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MIXING INDUCED BY AIR SLUGS RISING IN NARROW COLUMNS OF WATER

J. B. L. M. CAMPOS and J. R. F. GUEDES DE CARVALHO
Centro de Engenharia Química da Universidade do Porto, 4099 Porto Codex, Portugal

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Abstract—A series of equal-sized slugs of air was injected at the bottom of water columns of internal diameters 19 and 32 mm, respectively. The dispersion of tracer initially contained in the lower half of the columns was measured. A simple physical model was developed based on the idea that dispersion is mainly due to the action of the wakes of slugs, considered to be fully mixed; this model accounts remarkably well for the experimental results observed. An alternative dispersion model, based on the analogy with one-dimensional molecular diffusion, is also presented and the relationship between the two models is discussed. Experiments were performed for a range of values of slug volume, the number of slugs injected and the frequency of slug injection.

INTRODUCTION

Mixing in bubble columns is a topic of great practical relevance, and considerable research effort has been devoted to its study in recent years. Mixing is generally recognized to be the result of the two factors: (a) large-scale movement of the liquid bulk, also known as liquid circulation; and (b) liquid displacement in the vicinity of rising bubbles.

Given the complexity of the phenomena involved, an empirical dispersion coefficient (axial) is normally defined and the experiments are designed to yield the value of this coefficient over a range of experimental conditions.

In the present paper, mixing in slugging columns is considered from a fundamental point of view, in an attempt to clarify some of the factors in category (b) above. The study is confined to mixing induced by gas slugs (also known as Taylor bubbles) rising in narrow columns of water. Overall liquid circulation is avoided in this way, and the importance of bubble wakes in promoting mixing is better brought to light. This study is also expected to be of direct relevance to the understanding of those practical situations in which slug flow occurs (e.g. air lift contactors and gas fluidized beds of small diameter).

EXPERIMENTAL

In each experiment, a number of equal-sized slugs of air were injected in succession at the bottom of a long and narrow column containing water. Mixing of the liquid could be followed by means of a coloured tracer, "Rouge Solophenyle 6BL" (a water-soluble dye). Before injection of the first slug, the tracer concentration profile along the column was in the shape of a step (see Fig. 1): concentration $C = C^\circ$ uniformly from the bottom of the column, $z = -H_B$, to some level $z = 0$ near its middle; and concentration $C = 0$ from there to the free surface, $z = H_T$. After injection of a number (N) of slugs, samples of liquid were collected at various heights and analysed to obtain the final tracer concentration profile (see, for example, Fig. 6).

Columns with internal diameters of 19 and 32 mm were used in the experiments and the total liquid heights in the columns were around 2.0 and 2.6 m, respectively. The 32 mm column had one 32 mm bore ball valve near its middle, fitted so that when the valve was open the whole column presented a virtually straight bore from top to bottom. This ball valve was necessary to prevent mixing between tracer solution $C = C^\circ$, and water, $C = 0$, when the column was being filled with liquid for each experiment. In the 32 mm column, samples of liquid were drawn through small holes along the column wall, at the end of each experiment. Care was taken to sample from top to bottom so as to minimize distortion of the concentration profile in the process of sampling.

In the 19 mm column, distortion of the concentration profile in sampling would be more severe and a different technique was adopted. The column was built as a succession of ball valves and separating sections all with 19 mm internal bore (see Fig. 2). After injection of the required number of slugs, all the ball valves were closed and the column was placed horizontally. All the liquid between each two consecutive valves was then passed through side holes to the sampling bottles. The tracer concentration in all samples was determined by spectrophotometry.

Although the ball valves have a good finish on the inside, they are likely to disturb the liquid layer surrounding the slug and therefore promote mixing. No attempt was made to measure this effect, which is likely to be small considering that mixing in the wake is dominant.

The slugs were injected through a solenoid valve connected to a source of compressed air. Adjustment of the air pressure and the valve opening time allowed easy regulation of the slug size, and this was measured as the overall drop in liquid level, ΔH_s, after the slug erupted at the free surface. It should be noted that, if $(\Delta H_s/D) > 0.1$, the bubbles can be considered as slugs, as may be derived from a criterion suggested by Hovmand and Davidson (1971).

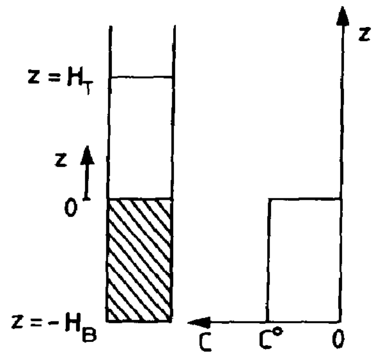


Fig. 1. Initial tracer distribution in the test column.

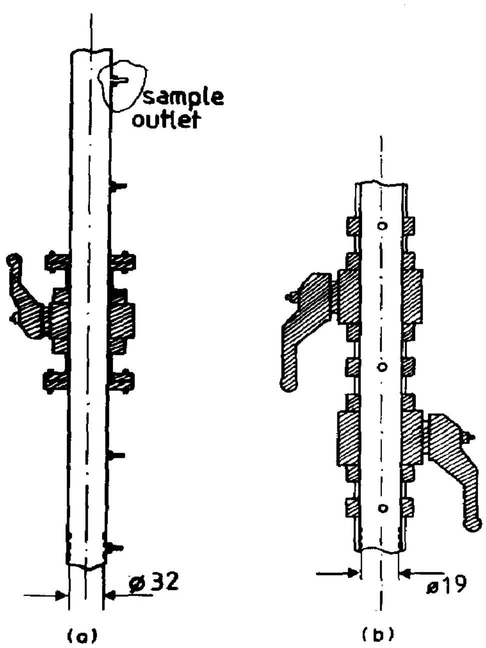


Fig. 2. View of cut along axis of columns. (a) 32 mm i.d.; (b) 19 mm i.d.

The frequency of slug injection, f, could be adjusted in the range $0 < f < 1.2\ s^{-1}$. In this paper, $f = 0$ is taken to mean that each slug was injected some time after eruption of the previous slug at the free surface (this corresponds to $f < 0.075\ s^{-1}$).

A photographic study was also made to help visualize the movement of liquid around the slug and in the wake region. Details of that study are provided elsewhere (Campos and Guedes de Carvalho, 1988).

MIXING MECHANISM

Visual inspection of slugs rising in water (confirmed by a photographic study) shows an orderly movement of liquid in the region around the nose and side of the slug and a turbulent movement in its wake.

An immediate consequence of this turbulence is that no significant radial concentration gradients are left behind a rising slug; the tracer is evenly distributed over each cross-section, and concentration gradients are significant only along the axial direction.

The contribution of molecular diffusion to tracer dispersion in this direction may be neglected in the time scale of one experiment (typically 5–20 min). For a time $t = 10^3$ s and a typical molecular diffusion coefficient $D'_m = 10^{-9}\ m^2/s$, the penetration of tracer by diffusion alone would be of the order of $(D'_m t)^{1/2} = 1$ mm, significantly below the measured length of dispersion, of the order of 1 m.

The sequence of events sketched in Fig. 3 helps to illustrate the way in which liquid movement around a rising slug brings about axial dispersion of the tracer. In Fig. 3(a), the slug is far below the horizontal surface separating the two layers of liquid (for graphical convenience the clear liquid is represented as dashed). In Fig. 3(b), the surface of separation has been distorted by the movement near the nose of the slug. In Fig. 3(c), the movement associated with the liquid film in free fall along the side of the slug is apparent, and the tracer is about to enter the wake. In Fig. 3(d), the tracer has reached the wake of the slug where it is dispersed by the turbulent recirculating flow observed there.

An approximate quantitative analysis of the extent of mixing associated with the flow field around the nose and side surface of the slug [suggested by (b) and (c) in Fig. 3] is presented in the Appendix; there, it is suggested that these contributions to mixing may be neglected.

THE "FULLY MIXED WAKE" MODEL OF MIXING

The basic assumption in this model is that mixing induced by a rising slug is due solely to the liquid movement in a perfectly mixed wake of volume V_w (to be determined). This is an oversimplified assumption because turbulence behind the slug dies out gradually with increasing axial distance from the slug. However, the existence of a region of intense recirculation in the wake is strongly suggested by photographs as in Fig. 4,

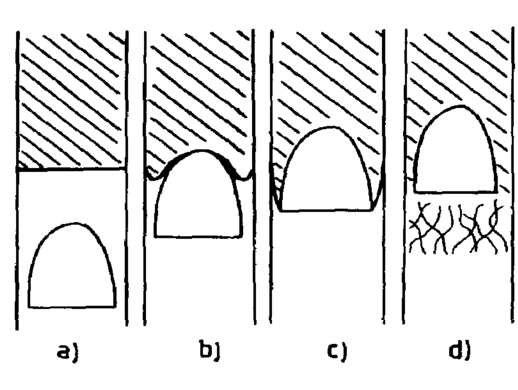


Fig. 3. Dispersion due to liquid movement around a rising slug.

