Advanced Digital Signal Processing (ADSP)

徐林

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About the lecture: Xu Lin / 徐林

Biography

- 2011.09 – 2016.01， Ph.D. in Electrical Engineering, Eindhoven University of Technology, Eindhoven, the Netherlands;
- 2016.01 – 2019.08, postdoc researcher, Eindhoven University of Technology, Eindhoven, the Netherlands;
- 2019.09 – now, Assistant professor, ShanghaiTech University, Shanghai, China.

Research interests

- Biomedical signal processing, e.g., Electromyography and Electrocardiography.
- Adaptive filtering, Blind Source Separation, Machine learning.
- Wearable medical devices.
- Neuromuscular rehabilitation.
- Ambulatory monitoring.

Lab

- Biomedical Signal Processing and Instrumentation

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Course Introduction

- Main purpose and content
- Motivation
- Exams and credits
- Textbook and materials

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Course Introduction

Main purpose ADSP course

- Describe Stochastic Signal processing and Adaptive Array Signal Processing from a signal processing perspective
- Hand-on Matlab experience

Main content ADSP course

- Part A: Stochastic Signal Processing
- Part B: Adaptive Signal Processing (Single channel, FIR)
- Part C: Array Signal Processing (ASP) (including DOA)
- Part D: Adaptive Array Signal Processing (AASP)

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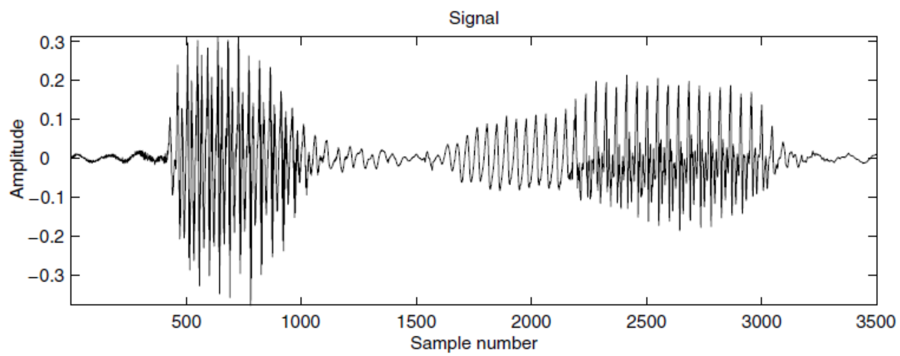
ADSP Course Introduction: motivation

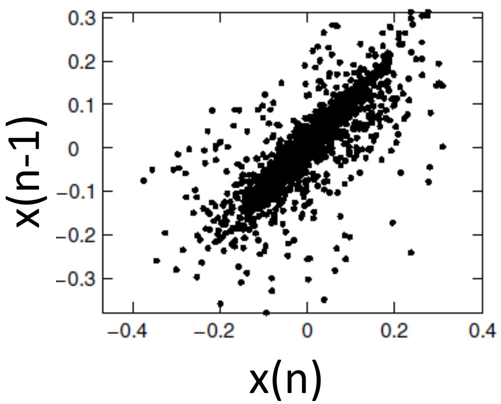
Stochastic signal processing

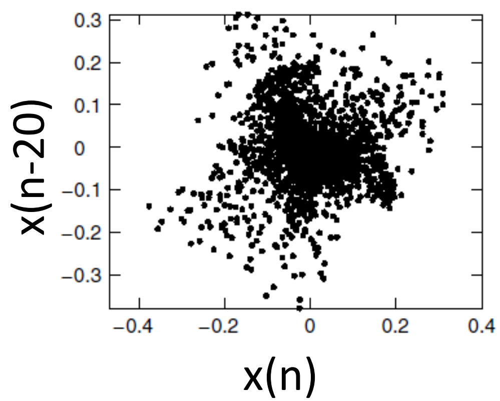
A discrete-time signal is obtained by periodically sampling a continuous-time signal and is also called a time series.

$$x(n) = x_c(nT),$$



- Ordered in time; adjacent observations may be dependent;
- Predict future values using past observations;
- Prediction is exact: Deterministic
- Prediction is inexact: Random or stochastic.
- Described by using the theory of stochastic processes.





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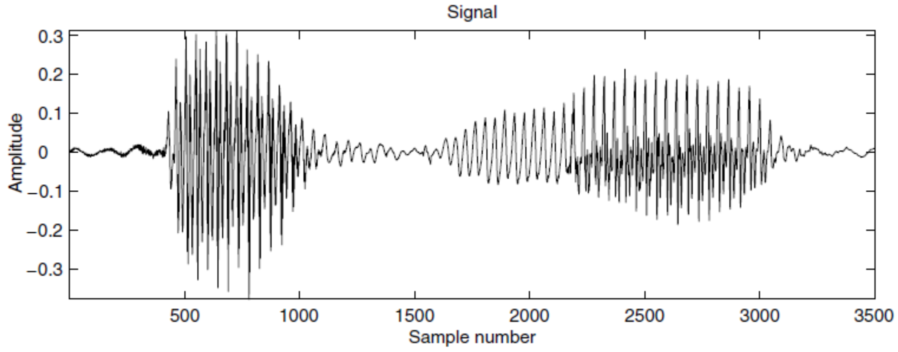
ADSP Course Introduction: motivation

Stochastic signal processing

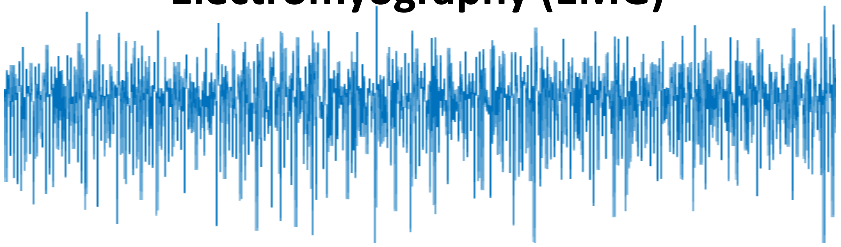
Typical stochastic signals:

- Speech signals
- Electrophysiological signals
- Geophysical signals

Speech signal



Electromyography (EMG)



Electrocardiography (ECG)

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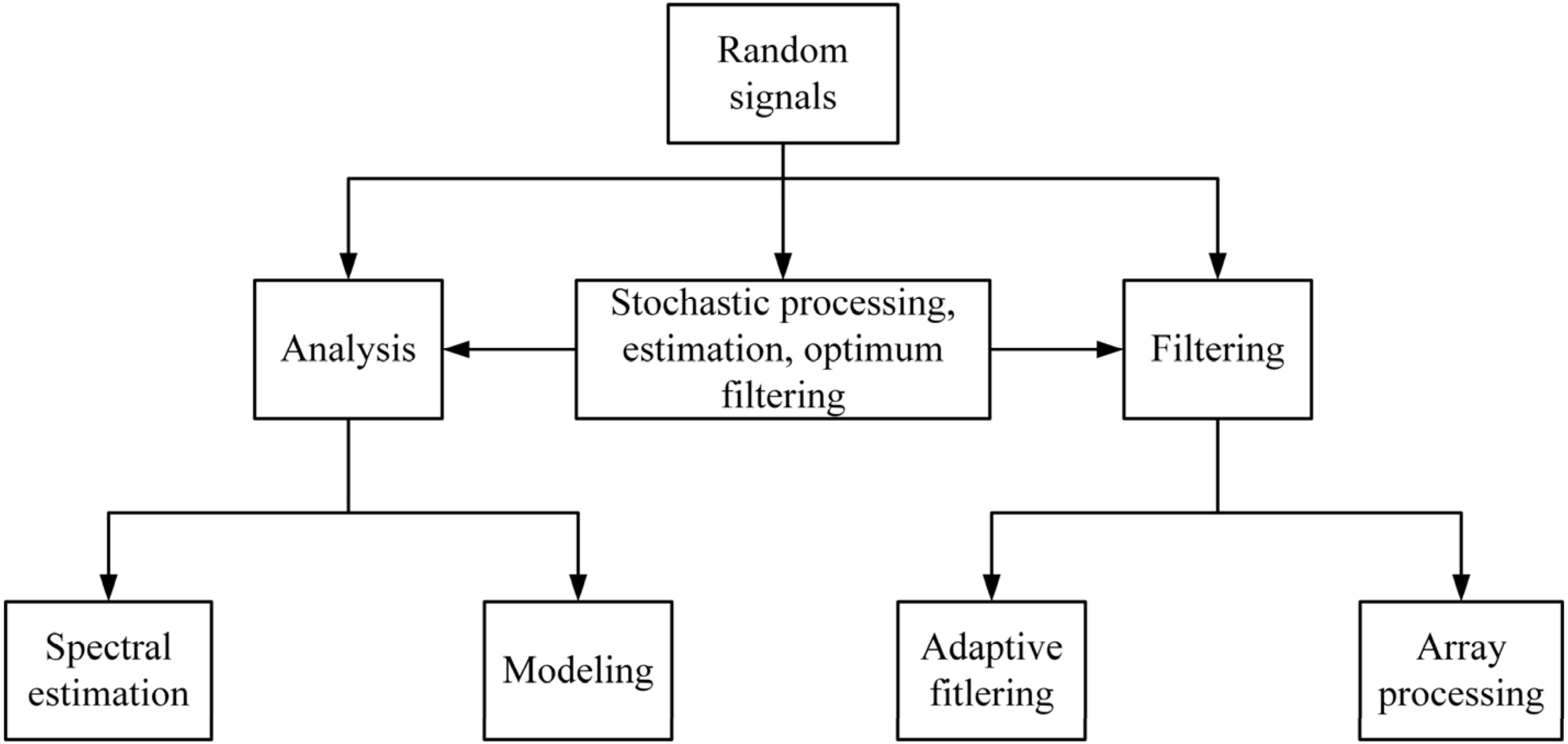
Course Introduction: motivation

