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Studies of falling liquid film flow

Film thickness on a smooth vertical plate

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Abstract—Investigating the film characteristics of thirteen liquids the author took some 450 hold-up measurements using a vertical column with a smooth plate. The average value of the rippled film thickness is less than the corresponding value with no ripples, i.e. when suitable amounts of surface active agents are used. The film thickness data are an excellent indication of the mode of flow of the falling film.

In all the cases considered KAPITSA's theory approximates to the experimental results better than NUSSELT's theory in the region for which these theories were intended. On the other hand both these theories compare unfavourably with the universal velocity profile treatment as originally proposed by VON KÁRMÁN and subsequently extended by DUKLER and BERGELIN. This is so far the best available approximation to the facts and the assumption of the original universal velocity profile based on full-pipe flow leads to the prediction of the average liquid film thickness in fair agreement with those experimentally determined. Nevertheless, systematic deviations still exist in consequence of the fact that the original velocity profile equations were proposed for full-pipe flow and are in need of modification in the light of the experimental data in falling film thickness.

INTRODUCTION

THE mechanics of falling liquid films are closely connected with the phenomena of wave-motion and ripples appearing quite naturally on liquid films at practically all flow rates. In the past few years there have been a number of attempts to investigate liquid film conditions of the wavy flow configuration both from the practical and theoretical points of approach [1–10]. A comparatively recent summary of the work published on falling liquid films is given in Ref. 35 [11–40].

In most cases the efforts of the investigators have been directed towards measuring the film thickness and this fact stresses sufficiently the importance of knowing the actual thickness of flowing liquid films.

Theory

Thickness is the most obvious characteristic of the falling film and in the past several different approaches have been tried in estimating this property. NUSSELT [41] presented the first important work on film flow and established the following equations:

the velocity at the surface,

$$V_s = \frac{gm^2\rho}{2\mu} \tag{1}$$

the mean velocity of the film,

$$V_{av} = \frac{gm^2\rho}{3\mu} \tag{2}$$

therefore,

$$\frac{V_s}{V_{av}} = \frac{3}{2} \tag{3}$$

also the mean film thickness,

$$m = \left[\frac{3\nu Q}{g}\right]^{1/3} \tag{4}$$

Later JEFFREYS [42, 43] analysed the case of the inclined plane, showing that the above relations held, provided that the gravity term is '$g \sin\theta$' (where θ is the inclination to the horizontal) instead of 'g'.

FALLAH *et al.* [19] have developed similar expressions and their equation for the mean film thickness is

$$m = \left\{ \frac{12\mu Q[1 - (V_s/2V_{av})]}{g(\rho_L - \rho_c)} \right\}^{1/3} \tag{5}$$

which reduces to equation (4) when the interfacial shear stress is zero. In cases where the core-fluid is a gas the term $(\rho_L - \rho_c)$ is nearly equal to ρ_L.

JACKSON [6] redeveloped these equations for liquid flowing down a circular tube. His equations are

$$V_s = \frac{\rho g}{4\mu} \left[R^2 - {}^2 + 2r^2 \ln \frac{r}{R} \right] \tag{6}$$

and

$$V_{av} = \frac{\rho g}{8\mu} \left[R^2 - 3r^2 - \frac{4r^4}{R^2 - r^2} \ln \frac{r}{R} \right] \tag{7}$$

so that

$$\frac{V_s}{V_{av}} = 2 \left[\frac{R^2 - r^2 + 2r^2 \ln r/R}{R^2 - 3r^2 - 4r^4/[R^2 - r^2] \ln r/R} \right] \tag{8}$$

The weakness of all the above equations is the fact that they were developed with the assumptions of perfect viscous flow and no wave-motion at the liquid surface. However, early work has shown that the liquid may assume a wave motion at extremely low flow rates well within the viscous flow region [20, 21]. There is comparatively little theoretical work on the problem of the inception of liquid surface instability in general, and of wave-motion on vertical liquid films in particular. A recent survey of postulated theories has been presented by URSELL [44]. The analysis of KELVIN [45] based on frictionless fluid streams and the "sheltering" theory of JEFFREYS [46] have to be discarded as they apply only to "deep water" conditions. Recently analyses of the problem based on the Orr–Sommerfeld equation [47] have been presented in the literature [8-10]. According to KAPITSA's theory [1, 2] of the wavy film flow, the mean film thickness,

$$a_0 = \left[\frac{2{\cdot}4\nu Q}{g} \right]^{1/3} \tag{9}$$

and the free surface function of the flowing film,

$$\phi = \alpha \sin n(x - kt) + 0{\cdot}28\alpha^2 \cos 2n(x - kt) - - \frac{g}{4a_0\,\delta n^3} \alpha^2 \sin 2n(x - kt) + \ldots \tag{10}$$

where

$$n = 2\pi/\lambda \tag{11}$$

This means, of course, that equation (9) is valid for Re > Re_i only.

The amplitude of the wave motion, α, and the value of the ratio of the phase velocity to the mean stream velocity, Z, may also be determined, [1, 35] i.e. at first approximation:

$$\alpha = 0{\cdot}46; \quad Z = 2{\cdot}4 \tag{12}$$

In addition, the value of λ is given by

$$\lambda = 7{\cdot}5 \left(\frac{\nu\delta}{Qg} \right)^{1/2} \tag{13}$$

provided that

$$\frac{\lambda}{a_0} > 13{\cdot}7 \tag{14}$$

One has to realize that the flow behaviour of liquid films on vertical surfaces is much more complicated than the flow of liquids in full pipes or channels. In the latter case there are certain more or less well defined regimes of flow, the laminar, turbulent and transitional zones. In film flow these zones are no longer so well defined and additional complications arise due to the wave motion or the presence of ripples on the surface of the liquid at quite low Reynolds numbers. For two–phase film flow the characteristic dimension of length in the equation for the Reynolds number is taken as the average film thickness, m; similarly the velocity used is the mean stream velocity, V_{av}, so that one can write

$$\text{Re}, = \frac{V_{av} m}{\nu} = \frac{Q}{\nu} \tag{15}$$

and, indeed, some authors [7, 8] use the above expression.

But usually one takes as the criterion the Reynolds number defined as follows:

$$\text{Re} = \frac{4Q\rho}{\mu} = \frac{4Q}{\nu} \tag{16}$$

Some investigators in the field of falling liquid films have regarded the Froude number as a very appropriate dimensionless group for correlating their experimental results [7, 6, 34]. JACKSON [6] used the Froude number

$$\mathrm{Fr}' = \frac{V}{(gm)^{1/2}} \tag{17}$$

as a criterion for wave inception on liquid films.

BELKIN *et al.* [34] accepting JACKSON's definition of the Froude number, have developed an equation expressing a dimensionless film thickness parameter as a function of the Reynolds and Froude numbers, thus

$$mg^{1/3}\nu^{-2/3} = 0{\cdot}397[\mathrm{Re}/\mathrm{Fr}']^{2/3} \tag{18}$$

BRAUER [7] has used another dimensionless criterion including the Weber group together with both the Froude and Reynolds groups, i.e. the reduced Weber number, We/Re.Fr, for correlation of film turbulence inception data. He has deduced that the critical Reynolds number of turbulence inception, Re_t, is given by

$$\mathrm{Re}_t = 9\left[\frac{\mathrm{Re}\,.\,\mathrm{Fr}}{\mathrm{We}}\right]^3 .\, K_F^{-1} \tag{19}$$

where K_F is yet another dimensionless group, known as the film number defined by

$$K_F = \frac{\rho\sigma^3}{g\mu^4} \tag{20}$$

It will be shown presently that one does not gain any advantage by introducing simultaneously both the Reynolds and Froude criteria for correlation of experimental data on falling film flow—and that in most cases it is sufficient to use one criterion only because in the laminar and the wavy regions of flow there exists a linear relationship between the Reynolds and Froude numbers.