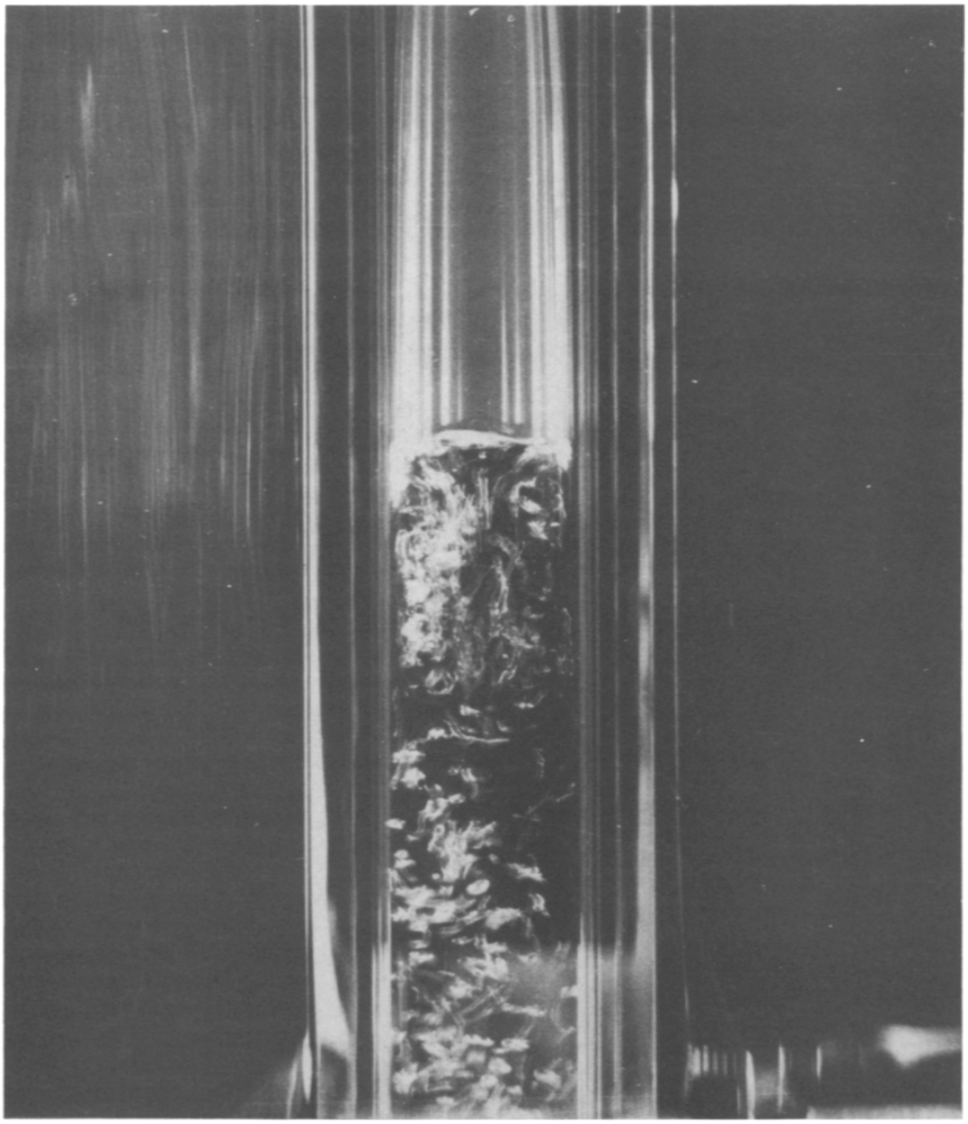


Fig. 4. Turbulent wake following air slug rising in water in a 52 mm i.d. column.

and the assumption adopted leads to a simple one-parameter model.

On a frame of reference moving with the slug, liquid flows down past it at a flow rate $v = U_s A$, where U_s is the rise velocity of the slug relative to the quiescent liquid and A is the cross-sectional area of the column. Therefore, the slug rise can be viewed (Fig. 5) as flow at a rate v through a perfectly mixed tank of volume V_w. In this way, the tracer concentration profile $C_i(z)$, following the passage of the i-th slug, may be calculated if the profile $C_{i-1}(z)$ prior to its injection is known.

Indeed, the material balance over a perfectly mixed wake leads to

$$v(C_{in} - C_{out}) = V_w \frac{dC_{out}}{dt} \tag{1}$$

where C_{in} and C_{out} represent the tracer concentrations at the inlet to and the outlet from the wake, respectively. At the level of the slug base, the liquid enters the wake and this is $l_w = V_w/A$ above the outlet from the wake, assuming the wake to occupy a cylindrical region immediately below the slug (see Fig. 5). Now, $U_s\,dt = dz$ and eq. (1) may be rewritten as

$$v[C_{i-1}(z+l_w) - C_i(z)] = U_s V_w \frac{dC_i(z)}{dz} \tag{2}$$

or

$$C_{i-1}(z+l_w) - C_i(z) = l_w \frac{dC_i(z)}{dz}. \tag{3}$$

For a known profile $C_{i-1}(z)$, this represents a differen-

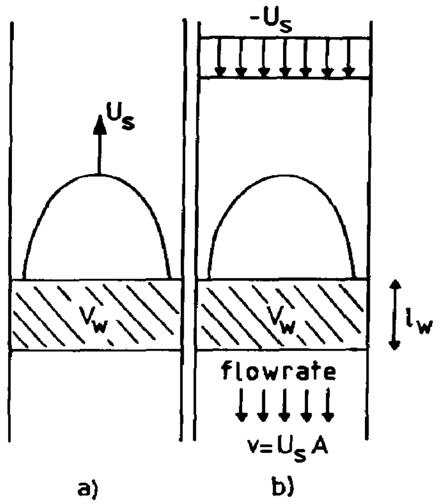


Fig. 5. Sketch of rising slug with attached wake. (a) Frame of reference fixed on the tube wall; (b) frame of reference moving with the slug.

tial equation which upon integration yields $C_i(z)$. The initial value $C_i(-H_B)$ is given by the obvious relation

$$C_i(-H_B) = \frac{1}{l_w}\int_{-H_B}^{-H_B+l_w} C_{i-1}(z)\,dz. \tag{4}$$

Equation (3) can be integrated only in the region $-H_B < z < (H_T - l_w)$, because there is no flow into the wake when the slug reaches the free surface. However, from the assumption of a perfectly mixed wake, there results $C_i(z) = C_i(H_T - l_w)$ for $(H_T - l_w) < z < H_T$.

For an assumed value of l_w (i.e. V_w), and for an initial concentration profile $C_0(z) = C^\circ$, $-H_B < z < 0$; $C_0(z) = 0$, $0 < z < H_T$, eq. (3) can be integrated N times, where N is the number of slugs injected, to yield the final calculated concentration profile $C_N(z)$. This procedure can be repeated for different values of l_w and the calculated profiles compared with the experimental values to determine the best value for l_w.

An important characteristic of the model is that for a given size of slug in a given column, l_w should be independent of the number N of slugs injected.

The analytical integration of eq. (3) is straightforward if the number N of slugs injected is small enough to have the tracer concentration at $z = -H_B$ and $z = H_T$ unchanged from the initial value. Under those conditions, the calculated concentration profile is similar to that resulting from a "step input" to a series of N well-mixed tanks of volume V_w:

$$C/C^\circ = \exp[-(z + Nl_w)/l_w] \times \left[1 + \sum_{j=1}^{N-1} \frac{1}{(j)!}\left(\frac{z + Nl_w}{l_w}\right)^j\right] \tag{5}$$

$$\text{for } -N l_w < z < H_T$$

and $C/C^\circ = 1$ for $-H_B < z < -Nl_w$.

When the number of slugs injected is large, eq. (3) has to be integrated numerically; a Runge–Kutta method was adopted.

THE "DIFFUSION MODEL" OF MIXING

When diffusion occurs in a narrow tube filled with liquid with initial tracer concentration distribution $C = C^\circ$ for $-H_B < z < 0$ and $C = 0$ for $0 < z < H_T$, the concentration profile varies with time, t, according to

$$\frac{C}{C^\circ} = \frac{H_B}{H_B + H_T} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)\sin\left(\frac{n\pi H_B}{H_B + H_T}\right) \times \exp\left[-\frac{(D't)n^2\pi^2}{(H_B + H_T)^2}\right]\cos\left[\frac{n\pi(H_B + z)}{H_B + H_T}\right] \tag{6}$$

where D' is the diffusion coefficient. Equation (6) satisfies Fick's law $\partial C/\partial t = D'(\partial^2 C/\partial z^2)$ with boundary conditions $\partial C/\partial z = 0$ at $z = -H_B$ and $z = H_T$, and gives the desired step concentration profile at $t = 0$ (Crank, 1975).