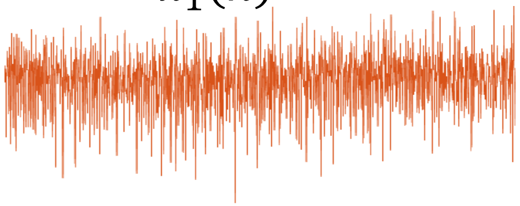
Stochastic signal processing



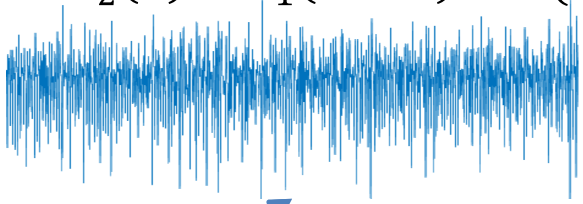
Course Introduction: motivation

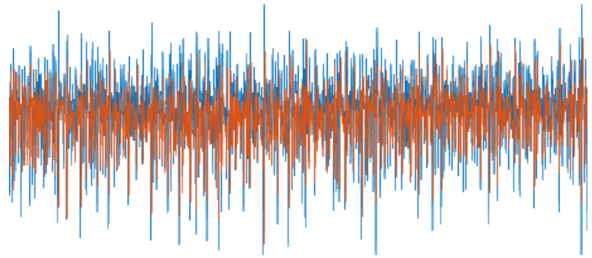
Stochastic signal processing: EMG conduction velocity estimation

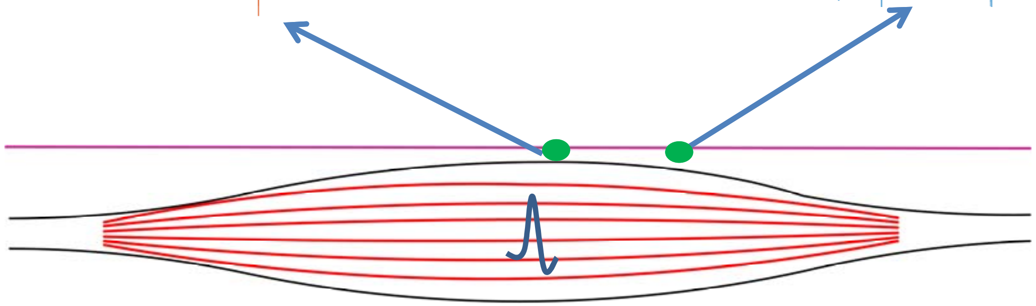
$x_1(n)$

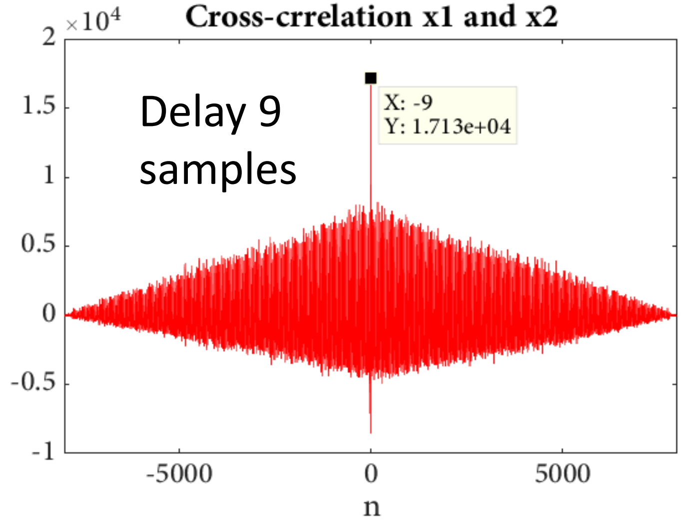


$x_2(n) = x_1(n - \tau) + \omega(n)$









$$R_{x_1x_2}(n) = \begin{cases} \sum_{m=0}^{M-n-1} x_1(m+n)x_2(m), & n \geq 0 \\ R_{x_2x_1}(n), & n < 0 \end{cases}$$

ADSP Course Introduction: motivation

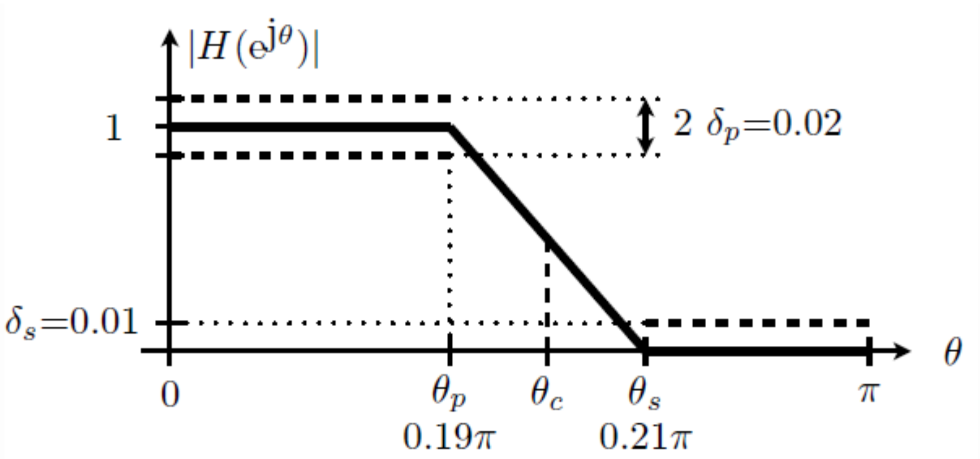
Adaptive filtering

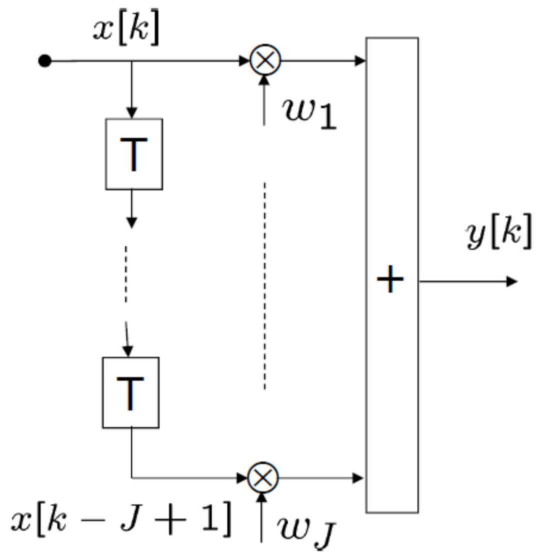
Frequency-selective digital filters:

- FIR or IIR;
- Predesigned, no need of the sample values;
- Filter coefficients constant;
- Signal and noise non-overlapping.

Adaptive filters:

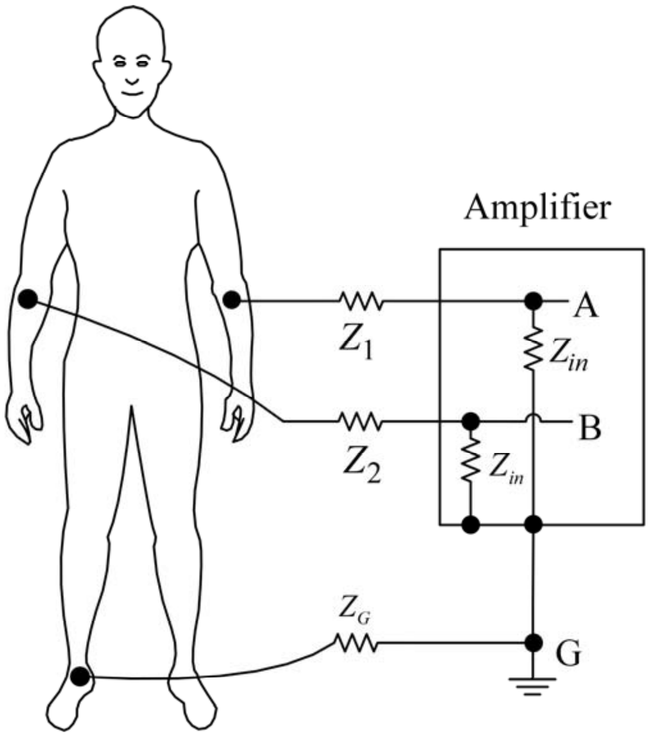
- Single channel, FIR;
- Not predesigned, need sample values;
- Filter coefficients update in each iteration.