A. *Laminar flow*

The Froude number for film flow is commonly taken as

$$\mathrm{Fr} = \frac{V^2}{gm} \tag{21}$$

Using NUSSELT's equations for laminar flow, we have on substituting for V (equation 2) in equation (21):

$$\mathrm{Fr} = \frac{\rho^2 g m^3}{9\mu^2} \tag{22}$$

and substituting for m (equation 4) in (22):

$$\mathrm{Fr} = \frac{Q\rho}{3\mu} = \frac{1}{3}\frac{Q}{\nu} \tag{23}$$

When equations (16) and (23) are combined, the Reynolds number can be expressed in terms of the Froude number

$$\mathrm{Re} = 12\ \mathrm{Fr} \tag{24}$$

B. *Wavy flow*

The flow behaviour in the wavy or pseudo-laminar region is different from the true laminar one and consequently the value of the Froude group is bound to be affected. The resulting change in the value of the Froude number may be established quantitatively in the following way:

By eliminating from equation (21) the mean film velocity, V, we have

$$\mathrm{Fr} = \frac{V^2}{gm} = \frac{Q^2}{m^3 g} \tag{25}$$

The above equation expresses the Froude group as a function of the average film thickness, m, which may be evaluated according to KAPITSA's theory [1, 2] of the wavy film flow, thus

$$m^3 = 2{\cdot}4\,\frac{\nu Q}{g} \tag{26}$$

substituting the above expression for m in equation (25), we have

$$\mathrm{Fr} = \frac{Q^2}{m^3 g} = \frac{1}{2{\cdot}4}\frac{Q}{\nu} \simeq \frac{1}{4}\frac{Q}{\nu} \tag{27}$$

and by combining equations (27) and (16), we obtain

$$\mathrm{Re} = 16\ \mathrm{Fr} \tag{28}$$

The above equation applies in the wavy flow up to the critical Reynolds number for the inception of turbulence in the film. The values given in the literature for the Reynolds number at which turbulence appears in the film flow seem to vary. For water-films the values quoted range between $Re_t \simeq 1000$ and $Re_t \simeq 2400$ where Re_t is the Reynolds number of transition between viscous and turbulent flow. A summary of the results of the critical Reynolds number of transition obtained by various workers for water films is given in Table 1.

Table 1. *Values of the transitional Reynolds number for water films*

Year	*Authors*	Re_t
1934	Kirkbride [18]	2000
1934	Cooper *et al.* [17]	2100
1945–47	Grimley [22, 23]	1000
1956	Brauer [7]	1600
1958	Thomas and Portalski [30]	1160
1960	Portalski [35]	1150

The experimental values for the Reynolds number of transition are normally obtained by plotting the mean film thickness against the Reynolds number of the film on a log–log scale. A number of characteristic straight lines can be drawn through the experimental data and the point of intersection of two of them marks the transitional Reynolds number.

The film thickness appears therefore to be quite an important characteristic of film flow, but the meaning liquid layer thickness varies somewhat in the published literature depending upon the particular method of measurement being used.

Experimental

Methods of film thickness measurement

Since waves and ripples are naturally present on most falling liquid films at all but the very lowest flow rates, the meaning of the phrase "liquid layer thickness" has to be carefully defined. There are three possibilities:
(a) maximum layer height, (b) average film thickness, (c) residual film thickness. The maximum layer height is, of course, the height of the highest wave crests; the residual film thickness represents the thinnest part of the film. The meaning of the average film thickness seems obvious, but it must be realised that the value of the mean film thickness may depend to a certain extent on the method of measurement being adopted.

The methods of film thickness measurement given in the literature are of two main types; (a) direct and (b) indirect.

(a) *The direct method of measurement.* This method involves the use of a micrometer screw and its different modifications. It was used by Hopf [11], Chwang [14] and Kirkbride [18] in the past and by Hanratty and Engen [29] recently. At its best it may be useful on extremely rare occasions when dealing with true laminar flow, but when waves are present the height of the highest wave crests is naturally obtained, rather than the mean film thickness. The feeler-probe method, used by Pennie and Belanger [24] and also by Brauer [7] is a refined modification of the micrometer screw method. Here a very fine needle is mounted on a micrometer and is adjusted so as to touch the liquid film, the contact being registered normally by an electronic counter.

The objection to this method is quite obvious: by touching the surface of the film by a probe, or even worse, by introducing the probe into the film, however fine the probe may be, one interferes with the natural flow of the film. One measures, therefore, a disturbed condition which may not be closely related to the original undisturbed mode of flow,

(b) *The indirect method.* In the indirect method the film thickness is calculated from some other measured quantity and it can thus be obtained in several ways, e.g. by
(1) a drainage or hold-up method, (2) shadow-photographs, (3) radio-active tracers, (4) a balancing tower method, (5) electrical capacitance [5] and electrical resistance [33] methods, (6) photometric [48, 49] and optical interference [40] method.

The drainage method is quite simple, at least in theory: the feed to the wetted-wall apparatus is suddenly stopped and simultaneously a measuring vessel is placed below the outlet, the liquid film layer being thus collected *in toto*. From the amount of liquid collected and the known wetted-wall area the average film thickness can be calculated. With a refined technique of draining and using a carefully designed apparatus this method is capable of very high accuracy.

The results shown in Table 2 show quite a wide measure of variation and the author considered it worth-while to carry out a preliminary investigation on water film flow using the most favoured type of apparatus for this purpose, viz. a vertical tube fitted with certain refinements [30]. The observations made concerning the average film thickness have shown that turbulence in water films begins quite early, i.e. at Re $\simeq$ 1160, and that the transition region may extend up to comparatively high value of Re $\simeq$ 1500 or more, before a fully turbulent condition is reached. It was realised also during the course of experiments with water films in vertical tubes that this type of apparatus suffers from certain disadvantages which preclude a thorough examination of the many phenomena relating to the hydrodynamics of film flow—and it would appear that at least some of these impediments have not been fully appreciated in the past. It is the experience of the author that the hold-up method of the film thickness measurement is very unlikely to produce highly accurate and reproducible results, unless the apparatus is so designed that it possesses such features as an automatic device for stoppage of the feed and simultaneous elimination of weir-head liquid as well as the collection of drainage from a fairly large wetted area. This cannot be achieved easily, if at all, using small-bore tubes of moderate length.

JACKSON [6] has employed the method of using radioactive tracers (yttrium 91) to establish the thickness of liquid films by the following expression:

$$m = C \, . \, A_c \tag{29}$$

where C is a constant and A_c is described as the activity of liquid film on a vertical tube apparatus, corrected for absorption and sample activity, in counts/sec. He had to evaluate first the constant C in equation (29). This was done, apparently "by operating the vertical tube apparatus under conditions of flow where the film thickness could be calculated from the flow rate" using NUSSELT's equation (4), i.e. the value of the constant was determined by plotting corrected activity, A_c, against the calculated film thickness, m.

The author's criticism of this technique is the fact that, even if this could be done for the region where the flow is truly viscous and waves are absent, this method automatically shifts the experimental film thickness results at low flow rates and pins them down on to the theoretical line proposed by Nusselt. But Nusselt's theory fails to predict the film thickness correctly, even for steady laminar conditions of flow [28, 31, 35]. On examination of his results JACKSON states that "the data permit few conclusions concerning the inception of turbulence" and that "for water and the lighter liquids no significant departure from viscous behaviour is indicated up to Reynolds numbers of 5000 although scattering of the data is observed above Re = 2000."