In an attempt to apply eq. (6) to the present study, it has to be realized that the passage of individual slugs rather than any random molecular motion in time is responsible for tracer dispersion. In other words, the final concentration profile after the injection of N slugs should be the same whether it takes 5 or 10 min to perform the experiment. Also, the injection of twice the number of slugs should have the same effect on the concentration profile as allowing a molecular diffusion process to run for twice as long. Therefore it makes sense to rewrite eq. (6) as

$$\frac{C}{C^\circ} = \frac{H_B}{H_B + H_T} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)\sin\left(\frac{n\pi H_B}{H_B + H_T}\right) \times \exp\left[-\frac{(\alpha N)n^2\pi^2}{(H_B + H_T)^2}\right]\cos\left[\frac{n\pi(H_B + z)}{H_B + H_T}\right] \tag{7}$$

where N is the number of slugs injected and α is the value of $(D't)$ equivalent to the action of one slug rising up the column. Again, if this model is to have any physical meaning, it is important that the value of α should depend only on the column diameter and the slug size, for a given liquid, and be independent of the number N of slugs injected.

For a low enough number of slugs injected, dispersion of the tracer is confined to the central portion of the column and eq. (7) gives the same profile as

$$C/C^\circ = \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left[\frac{z}{2\sqrt{(N\alpha)}}\right] \tag{8}$$

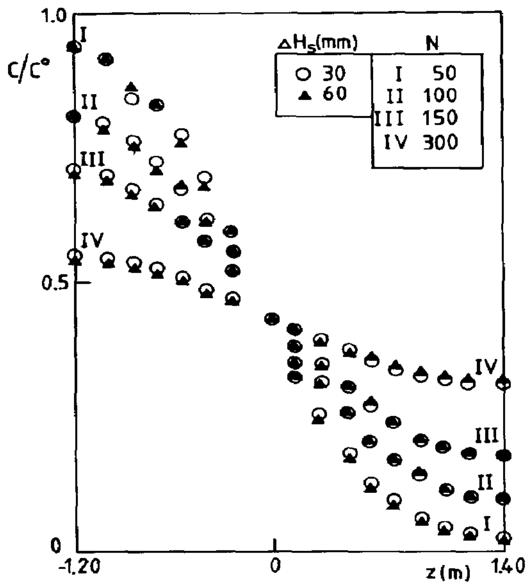
which is valid for dispersion in an infinite medium. A well-known analogy exists between the diffusion model and the tanks-in-series model in flowing systems (Levenspiel, 1972). This analogy may be adapted to the present situation and it leads to

$$\alpha = \frac{1}{2} l_w^2. \tag{9}$$

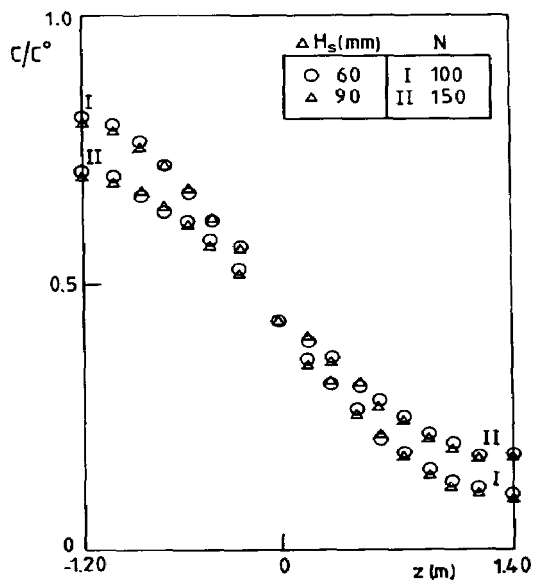
EXPERIMENTAL RESULTS FOR THE 32 mm I.D. COLUMN

(*i*) *Experiments with $f \simeq 0$*

In Figs 6(a)–6(c) a series of experimental concentration profiles is presented. These profiles suggest that the length of slugs injected in the column does not affect the extent of mixing observed. For the same number of slugs injected, the same final concentration profile is obtained with slug sizes in the range $30 < \Delta H_s < 125$ mm. This is a clear indication that flow in the wake dominates the process of mixing. Indeed, if flow around the side of the slugs were important in the process of mixing, longer slugs would lead to a greater extent of mixing. Furthermore, it may be concluded that the stirring action of the wake is the same for all

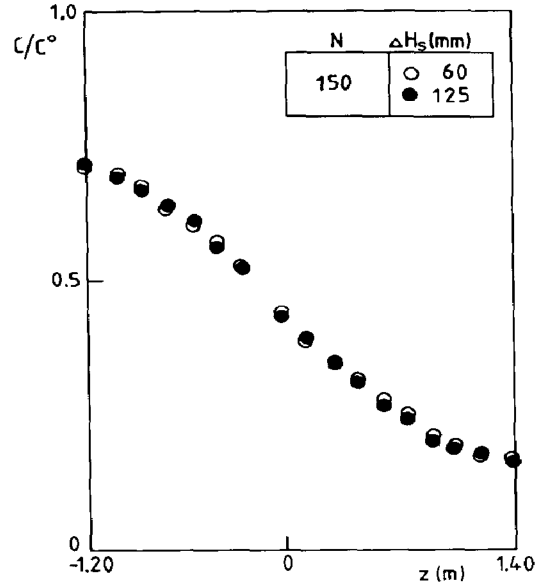


(a)



(b)

Fig. 6. (a) and (b).



(c)

Fig. 6. Experimental concentration profiles for the 32 mm i.d. column.

the sizes of slug compared. This is not surprising in view of the fact (Nicklin *et al.*, 1962) that the film of liquid running along the side of the slugs stabilizes (in velocity and thickness) above a certain distance from the slug nose. Therefore, the velocity profile in the liquid plunging in the wake is the same for all lengths of slug above a certain value, and the resulting wakes will then be insensitive to slug length; a discussion of this effect of slug length is presented elsewhere (Campos and Guedes de Carvalho, 1988). Further evidence of this aspect may be obtained from Fig. 7, where one of the concentration profiles from Fig. 6 is compared with that resulting from the injection of the same number of very short slugs ($\Delta H_s = 10$ mm). For the shorter slugs decreased mixing is observed and this is in line with the idea formulated above.

Model predictions are compared with experimental data for the long slugs in Fig. 8. The optimized value $l_w = 2.8D$ ($= 90$ mm), where D is the internal column diameter, is seen to fit the predictions of the perfectly mixed wake model to the experimental concentration profiles for all values of N. The predictions of the diffusion model for an optimum value $\alpha = 3.9 \times 10^3$ mm^2 are also seen to fit the experimental concentration profiles. The optimum values of l_w and α may be seen to agree with the prediction from eq. (9). The sensitivity of the predictions of the perfectly mixed wake model to the value of l_w adopted may be apprehended from Fig. 9. It may be stated with confidence that a value of l_w in the range $2.7D < l_w < 2.9D$ is a good representation of reality. This value should be compared with the suggestion of Dukler *et al.* (1985), namely $3.0D < l_w < 3.5D$, and with a value $l_w \simeq 2.0D$ obtained from our photographs of slugs

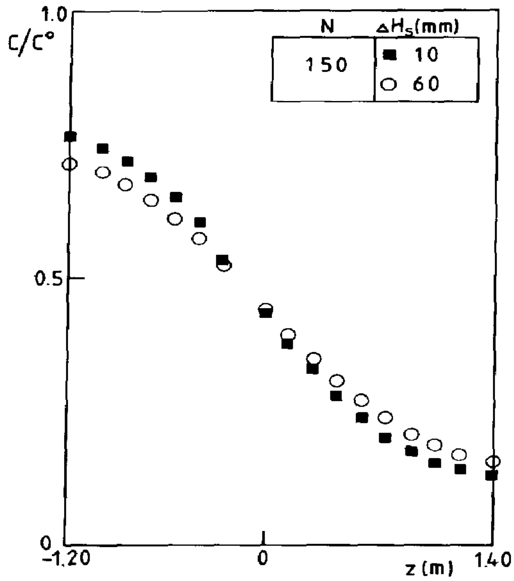


Fig. 7. Experimental concentration profiles for the 32 mm i.d. column.

