \theta) \left(by - \frac{y^2}{2} \right) \tag{11}$$

The surface velocity (at $y = b$) is therefore

$$u_s = \frac{gb^2}{2\nu} \sin \theta \tag{12}$$

By integrating (11) over the film thickness, the mean velocity is found to be

$$\overline{u} = \frac{gb^2}{3\nu} \sin \theta \tag{13}$$

whence

$$u_s/\overline{u} = 1.5 \tag{14}$$

The volumetric flow rate per wetted perimeter is

$$Q = b\overline{u} = \frac{gb^3}{3\nu} \sin \theta \tag{15}$$

In the absence of drag at the free surface, the wall shear stress must support the total body force on the film, so that

$$\tau_w = b\rho g \sin \theta = (3\rho^2 g^2 \mu Q \sin^2\theta)^{1/3} \tag{16}$$

Using the film Reynolds number defined as

$$N_{\mathrm{Re}} = b\overline{u}/\nu = Q/\nu \tag{17}$$

the results above can be rewritten to give

$$\overline{u} = \left[\frac{\nu g \sin \theta}{3} \right]^{1/3} N_{\mathrm{Re}}^{2/3} \tag{18}$$

$$b = \left[\frac{3\nu^2}{g \sin \theta} \right]^{1/3} N_{\mathrm{Re}}^{1/3} \tag{19}$$

$$\tau_w = \rho(3\nu^2 g^2 \sin^2\theta)^{1/3} N_{\mathrm{Re}}^{1/3} \tag{20}$$

If the friction factor for film flow is defined in the usual way, so that $\tau_w = f\rho(\bar{u})^2/2$, then, by substituting for τ_w and $\bar{u}$ from above,

$$f = 6/N_{\mathrm{Re}} \tag{21}$$

which can be compared with the analogous value for laminar flow in closed conduits, $f = 8/N_{\mathrm{Re}}$.

3. *Axisymmetric, Smooth, Laminar Film Flow*

When the film flows on a vertical cylindrical surface of radius R, Eq. (8) becomes

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} = -\frac{g}{\nu} \tag{22}$$

with r as the radial coordinate, which is to be solved with the boundary conditions

$$u = 0 \quad \text{at} \quad r = R \text{ (no slip at tube wall)}$$

$$du/dr = 0 \quad \text{at} \quad r = R + b \text{ (no drag at interface)}$$

(The film is assumed to flow on the outer surface of the tube; for flow inside the tube the interface is at $r = R - b$, but the derivation is similar otherwise.)

The velocity profile is found to be

$$u = \frac{g}{4\nu}(R^2 - r^2) + \frac{g}{2\nu}(R + b)^2 \ln\left(\frac{r}{R}\right) \tag{23}$$

and the flow rate per wetted perimeter is

$$Q = \frac{g}{16\nu}\left[\frac{4(R + b)^4}{R}\ln\left(\frac{R + b}{R}\right) - \frac{3(R + b)^4}{R} + 4R(R + b)^2 - R^3\right] \tag{24}$$

which is the result given by Feind (F2). The equation quoted by Jackson (J1) can be derived from (24) also. However, this equation is inconvenient for frequent use. By expanding the terms above in powers of (b/R), it is easily shown that

$$Q = \frac{gb^3}{3\nu}\left[1 + \frac{b}{R} + \frac{3}{20}\left(\frac{b}{R}\right)^2 - \frac{1}{40}\left(\frac{b}{R}\right)^3 + \frac{1}{140}\left(\frac{b}{R}\right)^4 \cdots\right] \tag{25}$$

which is a more general case of the equation used by Kamei and Oishi (K4).

As the tube radius R tends to infinity, it is seen that (25) tends towards (15) for the infinitely wide wall.

Considering a film 1 mm. thick flowing on a tube of 1 cm. diameter, it is seen that the b/R term in (25) constitutes a correction of 20% to the film thickness calculated by (15), while the next term makes only a very small correction. It can be seen that it is necessary to apply these curvature corrections in comparing film flow results on tubes with those on flat walls unless the tubes are of large diameter.

Grimley (G11) has shown experimentally that for tubes of very small diameter there is an additional capillary effect on the film thickness.

4. *Smooth, Laminar, Three-Dimensional Film Flow*

When the film flows in a channel of finite width, with side walls, the flow is no longer two-dimensional in nature, as in Section III, B, 2, but edge effects occur and must be taken into account. Two types of edge effects can occur: viscous edge effects, due to the drag of the side walls, and capillary edge effects, due to the capillary surface elevation at the side walls.

The viscous edge effect will be calculated first. It is assumed that the liquid possesses no surface tension, so that the liquid surface is flat from wall to wall of the channel. In this case, with the other assumptions of Section III, B, 2, Eq. (4) reduces to

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{\rho g}{\mu} \sin \theta \tag{26}$$

to be solved with zero velocity on the wall ($y = 0$) and at the side walls ($z = \pm w$) and zero drag at the interface ($y = b$). Several solutions of this equation have been given (C16, F7, H18, O1). These differ somewhat in form, but Owen (O1) has pointed out that, since this is a problem of the Dirichlet type, which is known from potential theory to possess only one regular solution, these must be merely different expressions for the same solution.

For the velocity distribution, Hopf gives (H18)

$$u = \sum_{n=0}^{\infty} (-1)^n \frac{16w^2 g \sin \theta}{(2n+1)^3 \pi^3 \nu} \left[\frac{\cos\left\{(2n+1)\frac{\pi}{2}\frac{y-b}{w}\right\}}{\cos\left\{(2n+1)\frac{\pi}{2}\frac{b}{w}\right\}} - 1 \right] \cos\left\{(2n+1)\frac{\pi}{2}\frac{z}{w}\right\} \tag{27}$$

from which the flow rate is given as

$$2wQ = \frac{4w^4 g \sin \theta}{\nu} \left[\sum_{n=0}^{\infty} \left(\left\{ \frac{2}{(2n+1)\pi} \right\}^5 \tan \frac{(2n+1)\pi b}{2w} \right) - \frac{b}{w} \sum_{n=0}^{\infty} \left\{ \frac{2}{(2n+1)\pi} \right\}^4 \right] \tag{28}$$

Provided that the channel width is considerably larger than the film thickness, which is usually the case, Hopf showed that (28) can be greatly simplified to give the mean flow per wetted perimeter as

$$Q = \frac{gb^3 \sin\theta}{3\nu}\left(1 - 0.63\,\frac{b}{w}\right) \tag{29}$$

As $w \to \infty$, (29) tends to the solution for the infinitely wide plate, (15), by comparison with which it is seen that the term $0.63b/w$ is the correction for the viscous edge effect. Since normally $w \gg b$, it is clear that this correction is very small.

The velocity distribution equation (27) indicates that in the absence of surface tension effects the maximum velocity in a film flowing in a flat channel of finite width should occur at the free surface of the film at the center of the channel. The surface velocity should then fall off to zero at the side walls. However, experimental observations have shown (B10, H18, H19, F7) that the surface velocity does not follow this pattern but shows a marked increase as the wall is approached, falling to zero only within a very narrow zone immediately adjacent to the walls. The explanation of this behavior is simple: because of surface tension forces, the liquid forms a meniscus near the side walls. Equation (12) shows that the surface velocity increases with the square of the local liquid depth, so the surface velocity increases sharply in the meniscus region until the side wall is approached so closely that the opposing viscous edge effect becomes dominant.

For this case the velocity equation is the same as (26), since the pressure term $\partial p/\partial x$ in Eq. (4) disappears in uniform flow in which the free surface is in contact with the atmosphere at all points. Equations (5) and (6) reduce to

$$\frac{dp}{dy} = \rho g \cos\theta \tag{30}$$

which shows that there is a hydrostatic pressure distribution across the film thickness and

$$\frac{dp}{dz} = 0 \tag{31}$$

If the film is flat, (31) is automatically satisfied, as in the calculation of the viscous edge effect above, but, with surface tension, the capillary pressure in the curved part of the meniscus must be taken into account, and Eq. (31) provides the condition from which the shape of the free surface can be calculated.

Numerical calculations of the velocity distributions in the meniscus region of a water film flowing in a glass channel have been made (F7) for a few cases. Figure 1 shows the isotaches in the meniscus region for a water

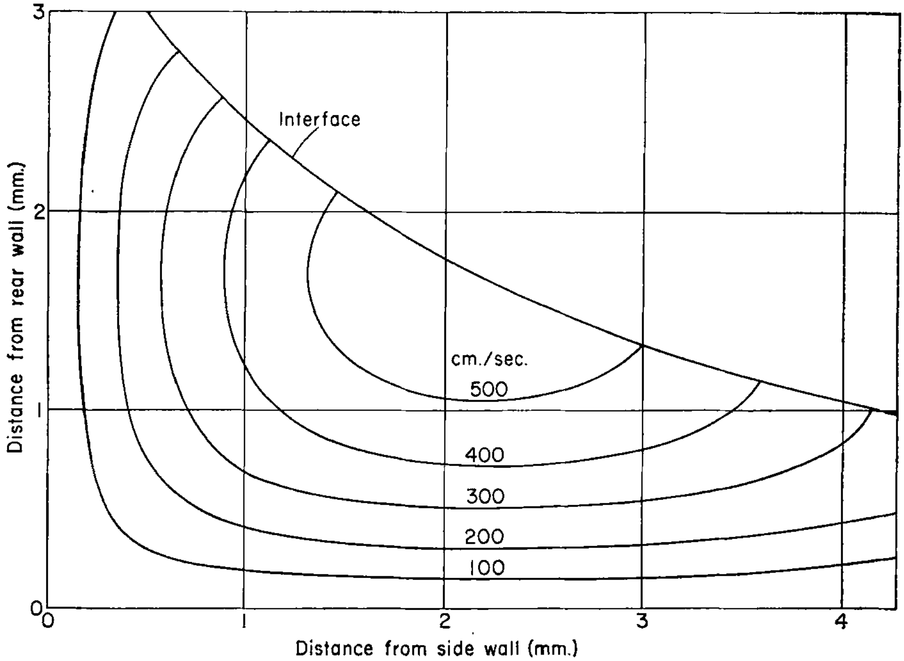

FIG. 1. Calculated lines of constant velocity in a water film flowing near the corner of a rectangular glass channel at 30° to the horizontal; film thickness far from side wall is $\frac{1}{3}$ mm. (F7).

film (at 20°C.) of undisturbed thickness $\frac{1}{3}$ mm. (far from the side walls) flowing in a glass channel at 30° to the horizontal. The calculated results for the local flow rates were in fair agreement with experimental data.

The calculations and experimental observations show that the local flow rates near the side walls are considerably larger than the local flow rates prevailing over the central part of the channel. It is clear that this capillary edge effect must be allowed for in film flow experiments in rectangular channels unless the channel is extremely wide (H19, B10). It is clear that the capillary effect noted here could be very important in the flow of films over packings, since meniscuses may be formed in the angles between packing elements; a relatively large part of the liquid might flow in such regions, due to the greater thickness of the film there, and, because of the greater velocity, the contact-time distribution of elements of the film surface could be greatly disturbed.

5. *Inertia Effects in Smooth Laminar Flow*

Kasimov and Zigmund (K11) have dealt with film flow on a vertical laterally unbounded surface for the case when the flow is steady but not necessarily uniform, i.e., taking the inertia terms into account. In this case, Eq. (4) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g \tag{32}$$

and (7) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{33}$$

Denoting the initial film thickness at $x = 0$ by b_i and assuming a semiparabolic velocity distribution, the following equation was obtained for the local film thickness b_x at $x = x$:

$$\frac{5gx}{6Q^2} = \frac{1}{6b_N^2}\ln\left[\frac{(b_x^2 + b_N b_x + b_N^2)}{(b_x - b_N)^2}\,\frac{(b_i - b_N)^2}{(b_i^2 + b_N b_i + b_N^2)}\right] + \frac{1}{b_N^2\sqrt{3}}\arctan\frac{b_N\sqrt{3}(b_x - b_i)}{b_x(b_i - b_N) + b_N b_i + 2b_N^2} \tag{34}$$

where b_N is the value of the film thickness given by the Nusselt theory, neglecting inertia [Eq. (15) or (19)].

The flow can be divided into three zones:

(1) The stabilization zone, where $b_x < b_N$.
(2) The zone of stabilized flow, where $b_x = b_N$.
(3) The zone in which inertia forces are important, and $b_x > b_N$.

The usual Nusselt equations, Section III, B, 2, are strictly valid only in the second zone, which is usually only of short extent.

A second approximate solution was obtained without assuming the shape of the velocity profile. In this case the x component of the velocity was found to be

$$u = \frac{g}{\nu}\left(b_x y - \frac{y^2}{2}\right) + \frac{\nu}{6}\left\{\frac{g}{4\nu^2}\right\}^2\frac{b_x^2}{x}\left[y^4 - \frac{13}{210}\frac{g}{4\nu^2}\frac{b_x y^7}{x}\right] \tag{35}$$

for the region $x > 2.02$ cm., with a similar relation for v. It will be observed that the Nusselt solution (11) is a special case of this general result. Similar expressions were obtained for the case when a surface drag due to a gas stream was present.

The main prediction of this treatment is that the film thickness should increase gradually in the direction of flow due to the inertia effects. Substitution of values of the physical properties into these equations has shown that these effects are usually small for ordinary mobile liquids, such as water.

C. The Onset of Wavy Flow (Theoretical Stability Considerations)

In addition to the general treatments of wavy flow, a number of theories concerning the stability of film flow have been published; in these the flow conditions under which waves can appear are determined. The general method of dealing with the problem is to set up the main equations of flow (usually the Navier-Stokes equations or the simplified Nusselt equations), on which small perturbations are imposed, leading to an equation of the Orr-Sommerfeld type, which is then solved by various approximate means to determine the conditions for stability to exist. The various treatments are lengthy, and only the briefest summary of the results can be given here.

Shibuya (S10) dealt with the case of the onset of instability in film flow on the outer surface of a vertical tube. By assuming a mixed disturbing velocity of the cosine–hyperbolic cosine type, it was found that the numerical value of the Reynolds number for instability was approximately

$$N_{\mathrm{Re}_i} = 7 \tag{36}$$

regardless of the physical properties. The wavelength at the onset of rippling was calculated as approximately 2 mm.

In his early paper, Yih (Y1) carried out numerical calculations for stability in the case of flow on a vertical wall, from which it appears that

$$N_{\mathrm{Re}_i} = 1.5 \tag{37}$$

Both these treatments omitted surface tension forces.

The Kapitsa treatment of wavy film flow (K7), which is discussed in detail later, indicates that

$$N_{\mathrm{Re}_i} = 0.61(K_F \sin\theta)^{-1/11} \tag{38}$$

where K_F is the physical properties group $\mu^4 g/\rho\sigma^3$. The Kapitsa theory is strictly applicable to long waves only ($\lambda/\bar{b} \geq 13.7$), so this result may not be accurate if the waves at the point of inception are short. This equation gives a numerical result of $N_{\mathrm{Re}_i} \cong 5.8$ for water on a vertical wall.

Benjamin (B5) has given a detailed treatment of the onset of two-dimensional instability in film flow, taking capillary effects into account. The expression for neutral stability found in this work can be given as

$$\frac{4n^2}{(3N_{\mathrm{Re}})^{5/3}}(K_F \sin\theta)^{-1/3} + \frac{4\cot\theta}{3N_{\mathrm{Re}}} - \frac{8}{5} + 5.5289142n^2 - 0.0000639(nN_{\mathrm{Re}})^2 - 14.6352952n^4 = 0 \tag{39}$$

where n is the wave number, defined as

$$n = 2\pi\bar{\delta}/\lambda \tag{40}$$

It was deduced that surface tension has a stabilizing effect, especially at short wavelengths, but instability cannot be converted to stability merely by increasing the surface tension. It was found that, for a vertical slope,

$$N_{\mathrm{Re}_i} = 0 \tag{41}$$

i.e., vertical film flow is always inherently unstable, though the instability at very low N_{Re} may not be physically manifest. At other slopes there is a small zone of stable flow.

Andreev (A5a) has recently considered the stability of viscous film flow with respect to infinitesimal disturbances. In this treatment, absolute instability was not found for laminar film flow along a vertical wall.

In a simplified treatment for longer waves, Benjamin derived an amplification factor $\mathcal{A}$, defined as the amplification experienced by the wave of maximum instability in traveling a distance of 10 cm. If neutral stability exists, $\mathcal{A} = 1$, and if $\mathcal{A} > 1$, wavy flow can be expected. For a vertical wall,

$$\mathcal{A} = \exp\left(1.7396\ [\mathrm{cm.}]\ \nu^{2/3}g^{2/3}\ \frac{\rho}{\sigma} N_{\mathrm{Re}}^{7/3}\right) \tag{42}$$

Binnie (B10) extended this treatment to cover angles other than the vertical; thus, for water films (~20°C.),

$$\mathcal{A} = \exp\left\{0.0434\left(\frac{8}{5} - \frac{4\cot\theta}{3N_{\mathrm{Re}}}\right)^2 (\sin^{2/3}\theta) N_{\mathrm{Re}}^{7/3}\right\} \tag{43}$$

Substituting the condition $\mathcal{A} = 1$ for neutral stability, it is seen that

$$N_{\mathrm{Re}_i} = \tfrac{5}{6}\cot\theta \tag{44}$$

This work was later extended to cover the case of three-dimensional instability (B6). The results tended to confirm the earlier results.

Hanratty and Hershman (H2) have given an interesting treatment of the stability problem. For the case of free film flow (i.e., film flow without a bounding gas flow), the condition for neutral stability becomes

$$\frac{\cos\theta}{(N_{\mathrm{Fr}})^2} = 3 - n^2(N_{\mathrm{We}})^2 \tag{45}$$

where n is given by (40). It can be shown that Benjamin's result can be expressed in an exactly similar form, apart from replacing the term 3 in Eq. (45) by 3.6.

The treatment of Ishihara *et al.* (I2) leads to results similar to those of the simplified Benjamin theory.

The stability problem in the presence of a bounding gas stream (cocurrent and countercurrent) has been considered for a number of special cases (F3, G6, M10, Z1). The countercurrent solutions tend to confirm Benjamin's result that the flow is always inherently unstable on a vertical wall.

Recently Yih (Y2) has given a detailed treatment of the stability of film flow on an inclined plane. Three cases are considered in detail: small wave numbers (n), small Reynolds numbers, and large wave numbers. In the first case the results are in agreement with the results of Benjamin noted above, but for large wave numbers and zero surface tension, Benjamin's tentative conclusions are shown to be invalid. The stability curves are considered for film flows on vertical and sloped walls for liquids with and without surface tension.

The stability of flow in open channels has been investigated theoretically from a more macroscopic or hydraulic point of view by several workers (C17, D9, D10, D11, I4, J4, K16, V2). Most of these stability criteria are expressed in the form of a numerical value for the critical Froude number. Unfortunately, most of these treatments refer to flow in channels of very small slope, and, under these circumstances, surface instability usually commences in the turbulent regime. Hence, the results, which are based mainly on the Chézy or Manning coefficient for turbulent flow, are not directly applicable in the case of thin film flow on steep surfaces, where the instability of laminar flow is usually in question. The values of the critical Froude numbers vary from 0.58 to 2.2, depending on the resistance coefficient used. Dressler and Pohle (D11) have used a general resistance coefficient, and Benjamin (B5) showed that the results of such analyses are not basically incompatible with those of the more exact investigations based on the differential rather than the integral ("hydraulic") equations of motion. The hydraulic treatment of the stability of laminar flow by Ishihara *et al.* (I2) has been mentioned already.

It is to be noted that, for laminar open channel flow, the Froude and Reynolds numbers are interrelated. Making use of (1), (3), and (19), it is easily shown that

$$N_{\mathrm{Fr}} = (N_{\mathrm{Re}}/3)^{1/2} \sin^{1/2}\theta \qquad (46)$$

so that for this regime the Reynolds number of instability N_{Re_i} can be expressed as a critical Froude number, if desired. Similar equations can also be obtained for other flow regimes [see, for instance, (P4), but it is necessary to note that N_{Re} and N_{Fr} are defined differently in this publication].

D. Wavy Laminar Film Flow

For the case of two-dimensional wavy film flow, Levich (L9) has shown that Eqs. (4) and (5) reduce to the familiar form of the boundary layer equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + g\sin\theta \tag{47}$$

$$\frac{\partial p}{\partial y} = 0 \tag{48}$$

with the following boundary conditions: at the free surface of the film, $y = b$, where $b = f(x, t)$,

$$p = p_\sigma \cong -\sigma\frac{d^2b}{dx^2} \qquad \text{(capillary pressure)} \tag{49}$$

$$\mu\frac{\partial u}{\partial y} = 0 \qquad \text{(zero interfacial shear)} \tag{50}$$

at the solid wall ($y = 0$),

$$u = v = 0 \qquad \text{(no slip)} \tag{51}$$

The continuity condition can be expressed as

$$\frac{\partial b}{\partial t} = -\frac{\partial}{\partial x}\left(\int u\,dy\right) \tag{52}$$

Kapitsa was the first to attempt the solution of this system of equations (K7). In this original solution the term $v\,\partial u/\partial y$ in Eq. (47) was omitted, as also in the analysis by Portalski (P3) who resolved Kapitsa's equations to a higher degree of accuracy. By comparing the corrected solution by Bushmanov (B22, L9) with the original result, it appears that the errors caused by the omission of this term are not large.

