Advanced Digital Signal Processing (ADSP)

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Main content ADSP course

- Part A: Stochastic Signal Processing
- Part B: Adaptive Signal Processing
- Part C: Array Signal Processing (ASP) (including DOA)
- **Part D: Adaptive Array Signal Processing (AASP)**

Contents

- Direction of Arrival (DOA) estimation
- Optimum (data-dependent) beamforming
  - Minimum mean squared error (MMSE)
  - Multiple sidelobe canceller (MSC)
  - Linearly constrained minimum variance (LCMV)
  - Minimum variance distortionless response (MVDR)
  - Generalized sidelobe canceller (GSC)
- Adaptive beamforming

  Adaptive versions of the MMSE, MSC, MVDR, LCMV and GSC

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Introduction to DoA estimation

- Estimate direction of arrival of sources (desired and/or interfering) from noisy observations

- Applications

  E.g., Tracking and presence detection for smart lighting;

  Detecting active talker in an video-conference to automatically steer a video camera;

  Surveillance applications, etc.

Techniques to DoA estimation

- Maximize steered response power
- Using high resolution spectral estimation concepts
- Using time difference of arrival

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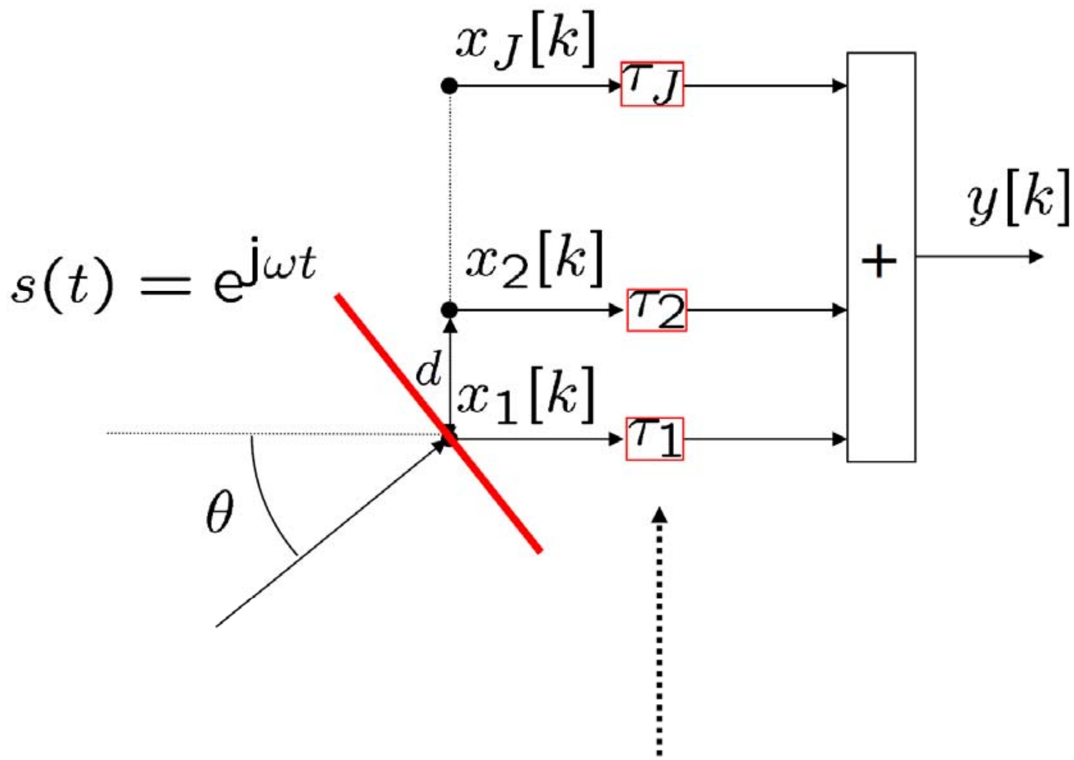
Data independent beamforming



For ULA choose: $\tau_i = (i-1)\tau \;\leftrightarrow\; w_i^* = e^{j(i-1)\omega\tau}$

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(DoA) Maximizing steered response power