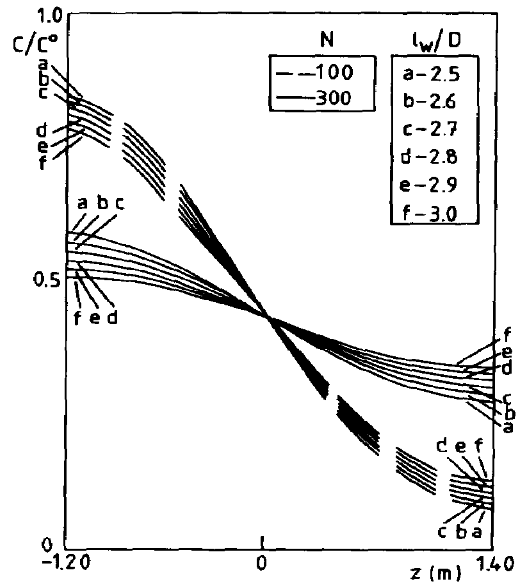


Fig. 9. Sensitivity of the model predictions to the value of l_w/D.

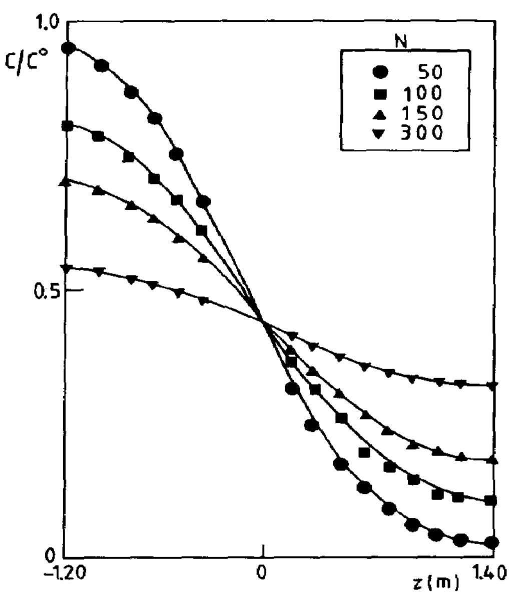


Fig. 8. Comparison model predictions with $l_w = 2.8D$, with experimental values for the 32 mm i.d. column.

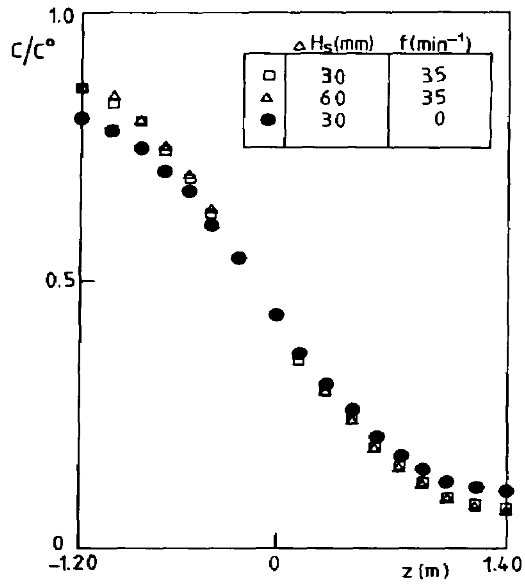


Fig. 10. Experimental concentration profiles for two frequencies of slug injection in the 32 mm i.d. column.

rising in a 52 mm i.d. column of liquid at a similar Reynolds number. The fact that the value obtained by photography is below the range suggested by the experiments on mixing is not surprising since turbulence dies out gradually as the recirculating zone of the wake is left behind, whereas in the present mixing model all mixing is assumed to take place in the recirculating wake.

(ii) *Experiments at higher values of f*

All the experimental results presented so far refer to a very low frequency of slug injection (i.e. $f \simeq 0$), each slug being injected after eruption at the surface of the previous one. Experimental results are reported in Figs 10 and 12 for slug injection at higher frequencies. Figure 10 shows that an increased injection frequency leads to a decreased extent of mixing for the same size and number of slugs injected. It is also apparent that for different sizes of slug ($\Delta H_s = 30$ and 60 mm) injected at the same frequency, the same final concentration profile of tracer is obtained. It is important to understand how the mixing intensity is affected by the injection frequency. The suggestion is that slugs rising in the column "detach" from their recirculating wakes when a new slug is injected at the bottom, and this leads to decreased axial dispersion. This idea was first conceived after observation of columns of viscous liquids in which slugs were being injected at a moderate frequency. Elsewhere, Campos and Guedes de Carvalho (1988) report that slugs rising in liquids with a viscosity within a certain range carry behind them a closed wake, as shown in Fig. 11(a). Detachment of the wake is obvious in Fig. 11(b), taken shortly after injection of a new slug at the bottom of the liquid column. Unfortunately, this type of photographic study is not possible in water columns, because the wakes are turbulent in low viscosity liquids and they do not carry enough dye along with them to give contrast to a picture. Indirect evidence had to be sought then for this idea of wake detachment, and it was decided to carry out a particular type of experiment in slugging columns of water, similar in all respects to those described previously, but with one particularity: the frequency of slug injection was adjusted so that only one slug was present in the column when the next one was injected. At that frequency of injection, each slug rose in the column to a level, z_{inj} (the same for all slugs), before injection of the next slug, and the value z_{inj} could be chosen by appropriate selection of the injection frequency. The results of such experiments are presented in Figs 12(a)–12(d) in which the final concentration profiles after the injection of 150 slugs at frequencies $f = 0$, 6 and 7 min^{-1} are compared. In each of these plots it is apparent that the profiles obtained for $f = 6$ and 7 min^{-1} intersect the profile for $f = 0$ at about the level z_{inj}.

This agrees well with the suggestion made above that the tracer-rich wake is left behind at level z_{inj} upon injection of each new slug; the extent of tracer transport above that level decreases then, as illustrated in Fig. 12. The practical importance of this phenomenon of decreased mixing in pulsating flow is obvious but no further attempt was made to model it.

Following the idea put forward by one referee of the present paper, that "wake detachment can have other hydrodynamic implications. For example, the rise velocity of the slugs can be affected; especially that of the trailing one might exhibit a hindered rise", we undertook to measure the velocity of rise of the slugs with pulsated gas feed and compared it with that of the string of slugs when the gas feed was interrupted. Absolute velocities were measured by timing the rise of individual slugs between two lines 1.5 m apart, and relative velocities were calculated from these by subtraction of the average liquid velocity between slugs. Although the values obtained are not very accurate, a "hindrance effect" was detected in some cases with a decrease in slug velocities of up to 4%. The relatively large distance between consecutive slugs no doubt explains why the effect is small.

EXPERIMENTAL RESULTS FOR THE 19 mm I.D. COLUMN

Phenomena other than flow in the wake are important in determining the extent of mixing in this narrower column. This is apparent from inspection of the profiles in Fig. 13, which shows that for the same number of slugs injected, longer slugs lead to a higher degree of mixing at $f = 0$. This was not the case in the 32 mm column, and an explanation is needed here. It is possible that mixing is induced in the process of slug injection, and also that mixing occurring during flow of the liquid alongside the slug surface is not negligible. The latter aspect will be considered in the Appendix.

(i) *Experiments with $f = 0$ and the importance of mixing during slug injection*

In the process of slug injection the liquid in the column is displaced forward, but the profile associated with this displacement is not uniform, as suggested by Fig. 14. This movement of liquid is bound to bring about axial dispersion, much in the same way as considered by Taylor (1954) for flow in tubes, and the analysis has to be adapted to the present situation.

Consider a liquid column with an axial concentration profile of tracer $C_i(z)$ before injection of slug $i + 1$. No radial concentration gradients are assumed to exist because the rise of the i-th slug has brought about radial uniformity at all levels. The liquid undergoes an average forward displacement, say ΔH, as a result of slug injection, and this will lead to radial concentration profiles which are not negligible because the forward displacement of liquid elements will depend on their radial coordinate. It is required to calculate the average concentration at each level z. This is given as

$$\overline{C_i^*}(z) = \frac{1}{\pi R^2}\int_0^R C_i^*(r, z)\,2\pi r\,\mathrm{d}r = \int_0^1 2C_i^*(r', z)\,r'\,\mathrm{d}r' \tag{10}$$

where $r' = r/R$ and $C_i^*(r, z)$ is the concentration at the point with coordinates (r, z) after injection of the $(i+1)$-th slug and before its rise up the column.