It is assumed that the velocity distribution in the film can be given by the usual semiparabolic expression [Eqs. (11) and (13)]. Substituting this value of the velocity into (47), making use of (49), and then averaging over the film thickness by integrating with respect to y and dividing by b, it is found that

$$\frac{\partial \bar{u}}{\partial t} + \frac{9}{10}\bar{u}\frac{\partial \bar{u}}{\partial x} = \frac{\sigma}{\rho}\frac{d^3b}{dx^3} - \frac{3\nu\bar{u}}{b^2} + g\sin\theta \tag{53}$$

while (52) becomes

$$\frac{\partial b}{\partial t} = -\frac{\partial(\bar{u}b)}{\partial x} \tag{54}$$

where $\bar{u}$ is the mean velocity over the film thickness.

It is then further assumed that the waves are long compared to the film thickness, so that the film thickness can be represented as

$$b = \bar{b}(1 + \phi) \tag{55}$$

where $\bar{b}$ is the mean film thickness and ϕ is the local deviation from the mean, and the mean velocity and thickness are then referred to the new variable $(x - ct)$, where c is the phase velocity of the waves. After various rearrangements, the equation for solution becomes, finally, to the first approximation,

$$\frac{\sigma\bar{b}}{\rho}\frac{d^3\phi}{dx^3} - (c - \bar{u})\left(\frac{9\bar{u}}{10} - c\right)\frac{d\phi}{dx} - \frac{3\nu}{(\bar{b})^2}(c - 3\bar{u})\phi + g \sin\theta - \frac{3\nu\bar{u}}{(\bar{b})^2} = 0 \tag{56}$$

where $\bar{u}$ is given by $Q = \bar{b}\bar{u}$.

In order for an undamped periodic solution to exist it is necessary that the constant term in this equation and the coefficient of ϕ be equal to zero, so that, to the first approximation,

$$\bar{u} = \frac{(\bar{b})^2 g \sin\theta}{3\nu} \tag{57}$$

$$c = 3\bar{u} \tag{58}$$

Comparison of (57) with (13) shows that, to the first approximation, the film thickness is not affected by the waviness of the flow.

It is of interest to note that the wave-velocity condition obtained by Benjamin (B5) can be written as

$$c = 2(1 - n^2 + \tfrac{11}{6}n^4 + 0.0077581n^2(N_{\mathrm{Re}})^4 - 3.3555556n^6)u_s \tag{59}$$

where u_s is the surface velocity (equal to $1.5\bar{u}$), and n is given by (40). For long waves, (59) reduces to

$$c = 2u_s = 3\bar{u} \tag{60}$$

which is in agreement with the first result of Kapitsa above. The same value of c has been obtained by Hanratty and Hershman (H2) and can also be deduced from the general wave theory of Lighthill and Whitham (L11).

Using (57) and (58), Eq. (56) becomes

$$\frac{\sigma\bar{b}}{\rho}\frac{d^3\phi}{dx^3} + 4.2\bar{u}^2\frac{d\phi}{dx} = 0 \tag{61}$$

from which the function ϕ defining the surface shape (55) is

$$\phi = \alpha \sin\left[\bar{u}(4.2\rho/\sigma\bar{b})^{1/2}(x - ct)\right] \tag{62}$$

where $\alpha\bar{b}$ is the wave amplitude. To this first approximation, it is seen that the surface disturbances are sinusoidal.

In the second approximation, the equation analogous to (56) contains terms to the second order in ϕ. The details of the calculation cannot be included here, but briefly, it is found that in this case the mean film thickness of the wavy flow depends on the wave amplitude α, or

$$(\bar{b})^3 = \frac{3\nu Q}{g}\Phi \tag{63}$$

where Φ is a function of α and $c/\bar{u}$. In order to estimate the wave amplitude, Kapitsa (K7) carried out an interesting analysis of the stability of the wavy flow and determined the conditions under which the energy supplied to the film by the gravity force is balanced by the dissipation of energy by viscous forces. From this it was deduced in general terms that the film will assume a configuration in which its mean thickness is a minimum for the given flow rate; that is, the function Φ in (63), which is unity for smooth flow, but which is always less than unity for wavy flow, must be a minimum. For the particular second-order conditions of this treatment, the equations for Φ as a function of α^2 and of $c/\bar{u}$ are solved graphically, making use of the fact that Φ must be a minimum. In this way, Levich (L9) shows that

$$\alpha = 0.46 \tag{64}$$

$$c = 2.4\bar{u} \tag{65}$$

$$\Phi = 0.8$$

so that, substituting in (63),

$$\bar{b} = 1.34(\nu Q/g)^{1/3} \tag{66}$$

$$\lambda = \frac{2\pi}{\bar{u}}\left\{\frac{\sigma\bar{b}}{\rho(c/\bar{u} - 1)(c/\bar{u} - 0.9)}\right\}^{1/2} \tag{67}$$

For flow on nonvertical slopes, g is replaced throughout by $g \sin\theta$, where θ is the angle of inclination of the wall, measured from the horizontal.

It is seen from (65) that the wave velocity is considerably smaller than the value given by the first approximation, (58). From (63), the ratio of the mean film thickness in wavy flow to the thickness of a smooth film at the same flow rate is given by $\Phi^{1/3}$ or, from above, 0.93. The corresponding value obtained by Portalski (P3) was 0.94. It is thus seen that for wavy flow of the type assumed here, the mean thickness of the wavy film is 6–7% smaller than the corresponding smooth film. It is pointed out by Kapitsa that it does not follow that there may not be some other type of film surface configuration which would lead to a greater reduction in thickness and, therefore, to greater stability of flow.

Based on a treatment like that above, Tailby and Portalski (T2) have derived an expression for the increase in the surface area of the film due to the waves at the interface:

$$\Delta S = \left(\frac{\bar{b}\pi\alpha}{\lambda}\right)^2 \left[1 + \left(\frac{g\alpha\rho\lambda^3}{2^4\pi^3\bar{b}\sigma}\right)^2\right] \tag{68}$$

It is claimed that this relationship is valid up to approximately $N_{\mathrm{Re}} = 300$. However, the Kapitsa treatment (K7) is shown to be valid only up to the condition that the wavelength is more than 13.7 times as great as the film thickness, which corresponds to $N_{\mathrm{Re}} \cong 50$ for a vertical water film.

Semenov (S7) simplified the equations of wavy flow for the case of very thin films, and this approach has also been followed by others (K20, K21). These treatments refer mainly to the case of film flow with an adjoining gas stream and will be considered in Section III, F, 3.

Kapitsa and Kapitsa (K10) have shown that the relative change in wavelength for flow on a tube of radius R compared with that on a flat surface at the same flow rate is

$$\Delta\lambda/\lambda = \tfrac{1}{2}(\lambda/2\pi R)^2 \tag{69}$$

which is quite small except for small values of R.

Recently, Kasimov and Zigmund (K12) have published the first part of a new theoretical treatment of wavy film flow, extending their recent work on smooth laminar film flow (Section III, B, 5) to this case also. It is shown that, with appropriate assumptions, the new theory reduces to the Nusselt solution for smooth films, or to a result similar to the corrected Kapitsa solution. The most interesting conclusions to be drawn from the part of the theory so far published are:

(1) The mean film thickness and the wave amplitude should increase in the direction of flow.

(2) The wave amplitude should decrease with increase of the liquid viscosity.

(3) The wavelength is proportional to $(N_{\mathrm{Re}})^{1/9}\sigma^{1/3}\nu^{2/9}$.

(4) The wave amplitude on an inclined surface should be smaller than on a vertical surface for the same flow rate and liquid properties, while the wavelength should be greater on the inclined surface.

In addition to the theories reviewed above, there are many treatments in the literature which deal with the hydraulics of wavy flow in open channels. Most of these refer to very small channel slopes (less than 5°) and relatively large water depths. Under these conditions, surface tension plays a relatively minor part and is customarily neglected, so that only gravity waves are considered. For thin film flows, however, capillary forces play an important part (K7, H2). In addition, most of these treatments consider a turbulent main flow, while in thin films the wavy flow is often laminar. It is of interest to note, however, that Dressler (D9) has shown that gravity waves in steep channels became nonsymmetrical, with steep fronts and gently sloping tails (roll waves). The breakup of an initially uniform wave pattern to form irregular roll waves is also considered by Lighthill (L10) and Mayer (M7). Ishihara *et al.* (I1) have considered gravity roll waves on a laminar main flow and deduced that, for small channel slopes and waves of small amplitude, the wave velocity is

$$c = \tfrac{6}{5}\bar{u} + [g\bar{b} + \tfrac{6}{25}(\bar{u})^2]^{1/2} \tag{70}$$

E. Turbulent Film Flow

The problem of turbulent flow in thin films has received comparatively little attention. Because of the great complexity of the flow processes involved, there are no theoretical treatments of the problem of wavy turbulent flow, and the usual procedure is to neglect the surface waves and obtain solutions for the case of smooth turbulent flow.

Keulegan (K13) applied the semiempirical boundary-layer concepts of Prandtl and von Kármán to the case of turbulent flow in open channels, taking into account the effects of channel cross-sectional shape, roughness of the wetted walls, and the free surface. Most of the results are applicable mainly to deep rough channels and bear little relation to the flow of thin films.

Levich (L8, L9) has given an interesting treatment of fully turbulent film flow. In the absence of a flowing gas stream at the interface, Levich deduced that the scale of turbulence and the turbulent velocity normal to the interface must be proportional to the distance from the interface, so that all turbulent pulsations must disappear at the interface itself, leaving there a nonturbulent layer of thickness

$$\delta' \propto b/(N_{\mathrm{Re}})^{3/4}$$

A theory of gas absorption by turbulent liquid films has been developed on the basis of this conclusion (L9).

For fully developed turbulent flow with zero interfacial drag, it is shown (L8) that the mean film velocity is

$$\bar{u} = \sqrt{\frac{gb \sin\theta}{K}} \ln\left[\frac{b}{\nu}\sqrt{\frac{gb\sin\theta}{K}}\right] \tag{71}$$

where K is a constant. No numerical value is given for K, but substitution of the value $b = 0.076$ cm. for $N_{\mathrm{Re}} = 1000$ obtained experimentally (B14) for a water film ($\nu = 0.01$ cm.^{2}/sec.) on a vertical wall ($\sin\theta = 1$) shows that K should be in the region of $\frac{1}{8}$.

Nearly all of the remaining treatments of turbulent film flow are based on the assumption that the film can be regarded as smooth, and that some form of the dimensionless velocity profile valid for single-phase flow (pipe flow) can be applied to the flow of films. In principle the treatments differ mainly in the form of the dimensionless velocity profile used, and only one will be discussed in detail.

Dukler and Bergelin (D16) used the universal velocity profile equations of Nikuradse:

$$u^+ = y^+ \quad \text{for} \quad 0 \le y^+ \le 5 \text{ (laminar sublayer)} \tag{72}$$

$$u^+ = -3.05 + 5.0 \ln y^+ \quad \text{for} \quad 5 < y^+ \le 30 \text{ (buffer layer)} \tag{73}$$

$$u^+ = 5.5 + 2.5 \ln y^+ \quad \text{for} \quad 30 < y^+ \le b^+ \text{ (turbulent zone)} \tag{74}$$

where

$$u^+ = u/u^* \quad \text{(dimensionless velocity)} \tag{75}$$

$$y^+ = yu^*/\nu \quad \text{(dimensionless distance from wall)} \tag{76}$$

$$b^+ = bu^*/\nu \quad \text{(dimensionless film thickness)} \tag{77}$$

$$u^* = (\tau_w/\rho)^{1/2} \quad \text{(friction velocity)} \tag{78}$$

By integrating the dimensionless velocities over the film thickness, it is found for the case of turbulent flow ($b^+ \ge 30$) that

$$N_{\mathrm{Re}} = b^+(3.0 + 2.5 \ln b^+) - 64 \tag{79}$$

[For $b^+ < 30$, Portalski (P3, P4) has given a corrected form of the original treatment.] In the case of zero interfacial shear, the wall shear stress is given by (16), so from (77) and (78)

$$b^+ = (g^{1/2}\rho \sin^{1/2}\theta)b^{3/2}/\mu \tag{80}$$

Hence, knowing the values of the physical properties appearing in (80), it is possible to calculate the value of b for any value of N_{Re} using (79) and (80). The values obtained in this way for the film thickness are in good agreement in the laminar zone with the values given by (19).

In the case of flow of the film in a tube of radius R, with a pressure drop per unit length of ψ, a force balance shows that, approximately,

$$\tau_w = (\rho_{\mathrm{gas}} g \sin\theta \pm \psi)\frac{R}{2} + b\rho g \sin\theta \tag{81}$$

in place of (80), and in this case a trial-and-error or graphical solution is necessary in order to find b at a given N_{Re} and ψ.

According to this theory, the film becomes turbulent when $b^+ = 30$. Substitution of this value into (79) gives

$$N_{\mathrm{Re}_{\mathrm{crit}}} = 270 \tag{82}$$

This result has been criticized (B14) on the grounds that recent investigations do not support the view of well-defined zones within the boundary layer. The critical Reynolds number will be discussed further in Section IV, B.

Kutateladze and Styrikovich (K25) have presented a treatment very similar to that outlined above. Thomas and Portalski (T14) have also used the universal velocity profile concept, but for the case of countercurrent gas flow they have used instead of (81) the more exact form (for a vertical tube):

$$\tau = (b - y)(\rho - \psi) - \frac{(R - b)}{2}(\psi - \rho_{\text{gas}}) \tag{81a}$$

The treatments of Anderson *et al.* (A4, A5), Charvonia (C4), Calvert (C1, C2), and others differ mainly in the manner in which the gas stream pressure drop is taken into account for various cases of gas flow. Labuntsov (L2) has carried out an analysis using two forms of the velocity profile which do not involve the assumption of sharply differentiated zones in the boundary layer; this analysis is mainly concerned with heat transfer in films.

Dukler (D12) has carried out a superior analysis of this type, based on the dimensionless velocity profile of Deissler (D7), but as this is also concerned with the effects of gas flow, it will be considered in the next section.

F. Film Flow in the Presence of an Adjoining Stream

In most of the treatments considered above it has been assumed that the phase adjoining the free surface of the film is stationary. In the case of an adjoining stationary gas phase, the buoyancy effect will be small and can be neglected, and, because of the small viscosity of a gas compared with that of the film liquid, it is reasonable to assume that the drag caused by the stationary gas is small. This was confirmed experimentally by Grimley (G11) who showed that similar results were obtained with various degrees of evacuation of the gas space of a wetted-wall column. On the other hand, since there can be no relative slip at the interface, it follows that a thin layer of the "stagnant" gas phase must be entrained by the film surface. This gas-pumping effect of the film surface has been dealt with theoretically and experimentally by Mazyukevich (M8) for the case of film flow inside a tube and has also been noted elsewhere (F2, F7).

1. *Flow in the Presence of an Adjoining Liquid Phase*

In the case of film flow in the presence of a heavy adjoining phase (density ρ_c), it has been shown (F1) that the results obtained in Section III, B, 2 for laminar flow remain valid provided there is negligible interfacial drag and the quantity $g\rho$ is replaced by the effective value $g(\rho - \rho_c)$

throughout the derivations. In particular, Eqs. (15) and (19) assume the form

$$b = \left[\frac{3Q}{g(\rho - \rho_c)\sin\theta}\right]^{1/3} = (3N_{\mathrm{Re}})^{1/3}\left[\frac{\rho}{\rho - \rho_c}\right]^{1/3}\left[\frac{\nu^2}{g\sin\theta}\right]^{1/3} \tag{83}$$

Even when the adjacent liquid phase is stationary it is clear that the interfacial shear is likely to be considerable because of the large viscosity of the adjoining phase. Film flow with interfacial drag is considered below.

2. *Smooth Laminar Film Flow with Interfacial Shear*

For smooth laminar film flow with an interfacial shear the equations of motion remain as in Section III, B, 2, but the boundary condition $du/dy = 0$ at $y = b$ must be replaced either by

$$(du/dy)_{y=b} = -\tau_i/\mu$$

with τ_i as the interfacial shear, in which case,

$$u = \frac{g}{\nu}\left(by - \frac{y^2}{2}\right)\sin\theta - \frac{\tau_i y}{\mu} \tag{84}$$

or by

$$u = u_s \qquad \text{at} \quad y = b$$

where u_s is no longer equal to $1.5\bar{u}$ as in Section III, B, 2, which leads to

$$u = \frac{\rho g}{2\mu}(by - y^2)\sin\theta + \frac{yu_s}{b} \tag{85}$$

From these, two series of expressions similar to those obtained in Section III, B, 2 can be obtained for use where τ_i or u_s are known.

Semenov (S6) considered the case of smooth film flow with an interfacial shear τ_i (which may be either positive or negative) and a pressure drop in the gas stream of ψ, and it was shown that no fewer than eight regimes of gas–liquid flow may exist for an upward gas stream; however, half of these are unstable and do not normally occur. Semenov's expression for the volumetric flow rate per wetted perimeter was

$$Q = \frac{(\rho g\sin\theta - \psi)}{3\mu}b^3 - \frac{\tau_i b^2}{2\mu} \tag{86}$$

At the commencement of flooding in a wetted-wall column, the net flow is zero ($Q = 0$), and, with this value, various solutions of (86) can be studied. The trivial solution is that $b = 0$. If τ_i is negative (downward cocurrent flow) the equation can only be solved to give a negative value of b, which is not physically meaningful, indicating that flooding cannot occur in this case. With τ_i positive, however (countercurrent flow), flooding commences at a film thickness of

$$b = \frac{3\tau_i}{2(\rho g \sin\theta - \psi)} \tag{87}$$

More recently, Brauer (B18) carried out a detailed analysis of the flow of smooth films and gas streams inside vertical tubes; this work was subsequently extended by Feind (F2). In this treatment, all the possible cases of film/gas flow (countercurrent, upward cocurrent, and downward cocurrent) are dealt with in a unified manner by plotting the calculated results in the form of $|f|$ as a function of $N_{\mathrm{Re_{gas}}}$. Here $|f|$ is the absolute value of the dimensionless pressure drop in the gas stream:

$$|f| = \frac{2\tau_i}{\rho_{\mathrm{gas}}(\bar{u}_{\mathrm{gas}})^2} = \frac{2(\Delta P)(R - b)}{\rho_{\mathrm{gas}}(\bar{u}_{\mathrm{gas}})^2 L} \tag{88}$$

where $\Delta P/L$ is the pressure drop per unit length of the wetted tube. The gas stream Reynolds number is defined as

$$N_{\mathrm{Re_{gas}}} = (\bar{u}_{\mathrm{gas}})2(R - b)/\nu_{\mathrm{gas}} \tag{89}$$

Limiting values of the gas stream pressure drops have been obtained for the various flow regimes.

Feind (F2) has shown that the effect of an interfacial shear τ_i due to a countercurrent gas stream is to increase the film thickness in the ratio

$$\frac{b}{b_0} = 1 \Big/ \left(1 - \frac{3\tau_i}{2b\rho g}\right)^{1/3} \tag{90}$$

(for a vertical wall), where b_0 denotes the value in the absence of a gas stream. Flooding commences when

$$\frac{\tau_i}{b\rho g} = \frac{2}{3} \tag{91}$$

It is found that the ratio of the wall shear stresses τ_w/τ_{w0} decreases until $\tau_i/b\rho g = \frac{1}{2}$ and then increases sharply. The surface velocity in the presence of the gas stream is shown to be

$$u_s = \frac{g}{\nu} b^2 \left(\frac{1}{2} - \frac{\tau_i}{bg\rho}\right) \tag{92}$$

and it can be deduced that the ratio of the surface velocity to the mean velocity of the film is given by

$$u_s/\bar{u} = 3\left(1 - \frac{2\tau_i}{bg\rho \sin\theta}\right) \Big/ \left(2 - \frac{3\tau_i}{bg\rho \sin\theta}\right) \tag{93}$$

which reduces to the value given by (14) if $\tau_i = 0$.

It is to be noted that these treatments assume that the pressure drop per unit length is constant over the length of the wetted tube and that the film is in smooth laminar flow, which is usually the case only at very low flow rates. The results above, therefore, represent mainly a limiting case, as recognized by Brauer and Feind.

