Advanced Digital Signal Processing (ADSP)

徐林

上海科技大学
ShanghaiTech University

ADSP Advanced digital signal processing

Main content ADSP course

- Part A: Stochastic Signal Processing
- Part B: Adaptive Signal Processing
- Part C: Array Signal Processing (ASP) (including DOA)
- **Part D: Adaptive Array Signal Processing (AASP)** 依赖 数据

Contents

- Direction of Arrival (DOA) estimation 通过数据估计
- Optimum (data-dependent) beamforming
  - Minimum mean squared error (MMSE) 结合统计量
  - Multiple sidelobe canceller (MSC)
  - Linearly constrained minimum variance (LCMV)
  - Minimum variance distortionless response (MVDR)
  - Generalized sidelobe canceller (GSC)
- Adaptive beamforming 通过迭代

  Adaptive versions of the MMSE, MSC, MVDR, LCMV and GSC

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Introduction to DoA estimation

- Estimate direction of arrival of sources (desired and/or interfering) from noisy observations

- Applications

  E.g., Tracking and presence detection for smart lighting;

  Detecting active talker in an video-conference to automatically steer a video camera;

  Surveillance applications, etc.

Techniques to DoA estimation

- Maximize steered response power
- Using high resolution spectral estimation concepts
- Using time difference of arrival

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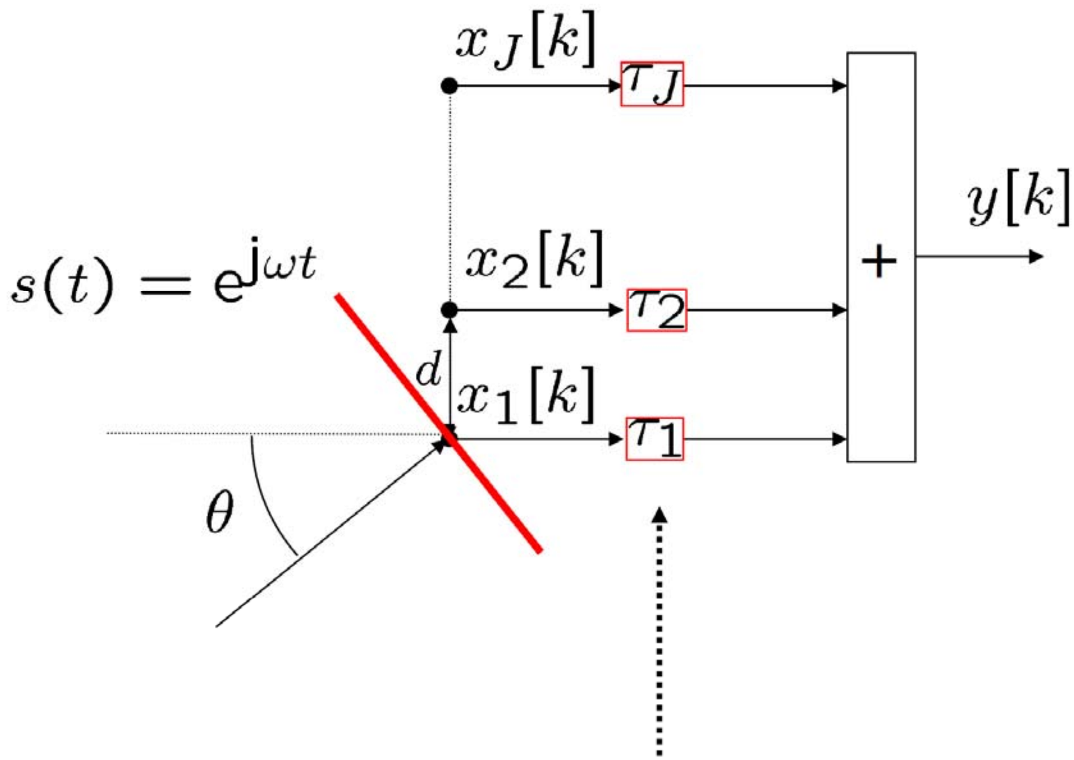
Data independent beamforming



For ULA choose: $\tau_i = (i-1)\tau \;\leftrightarrow\; w_i^* = e^{j(i-1)\omega\tau}$

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(DoA) Maximizing steered response power

- Consider one source located at θ^* and J sensors
- Input signals: $\underline{\mathbf{x}}[k] = \underline{\mathbf{a}}(\theta^*) \cdot s[k] + \underline{\mathbf{n}}[k]$