In view of the above criticism JACKSON's results are not entirely surprising. KAMEI and OISHI [28] have devised a balancing tower method for weighing of the hold-up. They claim that the sensitivity of this method without gas flow is 0·5 g. However they admit that "with counter-current gas flow the weight necessary to balance the tower was unstable and so the measurements were rougher than those without gas flow". In fact they mention that "with the air flowing, the tower swings violently and irregularly and the accuracy of the balance decreases. It is inevitable that the observed film thickness is more or less in error under such conditions".

It would thus appear that the balancing tower method of film thickness measurement is quite sensitive for the "no air-blow" condition but it is much less reliable with the air blow. It is perhaps not inappropriate to mention here that by using the technique and the wetted-column designed by the author one can measure the hold-up quite comfortably to the nearest 0·5 ml with the apparatus of approximately four times the wetted area of the largest tower used by KAMEI and OISHI; there is no difficulty at all with the measurements during air blow in either countercurrent or cocurrent flow. At the same time one has to stress that the new method of KAMEI and OISHI is a real improvement in the technique of film thickness measurement in wetted-wall tubes and this is reflected no doubt in the experimental results which they obtained for no air-blow conditions. These workers note that "by the use of this method of measurement it became obvious that the observed thickness of the liquid film was smaller than the theoretical" i.e. in the perfect laminar region. Their observations agree with DUKLER's work [4, 5] and with the author's experimental results on the thickness of vertical liquid films with no air blow.

BELKIN and associates [34] have published recently a paper on turbulent liquid film flow. Two related photographic techniques were used by them for water flow down the outer surface of 1 in. vertical glass rods. High-speed photographs of the rod with water flow were compared with photographs of the dry rod, using a planimeter on enlarged photographs. Their method of film thickness measurement is essentially the same as that used by BRAUER [7], except that BRAUER fully realized the limitations of his method and was taking great care (seven photographs of the water film for each value of the Reynolds number!) to obtain the best results possible with his technique.

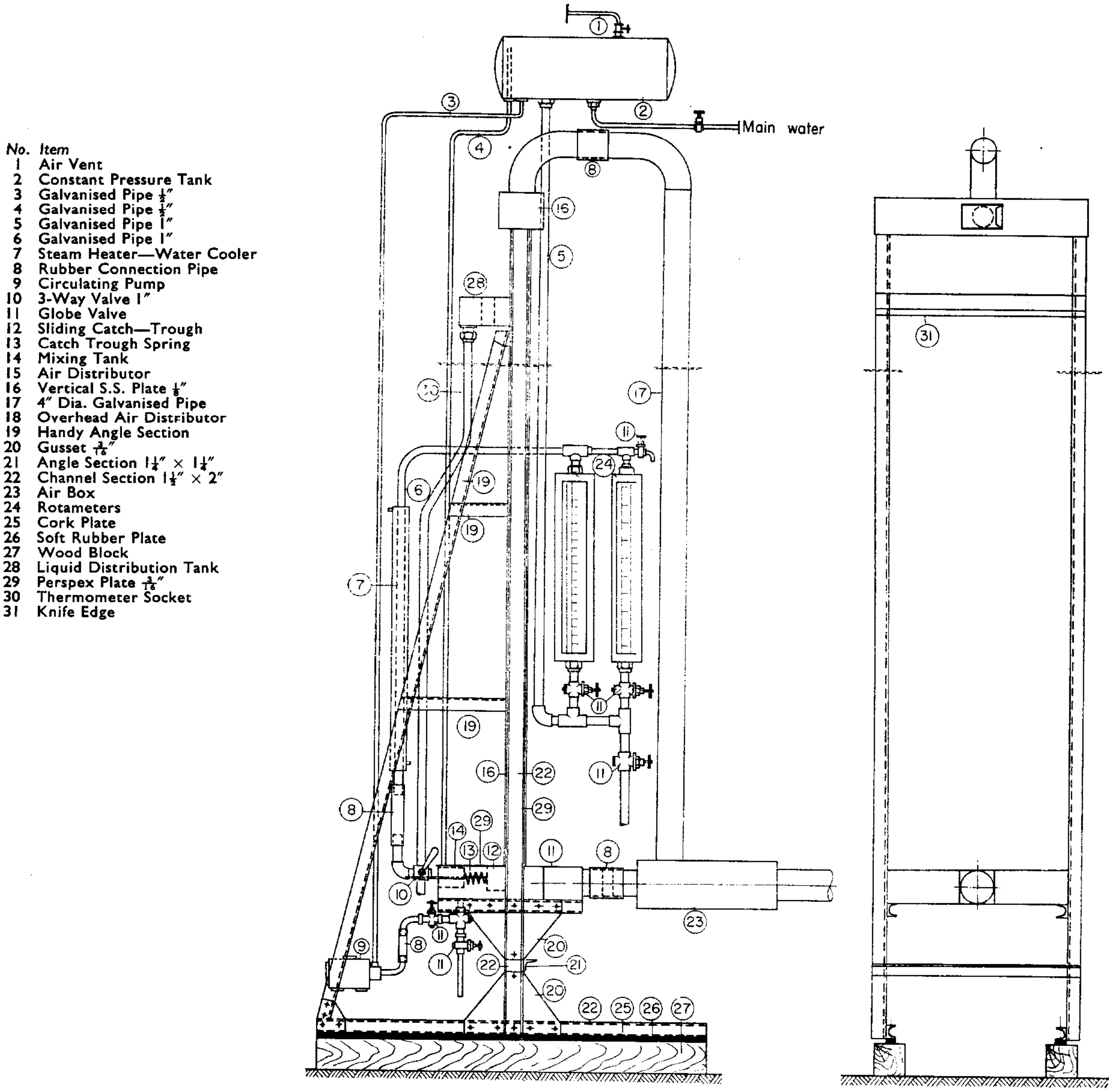

FIG. 1. General arrangement of apparatus.

In the author's opinion it is extremely difficult to enlarge photographs of apparatus to exactly the same length—especially if two cameras are used for the purpose—unless the photographed portions are very accurately marked beforehand, as in BRAUER's work [7] for instance. In any case this photographic technique does not produce a true silhouette of the liquid film, but a three-dimensional effect which is quite obvious from the photographs submitted. In other words BELKIN *et al.* did not obtain the exact orthogonal view or elevation photograph they had hoped for, but an isometric view which is a definite disadvantage and, of course, cannot be corrected for by subsequently employing "enlarged images on large sheets of paper" [34]. It is not surprising therefore that the spread of the data presented in the paper is quite considerable and is no doubt due to the reasons discussed above and possibly also due to the tank-level effects (no constant head) and persistant entrance disturbances (no knife-edge feed but annular orifices at the top of the test section) reported by the authors.

Apparatus

A very large wetted wall area seemed absolutely imperative for an accurate determination of the mean film thickness from the hold-up measurements. These, it would seem, could be carried properly out only with the provision of such features as an automatic operation for stoppage of feed and simultaneous elimination of weir-head liquid and collection of drainage into a receiving vessel suitably coupled with a stop-flow device.

The apparatus used has been partly described previously [38, 39]. A drawing of the equipment is given in Fig. 1. This consisted of a flat steel plate which could be bolted to a steel framework so that a channel 21 in. wide and 7 ft. high was formed. The frame was built up from two rolled steel sections (2 × 1½ in.) bolted to wooden blocks which rested on the floor. The blocks themselves rested on soft rubber and cork slabs to insulate them from stray mechanical vibrations. To provide flexibility two flat steel plates were made up in the first instance, the first being a plate of highly polished stainless steel, the second a rough mild steel one.