3. *Wavy Flow with an Adjoining Gas Stream*

Kapitsa (K8) extended his treatment of wavy free film flow to cover this case also. For the simplest case, in which the gas stream does not seriously affect the wavelength, it was found to a first approximation that the mean film thickness $\bar{b}$ could be given in terms of the flow rate per wetted perimeter Q and the mean gas velocity $\bar{u}_{\text{gas}}$ by means of the equation

$$(\bar{b})^3 \pm 0.27 \frac{\rho_{\text{gas}}}{\sigma g \sin\theta} (\bar{u}_{\text{gas}} - c)^2 Q^2 \bar{b} - \frac{2.4\nu Q}{g \sin\theta} = 0 \tag{94}$$

where the $\pm$ sign refers to countercurrent and cocurrent flow, respectively. Hence, in downward cocurrent flow, when increasing $\bar{u}_{\text{gas}}$ decreases $\bar{b}$ for a given Q, it follows from the stability considerations mentioned in Section III, D that increased stability will result, while for countercurrent flow an increase of $\bar{u}_{\text{gas}}$ leads to an increase of $\bar{b}$ for constant Q; hence, the film eventually becomes unstable and flooding commences, as observed in practice. Kapitsa derived a condition for the onset of flooding from (94) which was in excellent agreement with published experimental data (F4).

Semenov (S7) simplified the wavy flow equations by omitting the inertia terms, which is permissible in the case of very thin films. Expressions are obtained for the wavelength, wave velocity, surface shape, stability, etc., with an adjoining gas stream; the treatment refers mainly to the case of upward cocurrent flow of the gas and wavy film in a vertical tube.

Konobeev *et al.* (K20, K21) have generalized the Semenov and Kapitsa results for the case of wavy film flow with an adjoining gas stream. In particular, the wavelength expression is obtained as

$$\lambda = \frac{2\pi}{\bar{u}} \left[\frac{\bar{b}\sigma}{\rho(c/\bar{u} - 1)(c/\bar{u} - T)} \right]^{1/2} \tag{95}$$

In the absence of a moving gas stream, the velocity profile is semiparabolic, and $T = 0.9$, so that (95) reduces to (67). For a linear velocity profile, such as might occur in very high-speed cocurrent gas/film flow, $T = \frac{2}{3}$. It has been shown (K21) that the use of $T = 0.8$ gives excellent agreement with wavelengths measured experimentally with very low liquid flow rates and moderate cocurrent gas velocities. It seems that countercurrent flow of the gas and film would require values of T greater than 0.9. Konobeev *et al.* obtained experimental wave velocities on a vertical wall which were in excellent agreement with (95) up to film thicknesses of about 0.28 mm.

It is of interest to note that Zhivaĭkin and Volgin (Z4) have questioned the utility of the wavy flow theories of the Kapitsa-Semenov type on the grounds that the regular "sine wave" behavior assumed by these theories is not usually observed in practice. However, these are the only comprehensive wavy flow theories for thin films available at present, and they represent an interesting limiting case of film flow.

4. *Turbulent Film Flow with an Adjoining Gas Stream*

Attention in this section will be confined to the analysis of turbulent film flow by Dukler (D12, D13, D14), which includes the effect of a cocurrent downward gas stream. Film heat transfer under these circumstances is also considered. The Dukler analysis was later extended to cover the case of upward cocurrent gas/film flow by Hewitt (H7).

Briefly, in this treatment a force balance is carried out on an element of the film, taking into account the gas shear on the interface, which is expressed in terms of the pressure drop per unit length in the gas stream (assumed constant). As usual, the interface is assumed to be smooth. In this way an expression is obtained for the shear-stress distribution in the film, and this is converted to a velocity distribution by making use of the dimensionless velocity profiles proposed for channel flow by Deissler (D7) in the zone $0 < y^+ \leq 20$ and by von Kármân (V6) for the zone $y^+ > 20$. The complicated equations obtained in this way were solved on a computer, and the numerical results have been presented graphically and in tabular form (D12, H7).

An important result of this treatment is that the dimensionless velocity u^+ within the film is no longer a unique function of the dimensionless distance from the wall y^+, as in single-phase pipe flow, but depends significantly on a parameter involving the interfacial shear.

Neither of the treatments above covers the case of countercurrent gas/film flow, but the behavior of a downward film flow in the absence of a gas stream can be obtained from the original Dukler analysis. It is found that in the laminar zone the film thicknesses predicted by the Dukler theory are in agreement with (19). A comparison of the theory with experimental results will be carried out in Section IV, A.

Finally, it may be noted that the error in Dukler's original treatment (D12) pointed out by Hewitt (H7, p. 4) is only apparent, since Hewitt's result can be reduced to a form exactly analogous to the corresponding Dukler equation.

IV. Experimental Results and Comparison with Theory

The results of the more important experimental studies of the flow of thin films will be reviewed briefly in this section, and these results will be compared with the predictions of the theoretical treatments of film flow which have been outlined in Section III.

A brief chronological review of the more important theoretical and experimental investigations pertinent to the flow of liquids in thin films is given in the Appendix. In general, only those heat and mass transfer investigations which throw light on some aspect of the film flow problem have been included.

For a more detailed survey, it will be most convenient to consider the experimental results under separate headings, e.g., film thicknesses, film velocities, etc., even though it will be apparent that many of the investigations cover several of these topics.

A. Mean Film Thicknesses

1. *Introduction*

A knowledge of the thicknesses of flowing liquid films is of importance in a wide range of practical problems involving film flow. Such problems include the calculation of heat transfer in evaporators and condensers, mass transfer in film-type equipment, the design of overflows and downcomers, etc.

As a result, much experimental work has been carried out to determine the thicknesses of flowing films. The experimental techniques for measuring the film thickness, the most convenient manner of presenting the results, and finally the results with and without a gas stream adjoining the film will be discussed here.

2. *Experimental Techniques and Presentation of Results*

Many techniques have been proposed in the literature for the measurement of film thicknesses, and most have been used by various investigators. These techniques may be classified as:

(1) Direct determination of the position of the surface by means of a micrometer gauge and pointer, e.g. (C9, H18, J3, J4, K17, R4).

(2) Improved probe methods, in which contact of the probe with the surface is determined by some nonvisual method (B14, H9, P1, etc.).

(3) Photography of the film and channel (B4, C6, C11, K10, R3, W4).

(4) Weighing the channel and film continuously (K4).

(5) Drainage technique: the feed of liquid to the channel is shut off, and simultaneously the film liquid flowing from the channel is collected and measured. Knowing the wetted area of the channel, the mean film thickness can be determined from the volume of liquid (B21, C10, C15, F1, F5, W1). Improved drainage techniques have been used by (F2, F7, P3, T14).

(6) Measurement of the electrical resistance between two probes set in the channel wall (A1, B8, C13, G11, H9, K21, M7, V1).

(7) Measurement of the electrical capacitance between a probe placed above the film surface and the channel wall (B12, D16, H9, P3, S11).

(8) The light-beam extinction technique, using a dyed film liquid (C4, C14, G7, H11, L13).

(9) Radioactive tracer method (J1).

Of these methods, the first can give accurate values of the mean film thickness only in the absence of surface waves. The fourth and fifth methods can be used only for the mean film thickness, while methods (2), (3), (6), (7), and (8) may be used for measuring either the local or mean film thicknesses. Black (B12) and Portalski (P4) have discussed the advantages and disadvantages of most of these measurement techniques, and Hewitt and Lovegrove (H11a) have compared the film thickness values measured by three different methods.

Several methods of correlating film thickness data have been published in the literature. Thus, several investigators have plotted the film thickness b as a function of the Reynolds number N_{Re}; this yields a different curve for each liquid and is not very convenient. Hopf (H18) presented his film thickness results in the form of an apparent liquid viscosity as a function of N_{Re}. This type of plot makes it possible to see very clearly the value of N_{Re} at which turbulence commences, since at this point the apparent viscosity begins to differ from the true viscosity.

Two of the more general correlations may be considered here. The first of these is the film friction-factor correlation. From Eqs. (15), (17), and (21), it can be shown that

$$f = \frac{2b^3 g \sin \theta}{Q^2} \tag{96}$$

which indicates that the friction factor defined in this way can be regarded as a form of dimensionless film thickness, as pointed out by Brauer (B14). A plot of f as a function of N_{Re} should therefore correlate all film thickness data, and this method of correlation has been applied by several workers (B14, C15, F1, S13, Z3). However, it is to be noted that (96) gives a true value of the friction factor only for steady uniform laminar film flow in a wide channel. For other flow regimes the value of f calculated from (96) can be regarded only as a dimensionless film thickness, and it is somewhat misleading to take these values of f as friction factors in these cases. It has been shown that the true value of the friction factor as calculated from wall shear stress measurements in the wavy flow regime is not the same as that given by (96) (F7). For this reason, the second method of correlation given below seems to be preferable.

By rearranging Eq. (83), which is a generalized form of Eq. (19), it can be shown that

$$b(g \sin \theta/\nu^2)^{1/3} \left(\frac{\rho - \rho_c}{\rho}\right)^{1/3} = (3N_{Re})^{1/3} \tag{97}$$

The group on the left side of this equation is a form of dimensionless film thickness and has been termed the Nusselt film thickness parameter N_T (D12). Equation (97) indicates that a plot of N_T against N_{Re} on double-logarithmic coordinates should give a straight line of slope $\frac{1}{3}$ for the regime of smooth laminar flow. It has been found that this type of plot successfully correlates film thickness data obtained in the other flow regimes also (though the lines have a slope different from $\frac{1}{3}$ for the other regimes), and also data obtained with various liquids and on walls of various slopes. When the fluid phase adjoining the free surface of the film is gaseous the density correction term in (97) can be omitted, since then $\rho \gg \rho_c$.

3. *Discussion of Film Thickness Data in the Absence of a Gas Stream*

A search of the literature up to 1959 revealed some 1013 values of the film thickness which were tabulated or plotted on graphs large enough to be read accurately. These measurements were obtained for a wide variety of liquids, varying from very mobile hydrocarbon oils to glycerol, for film flow on vertical walls and at slopes down to about 1° to the horizontal. These values of the film thickness were recalculated and plotted as the dimensionless thickness parameter N_T against N_{Re} (F7). Figure 2 shows the

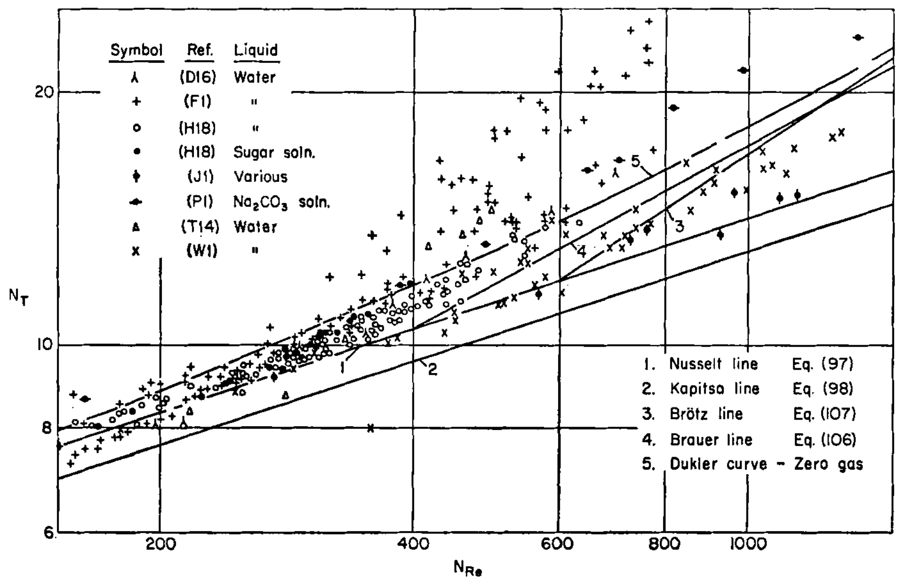

FIG. 2. Sample of earlier film thickness data near the critical Reynolds number plotted in terms of the Nusselt thickness parameter N_T and the Reynolds number N_{Re}, for the case of zero gas flow.

part of this plot in the region of the critical Reynolds number; the total range extended from (N_T, N_{Re}) of (0.5, 4×10^{-2}) to (50, 4×10^3). As can be seen from this figure, many of the early film thickness data, which were obtained mainly by the micrometer or simple drainage techniques, were not of a high degree of accuracy. However, in the laminar region the values fell near the line given by Eq. (97) for the most part. Above N_{Re} values of about 300–400, the experimental values deviated systematically from this line. In the laminar regime, even when waves were present, the N_T values appeared to agree better with Eq. (97) for a smooth film than with the predictions of the Kapitsa theory of wavy film flow, $N_{T(\mathrm{wavy})} = 0.93 N_{T(\mathrm{smooth})}$ (see Section III, D).

More recent film thickness measurements in the laminar wavy regime obtained by improved techniques (F2, F7, P3) have shown that there are appreciable reductions in the mean film thickness in the wavy regime for both vertical and sloped surfaces, as predicted by the Kapitsa theory.

Feind (F2) measured the thickness of various films of kinematic viscosities 1 to 19.7 centistokes flowing in a vertical tube. An improved drainage technique was used. At the lowest values of N_{Re} (smooth laminar flow regime) the values of N_T fell along the line given by Eq. (97). Once wavy flow commenced, the values deviated towards the Kapitsa line,

$$N_T = (2.4 N_{\mathrm{Re}})^{1/3} \tag{98}$$

At larger values of N_{Re} there was a gradual transition back towards the Nusselt line, Eq. (97), which was finally crossed at about $N_{\mathrm{Re_{crit}}} = 350$. In the turbulent zone the experimental values of N_T fell above the Nusselt line.

Feind's empirical relationships may be written as

$$N_T = (\nu')^{0.025} 3^{0.333} (N_{\mathrm{Re}})^{0.333(\nu')^{-0.11}} \tag{99}$$

for the region $0.72 K_F^{-0.1} < N_{\mathrm{Re}} \leq 1.35 K_F^{-0.1}$;

$$N_T = (\nu')^{-0.005} 3^{0.333} (N_{\mathrm{Re}})^{0.333(\nu')^{-0.03}} \tag{100}$$

for the region $1.35 K_F^{-0.1} < N_{\mathrm{Re}} \leq 6.0 K_F^{-0.1}$;

$$N_T = (\nu')^{-0.055} 3^{0.333} (N_{\mathrm{Re}})^{0.333(\nu')^{0.025}} \tag{101}$$

for the region $6.0 K_F^{-0.1} < N_{\mathrm{Re}} \leq 400$,

where K_F is the physical properties group given by

$$K_F = \mu^4 g / \rho \sigma^3 = (N_{\mathrm{We}})^6 / (N_{\mathrm{Re}})^4 (N_{\mathrm{Fr}})^2 \tag{102}$$

and ν' is a "relative viscosity" given by

$$\nu' = \nu \text{ (in m.}^2\text{/sec.)} / 0.6 \times 10^{-6} \tag{103}$$

so that $\nu' = 1.495$ for water at 25°C.

For the turbulent region ($N_{\mathrm{Re}} > 400$), it was found approximately that

$$N_T = (0.369) 3^{1/3} N_{\mathrm{Re}}^{1/2} \tag{104}$$

The results of recent measurements of the thicknesses of water films flowing in a channel of slope $7\frac{1}{2}°$ to 90° (F7) can be represented approximately over the range $30 < N_{Re} < 300$ by the empirical relationship

$$N_T = 1.28(\sin\theta)^{-0.065}N_{Re}^{0.337} \tag{105}$$

which indicates that the effect of the waves on the film thickness varies with the slope of the wall on which the film flows. The data of Kamei and Oishi (K4) also indicate that there is a reduction in the mean film thickness in the wavy laminar regime, but these data are not presented in a form suitable for exact comparison.

Portalski's experimental results (P4) for several liquids flowing as films on a vertical plate show that the measured film thicknesses are smaller than those calculated by the Nusselt theory for smooth laminar flow, except in the presence of surface-active materials. In certain cases the measured film thicknesses were also smaller than those predicted by the Kapitsa theory. Good agreement was also observed between the measured film thicknesses in the turbulent and laminar regimes (except at the lowest flow rates) and the predictions of the treatment based on the universal velocity profile [see Eq. (80), etc.]. At the lowest flow rates (in the smooth laminar regime), Portalski's experimental film thicknesses fall well below the Nusselt line and show a trend away from this line, and it is deduced that the Nusselt theory (Section III, B, 2) cannot be used for calculating the film thickness, even in this zone. However, these results are not in agreement with other recent measurements (F2, F7), which agree with or tend towards the Nusselt line in the smooth laminar zone.

Hence, the trend predicted by the Kapitsa theory is supported by the recent, more accurate, film thickness measurements. This does not indicate, however, that the Kapitsa theory will apply in detail over the whole wavy laminar regime of film flow, since Kapitsa (K7) pointed out that such a reduction in the mean thickness should result for other types of wavy flow besides the particular case considered in his theory.

Turning to the turbulent regime of film flow, there are several empirical relationships to be found in the literature. The experimental data of Brauer (B14) for the zone $N_{Re} > 400$ can be represented by the equation

$$N_T = 3^{1/3}N_{Re}^{8/15}(400)^{-1/5} \tag{106}$$

Feind's results (F2) have been given already by Eq. (104). Partly from dimensional considerations, Brötz (B21) showed that, in the turbulent region,

$$N_T = (3N_{Re}^2/590)^{1/3} \tag{107}$$

By analogy with pipe flow, Zhivaĭkin and Volgin (Z4) obtained for the turbulent region

$$N_T = 0.141(4N_{Re})^{7/12} \tag{108}$$

It can be seen that the empirical equations predict that N_T will vary with some power of the Reynolds number between $\frac{1}{2}$ and $\frac{2}{3}$. These equations are plotted in Fig. 3 on which is also shown the curve given by

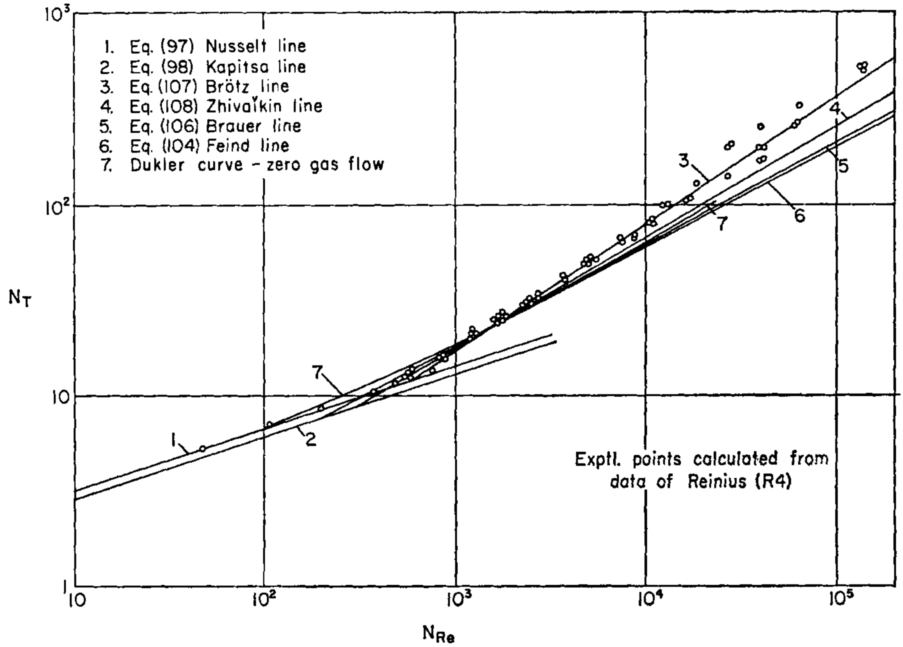

FIG. 3. Comparison of various correlations for the film thickness in the turbulent film flow regime in the absence of a gas flow.

the Dukler theory (Section III, F, 4) for the special case of zero interfacial drag. The experimental film thickness data obtained by Reinius (R4) for the flow of water in a smooth channel of small slope are also plotted on Fig. 3, and it is clear that the Brötz relationship, Eq. (107), is in excellent agreement with the experimental values over this very wide range of Reynolds numbers. However, at the smaller values of N_{Re} normally encountered in falling films, there is little difference between the predictions of these relationships; and the Dukler curve, which avoids a sharp transition between the laminar and turbulent regimes, lies close to many of the data in the transition and lower turbulent zones.

As noted earlier, the equation for turbulent film flow obtained by Levich (L8) [Eq. (71)] contains an unspecified constant and therefore cannot be readily compared with the other relationships for turbulent films.

4. *Film Thicknesses in the Presence of a Gas Stream*

There have been numerous studies of the effects of gas streams on the thicknesses of liquid films flowing in contact with them. Cases studied in the literature refer mainly to gas/film flow in vertical tubes or channels or horizontal pipes. The latter case will not be considered here. The gas/film flows may be countercurrent, cocurrent upwards, or cocurrent downwards. Only the most important studies can be discussed here.