  体现了数据的相关

  $R_x = E\{\underline{x}[k]\,\underline{x}^h[k]\}$
- Autocorrelation: $\mathbf{R}_x = \sigma_s^2 \underline{\mathbf{a}}(\theta^*)\underline{\mathbf{a}}^h(\theta^*) + \sigma_n^2 \mathbf{I}$
- We know that a beamformer steered towards θ^* maximizes the SNR, with weights $\underline{\mathbf{w}} \equiv \underline{\mathbf{a}}(\theta^*)$ (matched filter)

  纯虚数

  已知 σ_n^2 ?

$$
\begin{aligned}
P_y &= \sigma_s^2 \cdot \underline{\mathbf{w}}^h(\underline{\mathbf{a}}(\theta^*) \cdot \underline{\mathbf{a}}^h(\theta^*))\underline{\mathbf{w}} + \sigma_n^2 \cdot \underline{\mathbf{w}}^h \underline{\mathbf{w}} \\
&= P_s + P_n
\end{aligned}
$$

max. $= J^2\sigma_s^2 + J\sigma_n^2$

Maximum SNR ➡ Maximum P_y

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(DoA) Maximizing steered response power

- Steer the beamformer to a number of candidate directions θ.

$\underline{y}[k] = \underline{w}^h(\theta)\, \underline{x}[k]$

- Output power:
  $P_y(\theta) = E\{|y[k]|^2\} = \underline{\mathbf{w}}^h \cdot \mathbf{R}_x \cdot \underline{\mathbf{w}}$

找峰

$\underline{a}(\theta) = \underline{w}(\theta)$

导向量 $\underline{w}$

- Spatial spectrum: $P(\theta) = \dfrac{P_y(\theta)}{|\underline{\mathbf{w}}|^2} = \dfrac{\underline{\mathbf{a}}^h(\theta)\mathbf{R}_x\underline{\mathbf{a}}(\theta)}{J}$ 归一

$\underline{w} = \underline{a}(\theta^*)$ 纯虚数 when match. $P(\theta) = \dfrac{J^2\sigma_s^2 + J\sigma_n^2}{J}$

- Clearly $P(\theta)$ attains its maximum when $\theta = \theta^*$. Thus, the peak in $P(\theta)$ is the DoA estimate

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(DoA) Maximizing steered response power

ULA, $d/\lambda = 1/2$, $\underbrace{P = 3 \text{ source}}_{\text{?: 多源}}$, $J = 8$ sensors

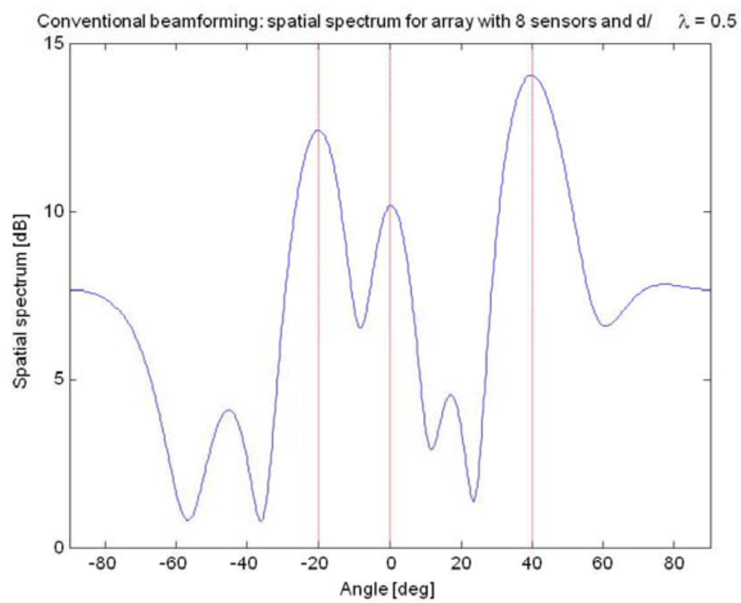

Figure: DoA estimation by steered response power method: sources at -20, 0 and 40 degrees

ADSP (DoA) Maximizing steered response power

- In practice, the search space is discretized depending on the desired accuracy
- Efficient search strategies may be implemented
- The autocorrelation matrix may be computed as $\hat{\mathbf{R}}_x = \frac{1}{T}\sum_{t=1}^{T} \underline{\mathbf{x}}[t] \cdot \underline{\mathbf{x}}^h[t]$