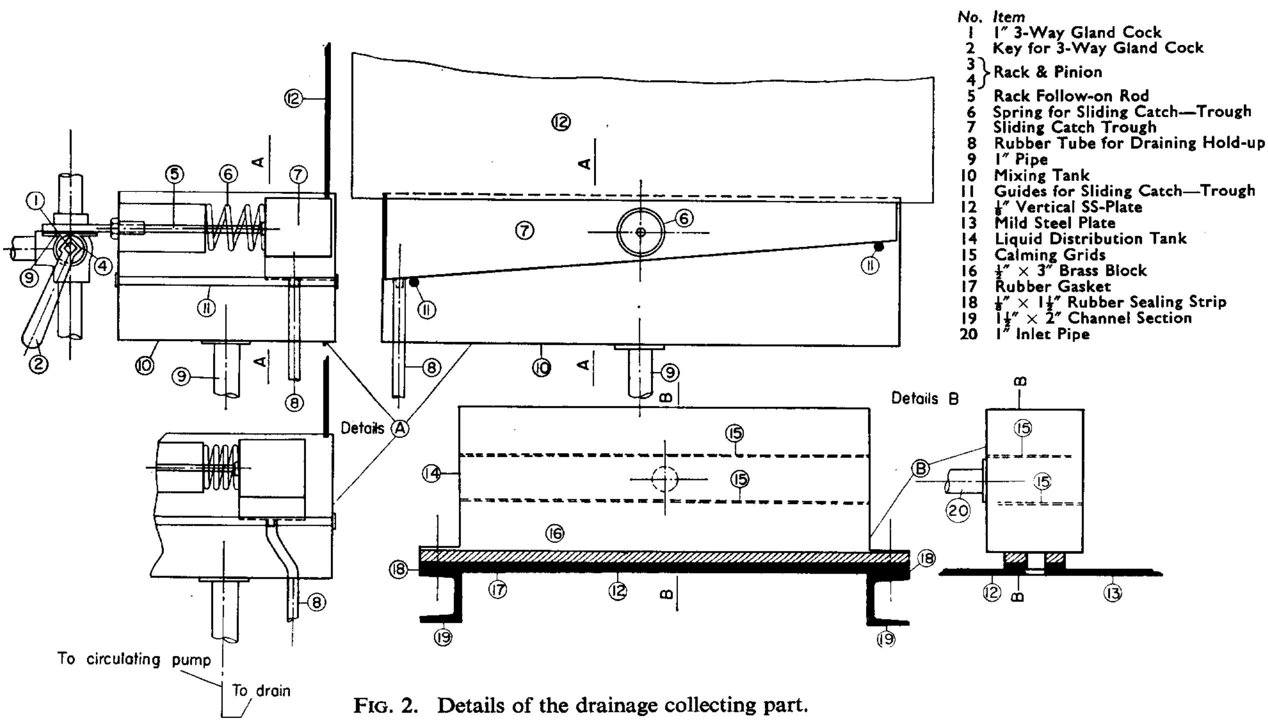

FIG. 2. Details of the drainage collecting part.

The wetted-plate was aligned with the vertical to within 0·05 in. on the front elevation and to within 0·1 in. on the end elevation over the entire length with a surveyor's transit. The symmetrical distribution of liquid about the central line of the wetted-plate was found to be very markedly dependent on the vertical alignment of the front elevation of the apparatus and the latter could therefore be checked readily and improved whenever necessary to an even higher degree of accuracy than that assured by using a surveyor's transit.

Liquid was pumped by means of a small centrifugal pump to an overhead constant head tank, wherefrom it passed through a selected rotameter, a steam heater/water cooler arrangement and to the liquid distribution tank, where the flow was calmed by means of screen baffles before flowing over the selected vertical plate, precision ground to a knife-edge of 45°. After flowing down the plate the liquid entered a mixing tank from which it was either recirculated or discharged down the drain, in the case of mains water.

Lengths of flexible rubber tubing installed in all the flow-lines leading to the apparatus provided insulation from the vibration transmitted by the ancillary equipment and the steam or water mains.

The collection of hold-up under any conditions of flow was made possible by designing a special semi-automatic device for this purpose. This part of the apparatus is shown in three views as "detail A" in Fig. 2, and Figs. 3 and 4 provide two photographs of the material parts of the equipment in question.

Technique

The wetted plate was thoroughly cleaned every time before use. Hot water and "Tide" were found to be the most useful washing agents. After cleaning, the column was flushed with water for about 20 min, when subsequently performing experiments with water films, or it was thoroughly dried first and then flushed with the liquid under examination, again for about 20 min, before any experimental runs. This procedure was always rigorously applied and as a result no trouble at all was experienced with the liquid film "channelling" on the wetted plate at flow rates above the minimum wetting rate.

Ordinary mains water was used for experiments with water films, similarly tap water was employed for preparing seven different glycerol solutions for experiments with liquids more viscous than water.

The column was made water-tight and air-tight by means of soft rubber bands, $\frac{1}{16}$ in. thick, laid on both faces of the vertical channel section adjoining the wetted plate and the Perspex plate (when used) respectively.

Two rotameters were used for the water runs: (a) low-range rotameter (0·025–0·25 ft^3/min water at 20°C) and (b) high-range rotameter (0·1–2·0 ft^3/min water at 20°C).

Nine other liquid films were investigated in this work (seven different glycerol solutions, isopropyl alcohol and methyl alcohol) and in order to cut down on the cost of instruments, two flowmeters of a "universal" type were employed. The manufacturers' calibration charts [50] were checked by calculation [58, 59] to have been accurately prepared and were subsequently used with confidence.

The absolute viscosity of the liquids used varied between 65 cP and 0·6 cP and their surface tension changed from 73 dyn/cm to 22 dyn/cm. The physical constants of the liquids used are given in Table 2.

Experimental measurements of the viscosity of some of the glycerol solutions (27 and 71·5 per cent) were made by using Standard U-tube viscometers, for other compositions of glycerol and water the results published by Sheely [51, 52] were used.

The glycerol solutions were prepared by mixing known volumes of pure glycerine and water and the calculated compositions were checked by direct density measurements by means of suitable hydrometers. The values obtained agreed with the handbook values [53] very closely.

Viscosities and densities of methyl alcohol and propyl alcohol were taken from the literature [52–54] and interpolated from a suitable graph for the required working temperatures when necessary [35]. Some surface tension data of the liquids used, i.e. glycerol solutions, iso–propyl alcohol, methyl alcohol and water were taken from the literature [52, 54, 55]. These were suitably plotted [35] and the pertinent surface tension values of liquids used (for the required working temperatures and con-

Table 2. The physical constants of the liquids used

Liquid	t (°C)	ρ (g/cm³)	μ (cP)	ν (cS)	σ (dyn/cm)	δ (CGS units)
Methyl alcohol	19	0·7927	0·602	0·759	22·6	28·5
Isopropyl alcohol	20	0·7877	2·37	3·009	21·7	27·6
Water	20	0·9982	1·000	1·002	72·5	72·5
	23	0·9986	0·936	0·938		
Glycerol soln.						
27% (wt.)	25	1·063	1·934	1·819	72·5	67·7
37%	22	1·091	3·097	2·839	71·3	65·3
45%	22	1·112	4·394	3·952	70·4	63·3
71·5%	25	1·186	20·3	17·12	67·4	56·8
75%	21	1·203	34·6	28·76	67·0	55·7
78%	21	1·204	45·6	37·87	66·7	55·4
82%	23	1·212	64·4	53·14	66·2	54·6

centrations) were obtained from the resulting graphs.

Where surface tension data could not be found in the literature direct measurements were made by the torsion balance method [60]. C.G.S. units were used throughout this work and all the experimental results and calculations were worked out accordingly.

Results

Seven glycerine solutions, two alcohols and water were used on a smooth stainless steel plate. To study the influence of surface-active agents on the film thickness, Manoxol OT, Teepol L and white commercial "Tide" were tried in aqueous solutions at optimum concentrations of the surf-actants for suppression of rippling [37].

The average film thickness was calculated from hold-up data. The collection of liquid in all the measurements was achieved by the use of the semi-automatic device described earlier.