Semenov (S6) considered generally the effects of a gas drag at the film interface for all the cases listed above for smooth laminar film flow (see Section III, F, 2), and later experimental work confirmed these results (K20, K10, S7) for the case when the film thickness is very small, with no waves present on the film surface, and at moderate gas flow rates. The early treatment by Nusselt (N6, N7) also gave results in agreement with the experimental data obtained under these restricted conditions. Brauer's treatment of the problem (Section III, F, 2) (B18) also assumed laminar flow of the film and absence of surface waves. The experimental work of Feind (F2), which refers to countercurrent gas/film flow in a vertical tube, showed that, although such a treatment was useful in predicting the qualitative effects of the gas stream on the film thickness and other properties, the Reynolds number range in which it applied strictly was very limited.

In general, the various treatments predict that, as the velocity of the gas stream is increased, the mean film thickness for a given liquid flow rate decreases in the case of downward cocurrent flow of the two phases, due to the acceleration of the surface by the gas stream drag. With countercurrent flow of the gas and film, the opposite effect occurs, until a point is reached at which the film no longer has a net downward flow, and "flooding" is imminent. If the liquid is introduced at the lower end of the channel and if the gas velocity is large enough, it is possible to have upward cocurrent flow of the film and gas, and, as in downward cocurrent flow, the mean thickness of the film at a given liquid flow rate decreases as the gas velocity increases. This general behavior has been confirmed experimentally (cocurrent upward: A5, C14, H9, H10, M5, K21, S6, Z3; cocurrent downward: C4, K5, K21, S6, Z3; countercurrent: F2, F4, F7, G10, H20, K4, N2, S6, S11, T14, Z3, among others).

For wavy flow the experimental results of Konobeev *et al.* (K20, K19) are generally in quite good agreement with the theoretical predictions outlined in Section III, F, 3 for very low liquid flow rates. At larger flow rates there are serious deviations from the predictions.

In the case of turbulent film flow with an adjoining gas stream, there is a wealth of experimental studies reported in the literature. Only the briefest review of these can be given here. In general, the difficulty encountered in such studies is to convert the measured pressure drop in the gas phase to a drag at the film/gas interface. It is frequently assumed that, if the gas stream is made to pass through a long calming section to form a fully developed flow profile before being introduced into the wetted section, the entry effect will be negligible, so that the interfacial shear stress can be calculated directly from the pressure drop per unit length. However, even if the gas stream is fully developed with respect to the dry entry tube, it will no longer be fully developed with respect to the wetted section, where the new boundaries of the gas stream are in motion relative to the inlet section walls. In general, it is necessary in such cases to make use of the more complicated relationship quoted by Eckert *et al.* (E1) for calculating the interfacial shear. In using this relationship, it is necessary to know the manner in which the gas stream velocity profile changes in the direction of flow, and this information is not often available.

In most of the work reported in the literature it is assumed that the wetted tube over which the pressure drop is measured is sufficiently long that such entry effects can be neglected. No account is taken of the waves at the film surface, or of the fact that they may move faster than the mean surface velocity of the film, and the energy lost or gained by the gas stream in accelerating or decelerating the liquid surface near the inlet is also neglected.

In spite of these apparently seriously restricting assumptions, Dukler (D15) has shown that his theory for downward cocurrent flow is in agreement with experimental gas/film thickness data reported by Charvonia (C4) and McManus (M3). Hewitt *et al.* (H10) and Collier and Hewitt (C13) have carried out analyses of numerous experimental data on the thicknesses of liquid films in upward cocurrent flow in terms of the Dukler theory and other simpler theories such as those due to Anderson and Mantzouranis (A5) and Calvert and Williams (C2) (Section III, F, 4). The experimental and calculated values are in moderate agreement in the case of the Dukler and the Anderson and Mantzouranis treatments. The discrepancies are probably due to the simplifying assumptions made in the theoretical treatments.

Zhivaĭkin (Z3) has recently published the results of a detailed investigation of the effects of a gas stream on the film thickness for upward cocurrent, downward cocurrent, and countercurrent flow of the phases over a wide range of flow rates. Thus, downward cocurrent flow is little affected by gas velocities up to about 4 m./sec. For gas velocities between 4 m./sec. and the velocity at which spray formation commences (which has been determined experimentally as a function of the flow rates of gas and liquid and their physical properties), the film thickness is given by

$$N_T = [1 - 0.022(v_g - 4)](3N_{\mathrm{Re}})^{1/3} \tag{109}$$

where v_g is the gas velocity, measured in meters per second.

For upward cocurrent flow, in the zone extending up to the formation of spray, it was found experimentally that

$$N_T = 1.18(\nu g)^{1/3}(N_{\mathrm{Re}})^{1/2}v_g^{-3/4} \tag{110}$$

where v_g is the gas velocity in meters per second as before. The conditions for flooding in countercurrent flow are also reported.

Feind (F2) has shown that, in countercurrent flow of the film and gas stream, there is a zone at low gas velocities in which the film thickness is hardly altered by the gas stream. At larger gas flow rates the film thickness increases; this point of initial increase of the film thickness occurs at lower gas Reynolds numbers as the film Reynolds number is increased, as might be expected. With countercurrent flow of liquid core streams, when the interfacial drag is much larger than with gas cores, Strang *et al.* (S13) and Treybal and Work (T16) have noted much larger increases in the film thicknesses.

From the remarks above, it can be seen that, although there are many experimental data available on the thicknesses of liquid films in the presence of gas streams, further work, both experimental and theoretical, will be required before film thicknesses can be predicted accurately under such conditions.

B. Onset of Turbulence in Films

There are several reports in the literature of the critical Reynolds number at which turbulent film flow commences. These values of $N_{\mathrm{Re}_{\mathrm{crit}}}$ are usually determined from the breaks which appear in the curves of film thickness, surface velocity of the film, heat or mass transfer coefficients in the film, etc., when plotted against N_{Re}. Some of the numerical values proposed by various investigators are listed in Table I.

Several of these investigators have quoted an upper and lower critical value, enclosing a transition region, and Schoklitsch (S3) has given three values, the lowest, $N_{\mathrm{Re}} = 144$, at which turbulence could be first detected, the second, $N_{\mathrm{Re}} = 389$, at which the turbulent part of the flow became important, and an upper value, $N_{\mathrm{Re}} = 900$, at which the film became "fully turbulent."

Dukler (D12) and Zhivaĭkin and Volgin (Z4) have pointed out that the transition to turbulence in a thin film is likely to be a gradual process, so that it is not reasonable to expect a single, sharply defined critical Reynolds number. The scatter in the experimental values of $N_{\mathrm{Re}_{\mathrm{crit}}}$ tabulated below can be explained in this way. However, the bulk of the evidence

TABLE I

VALUES OF THE CRITICAL REYNOLDS NUMBER

Investigator and Date	Ref.	$N_{Re_{crit}}$
Hopf, 1910	(H18)	250–300
Schoklitsch, 1920	(S3)	144, 389, 900
Jeffreys, 1925	(J4)	310
Monrad and Badger, 1930	(M12)	350
Schmidt *et al.*, 1930	(S2)	280–430
Kirkbride, 1934	(K17)	500
Cooper *et al.*, 1934	(C15)	525
Horton *et al.*, 1934	(H19)	550–750
Fallah *et al.*, 1934	(F1)	500
Kutateladze, 1939 (reported by Z4)	(Z4)	375
Grigull, 1942	(G8)	350
Dukler and Bergelin, 1952	(D16)	270
Grigull, 1952	(G9)	310–450
Emmert and Pigford, 1954	(E4)	300
Brötz, 1954	(B21)	590
Stirba and Hurt, 1955	(S12)	below 300
Brauer, 1956	(B14)	400±10
Thomas and Portalski, 1958	(T14)	290
Belkin *et al.*, 1959	(B4)	325–500
Feind, 1960	(F2)	400, 800
Kutateladze and Styrikovich, 1960	(K25)	100–400
Reinius, 1961 (smooth channel data)	(R4)	400
Zhivaĭkin and Volgin, 1961	(Z4)	400, 1000
Saveanu *et al.*, 1962	(S1)	362
Wilke, 1962	(W3)	400, 800
Fulford, 1962	(F7)	260–330

seems to support a lower value $N_{Re_{crit}}$ in the region of 250–400, with a less well-marked upper value of about 800.

Brauer (B14) and Kamei and Oishi (K4) have reported experimental values of the critical Reynolds numbers of films containing small quantities of surface-active materials in solution. In this case, the value of $N_{Re_{crit}}$ appeared to depend on the surface tension of the solution; this effect is probably due to the layer of surface-active material present at the interface.

C. ONSET OF RIPPLING IN FILM FLOW

As reported in Section III, C, there are numerous theoretical treatments of the problem of the onset of rippling in a falling film. By way of comparing these predictions, the theoretical lines giving the Reynolds number at the onset of rippling, N_{Re_i}, as a function of the channel slope are plotted in Fig. 4, using the Benjamin simplified theory (B5), Eq. (44), Kapitsa's theory [Eq. (38)], and the theory of Ishihara, *et al.* (I2), which reduces to a form similar to Eq. (44). The physical properties substituted in the equations in making this plot are those of water at room temperature.

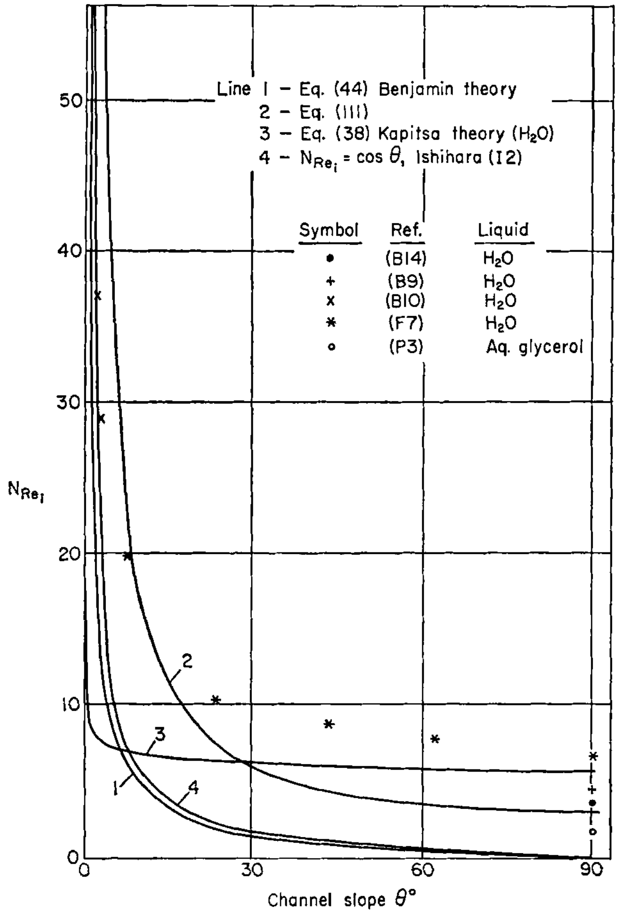

FIG. 4. Variation of the Reynolds number for wave inception, N_{Re_i}, with channel slope.

Making use of the condition that waves can only be stable if the Froude number of the flow is equal to unity (or exceeds unity), and using Eq. (46), it is seen that

$$N_{Re_i} = 3 \operatorname{cosec} \theta \tag{111}$$

The stability criterion of Hanratty and Hershman (H2) cannot be plotted on Fig. 4, since it contains the relative wavelength as a parameter. However, for long waves this reduces to a form very similar to the conditions of Benjamin and of Ishihara *et al.*

The various hydraulic stability analyses mentioned in Section III, C lead to the criterion $N_{\text{Fr}} \geq n$, where $n = 0.58$–2.0, instead of 1.0 as assumed above, and these reduce to conditions similar to Eq. (111) for the onset of rippling, but with numerical constants other than 3.

Also shown in Fig. 4 are the experimental values of the Reynolds number at which waves were first detected on water films flowing in channels of various slopes (B9, B10, B14, F7). These values were determined by careful visual observation of the free surface of the film (B9, F7), from photographs (B10), or by an electronic "feeler" (B14).

It is seen that in general the various theories are in reasonable agreement with each other, especially the Benjamin and the Ishihara theories. The simple condition $N_{\text{Fr}} = 1$ shows a similar trend with channel slope, but the values lie slightly higher at all slopes. The Kapitsa theory predicts that the values of N_{Re_i} will change little with slope except at very small slopes.

The experimental values lie above the theoretical curves but indicate a similar trend with channel slope. Binnie (B10) has shown that the exact value of N_{Re_i} obtained depends to some extent on the method used for detecting the waves; waves are detected at smaller values of N_{Re_i} when more sensitive methods are used. Since many of the experimental points in Fig. 4 were determined visually, which is not a particularly accurate method, it is reasonable to expect that they should fall high, for the Benjamin and Ishihara curves give a lower limit of N_{Re_i}. It can be seen that the condition $N_{\text{Fr}} = 1$ seems to be in moderate agreement with the experimental values of the Reynolds number at which waves are clearly visible for water films in channels of various slopes. It is interesting to note that, by analyzing his own and previously published experimental results on the onset of rippling in vertical columns using a number of liquids of widely different physical properties, Jackson (J1) also deduced that the criterion for wave formation was $N_{\text{Fr}} \geq 1$. Belkin *et al.* (B4) arrived at the same conclusion. It seems, therefore, that the condition $N_{\text{Fr}} = 1$, or its equivalent, Eq. (111), represents a useful working equation for calculating the onset of rippling.

Unfortunately, the experimental results mentioned above do not throw any light on an interesting aspect of the Benjamin theory of film stability, namely, the prediction that films flowing on vertical walls are unstable at all flow rates (see Section III, C). This is due to the fact that, although the theory predicts that vertical films will always be wavy, the waves may be very small at low flow rates. The fact that the use of more sensitive methods of wave detection leads to smaller experimental values of N_{Re_i} (see above) may be indirect confirmation of the theory.

Brauer (B14) carried out very careful determinations of the values of N_{Re_i} for films of water and aqueous diethylene glycol solutions flowing on vertical tubes. The results were correlated by the empirical relationship

$$N_{\mathrm{Re}_i} = 0.306(K_F)^{-0.1} \tag{112}$$

where K_F is the dimensionless group given by Eq. (102). Rather earlier, Grimley (G10), using less sensitive methods of detection, obtained a similar correlation,

$$N_{\mathrm{Re}_i} = 0.3(K_F)^{-1/8} \tag{113}$$

which gives slightly higher numerical values of N_{Re_i} for the liquids used.

A careful analysis of the experimental results of Brauer in terms of Benjamin's theory (Section III, C) indicated (F7) the interesting fact that, for all the pure liquids (i.e., liquids not containing surface-active materials in solution), the rate of increase of the Benjamin amplification factor with the Reynolds number, $d\mathcal{A}/d(N_{\mathrm{Re}})$, was the same at the point of onset of rippling.

Benjamin (B7) has also indicated that, if constant amplification of the most unstable wave per wavelength is assumed, it can be derived from the theory that

$$N_{\mathrm{Re}_i} \propto (K_F)^{-1/11} \tag{114}$$

and it is interesting to note that this relationship is quite similar to the experimental correlation obtained by Brauer, Eq. (112).

D. Surface Waves on Films

1. *General Observations of Wavy Flow*

Perhaps the most interesting features of film flow are the wave patterns which appear at the free surface of the flowing film at all except the lowest liquid flow rates. Numerous investigators have made observations of these waves under various flow conditions, e.g. (A2, A3, B6, B9, B10, B14, B15, B16, B20, C4, D17, E4, F2, F7, G10, H16, H19, H2, I2, J4, K1, K10, K18, L10, L11, L12, L13, M7, R2, S7, S11, T3, T5, T7, T13, T14, V1, V4, W2, W3), among others. It has been mentioned earlier (Section III, C) that wavy film flow is usually so complex that no satisfactory general theory of wavy film flow has yet been possible. For the same reason the experimental determination of the parameters characterizing the wave patterns is extremely difficult. Many of the earlier descriptions of wavy film flow are purely qualitative, and it is only in recent years that attempts have been made to measure mean wavelengths, amplitudes, etc., for any but the regular ("sine wave") regime of wavy flow.

In general, the wave patterns may be described qualitatively as follows. At very low flow rates ($N_{\mathrm{Re}} < N_{\mathrm{Re}_i}$), the film surface is completely smooth and mirror-like, troubled only occasionally by small random "dimples," which are rapidly damped out in the direction of flow of the film. At slightly larger Reynolds numbers, small, symmetrical, regular waves appear. The wave fronts are almost straight and perpendicular to the direction of flow. At still larger flow rates, the regular symmetrical waves tend to become less regular, and the cross section of the wave assumes the nonsymmetrical shape usually described as a "roll wave" (D9), with a steep front and a long, gently sloping tail. Frequently each roll wave is preceded by a number of small waves [termed "push waves" by Brauer (B14)] which move as a group with the main wave (B14, C6, F7, M7). (It appears likely that, although the wavelength of the main roll waves is large enough to make capillary effects negligible over most of the wave, the curvature of the steep front is sufficiently great to lead to the formation of capillary ripples at the toe of the main wave.) In this zone of flow the wave fronts are no longer straight at all times but show a tendency to form bulges or to split or overtake each other. Similar observations have been made recently for the case of upward cocurrent gas/film flow (T7).

As the liquid flow rate is increased a stage is reached when the main waves and their accompanying push waves have become so randomly mixed that the individual wave fronts can scarcely be distinguished, and the surface appears to be covered with a mass of small jagged "turbulent" waves. A number of photographs of this type of wave pattern have been published, e.g., by Bressler (B20), by Dukler and Bergelin (D16), and by Tailby and Portalski (T4). Certain observers (B14, T7) have noted larger wave rings or disturbance waves superimposed on this rough surface.

The presence of a gas stream appears to increase the size and randomness of the waves on the film surface (C4, F2, F7). Countercurrent gas flow leads to a decrease in the wave velocity, with the opposite effect for cocurrent flow.

2. *The Smooth Entry Zone*

In nearly all of the observations of wavy flow, it has been noted that there is initially a smooth region near the liquid inlet before waves appear on the film surface. The length of this smooth entry zone varies with the liquid flow rate, the flow rate of the adjoining gas stream, if present, and to some extent on the manner in which the liquid is introduced (T5). In general, the length of the smooth zone increases as the flow rate increases, but the increase is not in direct proportion to the flow rate (C6, J1, F2, M2, F7, T5, B14). Chien (C7) has noted that, as the flow rate is increased above a certain value, local disturbances occur within this initially smooth zone, leading to an apparent decrease in its length; however, it appears that the length of the zone without sustained wavy flow continues to increase with increasing flow rate. The observation made by Grimley (G11), that his films ceased to be wavy above a certain liquid flow rate, is probably due to the length of the entry region exceeding the total length of his wetted surfaces at the larger flow rates.

The cause of this initial smooth zone and the subsequent fairly sudden transition to wavy flow is not completely clear. Working on a much larger scale, with mostly turbulent flow of the liquid layers on dam spillways, Bauer (B1) has shown that the length of the smooth initial region is the same as the distance required for the turbulent boundary layer, which begins to form at the point of introduction of the liquid, to reach the thickness of the whole liquid layer, which in this initial zone accelerates and becomes thinner in the direction of flow. MacLeod (M2) arrived at a similar conclusion for flow rates normally encountered in wetted-wall columns, i.e., that the waves appeared at the end of the acceleration zone of the film, when the boundary layer (laminar or turbulent) had reached the same thickness as the film. Unfortunately, it is not a simple matter to calculate the film thickness in the acceleration zone; there is some dispute as to the length of the acceleration zone (S5, L14), and, in any case, the exact manner of introducing the liquid will affect the length of this zone to some extent (see F2, L14, W4). However, recent measurements of the film thicknesses near the liquid inlet have indicated a marked decrease in the film thickness in the direction of flow (C6, C7, F7, M7, T7), and it has been found that in many cases the film acceleration continued at an ever-decreasing rate as far as the point of wave inception (F7).

It seems possible, therefore, that in this initial region the upper part of the film, outside the growing boundary layer, is in potential-like flow, and that once the boundary layer reaches the free surface, its vorticity is sufficient to trigger the wave disturbances, which can then propagate or not, depending on whether the flow is unstable or stable ($N_{\mathrm{Fr}} > 1$ or $N_{\mathrm{Fr}} < 1$).

On the other hand, since the film is accelerating in this initial zone, it follows that the Froude number of the flow, which may be taken as the criterion for gravity-wave instability, increases from some very small value at the inlet to its equilibrium value for the particular flow rate and channel slope. Depending on the flow conditions, it is possible for the Froude number of the flow to reach the critical value before the end of the acceleration zone is reached. In this case it can be supposed that waves could occur before the end of the acceleration zone if some triggering mechanism were available. This appears to be the case in fact, for Tailby and Portalski (T5) have noted that when an adjacent gas stream (either cocurrent or countercurrent) is present, the length of the smooth entry zone decreases markedly.