- Consider one source located at θ^* and J sensors
- Input signals: $\underline{\mathbf{x}}[k] = \underline{\mathbf{a}}(\theta^*) \cdot s[k] + \underline{\mathbf{n}}[k]$
- Autocorrelation: $\mathbf{R}_x = \sigma_s^2 \underline{\mathbf{a}}(\theta^*)\underline{\mathbf{a}}^h(\theta^*) + \sigma_n^2 \mathbf{I}$
- We know that a beamformer steered towards θ^* maximizes the SNR, with weights $\underline{\mathbf{w}} \equiv \underline{\mathbf{a}}(\theta^*)$ (matched filter)

$$
\begin{aligned}
P_y &= \sigma_s^2 \cdot \underline{\mathbf{w}}^h(\underline{\mathbf{a}}(\theta^*) \cdot \underline{\mathbf{a}}^h(\theta^*))\underline{\mathbf{w}} + \sigma_n^2 \cdot \underline{\mathbf{w}}^h \underline{\mathbf{w}} \\
&= P_s + P_n
\end{aligned}
$$

Maximum SNR $\Rightarrow$ Maximum P_y

(DoA) Maximizing steered response power

- Steer the beamformer to a number of candidate directions θ.

- Output power:
  $P_y(\theta) = E\{|y[k]|^2\} = \underline{\mathbf{w}}^h \cdot \mathbf{R}_x \cdot \underline{\mathbf{w}}$

- Spatial spectrum: $P(\theta) = \frac{P_y(\theta)}{|\underline{\mathbf{w}}|^2} = \frac{\underline{\mathbf{a}}^h(\theta)\mathbf{R}_x\underline{\mathbf{a}}(\theta)}{J}$

- Clearly $P(\theta)$ attains its maximum when $\theta = \theta^*$. Thus, the peak in $P(\theta)$ is the DoA estimate

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(DoA) Maximizing steered response power

ULA, $d/\lambda = 1/2$, $P = 3$ source, $J = 8$ sensors

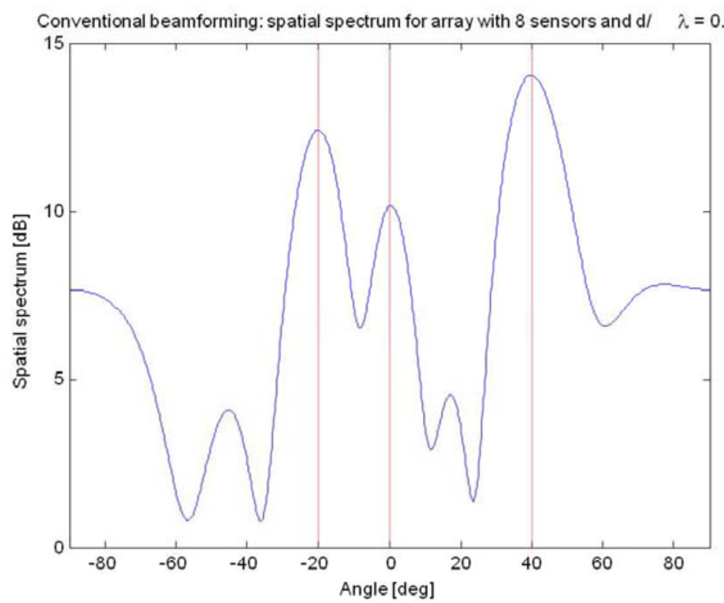

Figure: DoA estimation by steered response power method: sources at -20, 0 and 40 degrees

(DoA) Maximizing steered response power

- In practice, the search space is discretized depending on the desired accuracy

- Efficient search strategies may be implemented

- The autocorrelation matrix may be computed as $\hat{\mathbf{R}}_x = \frac{1}{T}\sum_{t=1}^{T} \underline{\mathbf{x}}[t] \cdot \underline{\mathbf{x}}^h[t]$

Techniques to DoA estimation

- Maximize steered response power
- Using high resolution spectral estimation concepts
- Using time difference of arrival

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(DoA) MUltiple SIgnal Classification: MUSIC

- DoA based on high-resolution spectral estimation
- MUSIC: MUltiple SIgnal Classification
- Employs signal subspace method

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(DoA) Multiple Signal Classification: MUSIC

- Consider J sensors and P sources, $P < J$
- $x_i[k] = \sum_{p=1}^{P} a_i[\theta_p] s_p[k] + n_i[k], \quad \mathrm{i} = 1 \ldots \mathrm{J}$
- $\underline{\mathbf{x}}[k] = \mathbf{A}\underline{\mathbf{s}}[k] + \underline{\mathbf{n}}[k]$
- $\mathbf{R}_x = E\{\underline{\mathbf{x}\mathbf{x}}^h\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^h + \mathbf{R}_n$
- $\mathbf{R}_s = \mathrm{diag}(\sigma_{s_1}^2, \ldots, \sigma_{s_P}^2)$
- $\mathbf{A}\mathbf{R}_s\mathbf{A}^h$ is a $J \times J$ matrix of rank P, and has $J - P$ zero eigenvalues