ADSP Course Introduction: motivation

Adaptive filtering: motion artefact removal in ECG measurements

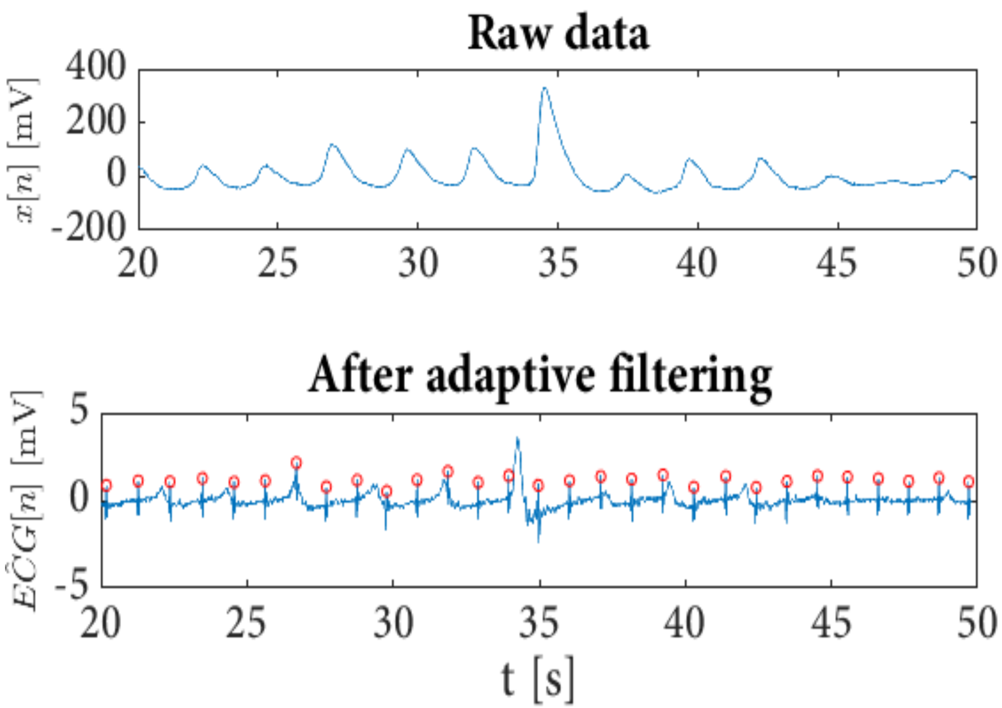


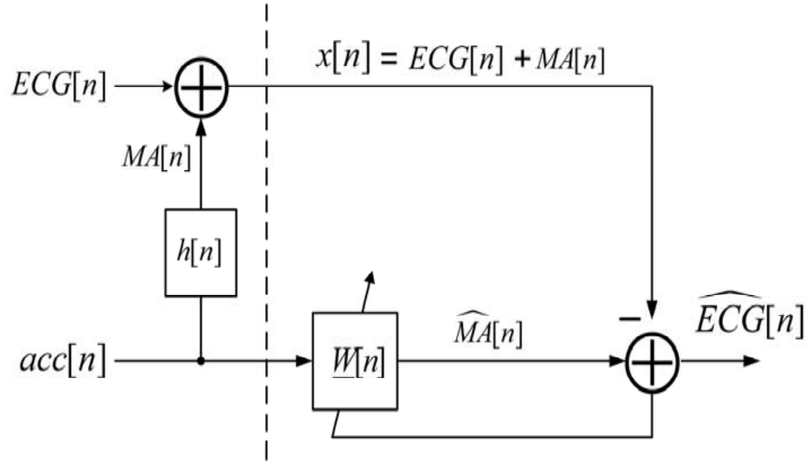
ECG measurement

Typical ECG Signals

ADSP Course Introduction: motivation

Adaptive filtering: motion artefact removal in ECG measurements





L Xu et al. Motion-Artifact Reduction in Capacitive Heart-Rate Measurements by Adaptive Filtering. *IEEE Trans Instrum Meas, 2018.*

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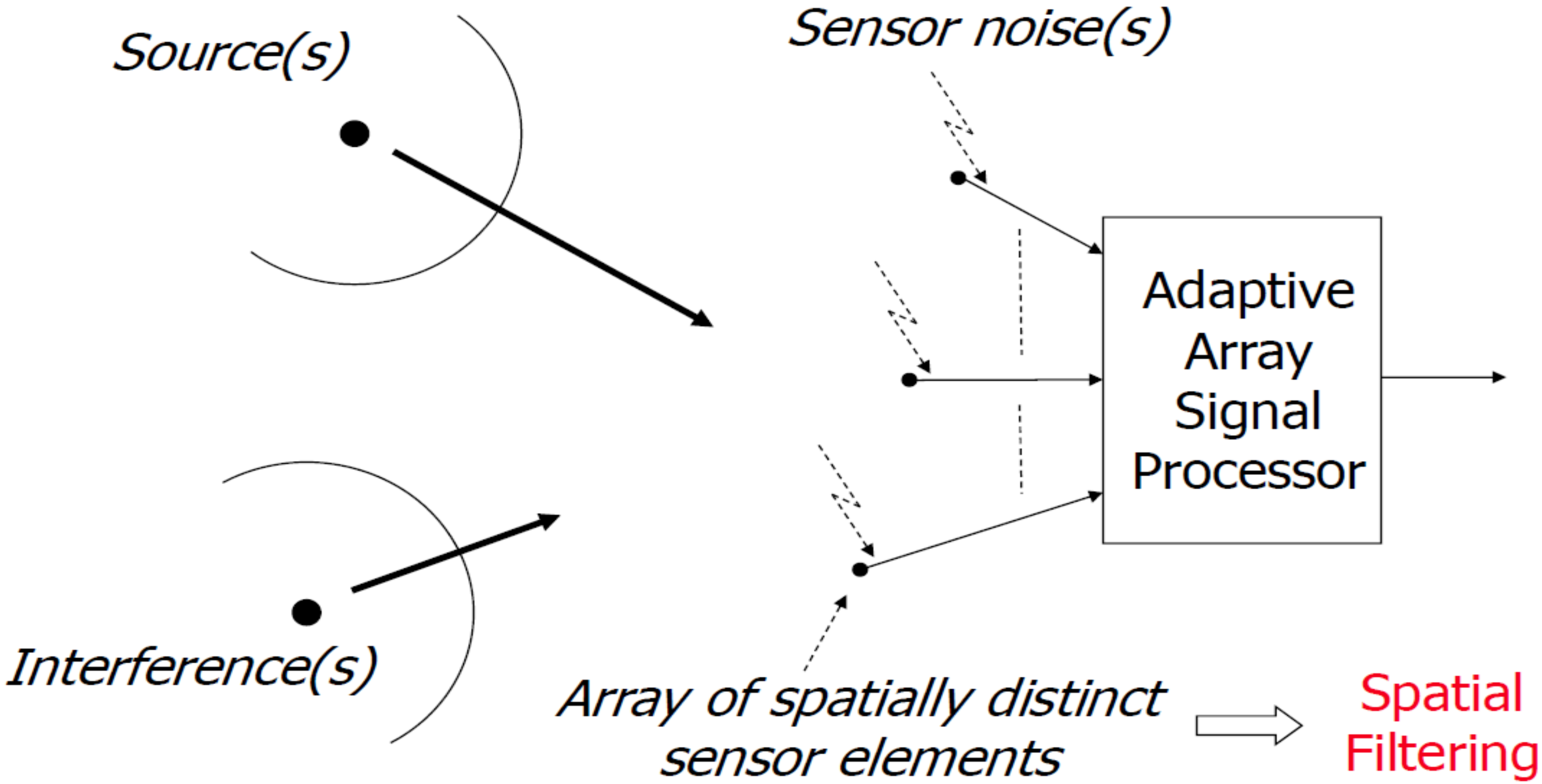
Course Introduction: motivation

Array signal processing



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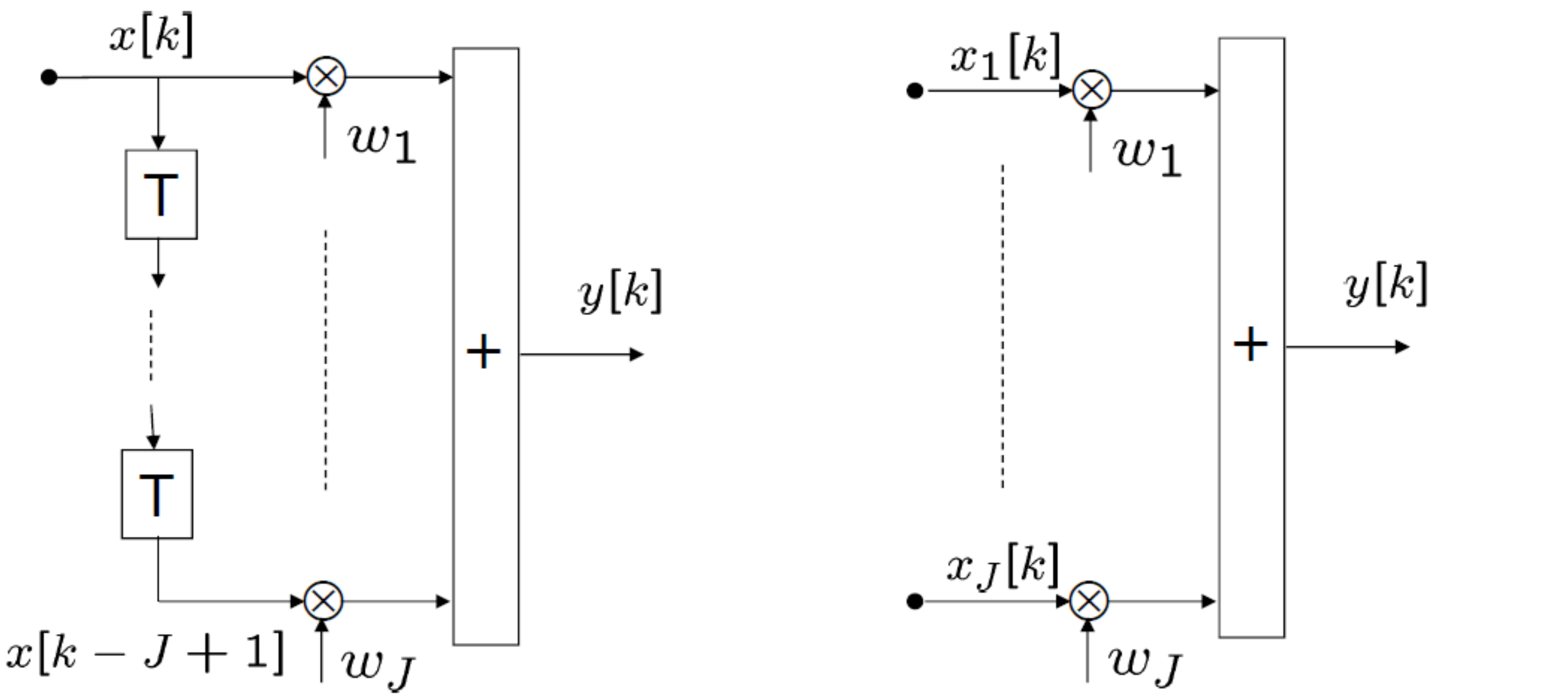
Course Introduction: motivation

Array signal processing

FIR (temporal processing):   Array (spatial processing):



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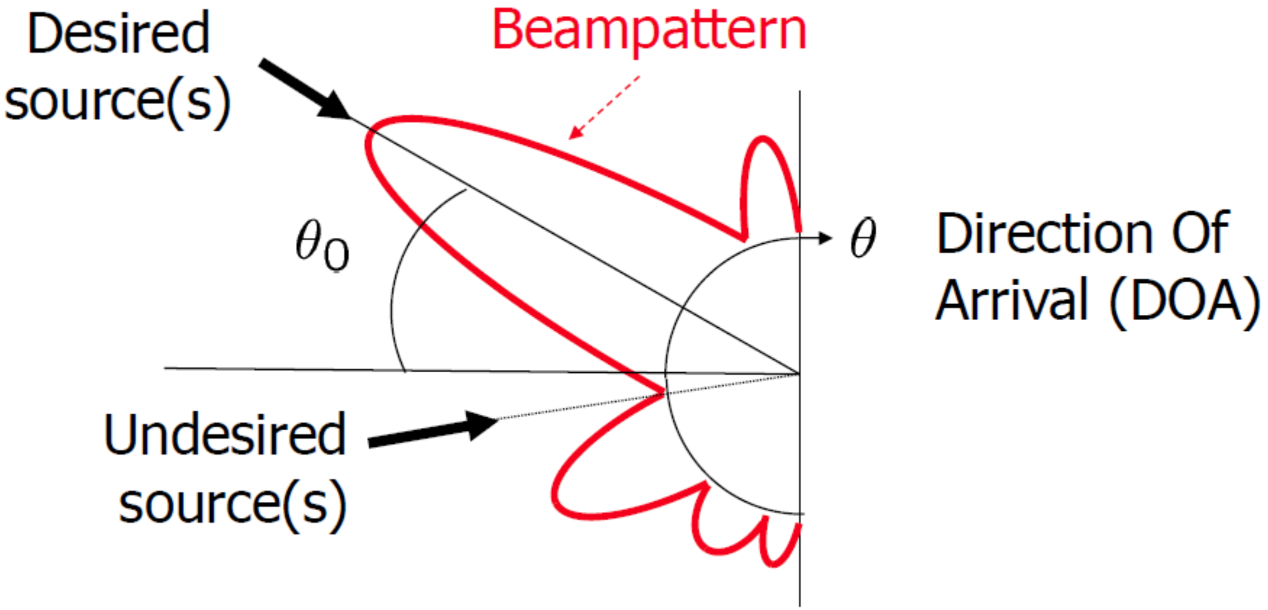
Array signal processing

Result of spatial filtering:



Separate signals with **overlapping** frequency content but from **different** spatial locations.