Chien (C7) has proposed an energy mechanism for explaining the initial decrease in film thickness and subsequent formation of waves. Mayer (M7) has dealt with the manner in which the waves, which are initially fairly regular near the inception line, later develop into roll waves, etc., downstream, and Lighthill (L10) has also considered the manner in which waveforms may change by amplitude and frequency dispersion.

It is clear that further detailed work is required in order to explain the initial smooth zone adequately.

3. *Effect of Surface-Active Materials on Wavy Films*

It is well known that many types of waves and ripples can be damped by interfacial films of surface-active materials, as shown theoretically by Levich (L6, L7). There have been a number of investigations into the effects of surface-active additives on the flow of wavy films (E4, H2, H20, I2, J1, L15, M7, S11, S12, T3). In addition, surface-active materials have also been used in various studies of mass and heat transfer to films, and some of these results throw light on the flow behavior of the films, e.g. (H13, M11, R1, T9, T10, T11, T12).

Most of the investigators have found that surface-active additives greatly reduce or completely eliminate the surface ripples. The mechanism by which the surface-active materials suppress the waves is discussed by Emmert and Pigford (E4). In the mass-transfer studies, it has been found that the addition of surface-active materials greatly reduces the rate of mass transfer at a given liquid flow rate. It is interesting to note that by adding surface-active material to an initially wavy water film, Ternovskaya and Belopol'skiĭ (T11) found that the rate of absorption of SO_2 at small Reynolds numbers decreased by 25–38%, while Kapitsa (K9) has calculated that, in the regime of regular waves, the increase in the mass-transfer rate due the waves is of the same order, namely 20–30%. Hence it seems that the change in the mass-transfer rates under these circumstances is due purely to the changes in the hydrodynamics of the flowing surface. It has been shown experimentally that the change in surface tension brought about by the surface-active additives has little effect as such on the mass-transfer rates (T9). Lynn *et al.* (L15, L16, L17) have shown that surface-active materials may lead to the formation of a stagnant "skin" over the lower part of a film surface, and this may become important in measuring rates of mass-transfer in short wetted-wall columns.

Emmert and Pigford (E4), Ternovskaya and Belopol'skiĭ (T9–T12), and Tailby and Portalski (T3) have carried out detailed investigations of the effects of surfactants, using several different surfactants, each at a number of concentrations. In nearly all cases it was found that, as the concentration of surfactant was increased, the waves were rapidly damped out as far as some optimum concentration, beyond which there was either little further damping of the waves, or the waviness increased again. Ternovskaya and Belopol'skiĭ calculated that the optimum concentrations for wave damping corresponded to quantities of surfactant just sufficient to form a saturated monolayer at the interface (T10).

It is of interest to note that Brötz (B21) and Jackson (J1) could find little effect of the addition of surface-active materials on film flow. It is possible that these experiments were carried out in a region where the waves were mainly of the gravity type, i.e., of fairly long wavelength, and hence less likely to be affected by capillary forces. In this connection, it may be noted that Keulegan (K15) has found that surface-active materials may prevent waves being formed in the first place, but that, once formed, they do little to prevent gravity waves from propagating in the usual way. In dealing with wave problems of the present sort it is clearly important for experimenters to specify whether the Froude and Weber numbers are above the critical values, so that it can be decided whether the waves are likely to be controlled mainly by gravity forces, capillary forces, or a combination of both; surface-active materials will have a much larger effect on the capillary waves with their shorter wavelengths and hence greater surface curvatures.

4. *Wavelengths, Wave Velocities, Maximum and Minimum Film Thicknesses*

a. Wavelengths. In recent years there have been several investigations of the wavelengths of the surface waves appearing on films or, alternatively, the wave frequencies, from which the wavelengths can be calculated if the wave velocities are known (A3, B10, C4, C6, F7, G7, G11, I1, I2, K10, K20, K21, M7, S7, T4, T7).

It has already been noted that the wavelengths are regular only near the line of wave inception, except at low liquid flow rates, when regular waves extend over the remainder of the film surface also.

Tailby and Portalski (T4) have reported measurements of the wavelengths near the point of wave inception on vertical films of various liquids. Even in this case, it was found that the wavelengths were considerably greater than predicted by the Kapitsa theory, Eq. (67), even in the cases in which the conditions of the theory were satisfied. Similar results have been obtained for water films on walls of various slopes (F7).

For most flow rates the wavelength in the region away from the line of inception varies considerably with time, and it is possible to obtain a mean wavelength only by averaging the distances between the fronts of a large number of waves. It has been found that after a zone at low Reynolds numbers in which the wavelength decreases with increasing N_{Re}, as predicted by the Kapitsa theory, there is a marked increase in the mean wavelength in the roll-wave zone (F7). The maximum wavelength is reached at $N_{\mathrm{Re}} = 100\text{–}160$ (depending on the channel slope), after which there is again a decrease in the mean wavelength. The locus of the maxima on the wavelength–Reynolds number curves has been found to agree closely with the line $N_{\mathrm{We}} = 0.7\text{–}1.0$, which indicates that the initial sharp increase in the wavelength is due to the action of gravity-type roll waves, while the subsequent decrease is probably due to the formation of shorter capillary waves near the critical Weber number of unity.

These results also show that, at a given liquid flow rate, the wavelength increases rapidly as the slope of the wetted wall is decreased.

The standard deviation of the individual waves from the mean wavelength was also calculated, giving some measure of the randomness of the wave patterns. (The wavelengths appeared to follow a Gaussian distribution quite closely.) The standard deviation increased in the roll-wave region, then decreased in the zone immediately above $N_{\mathrm{We}_{\mathrm{crit}}}$, and finally increased again, slowly, in the turbulent region.

Although there are numerous published investigations in which records of the wavy surface profile have been obtained, e.g. (H9, D16, S11), not many of these have been analyzed for information on wavelengths, most being concerned with wave-size (height) distributions. However, it may be noted that the experimental wavelengths of Kapitsa and Kapitsa (K10) show a trend in the direction of the data reported above, even at very small Reynolds numbers ($N_{\mathrm{Re}} < 25$). It seems, therefore, that the Kapitsa theory is applicable only at very small flow rates, as far as wave characteristics are concerned, in the case of the free flow of wavy films. Allen (A3) has reported a similar conclusion.

On the other hand, Konobeev and his co-workers (K20, K21) have shown that Eq. (95), which is a simplified and generalized form of the Kapitsa equation for the wavelength, is in good agreement with experimental data for the wavelengths appearing in upward cocurrent gas/film flow. However, these investigations were confined to very small liquid flow rates. At larger flow rates, Taylor *et al.* (T6, T7) have shown that the wave patterns in upward cocurrent gas/liquid flow are quite complex, though even at moderate liquid flow rates the wavelengths appear to be more uniform than in free film flow. It was found that the wave frequency was relatively insensitive to the air flow rate but increased with the liquid flow rate.

There appears to be little information available on the effect of a countercurrent gas stream on the wavelength.

b. Wave Velocities. The Kapitsa theory [Eq. (65)] predicts that the wave velocity should be 2.4 times the mean film velocity, while the theories of Benjamin (B5) and Hanratty and Hershman (H2) predict that $c = 3\bar{u}$. Ishihara *et al.* (I2) have shown that the wave velocity in the laminar region can be given as

$$c = \bar{u}\left\{\frac{6}{5} + \sqrt{\frac{6}{25} + \frac{3}{N_{\mathrm{Re}} \sin\theta}}\right\} \tag{115}$$

The experimental results of Mayer indicate that the wave velocity is proportional to $N_{\mathrm{Re}}^{1/3}$ (M7). Since $\bar{u}$ is proportional to $N_{\mathrm{Re}}^{2/3}$ [Eq. (18)], it follows that the ratio $c/\bar{u}$ must decrease with increasing Reynolds numbers. Similar experimental conclusions have been reported elsewhere also, e.g. (F7).

In order to compare the theoretical predictions with experimental values, the ratio $c/\bar{u}$ is plotted as a function of the Reynolds number in

Fig. 5. The lines $c/\bar{u} = 3$ and $c/\bar{u} = 2.4$ corresponding to the theories of Benjamin and Hanratty and Hershman and of Kapitsa are shown, together with the line given by Eq. (115), using $\theta = 7\frac{1}{2}°$. The remaining lines represent smoothed experimental wave velocity data (F7) for water films on wetted walls of slopes $7\frac{1}{2}$, 62, and 90°.

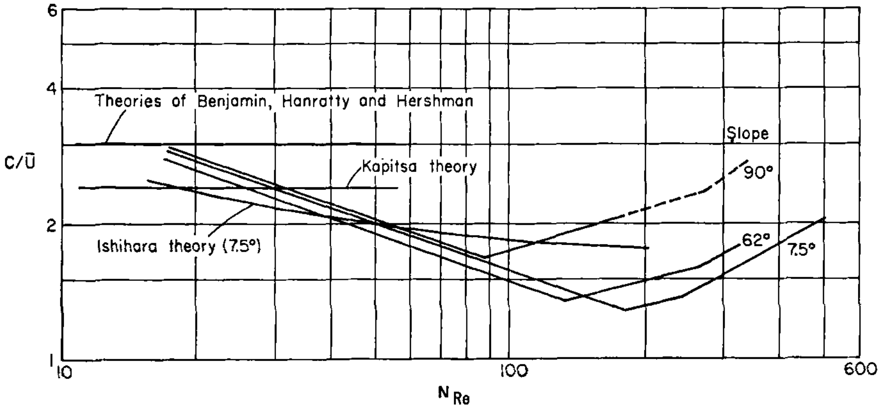

FIG. 5. Ratio of the wave velocity to mean film velocity, $c/\bar{u}$, in the absence of gas flow as a function of N_{Re}: comparison of various theories with smoothed experimental data (F7) for water films in inclined channel.

It can be seen immediately that the experimental values of $c/\bar{u}$ reach a value of 3 only at very small flow rates, near the flow rate for the onset of rippling, which is the zone for which Benjamin's theory is strictly applicable. The experimental values fall below the Kapitsa value of 2.4 at $N_{Re} = 30$. The theoretical relationship by Ishihara *et al.* predicts that $c/\bar{u}$ will decrease as N_{Re} increases, but less rapidly than observed experimentally. However, this theory is strictly applicable only at very small channel slopes and for waves of negligibly small amplitude, so that exact agreement cannot be expected.

The experimental results show that $c/\bar{u}$ decreases up to a certain flow rate at which it was found that $N_{We} = 0.8$, after which a different behavior is exhibited, with an increase in $c/\bar{u}$ as N_{Re} increases. At $N_{Re} = 280$, a further break was observed; this is due to the onset of turbulence in the film, which alters the manner in which $\bar{u}$ varies with N_{Re}. Hence, as in the case of the wavelengths, there appears to be an important change in the wave characteristics near the critical Weber number of unity.

The experimental wave velocities of Kapitsa and Kapitsa (K10) for vertical water films are in agreement with the results given above, but these investigators only covered the range up to $N_{Re} = 25$. The experimental wave velocities for water films given by Portalski (P3) (90°) and

Mayer (M7) (small slopes) show the same trend as these results, but the numerical values of c at a given N_{Re} are rather larger.

An interesting feature of the experimental results plotted in Fig. 5 is that the ratio $c/\bar{u}$ may be less than 1.5 under certain flow conditions. It will be shown later (Section IV, F) that the surface velocity of the film is equal to $1.5\bar{u}$, so that in this flow zone it appears that the surface waves move less rapidly than the surface of the film. Under these circumstances, the waves might tend to steepen at the upstream end, and the sudden transition from steep-fronted to steep-backed waves might explain the increase in the randomness of the waves in this flow zone, as illustrated by the standard deviation of the wavelengths mentioned in the last section.

Taylor *et al.* (T6, T7) have reported on wave velocities in upward cocurrent gas/film flow. It was found that the wave velocity increased rapidly with increasing gas flow rate but varied little with liquid flow rate. It was found, furthermore, that the individual wave velocities were not uniformly distributed around the mean value under given flow conditions, but that certain "preferred velocities" appeared to exist. The reasons for such behavior are not clear at present. More recently, Nedderman and Shearer (N1a) have carried out similar studies over a wider range of gas and liquid flow rates. Although the results were similar in many respects, it seems that the wave frequencies, of the large disturbance waves in particular, are dependent on the geometry of the apparatus. These results showed that, at constant water flow rate, the wave velocity for upward cocurrent flow varied with the square root of the air velocity relative to the waves.

There is again a lack of detailed measurements for the case of countercurrent gas/film flow. Qualitative observations (F7) indicate that the wave velocity first increases slightly as the gas flow rate increases, perhaps due to an increase in the size of the waves, and then decreases due to gas phase drag.

c. Maximum and Minimum Film Thicknesses. Brauer (B14) carried out the first detailed investigation of the distribution of wave sizes on a vertical film and reported on the maximum film thickness (height to the highest wave crest), the minimum film thickness (liquid thickness from the wall to the deepest wave trough), and also the frequencies of waves of various sizes, for the case of zero gas flow.

More recently, similar studies have been carried out for other cases of flow; Shirotsuka *et al.* (S11) have given empirical relationships for the mean wave heights as a function of the gas and liquid flow rates for the case of countercurrent gas/film flow. Lilleleht and Hanratty (L12, L13) and McManus (M3) have considered the amplitude characteristics of the waves at a horizontal liquid surface in the presence of a gas stream. Similar studies have been carried out for cocurrent gas/film flow by Hewitt *et al.*

(H9, H11) and Konobeev *et al.* (K20, K21) for the case of upward flow, and by Charvonia (C4), Chien (C7), and Konobeev *et al.* (K20, K21) for downward flow of the phases.

Brauer (B15, B16, B17) has pointed out that information on the frequency of the waves, regarded as surface disturbances, may be of considerable importance in calculating the rates of heat and mass transfer through the wavy film interface, and, in fact, Konobeev *et al.* (K20, K21) have shown that the rate of absorption of CO_2 by a water film in wavy cocurrent flow can be correlated in terms of the length and amplitude of the surface waves over the range of small liquid flow rates investigated.

In most cases, it is found that there is a considerable spread in the wave amplitudes, but that for given gas and liquid flow rates there is a certain wave height which occurs most frequently, and which can therefore be regarded as a characteristic of the wavy flow. The manner in which this characteristic frequency varies with the flow rates has been given in the literature, e.g. (B14, C4, H9).

In addition to the effects on heat and mass transfer, the wavy film surface also acts as "rough wall" to the adjoining gas phase (L12, C4, G3). This aspect of such flows will be considered more fully in Section IV, G.

In spite of the detailed investigations mentioned above, much work remains to be done on this aspect of film flow before it will be possible to characterize the wave amplitude behavior accurately for all the film flow regimes. Once wavelengths and wave amplitudes can be predicted accurately, it may be expected that a better understanding of transfer processes at wavy film interfaces will result.

5. *Increase of Interfacial Area Due to Waves*

Wetted-wall columns have been used for many years for determining mass-transfer coefficients on the assumption that the interfacial area across which mass transfer occurs can be obtained accurately from the dimensions of the column and a knowledge of the film thickness. It is therefore of considerable practical interest to determine whether the interfacial waves lead to an appreciable increase in the interfacial area of the film, which would introduce a grave uncertainty into such methods of determining mass-transfer coefficients.

Portalski (T2) has extended Kapitsa's treatment of wavy film flow to obtain an expression for the increase in interfacial area due to the waves [Eq. (68)]. For mobile liquids this relationship predicts that the increase in interfacial area will be very large, reaching 150% for 2-propanol at $N_{\mathrm{Re}} = 175$, for example, though the applicability of the Kapitsa theory at such large Reynolds numbers is in doubt. Experimental values of the increase in interfacial area were presented (T2) for films of 82% glycerol solution on a vertical wall and found to be in good agreement with the value predicted by Eq. (68) at $N_{Re} \cong 12$ (3% experimental, 3.3% calculated). Dukler and Bergelin (D16) also obtained records of the wavy surface profiles of vertical films and claimed that the increase in interfacial area was appreciable, though numerical values were not given.

Levich (L8) has shown that, for capillary waves, the relative increase in interfacial area is given by

$$\frac{\Delta S}{S} = \left(\frac{\alpha}{\lambda}\right)^2 \tag{116}$$

where α and λ are the wave amplitude and wavelength. Since, usually, $\lambda \gg \alpha$, this formula predicts that $\Delta S/S$ will be small.

The wave profile photographs published by Kapitsa and Kapitsa for vertical films of water and ethanol at small Reynolds numbers indicate that the increase in surface area can only be small. Brauer (B14) obtained numerous profile photographs of wavy films over the range $25 < N_{Re} < 1675$, and measured a maximum increase in the interfacial area of about 3%. From similar photographic studies, Belkin *et al.* (B4) and Vouyoucalos (V7) arrived at increases of not more than 10% and 8%, respectively. Portalski (T2, discussion) has criticized the results of Belkin and Brauer on the grounds that exact orthogonal views of the film profile were not obtained by the photographic technique used, and that the measured increases in interfacial area were too low, due to partial blocking of troughs by other crests in the photographs. Nevertheless, Shirotsuka *et al.* (S11), who obtained film profiles by means of a capacitance probe similar to Portalski's, have obtained increases in surface area of less than 0.2% up to liquid Reynolds numbers of 160, even in the presence of a countercurrent gas stream (water films). These workers have shown that it is difficult to obtain accurate values of the increase in surface area from a recorder chart of the wave profile, which is normally considerably exaggerated in the direction perpendicular to the film surface.

Ternovskaya and Belopol'skiĭ (T12) claim that the decrease in surface area due to damping of waves is small, and Allen (A3) has shown that the increase of surface area by the waves is smaller than predicted by Portalski.

In general, it seems that, although there must be a measurable increase in the interfacial area of a wavy film, this is not likely to be too important in practice.

6. *Mixing Effect of Surface Waves*

It is well known from the literature that the waves appearing at the surfaces of flowing films lead to increases in the rates of heat and mass transfer to such films. Zotikov and Bronskiĭ (Z5) have shown that surface waves lead to an increased rate of heat transfer, and Labuntsov (L1) has reported a correction to the Nusselt heat transfer equations for condensate films to take into account the effects of interfacial waves. Zhavoronkov *et al.* (Z2) and Kamei and Oishi (K3) have reported increases in mass-transfer rates to wavy films, which are up to several times as large as the values calculated for smooth flow of the same Reynolds number. Similarly Hikita (H12) and Stirba and Hurt (S12) have reported that the rates of mass transfer to wavy laminar films are much larger than those predicted from the theory for smooth laminar flow, but that when surface-active materials were added to damp out the waves, the results were in agreement with the theories of Pigford (S9) or Vyazovov (V8). Recently Heartinger (H5) has proposed a model to account for mass transfer in wavy films also, while Brauer (B14, B16) has reviewed a mass of experimental data and proposed a semi-empirical method of calculating mass and heat transfer in wavy films.

In order to explain these large increases in the transfer rates to wavy films, Jackson (J1) has postulated that the waves in a wetted-wall column behave as sources of localized mixing action moving over the film, which may otherwise be in laminar motion. Stirba and Hurt (S12) have carried out dye tracer experiments in a wetted-wall column, which indicated that even well within the laminar zone the dye streaks are broken up by the waves. When the waves were damped out by the use of surface-active materials the dye streaks remained undisturbed at the same liquid flow rates. Ishihara *et al.* (I1) have reported tracer studies in waves appearing on a laminar water flow at small slopes and concluded that there must be a considerable degree of turbulent mixing along the fronts of such waves. Mayer (M7) has published photographs of laminar roll waves and has shown that these waves are characterized by ridges of high vorticity with quiescent interwave zones. Allen (A3) has also suggested that the increased rates of heat and mass transfer in wavy films are due to mixing in the fluid streams.

On the theoretical side, Dmitriev and Bonchkovskaya (D8) have shown that in principle turbulence should spread from waves. Kapitsa (K9) has calculated a general tensor quantity, termed the coefficient of wavy transfer, which is applicable to any flow with periodic disturbances, such as pulsations or surface waves. This treatment predicts an appreciable increase in the rates of heat and mass transfer in wavy films, though this increase does not appear to be as large as that observed experimentally under certain conditions.