(DoA) Multiple Signal Classification: MUSIC

- Let $\underline{\mathbf{u}}_i$ be an eigenvector of $\mathbf{A}\mathbf{R}_s\mathbf{A}^h$ corresponding to one of the zero eigenvalues
- Then $\mathbf{A}\mathbf{R}_s\mathbf{A}^h\underline{\mathbf{u}}_i = 0$
- $\Rightarrow \underline{\mathbf{u}}_i^h\mathbf{A}\mathbf{R}_s\mathbf{A}^h \cdot \underline{\mathbf{u}}_i = 0$ or $(\mathbf{A}^h\underline{\mathbf{u}}_i)^h\mathbf{R}_s(\mathbf{A}^h\underline{\mathbf{u}}_i) = 0$
- $\Rightarrow \mathbf{A}^h\underline{\mathbf{u}}_i = 0$, as $\mathbf{R}_s$ is positive definite
- So, the eigenvectors corresponding to the $J - P$ eigenvalues that are zero are orthogonal to the steering vectors.

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(DoA) Multiple Signal Classification: MUSIC

- Let $\mathbf{U}_n$ denote the $J \times J - P$ matrix containing the $J - P$ eigenvectors corresponding to the eigenvalues that are zero
- Define $P_{SM}(\theta) = \frac{1}{\sum_{i=1}^{J-P} |\underline{\mathbf{a}}^h(\theta)\underline{\mathbf{u}}_i|^2} = \frac{1}{\underline{\mathbf{a}}^h(\theta)\mathbf{U}_n\mathbf{U}_n^h\underline{\mathbf{a}}(\theta)}$
- If θ corresponds to a source direction, then the denominator of $P_{SM}(\theta)$ becomes zero
- So, the P largest peaks of $P_{SM}(\theta)$ provide the source directions. $P_{SM}(\theta)$ is referred to as the pseudo-spectrum

(DoA) Multiple Signal Classification: MUSIC

- The analysis so far involved the EVD of $\mathbf{A}\mathbf{R}_s\mathbf{A}^h$, which is not available in practice; only $\mathbf{R}_x$ is. This turns out not to be a problem.
- For any $\underline{\mathbf{u}}_i \in \mathbf{U}_n$, $\mathbf{A}\mathbf{R}_s\mathbf{A}^h\underline{\mathbf{u}}_i = \lambda\underline{\mathbf{u}}_i$.
- $\Rightarrow \mathbf{R}_x\underline{\mathbf{u}}_i = (\mathbf{A}\mathbf{R}_s\mathbf{A}^h + \mathbf{R}_n)\underline{\mathbf{u}}_i = \lambda\underline{\mathbf{u}}_i + \sigma^2\underline{\mathbf{u}}_i = (\lambda + \sigma^2)\underline{\mathbf{u}}_i$
- So the eigen vectors of $\mathbf{R}_x$ are also the eigen vectors of $\mathbf{A}\mathbf{R}_s\mathbf{A}^h$! (note that we assumed the noise to be spatially white)

(DoA) Multiple Signal Classification: MUSIC

- Compute/Estimate $\mathbf{R}_x$
- Perform EVD of $\mathbf{R}_x$; determine $\mathbf{U}_n$ as the matrix containing the eigenvectors corresponding to the $J - P$ smallest eigenvalues
- Evaluate pseudo-spectrum:
  $P_{SM}(\theta) = \frac{1}{\underline{\mathbf{a}}^h(\theta)\mathbf{U}_n\mathbf{U}_n^h\underline{\mathbf{a}}(\theta)}$
- Locate P sharpest peaks in $P_{SM}(\theta)$

(DoA) Multiple Signal Classification: MUSIC

- The pseudo spectrum exhibits sharp peaks in the vicinity of the true DoAs
- P_{SM} averages $J - P$ pseudo spectra of individual noise sources. A large value of $J - P$ results in sharper peaks
- The name pseudo is used because P_{SM} contains no information about power
- In practice, $\hat{\mathbf{R}}_x = \frac{1}{T} \sum_{k=1}^{T} \underline{\mathbf{x}}[k] \underline{\mathbf{x}}^h[k]$