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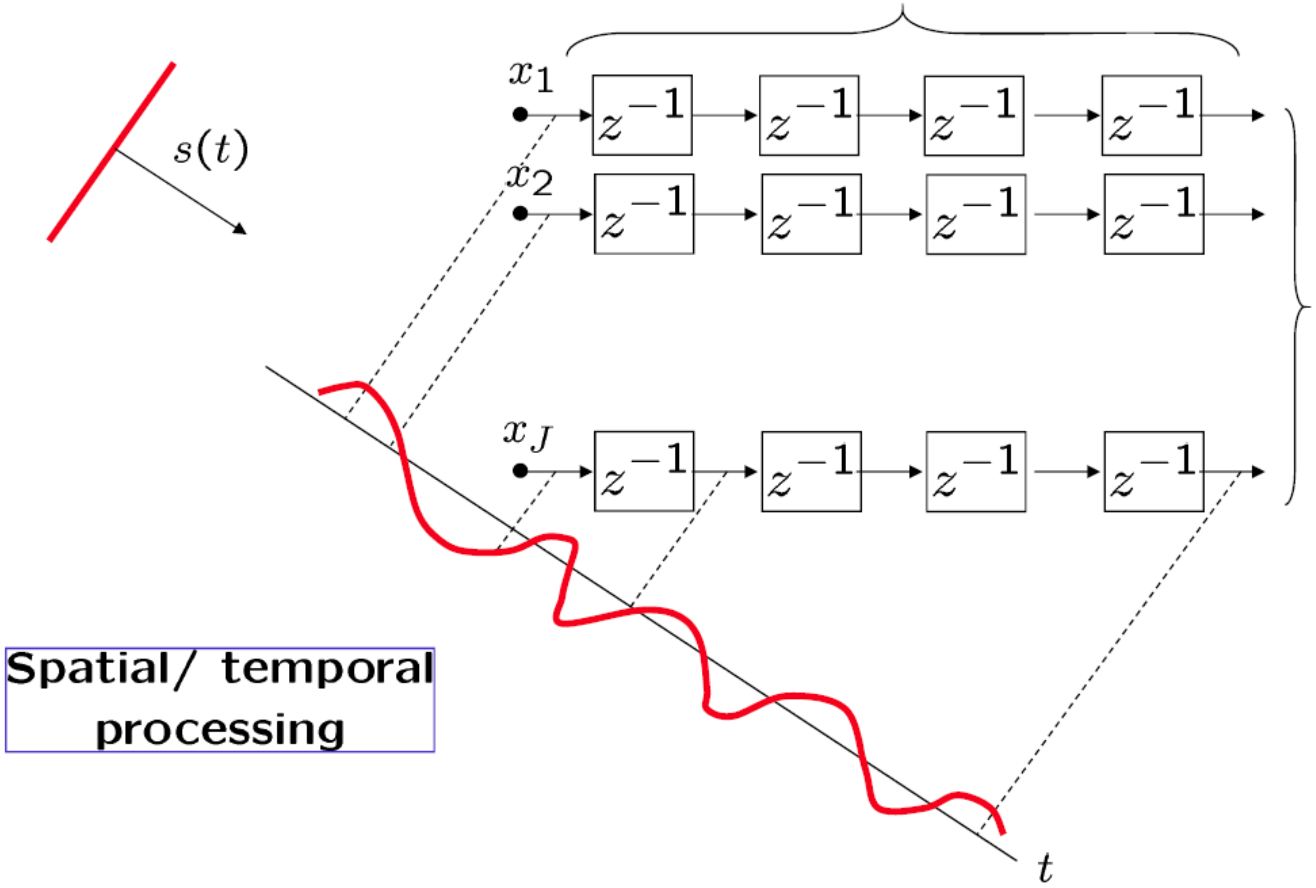
Course Introduction: motivation

Adaptive array signal processing



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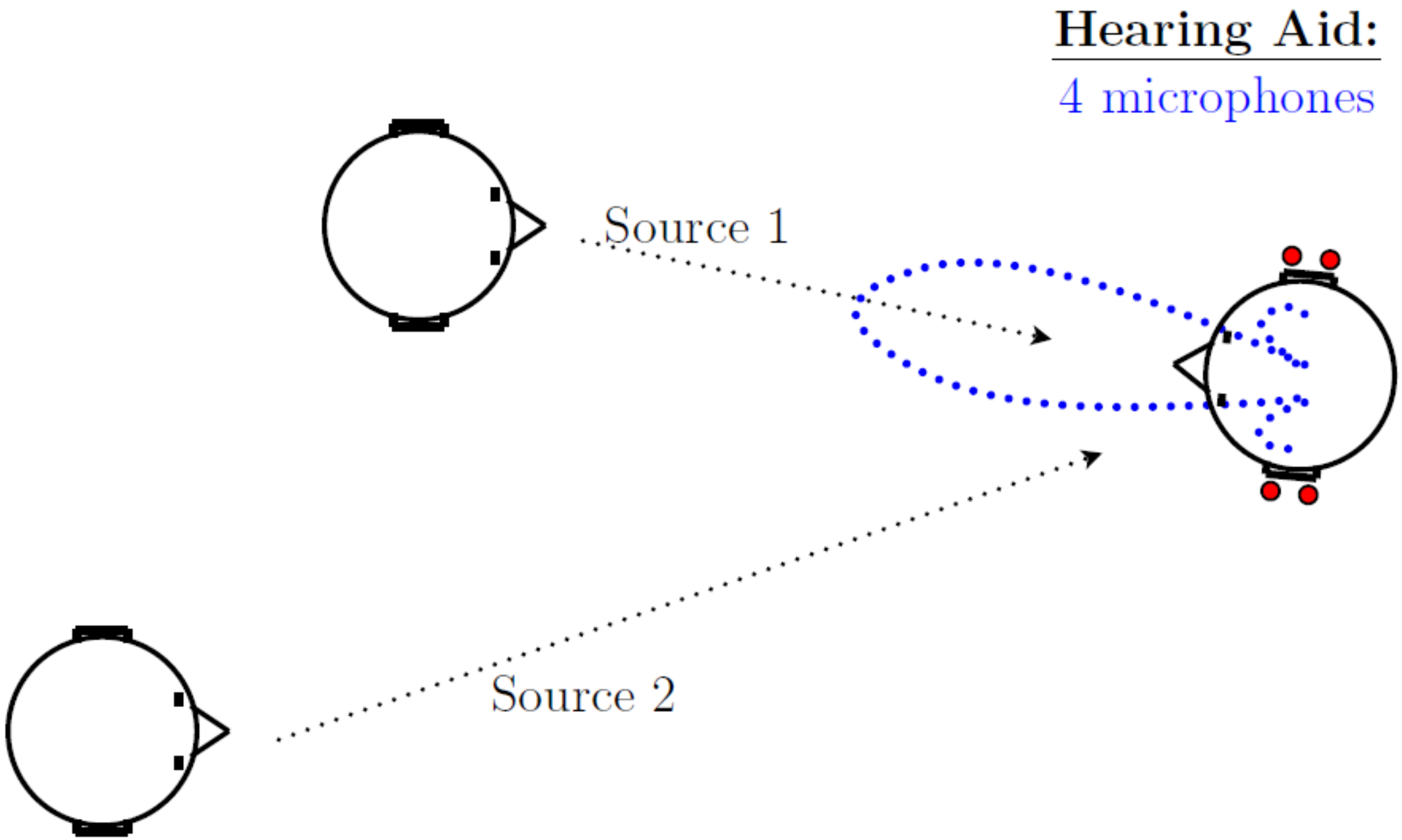
Adaptive array signal processing