Davies (D4, D5, D5a, D6) has reported that when a potassium permanganate dye streak was injected through a fine capillary tube below the surface of a water film in vertical wavy laminar flow, the surface waves appeared to have no effect on the width of the streak, indicating that there was no lateral mixing action in the wave fronts. From a consideration of the original Kapitsa treatment of wavy film flow, Portalski (P5) has shown theoretically that circulating eddies may arise as a result of the zones of reversed flow predicted in the wave troughs by this theory. Such eddies would explain the increased rates of heat and mass transfer to wavy films in the absence of lateral mixing in the wave fronts. However, recent experiments in which a potassium permanganate dye streak was injected carefully onto the surface of a water film have shown that the dye streak became appreciably wider in the wave fronts and eventually became very diffuse, even at $N_{\mathrm{Re}} < 90$, i.e., well within the laminar zone; the disappearance of the sharply defined dye streak was accelerated in the presence of a countercurrent gas stream (F7). It is possible that the persistence of the dye streaks in the earlier experiments is connected with the manner in which the dye trace was injected; in studying the flow of films over roughness elements, Vouyoucalos (V7) showed that dye streaks behaved differently if injected near the free surface (which is likely to be most affected by the waves) or deeper in the film.

Finally, brief mention must be made of a series of recent papers in which it is shown that the mixing action usually ascribed to discontinuities (e.g., mixing between packing elements in a packed column) may be, in fact, a result of the action of ripples present on the film flowing over the packing at the discontinuities (A7, R3, W5). It is clear that the flow patterns in wavy film flow and their effects on transfer processes merit a great deal of further study.

E. Effect of Wall Roughness on Film Flow

Although nearly all of the theoretical and experimental studies of film flow have dealt with the flow of films along hydrodynamically smooth surfaces, it is of interest to review the limited information available on the effects of wall roughness in view of their possible importance.

Using channels of glass and roughened brass, Hopf (H18) found that the critical Reynolds number appeared to be independent of the wall roughness in film flow. For vertical tubes of different roughnesses, Claassen (C10) found that the thicknesses of flowing films varied little, though the amount of liquid remaining on the wall after draining increased with the roughness of the surface.

There are numerous reports of investigations of the effects of roughness on flow in open channels. For instance, Reinius (R4) has reported on the effects of surfaces covered with various types of roughnesses (spheres, sand, etc.) on the flow of water in open channels, while Hama (H1) has reported the effects of other types of roughness elements on flow in a flume. Unfortunately, in most of these investigations the channel slopes are extremely small, and the conditions are not the same as in the flow of thin films on steep walls.

Vouyoucalos (V7) carried out experiments with film flow over regular transverse wall roughness elements of very large size compared with the film thickness and showed that interesting back-mixing patterns could occur in the zones between the roughness elements. Bressler (B20) has also shown that the use of a plate with horizontal ridges increases the turbulence in a film flowing over it and improves heat transfer to such a film. Saveanu *et al.* (S1) investigated the effects of introducing small roughness elements of various sizes on an otherwise smooth vertical wall. These elements were placed at intervals equal to the characteristic wavelength appearing at the free surface. It was found that, with elements of height only 0.2 mm., the critical Reynolds number at which turbulence became important was reduced from its value of 362 for a smooth wall to 287. Increase of the roughness to 0.5 mm. led to little additional change.

The most detailed investigation of the effects of roughness on film flow available at present is due to Brauer (B14). In these experiments fine wires of 0.1-, 0.2-, and 0.3-mm. diameter were placed around a wetted tube 20 mm. above a device for measuring the local heat transfer coefficient to the film. The critical Reynolds numbers obtained for the three sizes of disturbance were 380, 360, and 340, respectively, compared with the value of 400 for the smooth wall, and for the 0.2-mm. disturbance, the local heat transfer coefficient at $N_{\mathrm{Re}} = 800$ was at least 12% larger than for the smooth wall. When the roughness elements were placed 40 mm. or more above the heat transfer device, no change could be detected compared with the smooth wall, indicating that the effects of the roughness elements die out rapidly downstream.

It seems that it would be of practical interest to investigate the effects of wall roughness in greater detail, since it might be possible by means of suitably arranged small roughness elements to increase the rates of heat and mass transfer in film-type equipment.

F. Film Velocities and Velocity Profiles

A knowledge of the velocity profiles within falling films under various flow conditions would be of very great value, making it possible to calculate the rates of convective heat and mass transfer processes in flowing films without the need for the simplified models which must be used at present. For instance, the analyses of Hatta (H3, H4) and Vyazovov (V8, V9) indicate clearly the differences in the theoretical mass-transfer rates due to the assumption of linear or semiparabolic velocity profiles in smooth laminar films. The importance of the shape of the velocity profile in studies of heat transfer to films has also been stressed by Wilke (W2) and others; up to the present there have been no direct checks as to whether the various velocity profiles assumed for turbulent film flow (Sections III, E and III, F, 4) are in fact observed in the flow of films.

Unfortunately, the thinness of most liquid films makes it difficult to measure the velocity profiles experimentally, since it is practically impossible to introduce any of the usual fluid-velocity probes into a film which may be less than 1 mm. thick without grossly distorting the flow patterns. Nevertheless, film velocity profile measurements have been reported for a few special cases.

Grimley (G10, G11) used an ultramicroscope technique to determine the velocities of colloidal particles suspended in a falling film of tap water. It was assumed that the particles moved with the local liquid velocity, so that, by observing the velocities of particles at different distances from the wall, a complete velocity profile could be obtained. These results indicated that the velocity did not follow the semiparabolic pattern predicted by Eq. (11); instead, the maximum velocity occurred a short distance below the free surface, while nearer the wall the experimental results were lower than those given by Eq. (11). It was found, however, that the velocity profile approached the theoretical shape when surface-active material was added and the waves were damped out, and, in the light of later results, it seems probable that the discrepancies in the presence of wavy flow are due to the inclusion of the fluctuating wavy velocities near the free surface.

Clayton (C11) measured velocity profiles in vertical films of various liquids by a chronophotographic method, and this work was continued by Wilkes and Nedderman (W4), using improved techniques. Almost instantaneous values of the velocity are obtained at different distances from the wall in this method, so that the difficulty noted above is eliminated. It has been shown in this way (W4) that in the smooth laminar flow regime the velocity profiles are in very close agreement with the predictions of Eq. (11). In the presence of waves at the film surface the local velocities scatter about the theoretical curve, which, however, continues to give an excellent approximation to the time-averaged local velocities. This investigation was confined to quite small Reynolds numbers (regime of regular waves), and further results for other wavy regimes would be of great interest.

Wilke (W2, W3) obtained velocity profiles by means of a traversing scoop, which collected the liquid flowing within the part of the film between the free surface and the lip of the scoop. These results were complicated by the fact that the flow is periodic in the part of the film between the highest wave crests and deepest wave troughs, but, by applying suitable corrections, Wilke showed that useful information on the velocity profiles could be obtained.

However, for the most part it is necessary to concentrate on the determination of the mean film velocity (from the flow rate and the mean film thickness, see Section IV, A) and the true surface velocity of the film. If the ratio $u_s/\bar{u}$ is equal to 1.5 [see Eq. (14)] this will be an indication that the velocity profile is semiparabolic.

It is well known that in turbulent pipe flow the parabolic profile present in laminar flow becomes blunter, so that the ratio $u_{\max}/\bar{u}$ decreases. A similar effect has been found for the relatively deep flows in open channels at small slopes by Jeffreys (J4), who obtained values of $u_s/\bar{u}$ down to 1.06, and by Horton *et al.* (H19), who measured values as low as 1.1. It can be expected that in the flow of thin films the ratio will decrease in turbulent flow from the value of 1.5, but by a very much smaller amount than observed in the deep flows noted above.

Several workers have reported experimental values of the ratio $u_s/\bar{u}$ for film flow, e.g., Friedman and Miller (F5), Grimley (G11), and Chew (C6), who timed the movement of dye drops at the free surface, Brauer (B14) and Jaymond (J3) who used plastic "confetti" as surface tracers, and Asbjørnsen (A6), who used an interesting residence-time technique. Jackson *et al.* (J2) have deduced the effective film surface velocities from pressure drop measurements in an adjoining gas stream, neglecting the effects of the surface roughness due to the waves.

These studies show that up to the Reynolds number of wave inception, N_{Re_i}, the ratio $u_s/\bar{u}$ is equal to the theoretical value of 1.5 [Eq. (14)]. Above N_{Re_i}, all the investigators above found a sharp increase in $u_s/\bar{u}$ to a value between 1.9 and 2.25, followed by a more gradual decrease to a value of 1.5, again in the region of $N_{\mathrm{Re}_{\mathrm{crit}}}$ (except Jackson *et al.*, who did not observe the subsequent decrease to 1.5).

On the other hand, Portalski (P4) found values of $u_s/\bar{u}$ which fell near or below the theoretical value of 1.5 for all the flow rates investigated, including the range in which other workers found values of 2.0 or greater. It seems clear that the earlier workers have measured an "effective" surface velocity in the region of large waves, which lies between the true surface velocity of the film and the wave velocity. It is of interest to note that the maximum values of $u_s/\bar{u}$ have been reported in the zone in which the wavelengths appeared to pass through a maximum, i.e., near $N_{\mathrm{We}} = 1$ (see Section IV, D, 4, a), and the effective surface velocities tend towards the theoretical value of the surface velocity in the zone near $N_{\mathrm{Re}_{\mathrm{crit}}}$, where the wave velocities tend towards the value of the surface velocity (Section IV, D, 4, b) and where Kirkbride (K17) has reported a decrease in the wave heights.

Unfortunately, in the light of the paper of Tinney and Bassett (T15), it is not clear that the method used by Portalski for measuring the surface velocity of the film (measurement of the rate of propagation of a surge front at the given flow rate on a just-wetted surface) will give accurate values of this quantity. More recently (F7), other measurements of the surface velocities of water films have been made, in which the surface velocity has been calculated from the wavelengths of the standing waves on the film surface formed upstream of a stationary pointer just touching the surface. The theory underlying this method has been treated by Lamb (L5) under the title of Lord Rayleigh's "fishline problem." The experimental values, which covered the range $N_{\mathrm{Re}} = 180$–750, fell about the line $u_s/\bar{u} = 1.5$ and showed no tendency to increase towards the value of 2.0 at the lower Reynolds numbers. Although there was a fairly large experimental scatter, the average of 226 readings was $(u_s/\bar{u})_{\mathrm{av}} = 1.543$, quite close to the theoretical value.

It seems, therefore, that the true surface velocity of the film does not vary much from the theoretical value over the range of flow rates normally encountered in wetted-wall columns.

It was also found that, although u_s decreased in the presence of an air counterflow, the ratio $u_s/\bar{u}$ remained almost unchanged at moderate gas flow rates ($N_{\mathrm{Re}_{\mathrm{gas}}}$ up to 24,000), since the film thickness increased, and hence $\bar{u}$ decreased as well as u_s. This is in agreement with Eq. (93), which predicts that $u_s/\bar{u}$ will be only slightly smaller than 1.5, unless the interfacial shear stress is large.

Feind (F2) has indicated that, in determining the effect of a liquid film flow on an adjoining gas stream, the velocity of the gas relative to the film surface is an important parameter. At present there are few measurements of film surface velocities for the various cases of gas/film flow, and the problem remains as to whether the gas velocity should be considered relative to the true surface velocity of the film in the wavy regime, or to the wave velocity, or to some effective surface velocity.

G. Static Pressure Drop in the Gas Stream of a Wetted-Wall Column

Numerous pressure-drop measurements have been made in the gas streams of wetted-wall equipment under various geometric and flow conditions. For the case of vertical tubes, pressure drops have been reported for downward cocurrent gas/film flow by Charvonia (C4), Chien (C7), and Zhivaĭkin and Volgin (Z4a), for upward cocurrent flow by Bennett and

Thornton (B8), Calvert and Williams (C2), Hewitt and co-workers (C13, G3, H8, H9, H10), and Mahrenholtz (M5), and for countercurrent flow by Clayton (C11), Feind (F2), Jackson *et al.* (J2), Kamei and Oishi (K2), and Thomas and Portalski (T14). Pressure-drop measurements have also been reported for other geometries of the wetted channel (F7, H6, M3, among others). In addition, values have been reported for the small static pressure drops occurring in wetted-wall equipment in the absence of a net gas flow due to the entraining action of the liquid film surface (F2, F7, M8).

Attempts have been made to compare the experimentally measured pressure drops with various of the theories discussed in Section III, F and with the more generalized empirical two-phase friction-factor correlations of the type discussed by Dukler and Wicks (D17). Hewitt *et al.* (H10) have compared their experimental data for upward cocurrent flow with one such empirical correlation and with the theories of Anderson and Mantzouranis (A5) and of Dukler (D12) and Hewitt (H7). In most cases there was good qualitative agreement only.

The theoretical treatments assume that the pressure drop per unit length of the wetted-wall column is a constant quantity, that is, that the gas and liquid streams are both in steady-state flow, with no changes in their velocity profiles in the direction of flow of each phase. In many cases the experimental pressure gradients reported in the literature have been obtained by measuring the pressure drop over the whole length of a wetted wall column and dividing this value by the length of the column, which is clearly valid only so long as the pressure gradient is constant over the whole length of the column. Recent careful experimental measurements in countercurrent flow (F2, F7), in upward cocurrent flow (G3), and in downward cocurrent flow (Z4a) have shown that the pressure gradient is not always constant in the direction of flow of the gas, due to changes in the shape of the gas stream velocity profile and to changes in energy on accelerating or decelerating the liquid film near the inlet. Until it is possible to correct accurately for these effects, or until pressure gradient measurements referring specifically to the steady-state flow regions are available, it seems to serve little useful purpose to compare the theories with the experimental values, which may be valid only for the specific column dimensions investigated, and only a general discussion can be given here.

A particularly interesting feature of gas flow in a tube wetted by a wavy film is that the pressure drop for a given gas velocity is considerably larger than in the case of flow in a dry tube (M4), as shown very clearly by the data of Feind (F2). In an attempt to explain this effect, Laird (L3) investigated gas flows along tubes with flexible walls which performed sine wave oscillations. It was concluded that a large part of the increase in the pressure drop over the value for a smooth tube was due to changes in the gas stream profile. Later measurements by Laird *et al.* (L4) in tubes with wavy stationary walls showed that the pressure drop in this case was not greatly in excess of the value for a smooth tube, from which it was concluded that the boundary shape alone could not be the cause of the large increase in the pressure drop in columns wetted by wavy films. Konobeev and Zhavoronkov (K22) carried out similar studies for both long-wave and short-wave stationary wall roughnesses and pointed out that in wetted-wall columns the pressure drop increase would be greater than in a tube with stationary solid waves on the walls, due to the moving, irregular, and deformable nature of the roughness elements. In the case of countercurrent gas/film flow, Feind (F2) has suggested that the thin layer of gas entrained at the surface of the liquid film (to satisfy the condition of zero slip) must travel in a direction opposite to the main gas flow, so that the effective gas velocity gradient at the interface (and hence the pressure drop) will be increased.

Attempts have been made to characterize the effects of the interfacial waves on the gas stream by calculating an equivalent sand roughness for the wavy interface from velocity profile measurements in the gas stream (G2, H6, L13). Lilleleht and Hanratty (L12) have shown that a sand roughness calculated in this way is in good agreement with the root-mean-square wave heights. However, more recently Gill *et al.* (G3) have shown that the effect of a wavy film surface on a gas stream in contact with it is not the same in some respects as that of a solid rough wall. One grave complication in the approach of regarding the film surface as a rough surface relative to which the gas stream moves is that, in this case, the "roughness elements" are themselves in motion relative to the "wall," since it has been shown in Section IV, D, 4, b that the wave velocity usually differs from the film surface velocity.

From the brief discussion above, it can be seen that our knowledge of the pressure drop and the interfacial shear stress in the gas stream of a wetted-wall column is very unsatisfactory at present. Since the pressure drop is an important quantity in more complicated two-phase flows, of which gas/film flow is the simplest case, this is particularly unfortunate, and a great deal of detailed experimental work is necessary on this topic.

H. Wall Shear Stress in the Liquid Film

Experimental measurements of the wall shear stress exerted by a falling liquid film have been reported for the cases of film flow outside a vertical tube (B14) and in a channel of variable slope (F7). In both cases the experimental results in the zone of smooth laminar flow were in agreement with the predictions of Eq. (16) or (20), and the friction factors calculated from the wall shear stresses are therefore given by Eq. (21).

In the wavy laminar regime, however, the experimental values of the film wall shear stress, and hence of the film friction factor, are appreciably greater than predicted by the theoretical equations mentioned above. The deviation between the experimental and theoretical values at a given value of N_{Re} increased with increasing slope of the channel (F7). The increase in the wall shear stress is due at least partly to the decrease in the mean film thickness in the wavy flow regime, which leads to a greater mean velocity and velocity gradient at the wall. In the turbulent zone of film flow, Brauer (B14) has shown that the wall shear stress is given (for vertical film flow) by

$$\tau_w = 0.0465(N_{\mathrm{Re}})^{2/5}$$

which corresponds to a friction factor of

$$f = 0.408/(N_{\mathrm{Re}})^{8/15}$$

Preliminary results (F7) show that, in the case of countercurrent gas/film flow, the film wall shear stresses decrease with increasing gas velocity, due to the increase in the mean film thickness (and hence decrease in the velocity gradient at the wall) at a given liquid flow rate.

As yet there appear to be no reports of wall shear stress measurements in liquid films with cocurrent gas streams.

V. Conclusions

In the preceding sections an attempt has been made to summarize the information available at present on a number of aspects of film flow.

In his review of research in the field of film condensation, Colburn (C12) summed up the situation in 1951 by saying, ". . . the direction of most promising research is in studying fluid flow characteristics of liquids in layers, with and without a superimposed gas velocity. The types of turbulence in layers needs to be investigated, and also the nature of a laminar layer containing ripples. . . ." It is interesting to note that a significant part of the work on film flow carried out since that time has been concerned in one way or another with the wavy nature of flowing films and with the interactions between films and adjacent gas streams. However, in spite of several important advances, the field of gas/film flow is so vast that the remarks above remain quite valid today.

In recent years there has been a considerable increase in the amount of information available on the more macroscopic aspects of film flow under various conditions, such as the film thicknesses, general wave patterns, over-all pressure drops in the gas streams of wetted-wall columns, surface velocities of the film, the various critical flow rates at which changes occur in the flow behavior of the film, and the like. However, the situation is rather less satisfactory as regards the finer features of such flows, particularly in the wavy flow regime. For example, much work remains to be done in the investigation of the flow patterns and velocity profiles inside films and, in particular, inside the waves at the film surface; the interfacial shear stress exerted by a gas stream at a wavy film surface and the related local gas stream pressure drop also urgently require closer study, and a number of other features of film flow which seem to merit more detailed investigation have been mentioned in the earlier sections.

As regards the theoretical studies of film flow, it has been shown that it is possible to predict quite accurately the flow behavior in the smooth laminar flow regime of the film; unfortunately, this flow regime is not of great practical importance. The Kapitsa theory for wavy film flow appears to apply over only a very limited part of the total wavy flow regime (flow with regular waves). Apart from this theory, there appears to be no general theory available for the wavy flow of thin films on steep surfaces, and the mathematical difficulties in the way of developing such a theory appear to be enormous. It seems probable that it will be necessary to use empirical relationships, based on a suitably wide mass of experimental results, for describing the flow behavior in the wavy flow regime, but it will be some time before sufficient results are available.

It is of interest to note that several new experimental techniques for the study of film flow have been developed in recent years, including improved methods for studying local film thicknesses and wave profiles (e.g., H9, L13, S11), a method for obtaining instantaneous and undisturbed velocity profiles in films (W4) and for obtaining wall shear stresses and local heat transfer coefficients in films (B14, W3), to mention but a few. In this way it has become possible within the last few years to measure many of the features of film flow which were previously beyond the reach of experimentation, while the speed and accuracy of other measurements have been greatly increased. In view of the expanding interest in film flow (see the Appendix), it can be expected that this intense experimental activity will continue, eventually providing sufficient information to enable many of the puzzling features of film flow to be explained, thus laying a firm foundation for the study of more complex two-phase flows.