The drainage time varied between 20 min, for the very viscous 82% (wt.) glycerine solution, and 3 min for the very mobile methyl alcohol.

The collected drainage was corrected in all cases by the amount of liquid still adhering to the plate and the sliding catch-trough at the end of the specified time. The amount of the residual liquid left on the plate and the catch-trough was found directly by mopping it up with a linen swab and weighing. This was found to be independent of the flow rate. The results are given in Table 3.

The experimental film thickness values were compared with the calculated results based on the theories due to Kapitsa [1], Nusselt [41] and the application of the postulate of von Kármán [56] due to Dukler and Bergelin [3]. A brief mention of the work of Dukler and Bergelin has already been given. These workers made use of the hypothesis of von Kármán [56], who has suggested—as a result of Nikuradse's [57] experimental work on full-pipe flow—the existence of a universal velocity distribution equation in terms of two parameters, u^+ and y^+, which are defined as

$$u^+ = \frac{u}{u^*} \tag{30}$$

$$y^+ = \frac{u^* \rho_L y}{\mu_L} \tag{31}$$

where u^* is the friction velocity and is related to the wall shear by

$$u^* = [\tau_W g/\rho_L]^{1/2} \tag{32}$$

The resulting equations of von Kármán and their limits of application suggested by the data of Nikuradse on full-pipe flow are:

for the laminar sublayer,

$$u^+ = y^+, 0 < y^+ < 5 \tag{33}$$

Table 3.

Liquid used	*Water*	*Water and* 0·125% Teepol L	*Water and* 0·00125% Manoxol–OT	*Water and* 0·20% Tide	*Alcohols* PrOH	MeOH	*Glycerine solutions,* % w/w 27	37	45	71·5	75	78	82
Residual amount Wr, cm³	6·5	7	7	7	6	2	19	24	28	58	60	61	71

for the transition zone,

$$u^+ = -3{\cdot}05 + 5{\cdot}0 \ln y^+, 5 < y^+ < 30 \quad (34)$$

and for the turbulent layer,

$$u^+ = 5{\cdot}5 + 2{\cdot}5 \ln y^+, 30 < y^+ < \eta \quad (35)$$

where η = magnitude of the universal distance parameter at the liquid surface, viz.

$$\eta = \frac{u^* \rho_L m}{\mu_L} \quad (36)$$

DUKLER and BERGELIN have assumed that the above equations, which were developed for full-pipe flow, would apply to the liquid layer in two-phase film flow. With this assumption, the liquid flow rate per unit length of wetted periphery follows in terms of the universal parameters, thus:

$$\Gamma = \mu_L \int_0^\eta u^+ dy^+ \quad (37)$$

This integration may be accomplished by dividing the integral of equation (37) into three parts limited by the boundary conditions for the three regions of flow proposed by VON KÁRMÁN and substituting for each u^+ the proper defining equation:

$$\frac{\Gamma}{\mu_L} = \int_0^5 y^+ dy^+ + \int_5^{30} (-3{\cdot}05 + 5{\cdot}0 \ln y^+) dy^+ + \int_{30}^{\eta} (5{\cdot}5 + 2{\cdot}5 \ln y^+) dy^+ \quad (38)$$

On integrating and collecting terms one obtains

$$\frac{\Gamma}{\mu_L} + 64 = 3{\cdot}0\eta + 2{\cdot}5\eta \ln \eta \quad (39)$$

Having obtained this expression DUKLER and BERGELIN go on to claim that "for any flow rate Γ can be calculated and η evaluated from this equation. In order to eliminate trial and error solution Figure... (graphical solution of equation 39) has been prepared ..." This may be misleading. Because equation (39) was derived from a summation of the flow rate in each of the three regions of flow, it must not be used for $\eta < 30$ and therefore it does not apply "for any flow rate".

In fact it may be shown that for the laminar region, using VON KÁRMÁN's equations, as above

$$\frac{\Gamma}{\mu_L} = \eta^2/2, 0 < \eta \leqslant 5 \quad (40)$$

and for the laminar sublayer plus buffer layer:

$$\frac{\Gamma}{\mu_L} - 12{\cdot}5 = 5\eta \ln \eta - 8{\cdot}05\eta, 5 \leqslant \eta \leqslant 30 \quad (41)$$

The experimental film thickness results for the liquids used are presented in Figs. 5–11. Typical hold-up data for water (leading to the results of Fig. 5) are given in Table 4.

DISCUSSION OF RESULTS

It was shown above that according to the theory proposed by NUSSELT the mean film thickness,

$$m = \left[\frac{3\nu Q}{g}\right]^{1/3} \quad (4)$$

The above equation may be rewritten, thus:

$$m = \left[\frac{3\nu^2}{4g} \cdot \frac{4Q}{\nu}\right]^{1/3} = \left[\frac{3\nu^2}{4g}\right]^{1/3} \cdot \mathrm{Re}^{1/3} = \left[\frac{3\nu^2}{g}\right]^{1/3} \cdot \left[\frac{\Gamma}{\mu}\right]^{1/3} \quad (42)$$

Table 4. Average film thickness, water on SS plate

Mean temperature: 20°C. *Kinematic viscosity,* v_{20} = 1·0017 cS. *Drainage time:* 10·00 sec. *Residual liquid,* Wr = 6·5 cm^3

L	Q	$\frac{\Gamma}{\mu_L}$	Re_L	W_L	W_{Lm}	H_L	$m_p \times 10^2$	$Q\nu \times 10^3$	Theoretical film thickness			
									KAPITSA	NUSSELT	VON KÁRMÁN'S UVP developed in this work as proposed by DUKLER and BERGELIN	
(l/min)	(cm^3/sec cm)			(cm^3)	(cm^3)	(cm^3)	(cm)	(cm^4/ sec^2)	($a_0 \times 10^2$ cm)	($m_N \times 10^2$ cm	η	$m_K \times 10^2$ cm
1·42	0·444	44·3	177	220 / 220	220	226·5	1·99	4·45	2·21	2·39	9·6	2·11
1·70	0·531	53·0	212	235 / 232	233·5	240	2·11	5·32	2·35	2·53	10·7	2·27
1·98	0·619	61·8	247	253 / 248	250·5	257	2·26	6·20	2·48	2·67	11·6	2·40
2·27	0·709	70·8	283	270 / 270	270	276·5	2·43	7·10	2·59	2·79	12·7	2·55
2·55	0·797	79·6	318	286 / 281	283·5	290	2·55	7·98	2·69	2·90	13·5	2·65
2·83	0·884	88·2	353	293 / 293	293	299·5	2·63	8·86	2·79	3·00	14·4	2·77
3·11	0·972	97·0	388	313 / 311	312	318·5	2·80	9·74	2·88	3·10	15·3	2·88
3·40	1·062	106·0	424	320 / 320	320	326·5	2·87	10·64	2·96	3·19	16·1	2·98
3.96	1·237	123·5	494	346 / 342	344	350·5	3·08	12·39	3·12	3·36	17·6	3·16
4·53	1·415	141·3	565	365 / 366	365·5	372	3·27	14·17	3·26	3·51	19·2	3·35
5·66	1·769	176·6	706	391 / 400	395·5	402	3·53	17·72	3·51	3·78	22·2	3·69
6·80	2·125	212·1	848	434 / 434	434	440·5	3·87	21·29	3·74	4·02	24·9	3·99
7·93	2·478	247·4	990	462 / 460	461	467·5	4·11	24·82	3·93	4·23	27·8	4·29
9·06	2·831	282·6	1130	503 / 504	503·5	510	4·48	28·36	4·11	4·43	30·1	4·53
10·19	3·184	317·9	1272	527 / 529·5	528·5	535	4·70	31·89	4·27	4·60	32·6	4·77