列 空间沿着时间相加

RLS

N

L

$\sum_{l=} \underline{x}\,\underline{x}^h$

短时沿着长时间叠加

Techniques to DoA estimation

- Maximize steered response power
- Using high resolution spectral estimation concepts
- Using time difference of arrival

(DoA) Multiple Signal Classification: MUSIC

- DoA based on high-resolution spectral estimation
- MUSIC: MUltiple SIgnal Classification
- Employs signal subspace method

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(DoA) Multiple Signal Classification: MUSIC

只看这
不看些

- Consider J sensors and P sources, $P < J$
- $x_i[k] = \sum_{p=1}^{P} a_i[\theta_p] s_p[k] + n_i[k], \quad \mathrm{i} = 1 \ldots \mathrm{J}$

$$A = \begin{bmatrix} a_1(\theta_1) & a_1(\theta_2) & \cdots & a_1(\theta_p) \\ a_2(\theta_1) & \cdots & & a_2(\theta_p) \\ \vdots & & & \\ a_J(\theta_1) & \cdots & & a_J(\theta_p) \end{bmatrix}$$

- $\underset{J\times 1}{\underline{\mathbf{x}}[k]} = \underset{J\times P}{\mathbf{A}}\underset{P\times 1}{\underline{\mathbf{s}}[k]} + \underline{\mathbf{n}}[k]$
- $\mathbf{R}_x = E\{\underline{\mathbf{x}}\underline{\mathbf{x}}^h\} = \underset{J\times J}{\mathbf{A}\overset{P\times P}{\mathbf{R}_s}\mathbf{A}^h} + \mathbf{R}_n$
- $\mathbf{R}_s = \mathrm{diag}(\sigma_{s_1}^2, \ldots, \sigma_{s_P}^2)$ $\quad E\{\underline{s}[k]\underline{s}^h[k]\} = \begin{bmatrix} E\{s_i s_j^*\} & \\ & \ddots \end{bmatrix}$ $\quad i \neq j = 0$
- $\mathbf{A}\mathbf{R}_s\mathbf{A}^h$ is a $J \times J$ matrix of rank P, and has $J - P$ zero eigenvalues

$\mathrm{rank}(\underset{J\times J}{AR_sA^h}) = P$

设 $\underline{u}_i$ 是为0的特征值对应的特征向量

$AR_sA^h\underline{u}_i = \underline{0}_{J\times 1}$

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(DoA) Multiple Signal Classification: MUSIC

- Let $\underline{\mathbf{u}}_i$ be an eigenvector of $\mathbf{AR}_s\mathbf{A}^h$ corresponding to one of the zero eigenvalues

- Then $\mathbf{AR}_s\mathbf{A}^h\underline{\mathbf{u}}_i = \underline{0}$ $J\times 1$

- $\Rightarrow \underbrace{\underline{\mathbf{u}}_i^h}_{1\times J}\mathbf{AR}_s\mathbf{A}^h \cdot \underline{\mathbf{u}}_i = \underbrace{0}_{1\times 1}$ or $(\mathbf{A}^h\underline{\mathbf{u}}_i)^h\mathbf{R}_s(\mathbf{A}^h\underline{\mathbf{u}}_i) = 0$

  R_s 正定 $\Rightarrow R_s \neq 0$

- $\Rightarrow \underbrace{\mathbf{A}^h}_{P\times J}\underbrace{\underline{\mathbf{u}}_i}_{J\times 1} = \underbrace{\underline{0}}_{P\times 1}$, as $\mathbf{R}_s$ is positive definite

- So, the eigenvectors corresponding to the $J-P$ eigenvalues that are zero are orthogonal to the steering vectors.

  → 由(J-P)个特征值对应

  单源导向向量

  $\underbrace{\underline{a}(\theta_j)}_{J\times 1} \perp \underbrace{\underline{u}_i}_{J\times 1}$

  共有P个

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(DoA) Multiple Signal Classification: MUSIC

u_i 是 $J \times 1$

- Let $\mathbf{U}_n$ denote the $J \times (J-P)$ matrix containing the $J-P$ eigenvectors corresponding to the eigenvalues that are zero   noise space. $J \times (J-P)$