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Nomenclature

$\mathfrak{A}$	Amplification of most unstable wave in traveling 10 cm.
b	Film thickness
b_i	Initial film thickness at $x = 0$
b_N	Film thickness given by Nusselt equation [Eq. (15)]
b_x	Value of variable film thickness at $x = x$
b^+	Dimensionless film thickness, Eq. (77)
$\bar{b}$	Mean film thickness
c	Phase velocity of waves
f	Function of; or friction factor for film flow
g	Acceleration of gravity
K	Constant
L	Length of column
n	Integer (1, 2, 3 . . .) in summations; or wave number, Eq. (40)
p	Pressure
p_σ	Capillary pressure, Eq. (49)
Δp	Pressure drop
Q	Volumetric flow rate per unit wetted perimeter
r	Radial coordinate
R	Tube radius
S	Interfacial area
ΔS	Increase in interfacial area due to waves
t	Time
T	Velocity profile shape factor, Eq. (95)
u	Velocity in x-direction
u_s	Surface velocity of film
u^+	Dimensionless velocity in x-direction, Eq. (75)
u^*	Friction velocity, Eq. (78)
$\bar{u}$	Mean velocity in film
v	Velocity in y-direction
v_g	Gas stream velocity
w	Velocity in z-direction; or half-width of wetted channel (Section III, B, 4)
W	Weight flow rate per wetted perimeter
x	Coordinate in direction of flow
y	Coordinate measured from wall across thickness of film
y^+	Dimensionless distance from wall, Eq. (76)
z	Coordinate across channel

Greek letters

α	Wave amplitude
δ'	Thickness of nonturbulent surface layer
Δ	Change in quantity
θ	Angle of channel bed, measured from horizontal
λ	Wavelength

μ	Dynamic viscosity
ν	Kinematic viscosity
ν'	Relative viscosity, defined by Eq. (103)
ρ	Density
ρ_c	Density of adjacent phase
σ	Surface tension
τ	Shear stress
τ_i	Interfacial shear stress
τ_w	Wall shear stress
ϕ	Quantity defined in Eq. (55)
Φ	Quantity defined in Eq. (63)
ψ	Pressure drop per unit length
Ω	Force potential of field

NAMED DIMENSIONLESS GROUPS

K_F	$= \mu^4 g/\rho\sigma^3$, physical properties group
N_{Fr}	Froude number, defined by Eq. (3)
N_{Re}	Reynolds number of film, defined by Eq. (1)
$N_{\mathrm{Re}_{\mathrm{crit}}}$	Critical Reynolds number for onset of turbulence
$N_{\mathrm{Re}_{\mathrm{gas}}}$	Reynolds number of gas stream
N_{Re_i}	Reynolds number at onset of instability
$N_{T'}$	Nusselt dimensionless film thickness parameter, defined by Eq. (97)
N_{We}	Weber number, defined by Eq. (2).

SUPERSCRIPT

$+$	Dimensionless quantity

SUBSCRIPTS

crit	Critical value of quantity
gas	Gas phase quantity
0	Value of quantity in absence of gas stream.

OTHER SYMBOLS

∇^2	Laplacian operator
$\|q\|$	Absolute value of quantity q

APPENDIX. BRIEF CHRONOLOGICAL RÉSUMÉ OF PAPERS ON FILM FLOW AND RELATED TOPICS

Authors and Date	Remarks
Hopf (H18), 1910	Film flow experiments (water, sugar solution) in channel 5.2 × 40 cm., slope $\frac{1}{2}-3\frac{1}{2}°$; $N_{\mathrm{Re}} = 150–600$. Observations of film thickness, surface velocity, wave formation, effects of wall roughness. Theory of film flow in rectangular channel in absence of surface tension forces.

Authors and Date	Remarks
Nusselt (N6), 1916	Theoretical treatment of flow and heat transfer in smooth laminar films, with and without interfacial drag. Inertia forces neglected.
Claassen (C10), 1918	Experimental studies of film flow of water, NaCl solutions, and molasses on vertical tubes. Measurements of film thickness, liquid adhering after draining, effects of roughness.
Schoklitsch (S3), 1920	Flow studies in channel at small slopes (below 2°) with water; N_{Re} = 22–45,000. Thicknesses, critical Reynolds number, onset of turbulence studied.
Nusselt (N7), 1923	Extension of earlier work (N6), and comparison with experiments of Claassen.
Jeffreys (J4), 1925	Water flow in channel 10.2 × 364 cm. at small slopes, large range of N_{Re}. Measurements of velocities, ratio of mean to maximum velocity, thicknesses; $N_{Re_{crit}} = 310$. Dye tracer experiments used to deduce thickness of laminar sublayer, eddy viscosity, friction factors. Theoretical work on bores, waves, instability of flow.
Chwang (C9), 1928	Study of thicknesses of water, oil films on plates at slopes up to 13°. N_{Re} = 0.9–110.
Cornish (C16), 1928	Theory of flow with free surface in rectangular channel of finite width, neglecting interfacial drag.
Warden (W1), 1930	Flow of water films inside tubes, N_{Re} = 69–1830. Film thicknesses reported.
Cooper and Willey (C15), 1934	Flow of films of dilute H_2SO_4 inside tube, diameter 1.13 cm. N_{Re} = 1.53–192. Film thicknesses reported.
Hatta (H3), Hatta and Katori (H4), 1934	Theoretical and experimental work on absorption of CO_2 by water film on channel 1.5 cm. × various lengths, slopes 1°–90°. Shows inapplicability of "two film" theory to most cases of gas absorption by liquid films.
Horton *et al.* (H19), 1934	Laminar water flow studied in channel, 14.3 × 86.4 cm. at slopes up to 2°. Measurements of thickness, surface velocity, ratio $u_s/\bar{u}$. Capillary edge effect noted. Onset of turbulence discussed.
Kirkbride (K17), 1934	Flow of water and 4 oils outside tubes, N_{Re} = 0.04–2000. Film thicknesses (maximum wave heights) measured by micrometer. Wavy flow is described, and corrections to Nusselt theory derived for heat transfer in laminar wavy film flow.

Authors and Date	Remarks
Cooper *et al.* (C15), 1934	Earlier film thickness data correlated in form of film friction factor plot.
Fallah *et al.* (F1), 1934	Flow of water films inside tubes, with second phase of air, white oil, stationary and countercurrent kerosine. Film thicknesses of this and previous work correlated by film friction factor plot.
Holmes (H16), 1936	Observations of traveling waves in steep channels.
Bays and McAdams (B2), 1937	An improved correlation for heat transfer in laminar film flow is presented.
Strang *et al.* (S13), 1937	Flow of water films (N_{Re} = 6–150) inside tubes with stationary second phase of tetralin, kerosine, and 4 oils of various densities and viscosities. Effect of buoyancy forces on film thickness determined.
Keulegan (K13), 1938	Extension of Prandtl-von Kármán turbulent flow theories to turbulent flow in open channels. Effects of wall roughness, channel shape, and free surface on velocity distribution are considered.
Verschoor (V3), 1938	Hold-up in wetted-wall columns.
Sexauer (S8), 1939	Experimental study of heat transfer to and through condensate films.
Keulegan and Patterson (K16), 1940	Determination of criterion for wave formation in turbulent flow in steep channels: $N_{Fr} > \frac{2}{3}$ or $N_{Fr} > 2$, depending on use of Manning or Chézy coefficients for resistance term.
Levich (L6), 1940	Theory of the damping of waves by insoluble surface-active materials.
McAdams *et al.* (M1), 1940	Correlation of heat transfer coefficients in falling-film heaters, N_{Re} = 225–15,000.
Thomas (T13), 1940	Theory of wave propagation in steep channels. Experimental work on wave profiles in channel in which the wetted wall was moved upwards to keep wave profile stationary. Surface tension effects neglected.
Vyazovov (V8, V9), 1940	Observations of flow and absorption of CO_2 by water film (laminar flow) on plate 9.2 × 110 cm., various slopes. Gas absorption by smooth laminar film dealt with theoretically, assuming linear and semiparabolic velocity profiles.
Friedman and Miller (F5), 1941	Flow of films of water, oil, toluene, and kerosine inside tubes, N_{Re} = 0.02–115. Measurements of thickness, surface velocity, onset of wavy flow.

Authors and Date	Remarks
Levich (L7), 1941	Theory of damping of waves by soluble surface-active materials: shows that damping coefficient passes through a maximum as concentration of surfactant increases.
Grigull (G8), 1942	Treatment of heat transfer in condensate film on vertical surface, assuming applicability of Prandtl pipe-flow relationships. Comparison with experimental data.
Treybal and Work (T16), 1942	Thicknesses of aqueous acetic acid films inside tubes with moving second phase of benzene. Wide range of N_{Re}.
Semenov (S6), 1944	Experimental thicknesses of water films inside tube 1.38 $\times$ 23 cm.; N_{Re} = 2–77. Theoretical treatment of smooth laminar flow with interfacial drag due to cocurrent or countercurrent gas flow, and onset of flooding.
Grimley (G10), 1945	Film flow in tubes and channels (water, water + surfactant), co- and counter-flow of air. Wave observations, onset of rippling, surface velocity, velocity distribution, film thicknesses, effects of surface tension and surfactants.
Vedernikov (V2), 1946	Theoretical treatment of wavy flow in open channels. Wavy flow and turbulent flow clearly distinguished.
Fradkov (F4), 1947	Theory of laminar film flow with interfacial drag. Experimental work on film flow of liquid air with counter-flow of air; conditions for onset of flooding determined.
Grimley (G11), 1947	Amplification of earlier work (G10), and experimental work on CO_2 absorption by water film. Photos of wavy films.
Kapitsa (K7, K8), 1948	Theoretical treatment of wavy flow of thin films of viscous liquids, including capillary effects. Only regular waves considered. Wavy flow shown to be more stable than smooth film, and about 7% thinner than smooth film at same flow rate. Also calculates wave amplitudes, wavelengths, etc., onset of wavy flow, effects of countercurrent gas stream, heat transfer. Theory applicable only if wavelength exceeds 14 film thicknesses. Error in treatment pointed out by Levich (L9).
Levich (L8), 1948	Considers theoretically mass transfer across liquid/fluid interfaces, with special treatment of gas absorption by turbulent liquid films.

Authors and Date	Remarks
Dressler (D9), 1949	Mathematical treatment of roll waves in inclined open channel, including effects of slope, resistance to flow, but neglecting surface tension effects.
Kapitsa and Kapitsa (K10), 1949	Wavy flow of water and alcohol films on outside of tube of diameter 2 cm., $N_{Re} < 100$, studied photographically and stroboscopically. Experimental data at low flow rates in agreement with Kapitsa theory; waves become random at large flow rates.
Semenov (S7), 1950	Extension of earlier work to wavy film flow. Kapitsa theory simplified by omitting inertia terms, and applied to wavy film flow with co- or counter-flow of gas to give thickness, velocity, wavelength, wave velocity, stability, onset of flooding, etc.
Ternovskaya and Belopol'skiĭ (T9, T10), 1950	Experimental study of absorption of SO_2 in water film with various surfactant additives. Rate of absorption decreased rapidly then increased slowly as concentration of surfactant increased (cf. Levich, L7); this effect is shown to be due to damping of waves.
Tsien (T17), 1950	Describes applications of film cooling.
Carpenter and Colburn (C3, C12), 1951	Review of research on film condensation; shows importance of gas stream effects, waves, transition to turbulence, etc.
Jackson *et al.* (J2), 1951	Experimental study of film flow inside tube with counter-flow of gas. Film surface velocity deduced from pressure drop readings in gas stream, neglecting wave roughness effects. Surface velocities appear to exceed the theoretical values in the wavy flow regime.
Kapitsa (K9), 1951	Deals theoretically with heat and mass transfer to periodic flows, e.g., to wavy liquid films.
Shibuya (S10), 1951	Mathematical treatment of onset of wavy flow in liquid films on vertical tubes. Waves appear for $N_{Re} > 7$. Wavelength of first waves $\cong 3$ film thicknesses.
Zhavoronkov *et al.* (Z2), 1951	Mass transfer studies (CO_2 into water film) in two diameters of wetted-wall column and on wetted-plate packing (liquid mixed at intervals). Gas velocity had little effect on transfer rates.
Calvert (C1), 1952	Theory for case of upward cocurrent gas/film flow in tubes. Numerous experimental results on film thicknesses, pressure drops, etc.
Craya (C17), 1952	Treatment of stability problem in open channel flow.

Authors and Date	Remarks
Dressler (D10), 1952	Treatment of stability and roll-wave formation in open channels, using general resistance formula.
Grigull (G9), 1952	Correlation and discussion of heat transfer results for film condensation.
Ishihara *et al.* (I1), 1952	Theoretical treatment of stability of laminar film flow: waves possible for $N_{Fr} > 0.58$. Experiments in channel 20 × 500 cm., slopes up to 15°. Measurements of wavelength, wave velocities, and frequencies, onset of rippling and turbulence. Tracer studies of turbulence in wave fronts.
Pennie and Belanger (P1), 1952	Film thickness measurements in film of 5% aqueous sodium carbonate solution flowing inside tube (diameter 1.28 cm.). $N_{Re} = 13$–3250.
Dukler and Bergelin (D16), 1952	Study of water films on vertical plate, 61.4 × 204 cm. $N_{Re} = 120$–750. Film thicknesses (local and mean) by capacitance proximity meter; wave profiles; photos of wave patterns. Theoretical treatment assuming applicability to film flow of pipe-flow universal velocity profiles.
Sherwood and Pigford (S9), 1952	Much information on absorption, distillation, and vaporization in wetted-wall columns. Details of Pigford's theoretical treatment of gas absorption by smooth laminar films.
Ternovskaya and Belopol'skiĭ (T11, T12), 1952	Extension of earlier work (T10) on effects of surfactants on rates of gas absorption by water films, giving explanation of mechanism.
Chew (C6), 1953	Film thicknesses of water films on plates of various slopes measured. Observations of entry region and wave patterns. Surface velocity measurements.
Dressler and Pohle (D11), 1953	Treatment of hydraulic instability of turbulent flow in open channels, using general resistance law.
Brötz (B21), 1954	Study of films of water, pentadecane, and a refrigerating oil inside tubes of various diameters; kinematic viscosities 1.0–8.48 cs.; $N_{Re} = 100$–4300. Measurements of film thicknesses, rate of absorption of CO_2. Effective diffusivity in turbulent zone determined from dye tracer studies. Relationship derived for turbulent film thicknesses.
Emmert and Pigford (E4), 1954	Experimental work on mass transfer to water films and comparison with theory. O_2 and CO_2 absorption and desorption studied; $N_{Re} = 2$–250. Observations of

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	waves, onset of rippling, effect of surfactants. Mass transfer agrees with theory only in absence of waves.
Kamei and Oishi (K2), 1954	Experimental determination of pressure drops in air stream flowing countercurrently to liquid films inside columns of 4.5 and 20.3 cm. diameter. Liquids included water, soap solutions, glycerol solutions, $N_{\mathrm{Re}} = 0.2$–250; $N_{\mathrm{Re}_{\mathrm{gas}}} = 4000$–$200{,}000$.
Kamei *et al.* (K5), 1954	Thicknesses of films flowing inside tubes of diameters 1.90–5.09 cm. measured with zero and cocurrent gas flows.
Kamei *et al.* (K6), 1954	Experimental determination of flooding points in wetted-wall columns of i.d. 1.89–4.91 cm., using various liquids and air counter-flow.
Knuth (K18), 1954	Experimental study of film cooling; conditions for film attachment to wall, entrainment, instability.
Laird (L3), 1954	Experimental study of pressure drop in gas stream in tubes with sine-wave oscillations of tube wall. Shows that large pressure drop is partly due to change in shape of gas velocity profiles.
Owen (O1), 1954	Derives equations for flow in open rectangular channel of finite width; considers transition to turbulence.
Yih (Y1), 1954	Mathematical treatment of stability of laminar flow with free surface (neglecting surface tension). Numerical calculations for film on vertical surface give $N_{\mathrm{Re}_i} \cong 1.5$.
Calvert and Williams (C2), 1955	Presentation and discussion of work by Calvert (C1) on velocity distribution, flow rate, pressure drop, shear stresses, profile drag, etc., for upward cocurrent film/gas flow.
Garwin and Kelly (G1), 1955	Study of heat transfer between water film and heated plate at various inclinations.
Jackson (J1), 1955	Flow of films of ethyl acetate, methanol, water, water + surfactant, 2-propanol, glycerol solutions (with and without surfactant), inside tube of 3.6 cm. diameter. Film thicknesses by radioisotope tracer method; heights of waves measured. Surface tension had little effect.
Kamei and Oishi (K3), 1955	Experimental measurements of absorption of CO_2 by water film inside tubes, 4.76 $\times$ 250 cm., with zero and countercurrent gas flow. Large range of N_{Re}.
Lynn *et al.* (L15, L16, L17), 1955	Experimental work on absorption of SO_2 in water films, (1) in long wetted-wall column (rod, film outside); (2)

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	in very short wetted-wall columns (no rippling); (3) flowing over spheres. Entry and end effects studied, also effects of adding surfactant.
Malyusov *et al.* (M6), 1955	Study of distillation in wetted-wall columns, taking into account the effects of laminar and turbulent flow of vapor phase.
Stirba and Hurt (S12), 1955	Experimental work on CO_2 absorption by water films in vertical tubes of length 3 and 6 ft., and dissolution of tubes of solid organic acids by water films. Effective diffusivity exceeds molecular diffusivity, even at $N_{Re} = 300$. Dye streak experiments show that waves cause mixing; surfactants damp waves to give continuous dye streak and mass transfer results in agreement with theory.
Alimov (A2), 1956	Considers stable form of liquid film condensing on hot horizontal cylindrical surface, and shows that stable annular wave patterns are formed for certain flow rates and temperature differences.
Brauer (B14), 1956	Extensive experimental study of film flow outside tube 4.3×130 cm.; films of water, water + surfactant, aqueous diethylene glycol solutions, kinematic viscosity 0.9–12.7 cs.; $N_{Re} = 20$–1800. Data on film thicknesses, waves, maximum and minimum thicknesses, characteristic Reynolds numbers of flow, onset of rippling and turbulence, wall shear stress, etc.
Greenberg (G7), 1956	Flow of films (5 liquids) down tube 2.5×30 in. Data on film thickness, wave velocity, frequency, amplitude; analysis of roll waves.
Kamei and Oishi (K4), 1956	Flow of films of water, soap solution, millet-jelly solutions inside tubes of diameter 5.09–1.9 cm. $\times$ 100 cm., with zero and countercurrent air flow. Kinematic viscosities 1.1–40 cs.; $N_{Re} = 1$–4200. Surface tension found to affect holdup.
Kramers and Kreyger (K24), 1956	Experimental and theoretical study of mass transfer between a soluble wall surface and film flowing on it. Experiments carried out at low N_{Re} on inclined plane surface.
Labuntsov (L1), 1956	Heat transfer to falling films (laminar flow): effects of convective heat transfer and inertia forces (neglected in Nusselt theory) considered experimentally and theoretically.

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Mazyukevich (M8), 1956	Experimental study of entrainment of gas by the surface of a film (water, ethylene glycol) flowing inside a tube.
Vivian and Peaceman (V5), 1956	Experimental mass transfer work in short wetted-wall columns (1.9–4.3 cm. long); ripples absent at most flow rates. Rate of desorption of CO_2 independent of gas velocity up to $N_{\mathrm{Re}_{\mathrm{gas}}} = 2200$. Width and type of liquid inlet slot had little effect. Acceleration of film important.
Benjamin (B5), 1957	Theoretical study of wave formation in laminar flow down inclined plane, taking surface tension into account. Films on vertical surfaces shown to be always theoretically unstable. Simplified treatment for long waves presented.
Binnie (B9), 1957	Determination of onset of rippling in water film flowing outside vertical tube: $N_{\mathrm{Re}_i} \cong 4.4$.
Brauer (B15), 1957	Application of results on flow of films (B14) to case of heat transfer in film condensation.
Davidson and Cullen (D1), 1957	Consideration of the flow mechanics and diffusion in a liquid film flowing over a sphere.
Davidson and Howkins (D3), 1957	Calculations of shapes of standing waves formed on surface of film flow in the presence of an accelerating gas stream, and experimental results on same.
Feldman (F3), 1957	Mathematical consideration of stability of a liquid film on a wall with a cocurrent gas stream above it.
Konobeev *et al.* (K19), 1957	Theoretical and experimental studies of pressure drops and film thicknesses in upward cocurrent flow of gas and film in vertical tubes (1.35 × 122 cm.).
Konobeev *et al.* (K20), 1957	Study of CO_2 absorption in water films in upward and downward cocurrent flow; $N_{\mathrm{Re}} = 5$–200, gas velocities 11.6–39 m./sec. It is suggested that only wavelength and amplitude of interfacial waves affect the rate of mass transfer. Values of wavelength and amplitude measured and compared with previous theories.
Labuntsov (L2), 1957	Heat transfer to condensate films on vertical and horizontal surfaces. In laminar region, Nusselt equations are corrected for (a) inertia effects, (b) variation of physical properties with temperature, (c) effects of waves. In turbulent region various "universal" velocity profiles are used. Results compared with experimental data.
Michalik (M9), 1957	Study of hydraulics of laminar film flow of Newtonian fluid in tubes and on vertical plates. Optimum design parameters for wetted-wall columns derived.