The elements of liquid not too close to the top or bottom of the column will move up in a straight path during slug injection, and for negligible molecular diffusion the values of $C_i^*(r, z)$ may be calculated from $C_i(z)$ and knowledge of the "displacement profile". For "Poiseuille flow", the forward displacement, h, relative to the average displacement is

$$h = \Delta H\,[1 - 2\,(r/R)^2]. \tag{11}$$

Differentiation of this equality gives $\mathrm{d}h = -4\Delta H r'\,\mathrm{d}r'$ and substitution in eq. (10) leads to

$$\overline{C_i^*}(z) = -\frac{1}{2\Delta H}\int_{+\Delta H}^{-\Delta H} C_i^*(r', z)\,\mathrm{d}h. \tag{12}$$

The element at (r', z) after slug injection originates from level $z - h$ and therefore $C_i^*(r', z) = C_i(z - h)$

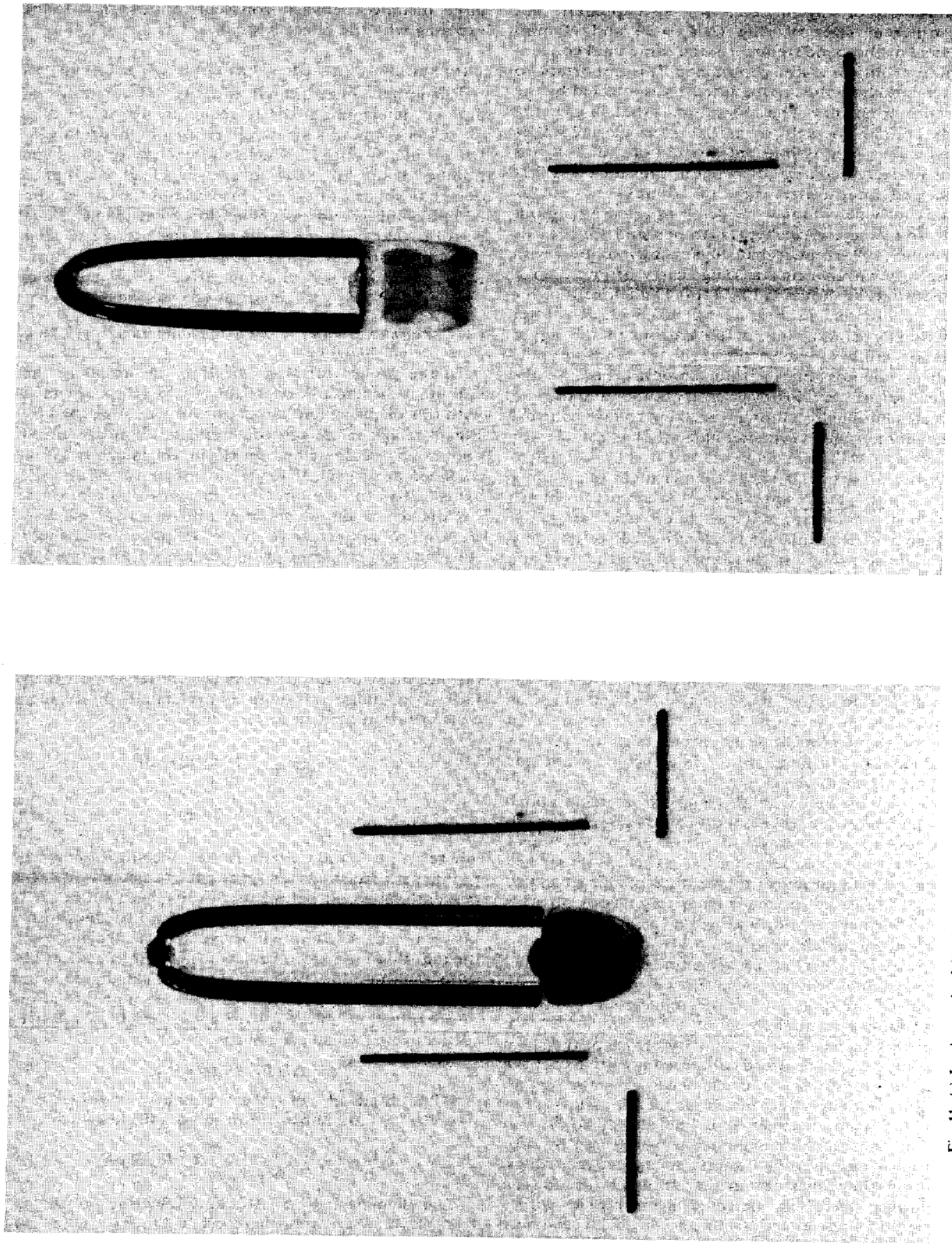


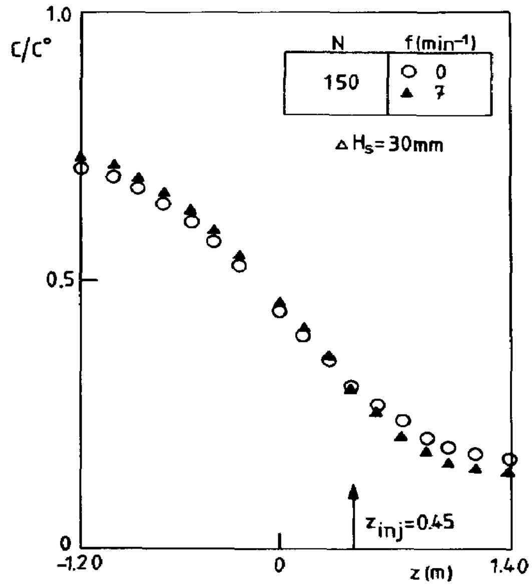
Fig. 11. (a) Laminar wake following air slug rising in a viscous liquid in the 19 mm i.d. column. (b) Detachment of laminar wake upon injection of a new slug.

may be used in eq. (12) to yield

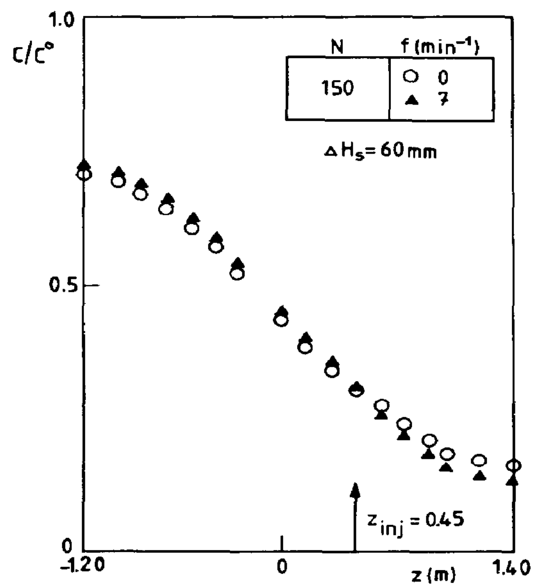
$$\overline{C_i^*}(z) = -\frac{1}{2\,\Delta H}\int_{\Delta H}^{-\Delta H} C_i(z-h)\,\mathrm{d}h. \tag{13}$$

This gives the average concentration distribution at any level immediately after slug injection, from knowledge of the concentration distribution $C_i(z)$ before slug injection.

Equation (13) is based on the assumption that the liquid moves forward in "Poiseuille flow" during slug injection, but this need not be so. Indeed, the forward

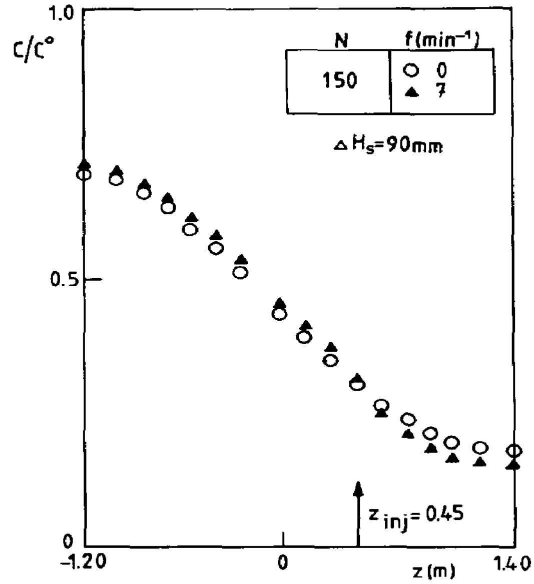


(a)



(b)

Fig. 12. (a) and (b).



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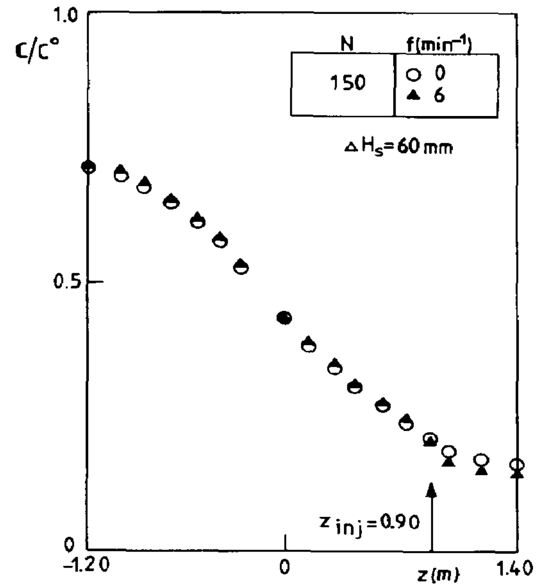

(d)

Fig. 12. Experimental concentration profiles suggestive of wake detachment (32 mm i.d. column).

movement of the liquid is not steady; it is an accelerated–decelerated movement starting from rest. An exact analysis of the transient displacement profile similar to that presented by Szymanski (1932) would be difficult here because the pressure at the bottom of the column is not a simple function of time. A semi-empirical approach was then adopted to describe the displacement of the liquid.