问：是否存在其它θ，使 $\underline{a}^h(\theta)\, u_i = 0$
↓
仍需要的DOA

- Define $P_{SM}(\theta) = \dfrac{1}{\sum_{i=1}^{J-P} |\underbrace{\underline{\mathbf{a}}^h(\theta)}_{1\times J}\underbrace{\underline{\mathbf{u}}_i}_{J\times 1}|^2} = \dfrac{1}{\underline{\mathbf{a}}^h(\theta)\mathbf{U}_n\mathbf{U}_n^h\underline{\mathbf{a}}(\theta)}$ 只需求为0的特征值对应的特征向量些

↓导向用 θ 遍历

- If θ corresponds to a source direction, then the denominator of $P_{SM}(\theta)$ becomes zero

- So, the P largest peaks of $P_{SM}(\theta)$ provide the source directions. $P_{SM}(\theta)$ is referred to as the pseudo-spectrum 伪谱

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(DoA) Multiple Signal Classification: MUSIC

- The analysis so far involved the EVD of $\mathbf{AR}_s\mathbf{A}^h$, which is not available in practice; only $\mathbf{R}_x$ is. This turns out not to be a problem. 实际情况，源未知

- For any $\underline{\mathbf{u}}_i \in \mathbf{U}_n$, $\mathbf{AR}_s\mathbf{A}^h\underline{\mathbf{u}}_i = \lambda\underline{\mathbf{u}}_i$. $\lambda=0$ $J\times(J-P)$

$R_n = \sigma_n^2 I$

- $\Rightarrow \mathbf{R}_x\underline{\mathbf{u}}_i = (\mathbf{AR}_s\mathbf{A}^h + \mathbf{R}_n)\underline{\mathbf{u}}_i = \lambda\underline{\mathbf{u}}_i + \sigma_n^2\underline{\mathbf{u}}_i = (\lambda + \sigma_n^2)\underline{\mathbf{u}}_i$ $= \sigma_n^2 \underline{u}_i$

根据 $\lambda \geq 0$，$\lambda=0$时对应 $\underline{u}_i$　所以 $\underline{u}_i$ 是 R_x 最小的特征值对应的特征向量

- So the eigen vectors of $\mathbf{R}_x$ are also the eigen vectors of $\mathbf{AR}_s\mathbf{A}^h$! (note that we assumed the noise to be spatially white)

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(DoA) Multiple Signal Classification: MUSIC

- Compute/Estimate $\mathbf{R}_x$ $\sum_{l=1}^{N} \underline{x}[k]\,\underline{x}^h[k]$ $\to$ $\sum_{l=0}^{N-1} \underline{x}[k+l]\,\underline{x}^h[k+l]$ $J\times J$ N段数据
- Perform EVD of $\mathbf{R}_x$; determine $\mathbf{U}_n$ as the matrix containing the eigenvectors corresponding to the $J-P$ smallest eigenvalues
- Evaluate pseudo-spectrum: $P_{SM}(\theta) = \frac{1}{\underline{\mathbf{a}}^h(\theta)\mathbf{U}_n\mathbf{U}_n^h\underline{\mathbf{a}}(\theta)}$
- Locate P sharpest peaks in $P_{SM}(\theta)$

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(DoA) Multiple Signal Classification: MUSIC

- The pseudo spectrum exhibits sharp peaks in the vicinity of the true DoAs

临近

- P_{SM} averages $J-P$ pseudo spectra of individual noise sources. A large value of $J-P$ results in sharper peaks

- The name pseudo is used because P_{SM} contains no information about power

- In practice, $\hat{\mathbf{R}}_x = \frac{1}{T}\sum_{k=1}^{T} \underline{\mathbf{x}}[k]\underline{\mathbf{x}}^h[k]$

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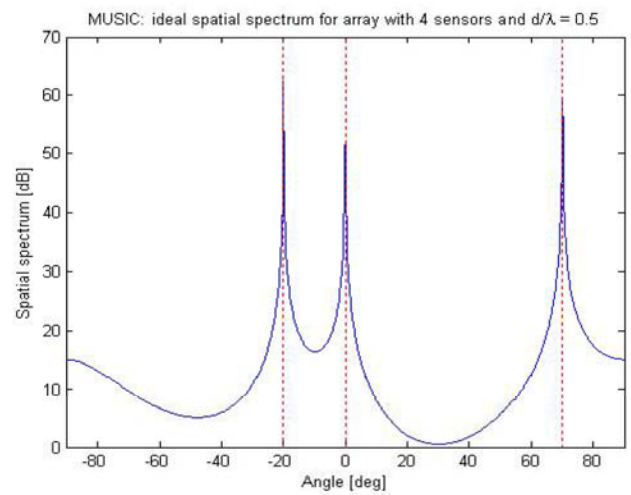
(DoA) Multiple Signal Classification: MUSIC