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Shirotsuka *et al.* (S11), 1957	Experiments on film flow in vertical channel, 8 cm. wide; N_{Re} = 100–1500; zero or countercurrent air flow. Data on local film thicknesses, wave heights, wave frequencies, increase in surface area due to rippling, with and without air flow.
Brauer (B17), 1958	Application of results on film flow (B14) to case of heat flow in filmwise condensation of pure vapors on vertical walls. Nomograms for practical use.
Brauer (B16), 1958	Same as previous citation, but application to mass transfer in liquid films; comparison with previous experimental work.
Bressler (B19), 1958	Review of research on evaporation from thin liquid films.
Clayton (C11), 1958	Experimental studies of flow of films of water, aqueous glycol solutions, inside tube, 1.27 cm. diameter; N_{Re} = 0.007–2100; zero or countercurrent air flow. Flooding conditions investigated; velocity profiles obtained within wavy film by chronophotographic method.
Collins (C14), 1958	Film thicknesses and CO_2 absorption by water film in cocurrent flow with gas stream in vertical tube, 2.05 $\times$ 36 in. Thicknesses by light-absorption technique.
Gay (G2), 1958	Experimental study of interfacial drag between liquid surface (nearly horizontal) and air stream flowing over it (co- or counter-flow). Distortion of gas stream profiles by rough liquid surface; determination of effective surface roughness.
Howkins and Davidson (H20), 1958	Investigation of stability of liquid film flowing over spheres in an upward airstream. Effects of surfactants studied.
Kutateladze and Styrikovich (K25), 1958	Sections deal with film flow in presence of zero, countercurrent, upward and downward cocurrent gas streams, including turbulent film flow.
Mahrenholtz (M5), 1958	Experimental and theoretical study of upward cocurrent gas/film flow. Data on mean film thicknesses, range of stability, efficiency of conveying liquid up wall in film, effects of physical properties of liquids.
Nikolaev (N2), 1958	Consideration of distillation in film columns. Includes observations of onset of flooding with counterflow of gas.
Poocza (P2), 1958	Theoretical treatment of binary thin-film distillation.
Scriven and Pigford (S5), 1958	Discussion of effect of acceleration of liquid film near inlet on gas absorption into film. See Lynn, 1960.

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Thomas and Portalski (T14), 1958	Experimental study of water film flowing inside tube 1.96 × 98 cm., N_{Re} = 141–493, counterflow of air. Data on film thicknesses, pressure drop, wave characteristics.
Belkin *et al.* (B4), 1959	Experimental studies of water films flowing outside tubes; observations of film thicknesses, onset of rippling. N_{Re} = 50–7500 (mostly turbulent zone).
Binnie (B10), 1959	Experimental studies of water films in channel 8.4 × 480 cm., small slopes, N_{Re} up to 2500. Data on onset of rippling and turbulence, capillary edge effect. Comparison with Benjamin stability theory (B5).
Charvonia (C4), 1959	Experimental studies of downward cocurrent flow of air and water films in vertical tubes, $2\frac{1}{2}$ × 30 in.; N_{Re} = 4–445. Data on local film thicknesses, pressure drops; analysis of amplitude and frequency spectra of surface waves.
Davidson *et al.* (D2), 1959	Study of film flow over spheres, and absorption of CO_2 by water films. Effects of adding surfactants on mixing between spheres noted.
Dukler (D12), 1959	Theoretical analysis of turbulent film flow (with and without downward cocurrent gas stream) with extension to film heat transfer. Interfacial disturbances are neglected; basic equations are solved by computer giving film thicknesses, velocity profiles, local and mean heat transfer coefficients. Interfacial shear is shown to be of great importance.
Ellis and Gay (E3), 1959	Presentation and discussion of results of Gay (G2).
Hikita (H12), 1959	Experimental study of effects of rippling on rate of absorption of CO_2 by water films containing surfactants (0.0005–0.05 wt.%), with film flow inside tubes 1.3 cm. × 15–101 cm. Results approach Emmert and Pigford theory (E4) as rippling is damped by surfactants.
Hikita and Nakanishi (H13), 1959	Film flow over spheres (1.42–4.12 cm. diameter) singly and in vertical groups of up to 5, spaced at 5-mm. intervals. CO_2 absorbed in water films with and without surfactant additives, N_{Re} = 1–200. With pure water results for multiple spheres agree with assumption of complete mixing between spheres; deviations with surfactants.
Hikita *et al.* (H14), 1959	Study of rate of dissolution of wall of vertical steel pipe by film of 0.01 N sulfuric acid flowing on inside wall. Comparison with theory.

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Hikita *et al.* (H15), 1959	Experimental studies of absorption of CO_2, H_2, Cl_2, H_2S, SO_2 by falling films of water, sugar solutions, butanol/methanol, and methanol/benzene solutions in tubes of 0.7–4.35 cm. i.d., 20–103 cm. long. $N_{Re} = 7.5$–500, $N_{Sc} = 73$–2600. No effect of gas rate noted up to $N_{Re_{gas}} = 7000$. Results correlated empirically.
Levich (L9), 1959	Final chapter deals with film flow theory (smooth, wavy laminar, turbulent) with and without gas flow. Also considers mass transfer to such films. Correction to theory of Kapitsa (K7).
van Rossum (V1), 1959	Experimental studies in horizontal channel, 15 $\times$ 310 cm., using water, kerosine, various oils, $N_{Re} = 0.1$–5000, cocurrent air flow. Data on local film thicknesses, onset of rippling and entrainment, effects of roughness and surfactants.
Anderson and Mantzouranis (A5), 1960	Experimental and theoretical study of hold-up and film thicknesses for upward cocurrent gas/film flow in vertical tubes. Theory based on use of universal velocity profile. Numerous experimental data on friction factors, etc.
Bird *et al.* (B11), 1960	Brief theoretical treatments of various cases of film flow (on cone, with variable viscosity, non-Newtonian liquids, etc.) and of heat and mass transfer to films.
Brauer (B18), 1960	Treatment of film flow in vertical tubes with gas streams (countercurrent, cocurrent up and down). Pressure drops and gas stream friction factors are calculated for the various cases. Fully developed gas flow and laminar film flow (smooth) are assumed throughout.
Bressler (B20), 1960	Experimental studies of film flow and heat transfer to films, work is extended to deal with swept-film evaporators.
Davies (D4), 1960	Considers, *inter alia*, dye-tracer experiments in wetted-wall columns.
Dukler (D13), 1960	Discussion and extension of earlier paper (D12).
Feind (F2), 1960	Experimental studies of film flow (water, aqueous glycol solutions) with countercurrent air flow in vertical tubes (2.0–5.0 cm. diameter). Data on mean film thicknesses, local heat transfer coefficients, pressure drop, wave heights, onset of flooding, gas/film interactions.
Fulford (F6), 1960	Brief review of heat and mass transfer to falling liquid films.

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Graebel (G6), 1960	Theoretical treatment of stability of countercurrent film/gas flow.
Heartinger (H5), 1960	Study of gas absorption in wavy liquid film; a simplified model is given and solved by computer to give rates of transfer under various conditions.
Lynn (L14), 1960	Theoretical consideration of acceleration of film near inlet slots of various types. Model studies suggested that acceleration should be complete in a very short distance.
Miles (M10), 1960	Considers stability problem of thin liquid film (linear velocity profile) bounded by a solid wall and a cocurrent gas stream.
Norman and Binns (N3), 1960	Experimental determination of minimum flow rates of liquid to ensure wetting in wetted-wall columns, and effects of surface tension on same.
Norman and McIntyre (N4), 1960	Investigation of effect of surface tension changes caused by heat transfer on minimum flow rates required to ensure wetting of wetted-wall columns.
Portalski (P3), 1960	Extensive study of film flow on vertical plates, with and without gas flow. Liquids included water, aqueous glycerol solutions, methanol. Data on effects of surface tension changes and surfactants, wave and surface velocities, increase in interfacial area by waves, etc.
Tailby and Portalski (T2), 1960	Reports on extension of Kapitsa theory (K7) to give increase in interfacial area due to waves; experimental measurements of N_{Re} for wave inception, entry length, and increase in interfacial area.
Zaitsev (Z1), 1960	Theoretical treatment of stability of thin film of viscous liquid (with surface tension) on wall in the presence of a cocurrent gas stream.
Adorni *et al.* (A1), 1961	Experimental studies of upward cocurrent flow of argon and water film in tube 0.987 in. × 3.3 ft.; gas velocities up to 86 ft./sec. Data on film thicknesses, entrance characteristics, droplets entrained in gas stream.
Anderson *et al.* (A4), 1961	Theoretical and experimental study of heat transfer to liquid films in vertical long-tube evaporators. Theory based on use of universal velocity profile.
Asbjørnsen (A6), 1961	Residence times in falling water films determined by a pulsed tracer technique. Mean residence time 2–7% greater than calculated from laminar film flow theory.

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	Minimum residence time in agreement with Brauer's (B14) effective surface velocity results up to $N_{Re} = 400$. Distribution function agrees with theory in presence of surfactants.
Beda (B3), 1961	Theoretical treatment of film flow of liquid permeating through porous wall, with heat transfer.
Benjamin (B6), 1961	Extension of earlier work (B5) to the case of three-dimensional disturbances on surface of film.
Bennett and Thornton (B8), 1961	Experimental work on annular film/gas flow in vertical tubes. Data on film thicknesses and pressure drops along the wetted tubes.
Black (B12), 1961	Discussion of various methods of measuring local film thicknesses.
Chien (C7), 1961	Investigation of liquid film structure and pressure drops in vertical downward cocurrent gas/film flow. Data on surface waves, entry length, energy dissipation in film, film thicknesses (local and mean), pressure drop.
Collier and Hewitt (C13), 1961	Experimental studies of film thickness, pressure drop, and entrainment in upward, cocurrent gas/film flow, using various liquids. Results compared with earlier theories.
Dukler (D14), 1961	Practical application of the theoretical calculations of film heat transfer coefficients (Dukler, D12, D13).
Dukler (D15), 1961	Comparison of calculated values of the film thickness (Dukler, D12, D13) with published experimental results.
Escoffier (E5), 1961	Analysis of onset of instability in open channel flow and origin of waves of instability. Discussion of earlier instability criteria.
Glaser (G4), 1961	Studies of heat transfer between a vertical tube and a liquid film flowing on it.
Hanratty and Hershman (H2) 1961	Jeffreys' theory (J4) for roll-wave transition on a liquid surface is applied to the cocurrent flow of gas and liquid at small slopes. Experimental data on the initiation and growth of waves on various liquids with and without surfactant additives.
Hewitt (H7), 1961	Extension of Dukler treatment (D12, D13) to case of upward cocurrent flow of gas and liquid film, with heat transfer. Computer solutions presented.

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Hewitt *et al.* (H8), 1961	Experimental study of upward cocurrent flow of air (200–500 lb./hr.) and water film (20–1000 lb./hr.) in $1\frac{1}{4}$-in. vertical tube. Results compared with theory by Hewitt (H7).
Ishihara *et al.* (I2), 1961	Gives summary of recent Japanese work on wavy flow in open channels, and semitheoretical analysis of problem (wave velocities, frequencies, heights, lengths). Mostly small channel slopes considered.
Jaymond (J3), 1961	Experimental absorption of CO_2 by laminar film of NaOH solution flowing on plate, 4 $\times$ 100 cm., slopes 1–15°, countercurrent gas flow. Film thicknesses and surface velocities also measured. Good agreement with theory in absence of waves.
Kaiser (K1), 1961	Distillation in wetted-wall column. Film theory corrected to allow for oscillations of film surface which were observed.
Konobeev *et al.* (K21), 1961	Experimental study of CO_2 absorption by water film, with upward and downward cocurrent gas/film flow, inside tubes 1.05–1.66 cm. i.d., 20–87 cm. long. Gas velocities 6–86 m./sec. $N_{Re} = 5$–105. Length and amplitude of surface ripples and local film thicknesses measured. Rate of mass transfer stated to be function of wave characteristics only.
Lilleleht and Hanratty (L12, L13), 1961	Local film thicknesses (wave profiles) measured in horizontal channel with cocurrent gas stream and interpreted statistically. Effect of interfacial roughness in increasing the interfacial stress also investigated.
Mayer (M7), 1961	Experimental and theoretical study of wavy flow of water in open channel (slopes up to 5°). Data on growth of turbulent spots, local depths, surface velocity, length of entry zone, wave velocities, heights, frequencies, effect of surface-active materials.
Mirev *et al.* (M11), 1961	Experimental studies of rates of absorption of C_2H_2, SO_2 in water films in wetted-wall columns. Experimental results not in agreement with Vyazovov (V8) and penetration theories. Surfactant reduced rippling but appeared to increase interfacial resistance to mass transfer.
Ratcliff and Reid (R2), 1961	Equations set up for flow of liquid film (water) over sphere in presence of second liquid (benzene). Drag taken into account, surface tension neglected. Experimental data on film thicknesses and mass transfer.

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Ratcliff and Holdcroft (R1), 1961	Mass transfer into liquid film flowing on sphere with first-order chemical reaction (theory and experiment).
Reinius (R4), 1961	Studies of water flows in open channels at small slopes, $N_{Re} = 50\text{–}13{,}000$. Data on film thicknesses, film friction factors, effects of wall roughness.
Tailby and Portalski (T3), 1961	Experimental work on the damping of waves on water films flowing on vertical plate 21×84 in. by surface-active materials. Three surfactants used at various concentrations; optimum concentration for damping was observed in each case.
Taylor and Kennedy (T8), 1961	Discussion of onset of wave formation and wave behavior in open-channel flow.
Vouyoucalos (V7), 1961	Experimental study of absorption and desorption of SO_2 by water film in channel of 10×40 cm., slope 25°. Channel floor covered with regular transverse rugosities of large dimensions. Also dye-tracer studies of flow patterns, wave profiles.
Wilke (W2), 1961	Heat transfer to falling films. More detail in later paper (W3).
Zhivaĭkin and Volgin (Z4), 1961	Review of film flow literature and experimental work on film thicknesses in tube 2×98 cm., $N_{Re} = 150\text{–}3500$, and surface velocities.
Zucrow and Sellars (Z6), 1961	Experimental study of film cooling of rocket motors. Liquids of various N_{Pr} used, with and without chemical reaction.
Allen (A3), 1962	Investigation of characteristics of liquid films on vertical surface, with emphasis on surface features. Kapitsa theory shown to be applicable only at low flow rates. Increase in interfacial area reported to be smaller than predicted by Portalski theory.
Boyarchuk and Planovskiĭ (B13), 1962	Study of the kinetics of mass transfer in film-type distillation equipment.
Fulford (F7), 1962	Experimental study of countercurrent flow of water film and air stream in channel ($5\frac{1}{2} \times 30$ in.) at slopes 7–90°.
Hewitt *et al.* (H9), 1962	Description of techniques for measuring properties of liquid films and pressure drops in vertical upward co-current flow of gas and film. Numerous data reported for air/water system in $1\frac{1}{4}$-in. i.d. tube.
Hewitt and Lovegrove (H11), 1962	Data and techniques for continuous film thickness recording in vertical gas/film flow.

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Kasimov and Zigmund (K11), 1962	Theoretical treatment of smooth laminar film flow on vertical surface, with and without gas flow, including inertia effects. Nusselt equations (N6, N7) are shown to be special cases of the present solutions.
Konobeev and Zhavoronkov (K22), 1962	Theoretical and experimental studies of pressure drops in gas streams flowing in tubes with wavy walls (long-wave and short-wave roughnesses). Importance to case of flow of gas adjacent to wavy film pointed out.
Laird *et al.* (L4), 1962	Deals with laminar flow and transition to turbulence in flow along tubes with wavy walls of various wavelengths. Entry effects also studied.
Ratcliff and Reid (R3), 1962	Deals with smooth and rippling flow of a liquid over a sphere or series of spheres, especially with the problem of mixing in the liquid film at the junctions of spheres.
Saveanu *et al.* (S1), 1962	Determination of $N_{Re_{crit}}$ for film flow is made from measurements of absorption of CO_2 in water film. Effects of wall roughness on $N_{Re_{crit}}$ were also studied.
Saveanu and Tudose (S1a), 1962	Study of onset of flooding in tubes 8–15.8 mm. diameter, 1200 mm. long with downward water films, counter-current air stream of 0.5–12 m. sec. Gas velocity to cause flooding found to depend on liquid flow rate and tube diameter.
Tailby and Portalski (T4), 1962	Experimental determination of wavelengths near point of onset of rippling on films of various liquids flowing on a vertical wall.
Tailby and Portalski (T5), 1962	Experimental determination of the distance of the line of wave inception on a vertical liquid film from the inlet as a function of liquid flow rate, viscosity, and co- and counter-flows of air.
Taylor and Hewitt (T6), 1962	Considers the motion and frequency of large "disturbance" waves in upward cocurrent flow of air and water films.
Wilke (W3), 1962	Extensive survey of flow and heat transfer in liquid films flowing outside tubes. Measurements of temperature and velocity profiles in films of various liquids are reported, and a heat transfer mechanism is proposed.
Wilkes and Nedderman (W4), 1962	Experimental determination of velocity profiles in films flowing on vertical tube by stereoscopic chronophotographic method. Films included glycerol, aqueous glycerol solutions, liquid paraffin, glycerol + surfactant. In smooth flow, profiles agreed with theoretical semi-

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	parabola; with waves, profiles scattered about semi-parabola. Entry effects also studied.
Zhivaĭkin (Z3), 1962	Experimental work on film flow and upward and downward cocurrent gas/film flow (water and aqueous glycerol solutions). Data on onset of flooding and entrainment, effect of gas velocity.
Adorni *et al.* (A1a), 1963	An instrument is described for measuring the wall shear stress in a two-phase flow in the presence of a pressure gradient and a changing velocity profile.
Andreev (A5a), 1963	Considers stability of laminar flow of viscous incompressible liquid film on vertical wall with respect to infinitesimal disturbances. Absolute instability is not found.
Atkinson and Taylor (A7), 1963	Study of the effect of discontinuities on mass transfer to a liquid film. It is concluded that the mixing effect at discontinuities is a result of ripple action.
Dukler and Wicks (D17), 1963	General review of gas/liquid flow in conduits, including brief consideration of gas/film flows.
Gill *et al.* (G3), 1963	Experimental study of upward cocurrent flow of air/water system. Data on pressure drop and film thicknesses (and effects of liquid droplet entrainment) as functions of distance from inlet. Effects of waves on film surface considered.
Hewitt *et al.* (H10), 1963	Numerous data on pressure drops and liquid holdup in vertical upward cocurrent flow of air and water films, and comparisons with published theories.
Hewitt and Lovegrove (H11a), 1963	Measurements are reported of pressure gradient, holdup, film thicknesses in vertical, upward, cocurrent air/water streams in a tube of 1.25 in. diameter. Film thicknesses by three measurement methods are compared.
Kasimov and Zigmund (K12), 1963	General solution is given to equations of wavy film flow (with corrected continuity equation), using simple parabolic or higher approximation to velocity profile at given plane. It is shown that the film thickness and wave amplitude should increase in direction of flow. Effect of channel slope also considered.
Nedderman and Shearer (N1a), 1963	Deals with the motion and frequency of large disturbance waves in upward cocurrent flow of air and water film, extending work of Taylor and Hewitt (T6).

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Niebergall (N1), 1963	Deals with the effects of changes of surface tension during heat and mass transfer in wetted-wall equipment.
Norman and Sammak (N5), 1963	Deals with effects of mixing on rates of mass transfer to a liquid film flowing over packing elements.
Portalski (P4), 1963	Theories of film flow and methods of measuring film thickness are reviewed. Film thicknesses on vertical plate (zero gas flow) reported for glycerol solutions, methanol, isopropanol, water, and aqueous solutions of surfactants. Results compared with values calculated by Nusselt, Kapitsa, and corrected Dukler and Bergelin treatments.
Scott (S4), 1963	General survey of cocurrent gas/liquid flows, including gas/film flows.
Tadaki and Maeda (T1), 1963	Experimental study of absorption of CO_2 in downward cocurrent gas/film flow, and comparison with various theories.
Taylor *et al.* (T7), 1963	Study of large "disturbance" waves in upward cocurrent flow of air and water film in vertical tubes. A "preferred" wave velocity is reported.
Yih (Y2), 1963	A general consideration (theoretical) of the stability of film flow down an inclined plane, including the cases of long and short waves and small N_{Re}. Results for long waves are in agreement with theory of Benjamin (B5). The effects of surface tension and viscosity on stability are discussed.
Portalski (P5), 1964	From Kapitsa's theory of wavy film flow, it is shown that regions of reversed flow exist under the wave troughs, leading to the generation of circulating eddies which may explain the increased rates of heat and mass transfer to wavy laminar films.
Zhivaĭkin and Volgin (Z4a), 1964	Considers pressure drop in downward cocurrent flow of air and films of water, glycerol solutions, water + surfactant, in tubes of diameter 12.9 mm., lengths 150–830 mm. Air velocities 3–45 m./sec. Conditions for droplet entrainment from film also reported.

References

A1. Adorni, N., Casagrande, I., Cravarolo, L., Hassid, A., and Silvestri, M., "Experimental Data on Two-phase Adiabatic Flow: Liquid Film Thickness, Phase and Velocity Distribution, Pressure Drops in Vertical Gas-Liquid Flow". Centro Informazioni Studi Esperienze, Milan, Report R-35, March 1961.

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A6. Asbjørnsen, O. A., *Chem. Eng. Sci.* **14**, 211 (1961).

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