Photographs were taken of the surface of separation clear-coloured water (initially flat) immediately after

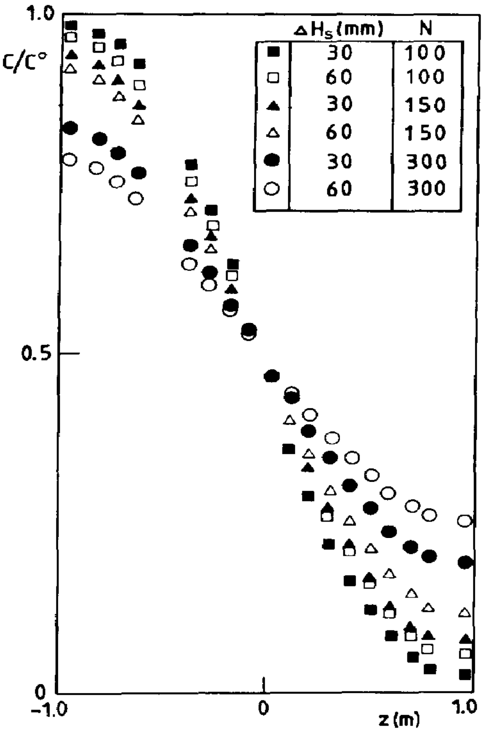

Fig. 13. Experimental concentration profiles in the 19 mm i.d. column.

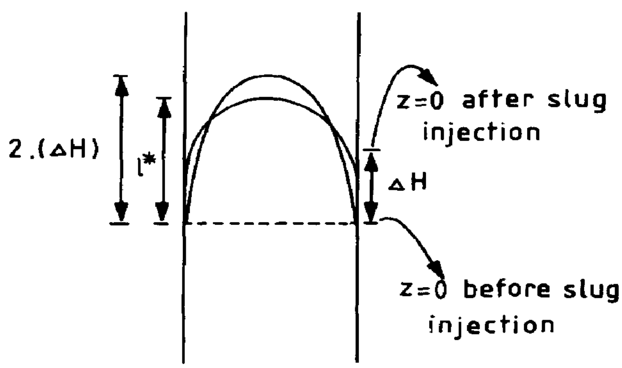

Fig. 14. Displacement of liquid due to slug injection.

the injection of one slug, in an attempt to determine its shape and therefore the displacement profile resulting from slug injection. From a series of photographs, for a range of volumes injected, the shape sketched in Fig. 14 was drawn. The effective displacement l^* at the axis fell invariably in the range $1.5\Delta H < l^* < 1.7\Delta H$ in contrast with the value $l^* = 2\Delta H$ which would be observed for "Poiseuille flow". In view of this, it was decided to adopt the approximate expression for the displacement:

$$h = \gamma \Delta H[1 - 2(r/R)^2] \tag{14}$$

where γ is a constant with a value between 0.5 and 0.7 ($\gamma = 0$ and $\gamma = 1$ would mean piston flow and "Poiseuille flow", respectively).

Equation (13) was accordingly rewritten as

$$\overline{C_i^*}(z) = -\frac{1}{2\gamma\Delta H}\int_{+\Delta H}^{-\Delta H} C_i(z-h)\,\mathrm{d}h. \tag{15}$$

The values of $\overline{C_i^*}(z)$ in the bottom and top sections of length $\gamma\Delta H$ were calculated assuming that the liquid which is forced into those sections is fully mixed. Hence

$$\text{(top)}\quad \overline{C_i^*}(z) = \frac{1}{\pi R^2 \gamma \Delta H}\int_0^R 2\pi r \times \int_{H_T - 2\gamma\Delta H[1-(r/R)^2]}^{H_T} C_i(z)\,\mathrm{d}z\,\mathrm{d}r, \tag{16}$$

$$(H_T - \gamma\Delta H) < z < H_T$$

$$\text{(bottom)}\quad \overline{C_i^*}(z) = \frac{1}{\pi R^2 \gamma \Delta H}\int_0^R 2\pi r \times \int_0^{2\gamma\Delta H (r/R)^2} C_i(z)\,\mathrm{d}z\,\mathrm{d}r, \tag{17}$$

$$-H_B < z < (-H_B + \gamma\Delta H).$$

The profile $C_{i+1}(z)$ observed after the rise of the slug up the column may now be calculated from $\overline{C_i^*}(z)$, by means of the perfectly mixed wake model described above. It should be noted at this stage that the values $\overline{C_i^*}(z)$ obtained from eq. (15) are only calculated averages since in reality the liquid is not mixed radially after slug injection. However, these calculated average values are appropriate if it is assumed that no further axial dispersion occurs in the liquid before the inlet to the wake (i.e. in the flow around the nose and side surface of the slug). This is because the wake is taken as fully mixed and consideration of perfect radial mixing at its inlet does not affect the calculations.

At this stage it is possible to calculate concentration profiles after injection of any number N of slugs, based on a model which accounts both for dispersion in slug injection and dispersion induced by the rising wake. Starting from an initial concentration profile ($C_0(z) = C^\circ, -H_B < z < 0; C_0(z) = 0, 0 < z < H_T$), the profile $\overline{C_0^*}(z)$ is calculated from eqs (15) and (17) after which the profile $C_1(z)$ is obtained by integration of eq. (3), where $C_{i-1}(z + l_w)$ is replaced by $\overline{C_0^*}(z + l_w)$. This procedure is then repeated as many times as the number of slugs injected to obtain the final concentration profile $C_N(z)$.

Computed concentration profiles are presented in Figs 15(a) and 15(b); from these, the contribution of slug injection to the overall mixing process may be assessed. Figure 15(a) shows the sensitivity of the calculated profile to γ to be very small for the 32 mm i.d. column. This indicates negligible contribution to mixing in the process of slug injection, and this had been found experimentally, as shown in Fig. 6. For the 19 mm i.d. column, Fig. 15(b) shows that mixing in the injection period contributes significantly to the overall

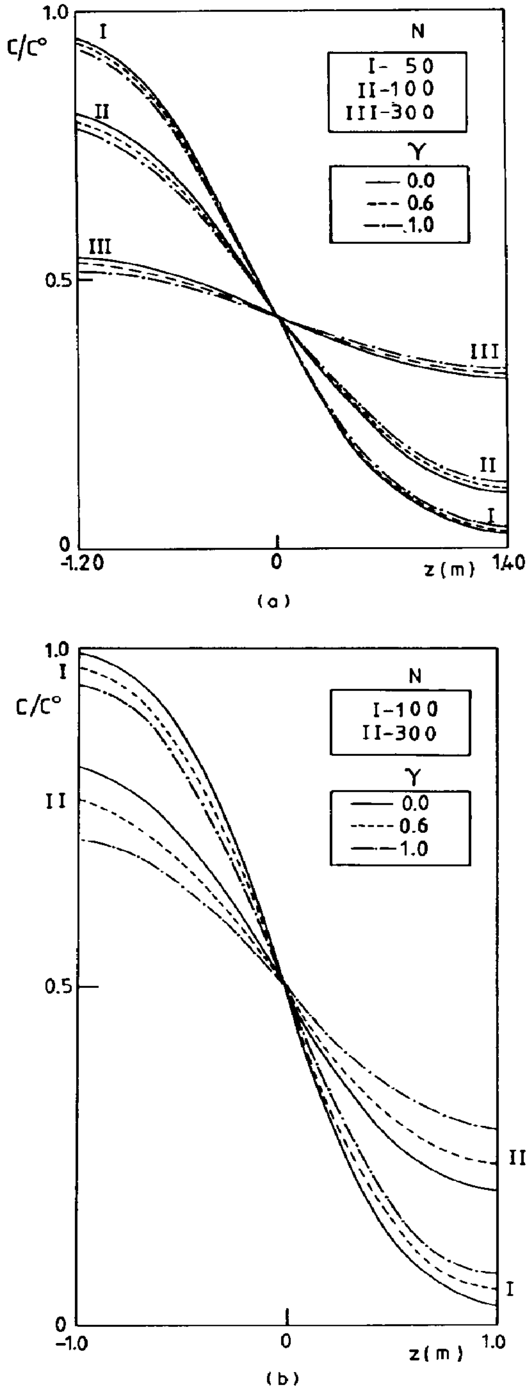

Fig. 15. Calculated concentration profiles showing the importance of mixing due to injection of slug ($\Delta H_s = 60$ mm). (a) 32 mm i.d. column, $l_w = 2.8D$; (b) 19 mm i.d. column, $l_w = 2.3D$.