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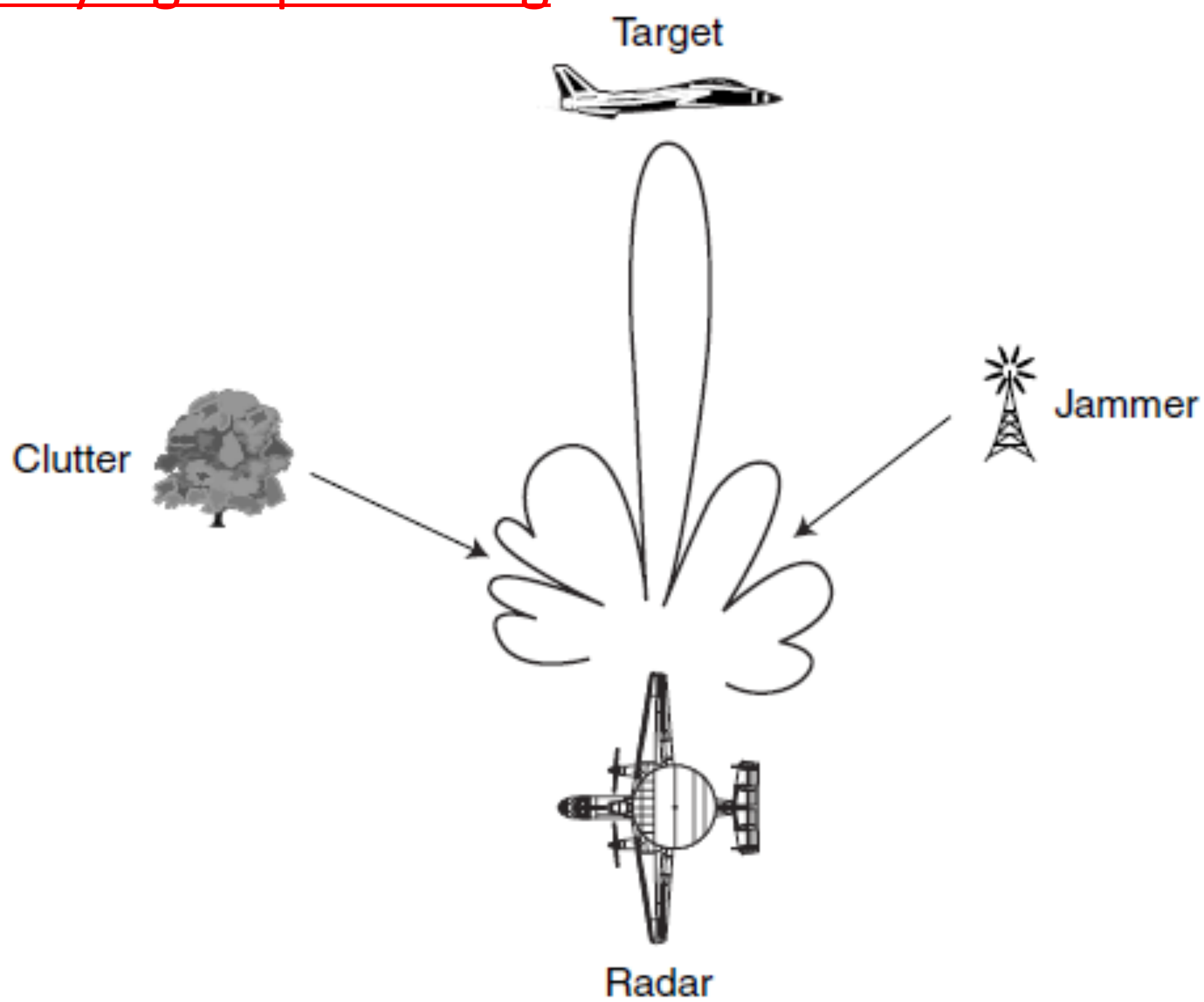
Adaptive array signal processing



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Course Introduction

Evaluation: Assignment + Project + Oral exam

Code	Assignment	Credits
A1	Stochastic signal processing	10
A2	Adaptive filtering	10
A3	Beam former design	10
A4	DOA estimation	10
B	Final project	50
	Oral exam	10
Total		100

Work in groups: two student each group

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Course Introduction

Weeks	Lectures 1	Lectures 2
1	Course introduction + Stochastic signal processing	Stochastic signal processing
2	**Exercises + Labs**	Stochastic signal processing
3	Stochastic signal processing	Exercises + Labs
4	Adaptive signal processing	Adaptive signal processing
5	Exercises + Labs	Adaptive signal processing
6	Adaptive signal processing	Exercises + Labs
7	Array signal processing	Array signal processing
8	Exercises + Labs	Array signal processing
9	Array signal processing	Exercises + Labs
10	Adaptive array signal processing	Adaptive array signal processing
11	Exercises + Labs	Adaptive array signal processing
12	Adaptive array signal processing	Exercises + Labs

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Course Introduction

- **Book:**

  "Statistical and Adaptive Array Signal Processing: Spectral estimation, signal modeling, adaptive filtering and array processing";
  Dimitris G. Manolakis, Vinay K. Ingle and Stephen M. Kogon; McGraw Hill; 2003

  - Random Variables, Vectors, and Sequences (Chapter 3)
  - Optimum linear filters (Chapter 6)
  - Adaptive filters (Chapter 10)
  - Array processing (Chapter 11)

- **These slides**
- **Predefined answer documents for 1A, 1B and 1C**
- **Necessary Matlab code**

Part A: Stochastic signal processing

Contents

- Random variable
- Random vector
- Stochastic process
- Second order statistics
- Power spectrum estimation

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Part A: Stochastic signal processing

Contents

- Random variable
- Random vector
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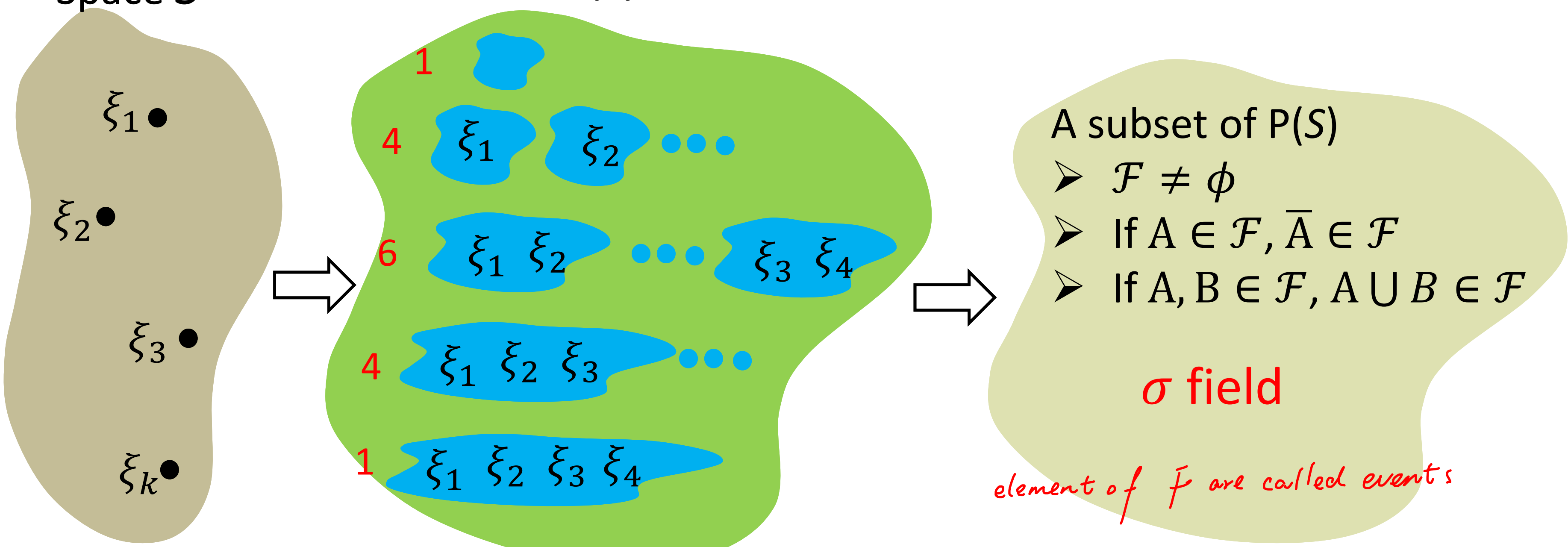
Random Variable: probability space

- Concept of random Variable begins with the definition of probability;
- Probability is defined on probability space $(S,\ \mathcal{F},\ \Pr)$.

Abstract Space S

P(S)

$\mathcal{F}$ (Closed)



$\xi_1 \ldots \xi_k$ Outcomes of an experiment

实验的结果

16 elements

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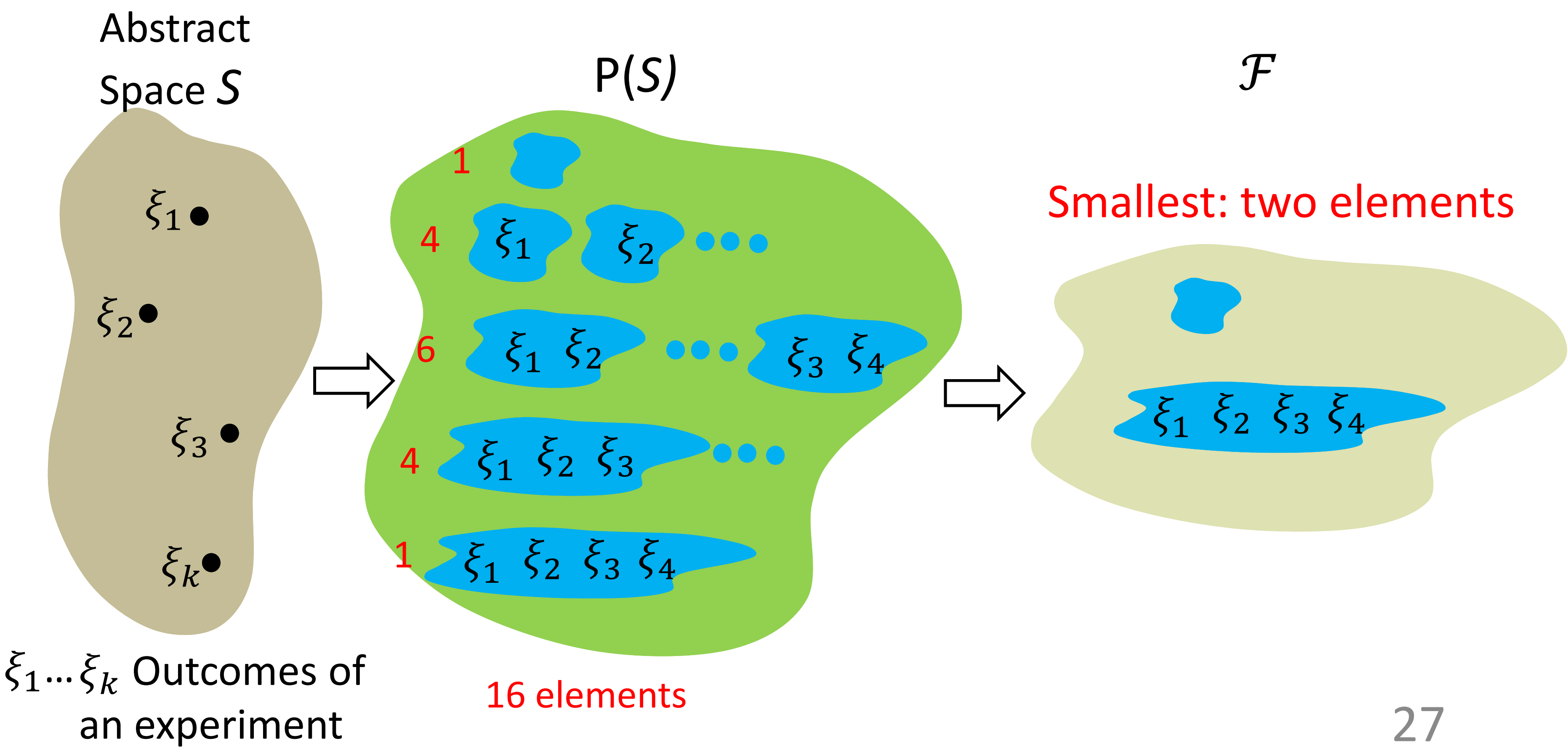
Random Variable: probability space

- Concept of random Variable begins with the definition of probability;
- Probability is defined on probability space (S, $\mathcal{F}$, Pr).