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(DoA) Multiple Signal Classification: MUSIC

ULA, $d/\lambda = 1/2$, $P = 3$ sources, $J = 4$ sensors

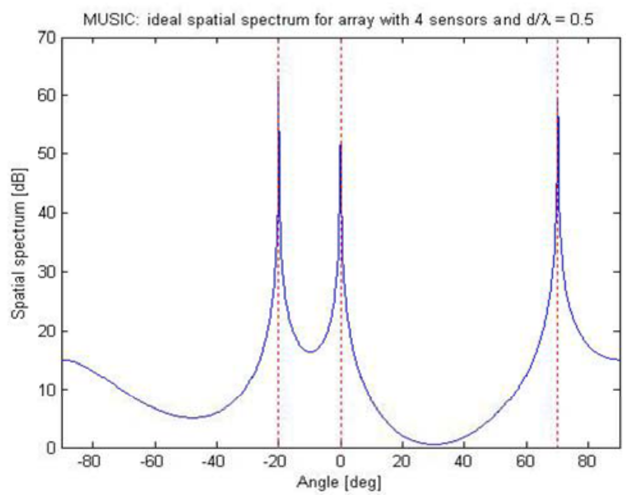

Figure: DoA estimation by Spectral MUSIC: sources at -20, 0 and 70 degrees

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(DoA) Multiple Signal Classification: MUSIC

ULA, $d/\lambda = 1/2$, $P = 3$ sources, $J = 4$ sensors

$P(\theta) = \underline{\mathbf{a}}^h(\theta)\mathbf{R}_x\underline{\mathbf{a}}(\theta)/J$

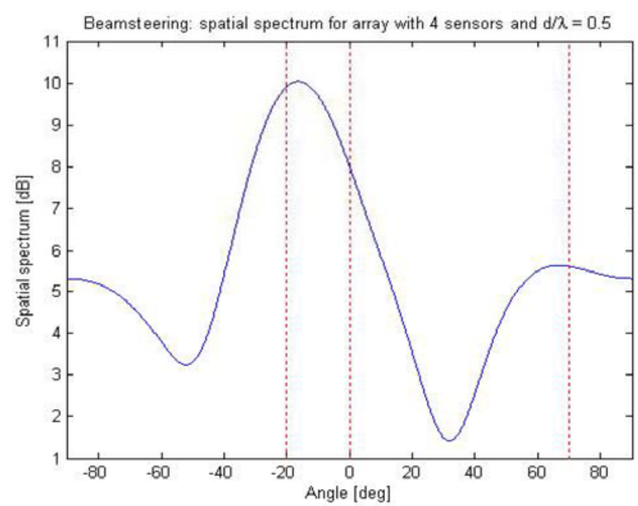

Figure: DoA estimation by beamsteering: sources at -20, 0 and 70 degrees

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(DoA) Multiple Signal Classification: MUSIC