Table 4 (continued)

									Theoretical film thickness			
L	Q	$\frac{\Gamma}{\mu_L}$	Re_L	W_L	W_{Lm}	H_L	$m_p \times 10^2$	$Q_v \times 10^3$	KAPITSA	NUSSELT	VON KÁRMÁN'S UVP developed in this work as proposed by DUKLER and BERGELIN	
(l/min)	(cm³/sec cm)			(cm³)	(cm³)	(cm³)	(cm)	(cm⁴/sec²)	($a_0 \times 10^2$ cm)	($m_N \times 10^2$ cm	η	$m_K \times 10^2$ cm
11·33	3·540	353·4	1414	559 559	559	565·5	4·97	35·46	4·43	4·77	35·1	5·01
12·46	3·893	388·6	1554	590 586	588	594·5	5·22	39·00	4·57	4·92	37·6	5·25
13·59	4·246	423·9	1696	621 621	621	627·5	5·51	42·53	4·70	5·07	39·9	5·46
14·73	4·603	459·5	1838	638 640	639	645·5	5·67	46·11	4·83	5·20	42·5	5·70
15·86	4·956	494·8	1979	668 670	669	675·5	5·94	49·64	4·95	5·33	44·8	5·90
16·99	5·309	530·0	2120	691 690	690·5	697	6·12	53·18	5·07	5·46	47·0	6·09
18·12	5·662	565·2	2261	720 716	718	724.5	6·37	56·72	5·18	5·58	49·3	6·29
19·26	6·018	600·8	2403	741 744	742·5	749	6·58	60·28	5·28	5·69	51·8	6·50
20·39	6·371	636·0	2544	780 780	780	786·5	6·91	63·82	5·39	5·80	54·0	6·68
21·52	6·724	671·3	2685	816 805	810·5	817	7·18	67·35	5·48	5·90	56·5	6·89
22·65	7·077	706·5	2826	835 845 838	839	845·5	7·43	70·89	5·58	6·01	58·7	7·06
23·79	7·433	742·0	2968	862 870 867	867	873·5	7·62	74·46	5·67	6·11	60·8	7·23
24·92	7·787	777·4	3110	885 891	888	894·5	7·86	78·00	5·76	6·20	62·6	7·37
26·05	8·140	812·6	3250	927 913 917 920	919	925·5	8·13	81·54	5·84	6·29	65·1	7·57

Table 4 (continued)

L	Q	$\frac{\Gamma}{\mu_L}$	Re_L	W_L	W_{Lm}	H_L	$m_p \times 10^2$	$Q\nu \times 10^3$	Theoretical film thickness			
									KAPITSA	NUSSELT	VON KÁRMÁN's UVP developed in this work as proposed by DUKLER and BERGELIN	
(l/min)	(cm³/sec cm)			(cm³)	(cm³)	(cm³)	(cm)	(cm⁴/sec²)	($a_0 \times 10^2$ cm)	($m_N \times 10^2$ cm	η	$m_K \times 10^2$ cm
27·19	8·496	848·2	3393	941 920 928 947	934	940·5	8·26	85·10	5·93	6·38	67·4	7·75
28·32	8·849	883·4	3534	965 961	963	969·5	8·52	88·64	6·01	6·47	69·8	7·93
29·45	9·202	918·6	3674	970 975	973	979·5	8·61	92·18	6·09	6·56	71·7	8·07
30·58	9·555	953·9	3816	1005 1001	1003	1009·5	8·87	95·71	6·16	6·64	73·6	8·21

The film thickness vs. Γ/μ_L relation for water on SS plate is given in Fig. 5.

In Figs. 5–11 the experimental film thickness, m_p, is accordingly plotted against Γ/μ_L. To compare the experimental results with those predicted by the theoretical equations proposed by NUSSELT [41], KAPITSA [1] and VON KÁRMÁN [56], three curves, marked N, K, and UVP respectively, have been added in each case for reference.

The following conclusions may be drawn from examination of the obtained results:

1. With the method used here only the average value of the falling liquid film thickness can be obtained; with a column of about 213 cm length and more than 50 cm width this average thickness would not be influenced by the "end effects", i.e. acceleration etc. of liquid flow (see Ref. [28]).

2. In all the cases considered, except possibly when surface-active agents were added to suppress rippling, KAPITSA's theory approximates to the experimental results better than NUSSELT's theory in the region for which these theories were intended; this means that the experimental film thickness, m_p, is significantly thinner than that calculated according to NUSSELT's theory and even somewhat thinner than that predicted by KAPITSA's theory within the specified limits.

3. The mode of film flow may be divided into the following regions: Steady laminar – pseudo-

$\uparrow$ $Re_1 = Re_i$

laminar – transitional – pseudoturbulent – turbu-

$\uparrow$ Re_2 $\uparrow$ $Re_3 = Re_t$ $\uparrow$ Re_4

lent

Appropriate Reynolds numbers of transition (as above) may be easily found on the relevant film thickness vs. Γ/μ_L graphs. Steady laminar flow terminates at the point of inception of wave motion and the critical Reynolds number for the change [36], Re_1, is therefore the same as the Reynolds number of wave inception, Re_i. In the case of water, or a liquid more mobile than water, Re_2 is somewhat masked by rippling but it may be determined easily by suppression of this phenomenon by the addition of surfactants at a suitable concentration [37].

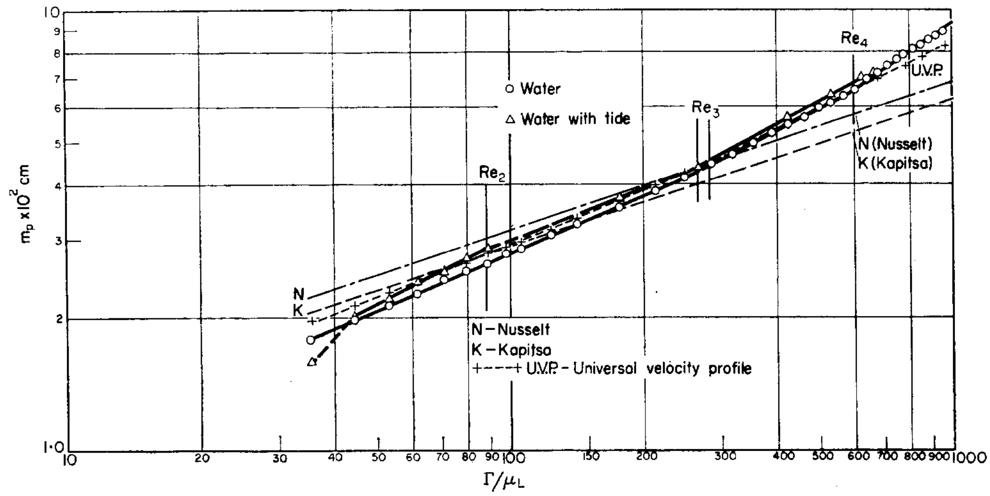

FIG. 5. Film thickness vs. Γ/μ_L relation for water and water with 0·20% (wt) Tide on the SS plate.