mixing observed. Photographs of the displacement profile during injection mentioned above suggest $0.5 < \gamma < 0.6$, and in this range the sensitivity of the computed concentration profiles to the value of γ is slight. Figures 16(a) and 16(b) show the final concen-

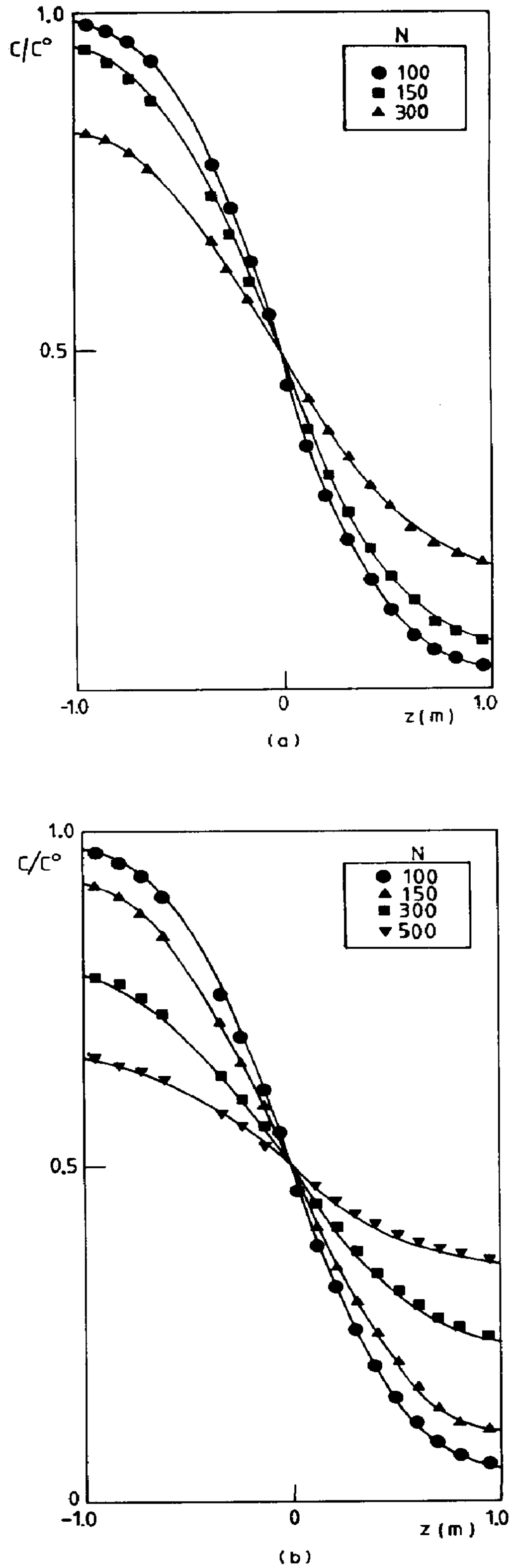

Fig. 16. Comparison of the model predictions, with $l_w = 2.3D$ and $\gamma = 0.6$, with the experimental values for the 19 mm i.d. column. (a) $\Delta H_s = 30$ mm; (b) $\Delta H_s = 60$ mm.

tration profiles calculated with $\gamma = 0.6$ for a range of slug lengths, with a constant optimized value $l_w = 2.3D$ for different values of N. Agreement between the model predictions and experiment is excellent with a value $l_w/D = 2.3 \pm 0.1$ for slug sizes $30 < \Delta H < 60$. For longer slug sizes, "Taylor dispersion" during injection dominates over the wake effect and the prediction of concentration profiles becomes less sensitive to the value of l_w.

An analysis of experimental data in terms of the dispersion model was also undertaken, but here the "best" values of α from curve fitting depend on the slug size as expected. The values 500, 900 and 1200 mm^2 were obtained for α, corresponding respectively to 10, 30 and 60 mm for ΔH.

(ii) *Experiments at higher f*

In Fig. 17 it is apparent that mixing is reduced when the frequency of slug injection is increased, up to a point. This is in line with the observations on the 32 mm i.d. column and the explanation advanced before is likely to hold here too.

CONCLUSIONS

The study of mixing presented above is thought to be a significant improvement over previous approaches to the subject in that a connection is established between the hydrodynamics of the wake and the "stirring" action of the slugs. A model of mixing was developed based on the idea of a fully mixed wake and this requires only one parameter, the wake length (l_w), to be adjusted.

Experiments were performed on columns of water with internal diameters of 32 and 19 mm, and the values of the Reynolds number ($D \cdot U_s/\nu$) for the rising slugs are 6.3×10^3 and 2.9×10^3, respectively. The corresponding values of l_w obtained by fitting model predictions to experimental profiles are $l_w = 2.8D$ and $l_w = 2.3D$, respectively, for each of the columns used. In the 19 mm i.d. column "Taylor dispersion" in the process of slug injection is not negligible, and a simple modification of the above model was developed to take this effect into account.

An alternative interpretation of the results in terms of a "diffusion model" was also presented.

A significant aspect of the models presented is that they lead to the determination of a parameter which quantifies the extent of mixing induced by individual slugs. This is important from the fundamental point of view and makes it possible to predict accurately the extent of mixing induced by the rise of any number of equal-sized slugs along a given column. This is expected to be an improvement over previous approaches. For example, van Heuven and Beek (1963), in their pioneering work, do not associate the extent of mixing with the number of slugs present in the column; Thiel and Potter (1978) come closer to the present approach.

Experiments were also performed on mixing of liquid with a pulsated gas feed and it was observed that an increased frequency of slug injection leads to decreased mixing of the liquid. A simple experiment suggests that this effect is related to detachment of the wakes of slugs in the pulsating liquid.

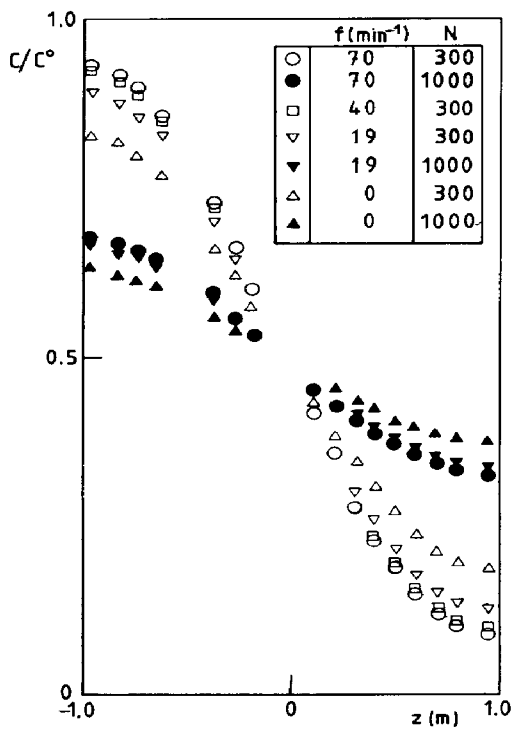

Fig. 17. Influence of the frequency of slug injection on the final concentration profile in the 19 mm i.d. column, with $\Delta H_s = 30$ mm.

NOTATION

A	cross-sectional area of column
C	concentration of tracer
C°	initial concentration of tracer
C_i	concentration profile of tracer following the passage of the i-th slug
C_i^*	concentration of tracer in each point of the column immediately after injection of slug $i+1$
$\overline{C_i^*}$	average concentration of tracer at a level z in the column after injection of slug $i+1$
C_{in}	concentration of tracer at inlet to the wake
C_{out}	concentration of tracer at outlet from the wake
D	internal diameter of column
D'	coefficient of molecular diffusion
f	frequency of slug injection
h	forward displacement relative to the average displacement (ΔH), resulting from slug injection (dependent on r)
H_B	height of coloured liquid initially present in the column
H_T	height of colourless liquid initially present in the column

l^* effective displacement of liquid on the axis resulting from slug injection
l displacement of liquid on the axis relative to the average displacement, resulting from slug injection ($= l^* - \Delta H$)
l_w length of column equivalent to fully mixed wake
L length of cylinder equivalent to slug—see Appendix
N number of slugs injected
Q liquid flow rate past the slug
r radial coordinate
r' dimensionless radial coordinate ($= r/R$)
R internal radius of column
Re_δ Reynolds number for flow in liquid film running between slug surface and tube wall
t time
u velocity in free falling laminar liquid film, relative to the slug
u' velocity in free falling laminar liquid film, relative to the wall
u'_m average velocity in free falling laminar liquid film, relative to the wall
U_s velocity of rise of slug
v flow rate of liquid through the wake
V_W volume of wake
y distance from the wall
z longitudinal coordinate
z_{inj} level at which slug is in the column when next one is injected