ULA, $d/\lambda = 1/2$, $P = 4$ sources, $J = 10$ sensors

Larger $J - P$ in this case results in deeper minima

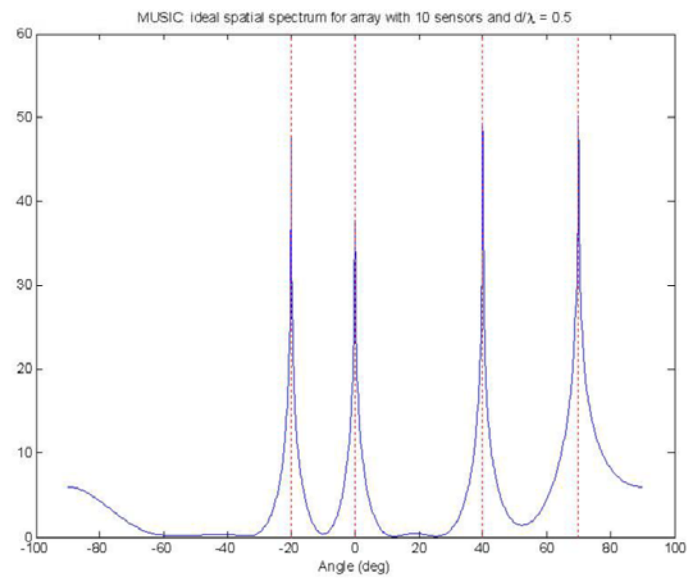

Figure: DoA estimation by Spectral MUSIC: sources at -20, 0, 40 and 70 degrees

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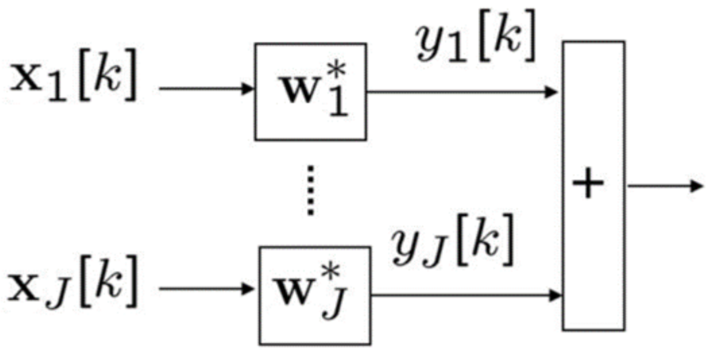
Data dependent beamforming

- The methods discussed so far compute beamformer weights regardless of data being processed
- Alternatively, weights can be based on the statistics of the data, obtained by optimizing a certain criterion

Data dependent beamforming

- Optimum: Assume knowledge of array data statistics. Optimal with respect to a certain optimization criterion (e.g., min MSE, max SINR)
- Adaptive: Estimate the required statistics as data becomes available (when statistics are unknown or time-varying). The derivation is done assuming known statistics. These statistics are estimated on-line.

Data dependent beamforming: MMSE



$$y[k] = \sum_{i=1}^{J} \mathbf{w}_i^h \mathbf{x}_i[k]$$

Short notation: $y[k] = \underline{\mathbf{w}}^h \cdot \underline{\mathbf{x}}[k]$

$$\underline{\mathbf{x}}[k] = (\mathbf{x}_1[k], \cdots \mathbf{x}_J[k])^t$$

$$\underline{\mathbf{w}} = (\mathbf{w}_1, \cdots, \mathbf{w}_J)^t$$

$$\underline{\mathbf{x}}[k] \longrightarrow \boxed{\underline{\mathbf{w}}^*} \longrightarrow y[k]$$

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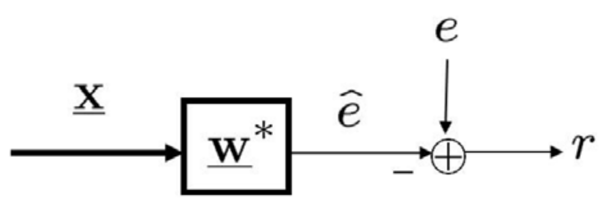
Data dependent beamforming: MMSE



- Cost function: $\xi = E\{|r|^2\}$
- Solution: $\underline{\mathbf{w}}_{\mathrm{m}se} = \arg\min_{\underline{\mathbf{w}}} \xi = \mathbf{R}_x^{-1} \cdot \underline{\mathbf{r}}_{xe^*}$

$$\mathbf{R}_x = E\{\underline{\mathbf{x}} \cdot \underline{\mathbf{x}}^h\} \qquad \underline{\mathbf{r}}_{xe^*} = E\{\underline{\mathbf{x}} \cdot e^*\}$$

- Need to know $\mathbf{R}_x$ and $\underline{\mathbf{r}}_{xe^*}$ (from measurements)

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Data dependent beamforming: MMSE

Example: ULA, one source, narrowband, farfield

With $\underline{\mathbf{x}} = \underline{\mathbf{a}} \cdot s + \underline{\mathbf{n}}$ and $e = s$

$\Rightarrow$

$\mathbf{R}_x = E\{\underline{\mathbf{x}} \cdot \underline{\mathbf{x}}^h\} = (\underline{\mathbf{a}} \cdot \underline{\mathbf{a}}^h) \cdot \sigma_s^2 + \sigma_n^2 \cdot \mathbf{I}$ and

$\underline{\mathbf{r}}_{xe^*} = E\{\underline{\mathbf{x}} \cdot e^*\} = \underline{\mathbf{a}} \cdot \sigma_s^2$

$\Rightarrow$

$$\underline{\mathbf{w}}_{mse} = \mathbf{R}_x^{-1} \cdot \underline{\mathbf{r}}_{xe^*} = \left(\underline{\mathbf{a}} \cdot \underline{\mathbf{a}}^h \cdot \sigma_s^2 + \sigma_n^2 \cdot \mathbf{I}\right)^{-1} \cdot \underline{\mathbf{a}} \cdot \sigma_s^2$$