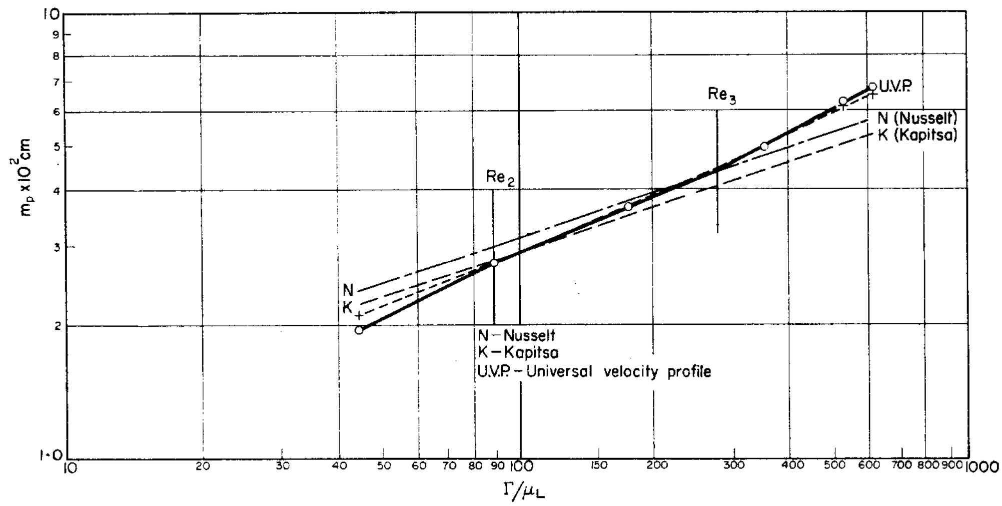

FIG. 6. Film thickness vs. Γ/μ_L relation for water with 0·125% (Vol) Teepol L on the SS plate.

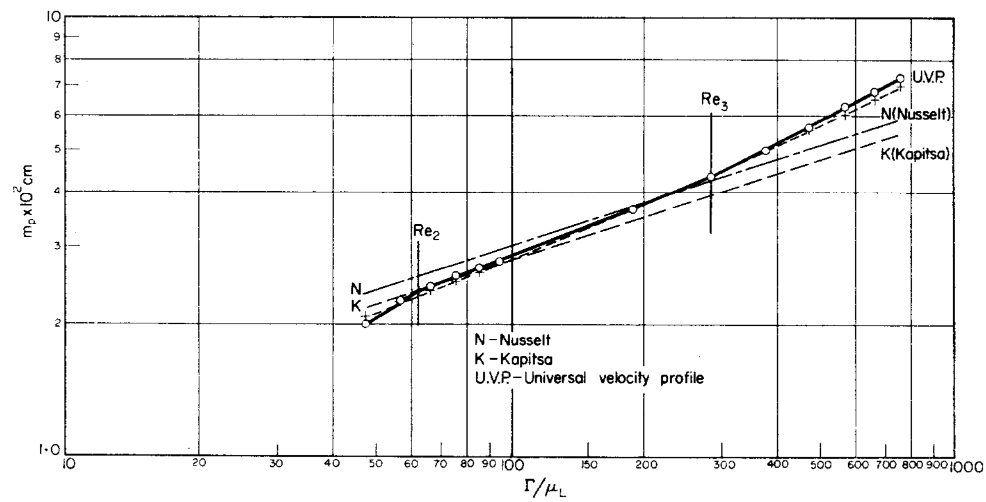

FIG. 7. Film thickness vs. Γ/μ_L relation for water with 0·00125% (wt) Manoxol–OT on the SS plate.

4. A liquid film on which the ripples have been suppressed by the addition of a surface active agent is slightly thicker than the corresponding one with ripples (at the same flow rate), as predicted by the theoretical considerations of KAPITSA.

5. The physical properties of the liquid, such as viscosity, specific gravity and surface tension, influence the falling film thickness. The higher the kinematic viscosity the thicker is the film and the more regular appears to be the wave motion associated with it. In fact, viscosity seems to play the role of a damping factor, not unlike the role of resistance in oscillating electrical circuits.

6. The plots of the points with the same kinematic viscosity, ν, are influenced by the surface tension, σ; this seems to determine the relevant ratio of the pseudolaminar to the transitional portions of flow.

7. The degree of deviation from the N and K lines in the pseudolaminar region decreases as σ becomes smaller.

8. When the film thickness, m_p, is plotted against

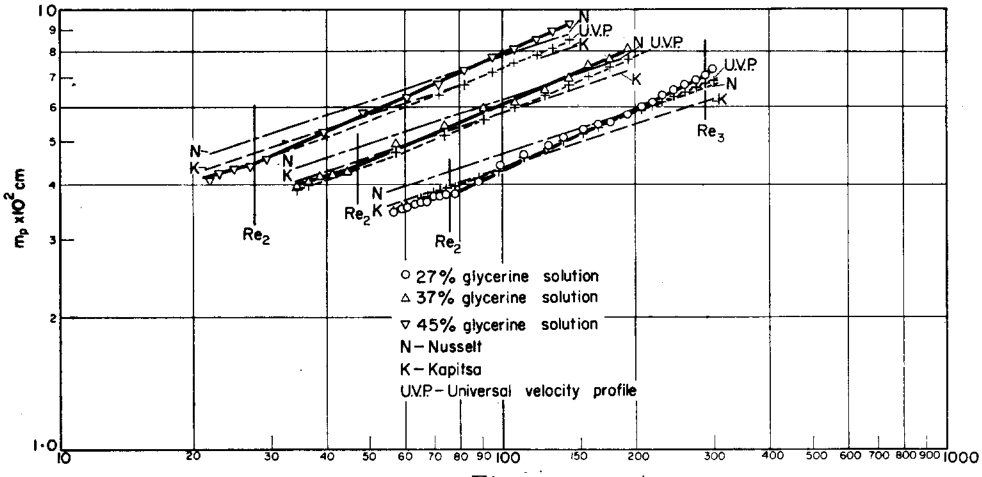

FIG. 8. Film thickness vs. Γ/μ_L relation for glycerine solutions on the SS plate.

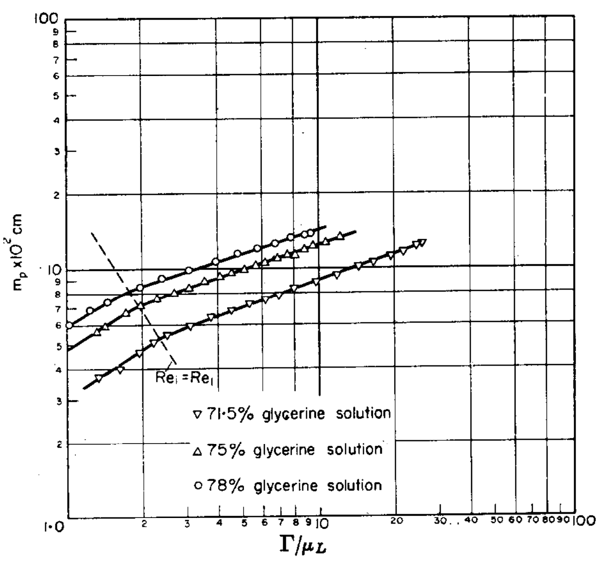

FIG. 9. Film thickness vs. Γ/μ_L relation for glycerine solutions.

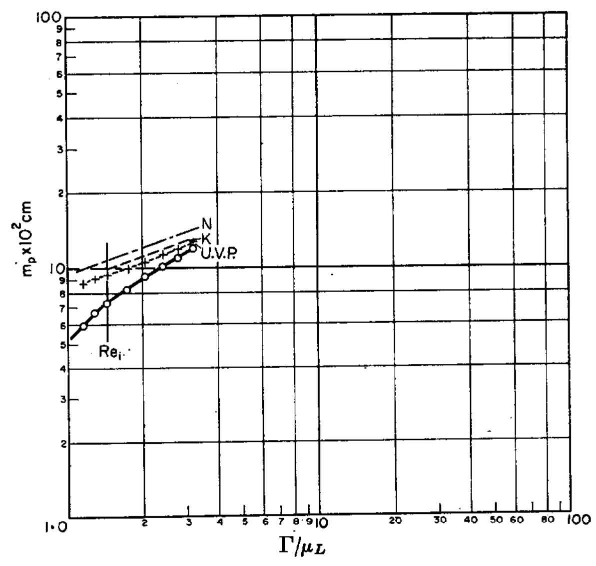

FIG. 10. Film thickness vs. Γ/μ_L relation for 82% (wt) glycerol solution on a smooth plate.