ULA, $d/\lambda = 1/2$, $P = 3$ sources, $J = 4$ sensors



$\sin(\theta) = \sin(\pi - \theta)$

$-90 \sim 90^\circ$

$-180 \sim 180$ 有镜像

Figure: DoA estimation by Spectral MUSIC: sources at -20, 0 and 70 degrees

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(DoA) Multiple Signal Classification: MUSIC

ULA, $d/\lambda = 1/2$, $P = 3$ sources, $J = 4$ sensors
$P(\theta) = \underline{\mathbf{a}}^h(\theta)\mathbf{R}_x\underline{\mathbf{a}}(\theta)/J$

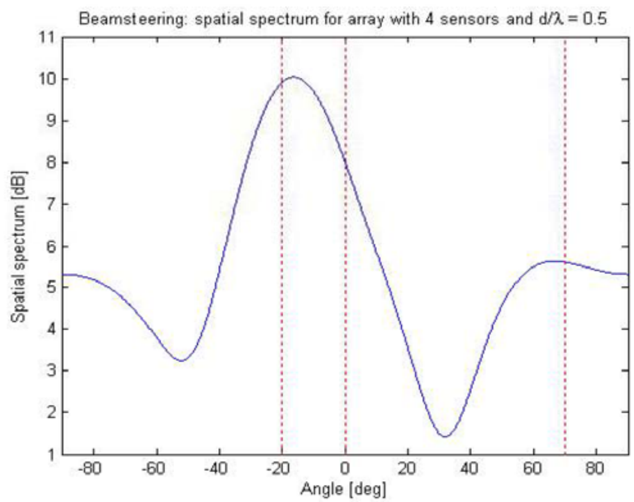

Figure: DoA estimation by beamsteering: sources at -20, 0 and 70 degrees

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(DoA) Multiple Signal Classification: MUSIC

ULA, $d/\lambda = 1/2$, $P = 4$ sources, $J = 10$ sensors
Larger $J - P$ in this case results in deeper minima

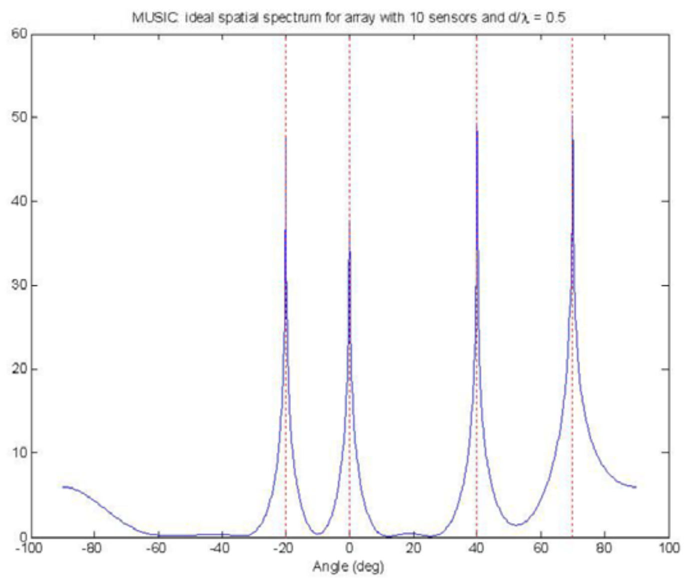

Figure: DoA estimation by Spectral MUSIC: sources at -20, 0, 40 and 70 degrees

Techniques to DoA estimation

- Maximize steered response power
- Using high resolution spectral estimation concepts
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Part D: Adaptive Array signal processing

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①需要权重(steer vector)，也需 R_x，match，以线性数之形式进行搜索

②不需要权重，仅需 $\underline{x}$ MUSIC

① · max steer response power

② · music

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Data dependent beamforming

- The methods discussed so far compute beamformer weights regardless of data being processed
- Alternatively, weights can be based on the statistics of the data, obtained by optimizing a certain criterion

Data dependent beamforming

- Optimum: Assume knowledge of array data statistics. Optimal with respect to a certain optimization criterion (e.g., min MSE, max SINR)
- Adaptive: Estimate the required statistics as data becomes available (when statistics are unknown or time-varying). The derivation is done assuming known statistics. These statistics are estimated on-line.