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Random Variable: probability space

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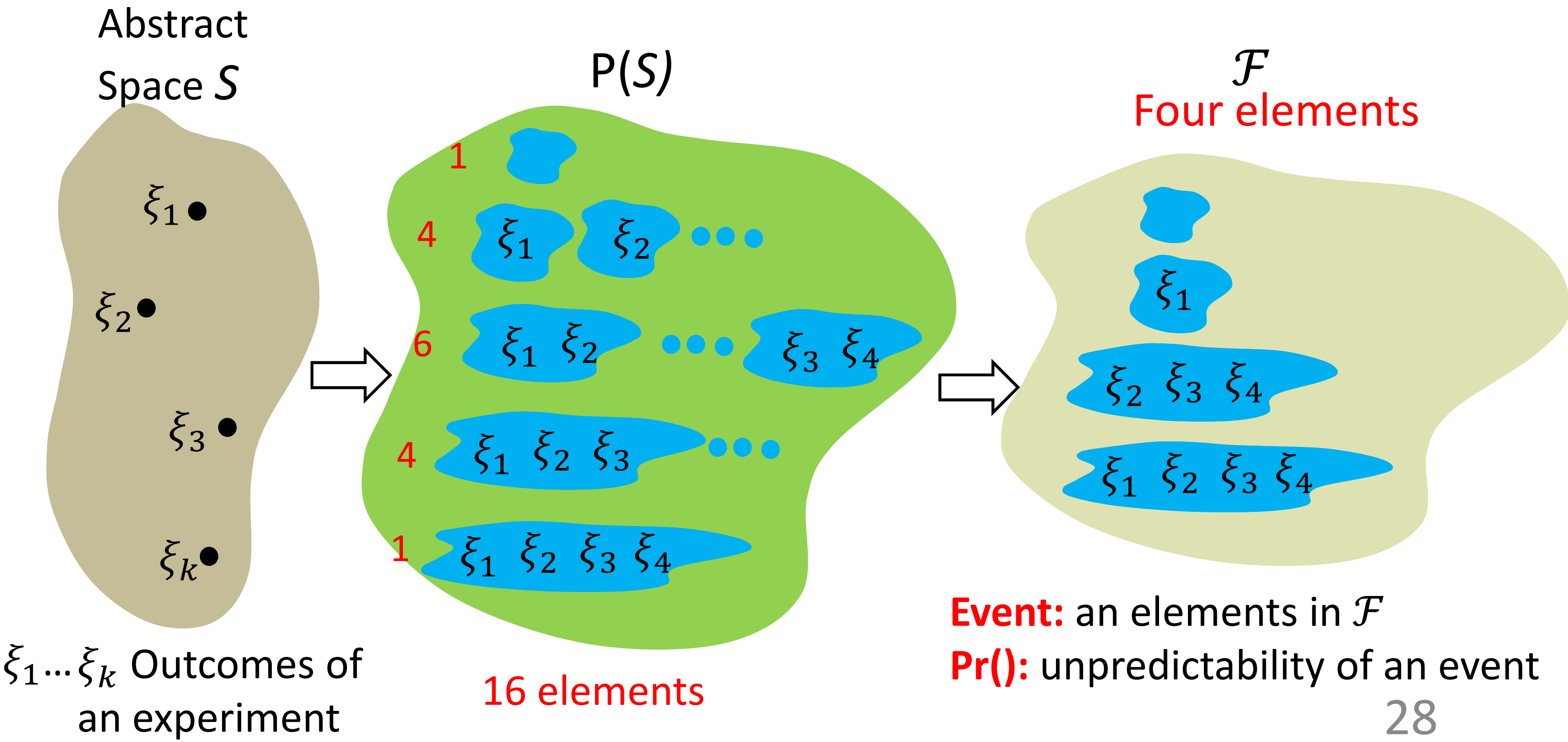

Event: an elements in $\mathcal{F}$

Pr(): unpredictability of an event

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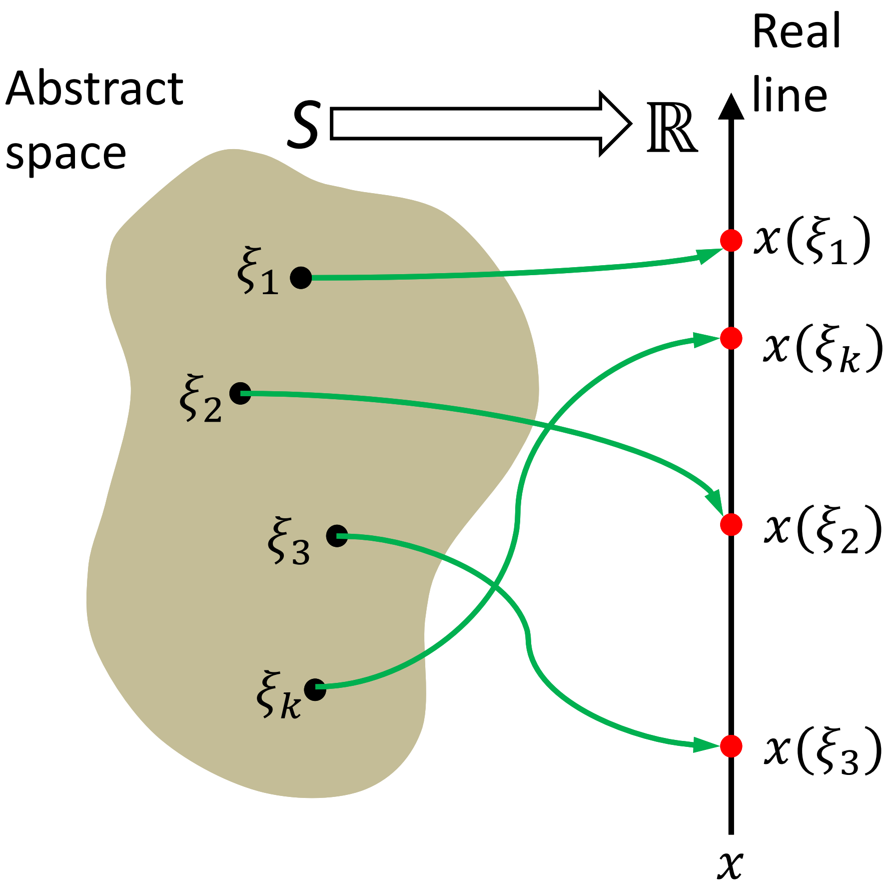
Random Variable: definition

Random Variable: a mapping that assigns a real number x to ξ. $\{x(\xi) < x\}$ is an event for every x; $\Pr\{x(\xi) = \infty\}$=0, $\Pr\{x(\xi) = -\infty\}$=0.



- $\xi_1, \xi_2, \dots, \xi_k$: Outcomes of a random experiment

  $\{\xi_1, \xi_2, \dots, \xi_k\} = S$

- x $(x \in R)$: Value of a random variable
- $x(\xi)$: Random Variable

x: discrete values $\Leftrightarrow x(\xi)$: *discrete*

x: continuous values $\Leftrightarrow x(\xi)$: continuous

ADSP Random Variable: distribution function

Distribution function: $F_x(x) \triangleq \Pr\{x(\xi) \leq x\}$

Probability density function

$$f_x(x) \triangleq \frac{\mathrm{d}F_x(x)}{\mathrm{d}x}, \text{ continue random variable}$$

$$p_k = \Pr\{x(\xi) = x_k\}, \text{ discrete random variable}$$

Note: $f_x(x)$ is not the probability, but must be multiplied by Δx.

$$f_x(x)\,\Delta x \approx \Delta F_x(x) \triangleq F_x(x+\Delta x) - F_x(x) = \Pr(x < x(\xi) \leq x + \Delta x\}$$

Probability: $\Pr(x_1 < x(\xi) \leq x_2\} = F_x(x_2) - F_x(x_1) = \int_{x_1}^{x_2} f_x\,(x)\mathrm{d}x$

Note: $\Pr(x(\xi) = x\}$=0.