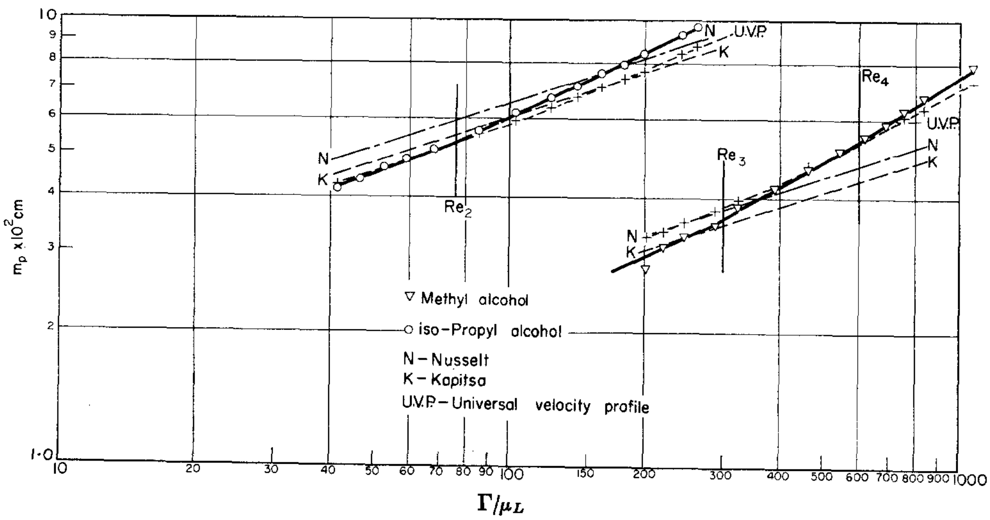

FIG. 11. Film thickness vs. Γ/μ_L relation for iso-propyl and methyl alcohols on the SS plate.

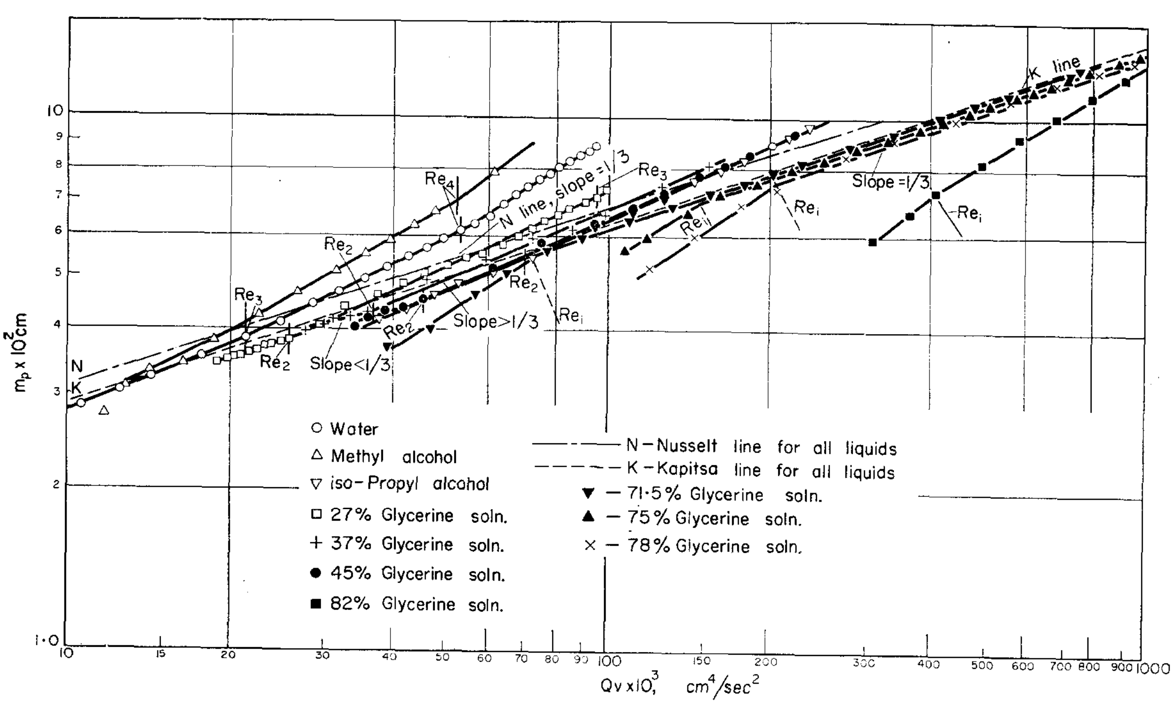

FIG. 12. Experimental film thickness, m_p, vs. Qv relation for the liquids used.

Qv, as in Fig. 12, a family of curves results, where, generally, the pseudolaminar portion of the curve has a slope of less than $\frac{1}{3}$ and the transitional segment has a slope greater than $\frac{1}{3}$. The theories proposed by NUSSELT and KAPITSA were intended only for the viscous flow and they hardly apply beyond the transitional region. For water the transitional region finishes at Re $\simeq$ 1160.

9. NUSSELT's theory cannot be used to predict falling film thickness accurately even for the steady laminar flow, i.e. before the onset of wave motion.

10. The universal velocity profile concept is, so far, the best available approximation to the facts and the assumption of the original universal velocity profile based on the full-pipe flow leads to the prediction of the average falling film thickness in a better agreement with those experimentally determined than is possible by any other existing theory, as shown by the calculated results in Table 4 and Figs. 5–11.

Nevertheless, systematic deviations still exist in consequence of the fact that the original velocity profile equations were proposed for full-pipe flow and are, therefore, in need of modification in the light of the experimental data in falling film thickness. This will be the subject of a future paper.

NOTATION

V	Average film velocity	cm/sec
g	Acceleration due to gravity	cm/sec^2
m	Film thickness in laminar flow	cm
a_0	Film thickness in wavy flow	cm
ρ	Density	g/cm^3
μ	Absolute viscosity	P
ν	Kinematic viscosity	S
Q	Liquid flow rate per unit length of wetted perimeter	cm^3/sec cm
R	Radius of tube	
r	Radius from centre of tube to the inner surface of the film	cm
k	Phase velocity of wave motion	cm/sec
λ	Wavelength	cm
ϕ	Free surface function of the flowing film	
δ	Kinematic surface tension $= \sigma/\rho$	
σ	Dynamic surface tension	dyn/cm
D	Diameter	cm
R_e	Reynolds number $= 4Q/\nu$,	dimensionless
F_r	Froude number $= V^2/gm$	dimensionless
W_e	Weber number $= mV^2\rho/\sigma$	dimensionless
u^+	Universal velocity parameter	dimensionless
u^*	Friction velocity	cm/sec
y^+	Universal distance parameter	dimensionless
η	Magnitude of the universal distance parameter at the film surface	dimensionless
τ	Shear stress	g/cm sec^2
Γ	Liquid flow rate per unit length of wetted perimeter	g/sec cm
L	Volumetric rate of flow	l/min
H_L	Liquid hold-up	cm^3
W_L	Liquid drainage	cm^3
W_{Lm}	Mean liquid surface	cm^3
W_r	Residual liquid on vertical plate	cm^3

Subscripts

S	Value at the surface
av	Average value
L	Liquid
C	Core fluid
eq	Equivalent value
h	Hydraulic
t	Value at the inception of turbulence
w	Value at the wall
i	Value at the point of wave inception
2	Value at the point of change from pseudolaminar to transitional flow
3	Value at the point of change from transitional to pseudoturbulent flow
4	Value at the point of change from pseudoturbulent to turbulent flow

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