Greek letters

α value of $(D't)$ equivalent to one slug
γ constant related to liquid flow in the column due to slug injection
δ thickness of free falling film
ΔH average forward displacement of liquid resulting from slug injection
ΔH_s drop in liquid level as slug erupts at the free surface
ν kinematic viscosity

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APPENDIX. MIXING DUE TO LIQUID MOVEMENT ALONG THE FRONT AND SIDE SURFACE OF THE SLUG

Figure A1 shows some of the features of the flow field seen by an observer moving with a rising slug. Sufficiently ahead of the slug, above A–A say, the velocity profile is uniform, $u = U_s$. For large enough values of z, say below B–B the liquid flows with a steady velocity profile corresponding to flow under gravity along a vertical cylindrical wall. In the region between A–A and B–B, it is not easy to describe the flow field accurately. However, in an attempt to predict the distortion of a time-line which is horizontal above A–A, this region may be neglected if the slug is long enough. This is because the distance from A–A to B–B is independent of the slug length whereas the distance from B–B to C–C increases linearly with the slug length and therefore distortion in this region will dominate for long enough slugs.

Now consider the elements of a fluid on a horizontal plane before injection of the slug in the column [line O–O in Fig. A2(a)]. Imagine the simplified picture in Fig. A2(b) to represent the situation some time after the injection of a slug of volume $V_s = A(\Delta H)$; the extreme case of "Poiseuille flow" of the liquid during slug injection is assumed. It is required to calculate the approximate shape of the line O–O after slug rise up the column, neglecting any mixing in the wake.

The velocity profile in a laminar liquid film of thickness δ is given (e.g. Kay and Nedderman, 1974) by $u' = (g/\nu)(y\delta - y^2/2)$ if wall curvature is neglected. Here, y is the distance from the wall and ν is the kinematic viscosity. Relative to the slug, rising at velocity U_s, the velocity profile reads

$$u = U_s + (g/\nu)(y\delta - y^2/2) \tag{A1}$$

and the liquid flow rate past the slug may be calculated as $Q = \pi D \int_0^\delta u\,dy$, with good approximation. Since, on the other

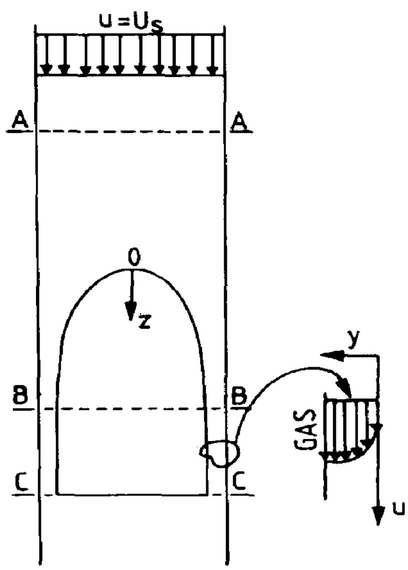

Fig. A1: Sketch of the velocity profile in the liquid ahead of the slug and in the falling film.

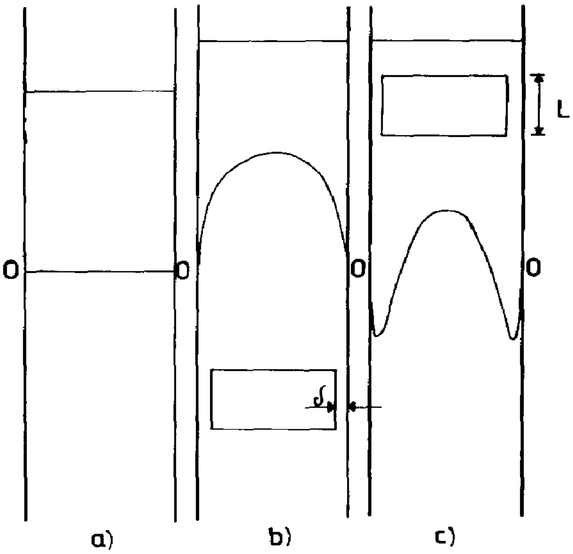

Fig. A2. Idealized drift of liquid elements initially on a cross-sectional surface due to successive action of injection and flow along the side surface of a slug.

hand, $Q = (\pi/4)D^2 \cdot U_s$, an equation giving δ may be obtained:

$$U_s = (2/3)(g/\nu)R^2(\delta/R)^3[1-2(\delta/R)]^{-1}. \qquad (A2)$$

The length L [Fig. A2(b)] may be obtained from δ as $L = V_s/[\pi(0.5D-\delta)^2]$.

Now the streamline at a distance r from the axis of the column far above the slug nose runs along the film at a distance y from the wall given by continuity as

$$\pi(0.5D-r)^2\, U_s = \pi D \int_0^y u \, dy'. \qquad (A3)$$

By means of eqs (A1) and (A3), it is possible to calculate the velocity at which elements at different initial radial positions on O–O run the distance L taken to represent the liquid film around the slug in Fig. A2(b). If the fluid elements are considered to move with velocity U_s ahead of and behind the slug, the shape of the line O–O after slug passage may be calculated and it is sketched in Fig. A2(c). It should be recalled that this line was obtained accounting only for distortion during slug injection ("Poiseuille flow" of the liquid) and in flow along the side of the slug; flow near the nose was neglected and flow in the wake is treated in the main text of this paper.

Figure A3 shows a magnified view of line O–O in Fig. A2(c) and the line which would correspond to distortion due to slug injection alone ("Poiseuille flow"). The close similarity between the two lines justifies the assumption made in this paper that mixing due to flow along the side surface of the slug is negligible in comparison with mixing induced by slug injection.

The arguments outlined above refer to the situation of laminar flow in the liquid film running between the slug and the tube wall. A Reynolds number, Re_δ, for liquid film flow down a vertical wall may be defined, $Re_\delta = 4u'_m\delta/\nu$, where u'_m is the average fluid velocity in the film of thickness δ, relative to the wall. According to Kay and Nedderman (1974), turbulent flow is likely to occur in the liquid film for $Re_\delta > 2000$. It is easy to prove that for the liquid film running alongside the slug $Re_\delta \simeq \rho U_s D/\mu$, and if $U_s \simeq 0.35\sqrt{gD}$ there results $0.35(gD^3)^{1/2}\rho/\mu > 2000$ as a criterion for turbulent flow in the liquid film. For cold water (i.e. $\rho = 10^3$ kg/m^3 and $\mu = 10^{-3}$ N s/m^2), the same criterion leads to $D > 15$ mm and this suggests that in the experiments reported the film could have been in the turbulent regime. This would mean a flatter velocity profile than is given by eq. (A1) and as a result a near coincidence between the dashed line and the solid line in Fig. A3.

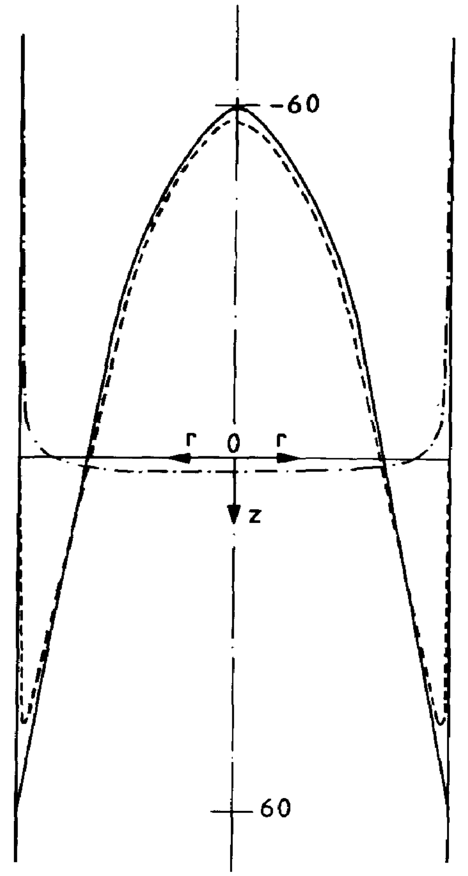

Fig. A3. Drift of liquid initially on the cross-sectional surface, calculated by the approximate method described in the Appendix, for the injection of a slug ($\Delta H_s = 60$ mm) in a 19 mm column of water. (———) Forward displacement: Poiseuille flow; backward displacement: piston flow. (– – – –) Forward displacement: Poiseuille flow; backward displacement: falling film flow. (— · — ·) Forward displacement: piston flow; backward displacement: falling film flow.