$$F_x(x) = \int_{-\infty}^{x} f_x\,(v)\mathrm{d}v,$$

Properties: $F_x(-\infty) = 0,\quad F_x(\infty) = 1,\quad 0 \leq F_x(x) \leq 1$

$f_x(x) \geq 0,\quad \int_{-\infty}^{\infty} f_x\,(x)\mathrm{d}x = 1$

$= F_x(\infty) = 1$

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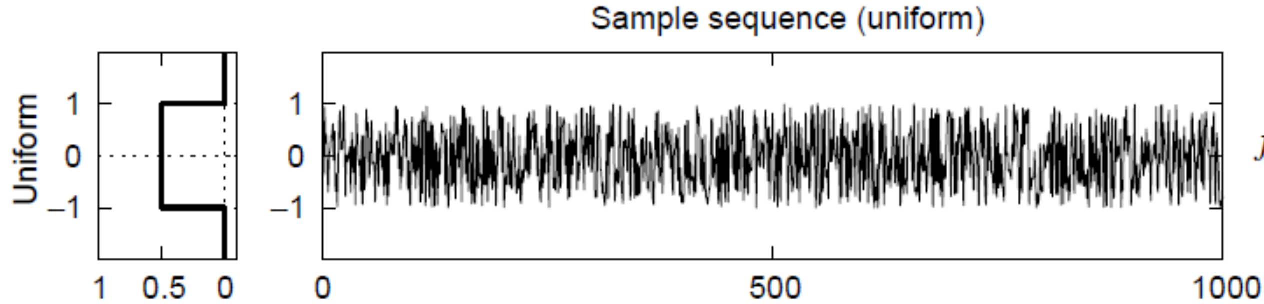
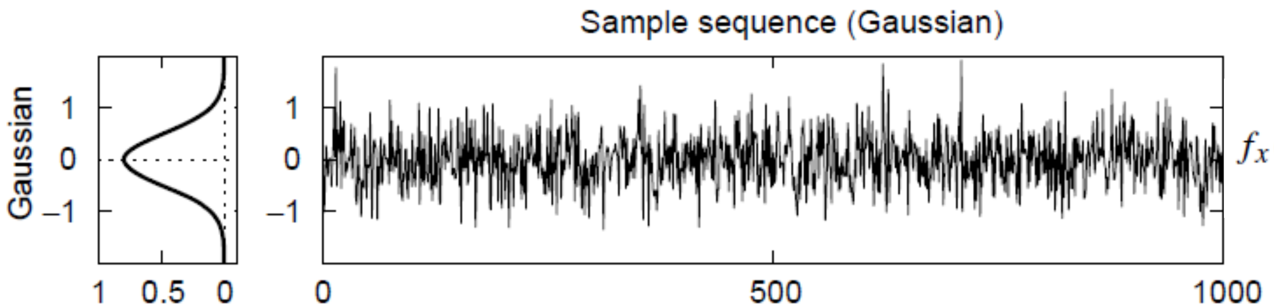
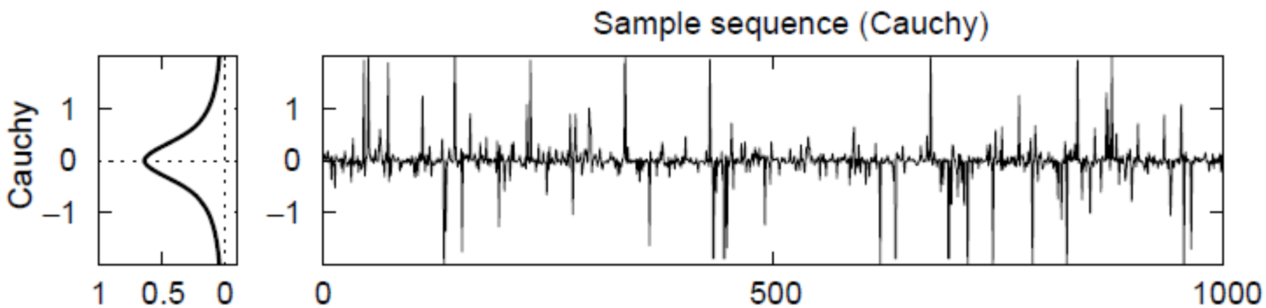
Random Variable: distribution



$$f_x(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$



$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2\right]$$



$$f_x(x) = \frac{\beta}{\pi} \frac{1}{(x-\mu)^2 + \beta^2}$$

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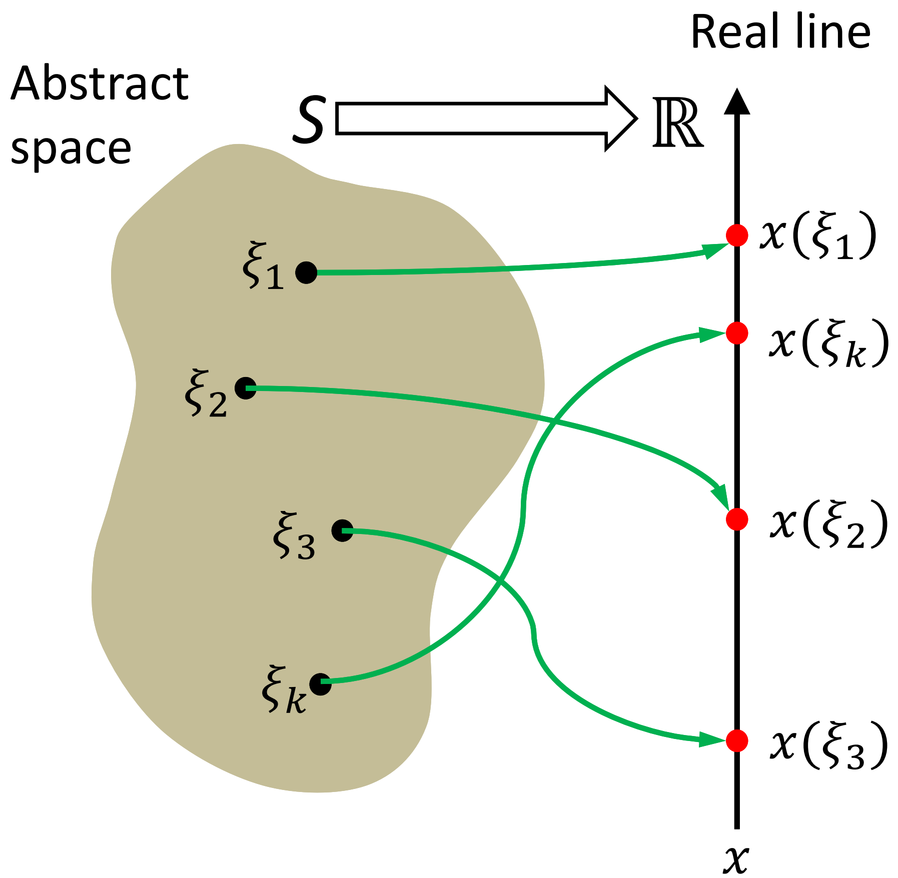
Random Variable: moments 矩（原点矩）

$$1^{st} - \text{order moment} \triangleq E\{x(\xi)\} = \int_{-\infty}^{\infty} x f_x(x) \mathrm{d}x$$



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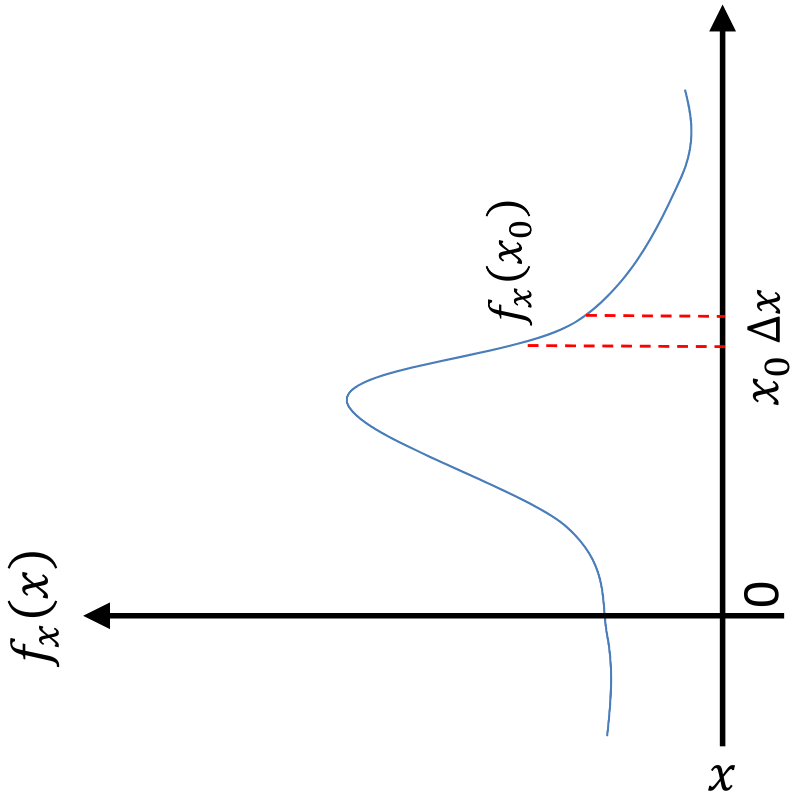
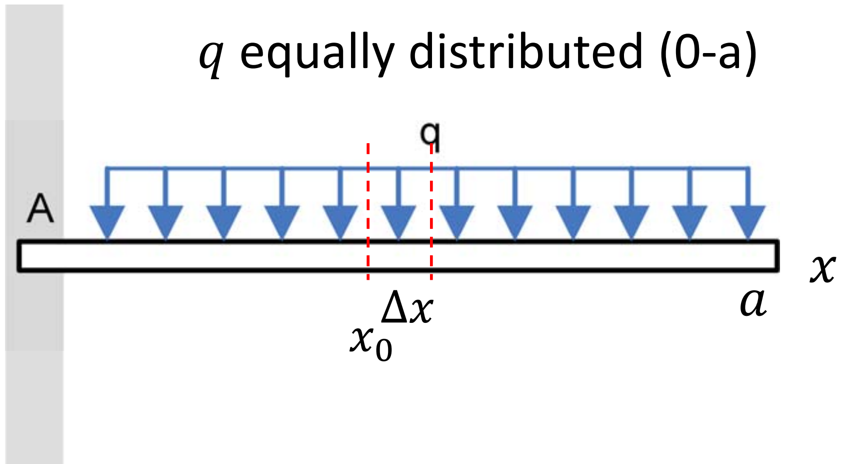
Random Variable: moments

$$1^{st} - \text{order moment} \triangleq E\{x(\xi)\} = \int_{-\infty}^{\infty} x f_x(x) \mathrm{d}x$$

$f_x(x_0)\,\Delta x \approx \Pr(x_0 < x(\xi) \le x_0 + \Delta x\}$



q equally distributed (0-a)