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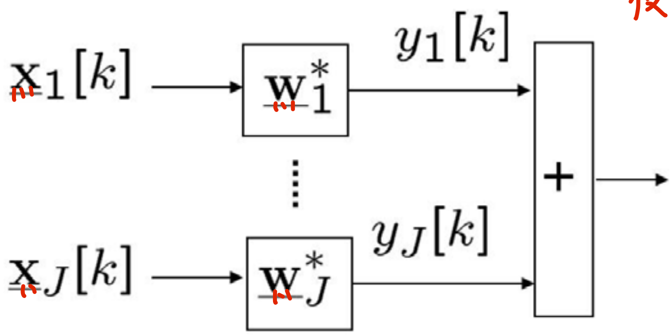
Data dependent beamforming: MMSE

仅有空间，没有时间



$$y[k] = \sum_{i=1}^{J} \underline{w}_i^h \underline{x}_i[k]$$

Short notation: $y[k] = \underline{w}^h \cdot \underline{x}[k]$

$$\underline{x}[k] = (\underline{x}_1[k], \cdots \underline{x}_J[k])^t$$

$$\underline{w} = (\underline{w}_1, \cdots, \underline{w}_J)^t$$

$$\underline{x}[k] \longrightarrow \boxed{\underline{w}^*} \longrightarrow y[k]$$

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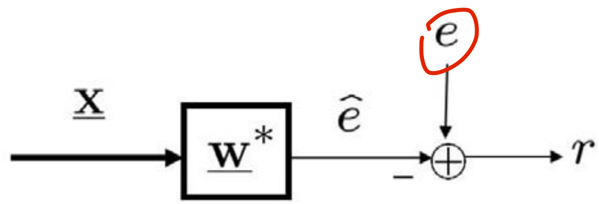
Data dependent beamforming: MMSE



- Cost function: $\xi = E\{|r|^2\}$
- Solution: $\underline{\mathbf{w}}_{\mathrm{mse}} = \arg\min_{\underline{\mathbf{w}}} \xi = \mathbf{R}_x^{-1} \cdot \underline{\mathbf{r}}_{xe^*}$,
  J×J
  where $\mathbf{R}_x = E\{\underline{\mathbf{x}} \cdot \underline{\mathbf{x}}^h\}$ and $\underline{\mathbf{r}}_{xe^*} = E\{\underline{\mathbf{x}} \cdot e^*\}$
- Need to know $\mathbf{R}_x$ and $\underline{\mathbf{r}}_{xe^*}$ (from measurements)

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Data dependent beamforming: MMSE

Example: ULA, one source, narrowband, farfield

假设.

With $\underline{\mathbf{x}} = \underline{\mathbf{a}} \cdot s + \underline{\mathbf{n}}$ and $e = s \Rightarrow$

$\mathbf{R}_x = E\{\underline{\mathbf{x}} \cdot \underline{\mathbf{x}}^h\} = (\underline{\mathbf{a}} \cdot \underline{\mathbf{a}}^h) \cdot \sigma_s^2 + \sigma_n^2 \cdot \mathbf{I}$ and

$\underline{\mathbf{r}}_{xe^*} = E\{\underline{\mathbf{x}} \cdot e^*\} = \underline{\mathbf{a}} \cdot \sigma_s^2$ 只需知道源s的统计量

$E\{(\underline{a} \cdot s + \underline{n}) \cdot s^h\}$

$\Rightarrow$

$$\underline{\mathbf{w}}_{mse} = \mathbf{R}_x^{-1} \cdot \underline{\mathbf{r}}_{xe^*} = \left(\underline{\mathbf{a}} \cdot \underline{\mathbf{a}}^h \cdot \sigma_s^2 + \sigma_n^2 \cdot \mathbf{I}\right)^{-1} \cdot \underline{\mathbf{a}} \cdot \sigma_s^2$$

$$= \left(\frac{(\sigma_s^2/\sigma_n^2)}{1 + J \cdot (\sigma_s^2/\sigma_n^2)}\right) \cdot \underline{\mathbf{a}} = \beta \cdot \underline{\mathbf{a}}$$

$$J_{min} = \left(\frac{(\sigma_s^2/\sigma_n^2)}{1 + J \cdot (\sigma_s^2/\sigma_n^2)}\right) \cdot \sigma_n^2$$