Force = $q\ \Delta x$

$$\text{moment} \triangleq \int_0^a x\, q\, \mathrm{d}x$$

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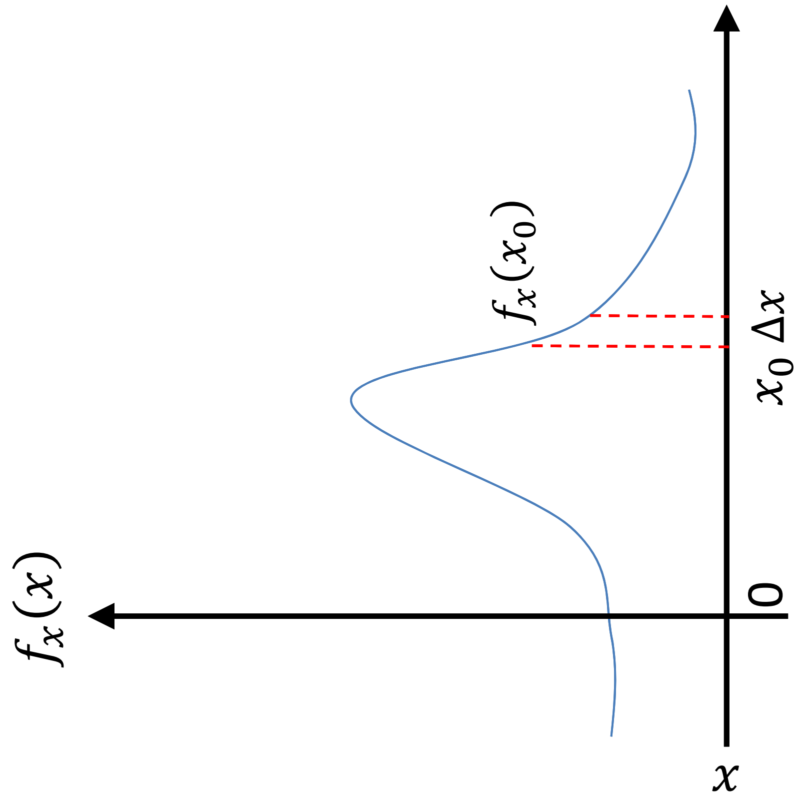
Random Variable: moments

$$1^{st} - \text{order moment} \triangleq E\{x(\xi)\} = \int_{-\infty}^{\infty} x f_x(x) \mathrm{d}x$$

$$f_x(x_0)\,\Delta x \approx \Pr(x_0 < x(\xi) \leq x_0 + \Delta x\}$$



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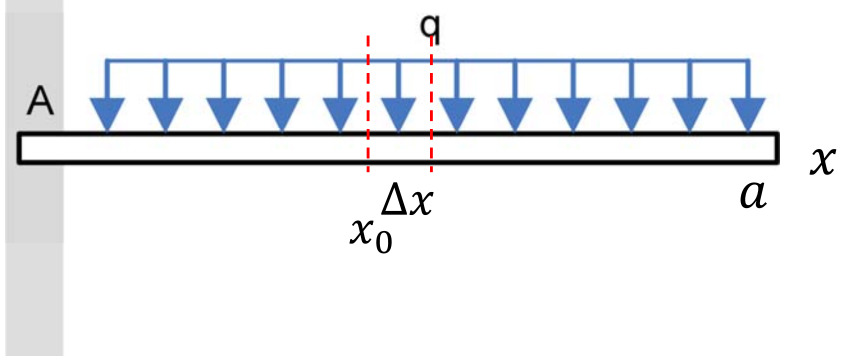

Force = $q\ \Delta x$

$$\text{moment} \triangleq \int_0^a x\ q\ \mathrm{d}x$$

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Random Variable: moments

$$1^{st} - \text{order moment:}\quad \mathrm{r}_x \triangleq E\{x(\xi)\} = \int_{-\infty}^{\infty} x f_x(x)\mathrm{d}x$$

$$m^{th}\text{—order moment:}\quad r_x^{(m)} \triangleq E\{x^m(\xi)\} = \int_{-\infty}^{\infty} x^m f_x(x)\mathrm{d}x$$

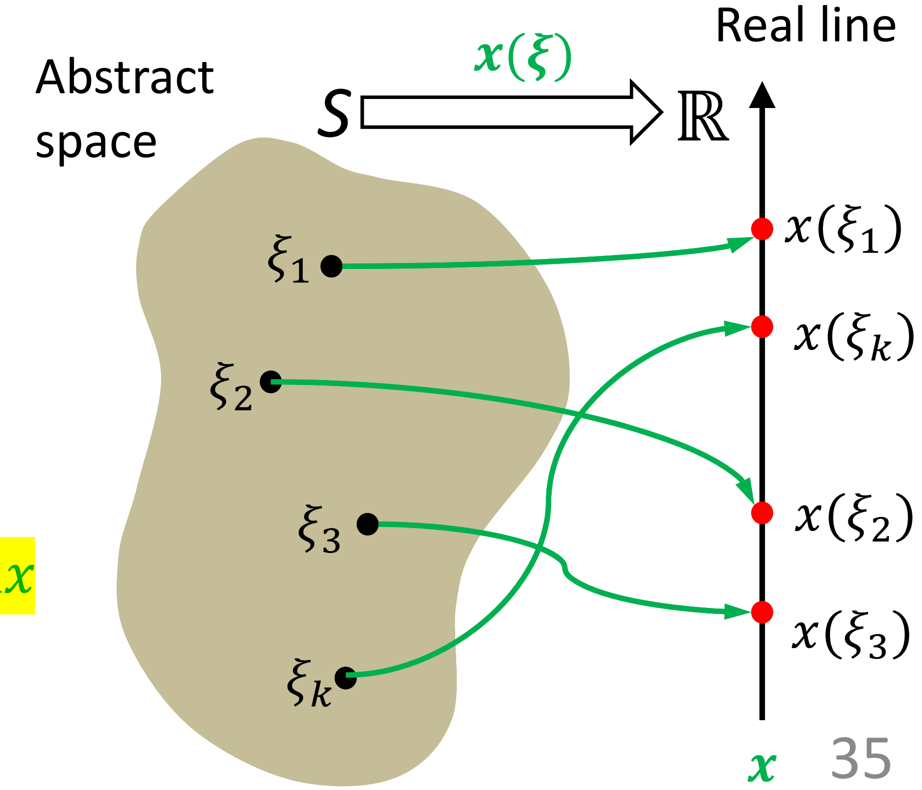
If $y(\xi) = g[x(\xi)]$,

$$E\{y(\xi)\} \triangleq \int_{-\infty}^{\infty} y f_y(y)\mathrm{d}y$$

$$E\{y(\xi)\} \triangleq E\{g[x(\xi)]\}$$

$$= \int_{-\infty}^{\infty} g(x) f_x(x)\mathrm{d}x$$

$$\mathrm{g} \triangleq (\cdots)^m$$



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Random Variable: moments

mth−order *moment*: $r_x^{(m)} \triangleq E\{x^m(\xi)\} = \int_{-\infty}^{\infty} x^m f_x(x)\mathrm{d}x$

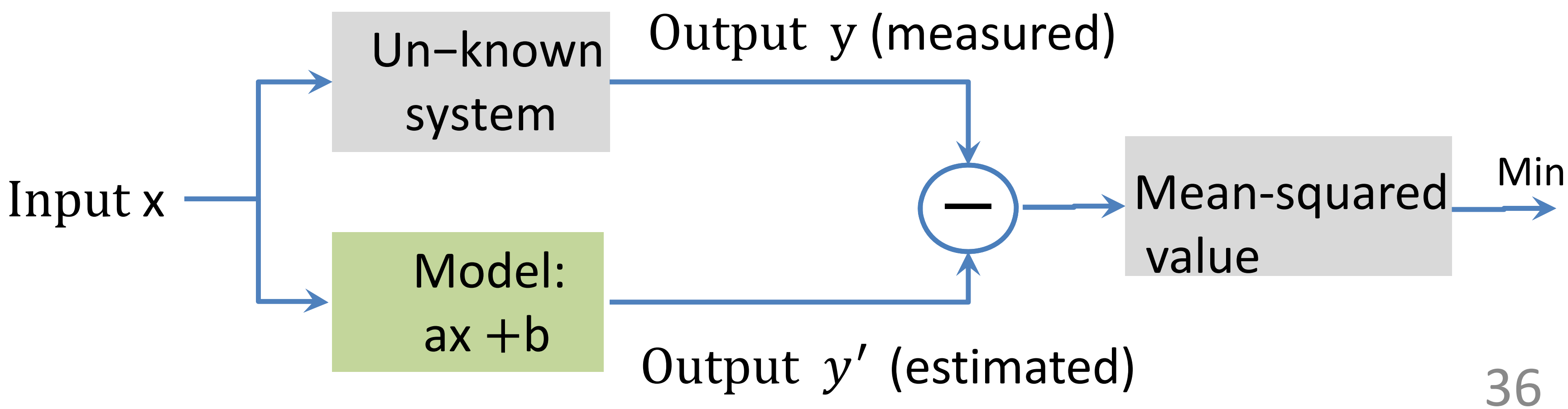
Statistical average (center of gravity):

$$r_x^{(1)} \triangleq E\{x(\xi)\} \triangleq \mu_x = \int_{-\infty}^{\infty} x\, f_x(x)\mathrm{d}x$$

Mean−squared value:

$$r_x^{(2)} \triangleq E\{x^2(\xi)\}$$

$$E\{x^2(\xi)\} \neq E^2\{x(\xi)\}$$



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Random Variable: moments

m^{th}–order central moment: $\gamma_x^{(m)} = E\{[x(\xi) - \mu_x]^m\} = \int_{-\infty}^{\infty} (x - \mu_x)^m f_x(x)\mathrm{d}x$

Variance: $\mathrm{var}[x(\xi)] \triangleq {\sigma_x}^2 \triangleq \gamma_x^{(2)} = E\{[x(\xi) - \mu_x]^2\} = r_x^{(2)} - \mu_x^2$

$= \int_{-\infty}^{\infty} (x - u_x)^2 f_x(x)\, dx$

$= \int_{-\infty}^{+\infty} x^2 f_x(x) dx + u_x^2 \int_{-\infty}^{+\infty} f_x(x) dx - 2u_x \int x f_x(x) dx$

$= r_x^2 + u_x^2 - 2u_x^2$

$= r_x^2 - u_x^2$

Standard deviation: $\sigma_x = \sqrt{\gamma_x^{(2)}}$

Skew $\triangleq {\tilde{k}_x}^3 \triangleq E\left\{\left[\frac{x(\xi) - \mu_x}{\sigma_x}\right]^3\right\} = \frac{1}{{\sigma_x}^3}\gamma_x^{(3)}$

Kurtosis $\triangleq {\tilde{k}_x}^4 \triangleq E\left\{\left[\frac{x(\xi) - \mu_x}{\sigma_x}\right]^4\right\} - 3 = \frac{1}{{\sigma_x}^4}\gamma_x^{(4)} - 3$