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 Primeira Prova

1.1) (2.5) Escreva em forma de tabela as matrizes dadas:

1.1) $A = (a_{ij})_{3 \times 4}$ tal que $a_{ij} = (-i^2) - (-j^2) - 3/5 (i \cdot j)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3 \times 4}; A = \begin{bmatrix} -3/5 & 9/5 & 31/5 & 63/5 \\ -21/5 & -12/5 & 7/5 & 36/5 \\ -49/5 & -43/5 & -27/5 & -1/5 \end{bmatrix}_{3 \times 4}$$

$$\begin{aligned} a_{11} &= (-1^2) - (-1^2) - 3/5 (1 \cdot 1) = -1 + 1 - 3/5 \cdot 1 = 0 - 3/5 = -3/5 \\ a_{12} &= (-1^2) - (-2^2) - 3/5 (1 \cdot 2) = -1 + 4 - 3/5 \cdot 2 = +3 - 3/5 \cdot 2/1 = +3 - 6/5 = 3/1 - 6/5 = 15/5 - 6/5 = +9/5 \end{aligned}$$

$$\begin{aligned} a_{13} &= (-1^2) - (-3^2) - 3/5 (1 \cdot 3) = -1 + 9 - 3/5 \cdot 3 = +8 - 3/5 \cdot 3/1 = +8/1 - 9/5 = +40/5 - 9/5 = +31/5 \\ a_{14} &= (-1^2) - (-4^2) - 3/5 (1 \cdot 4) = -1 + 16 - 3/5 \cdot 4 = +15 - 3/5 \cdot 4/1 = +15 - 12/5 = +15/1 - 12/5 = +75/5 - 12/5 = +63/5 \end{aligned}$$

$$\begin{aligned} a_{21} &= (-2^2) - (-1^2) - 3/5 (2 \cdot 1) = -4 + 1 - 3/5 \cdot 2 = -3 - 3/5 \cdot 2/1 = -3/1 - 6/5 = -15/5 - 6/5 = -21/5 \\ a_{22} &= (-2^2) - (-2^2) - 3/5 (2 \cdot 2) = -4 + 4 - 3/5 \cdot 4 = 0 - 3/5 \cdot 4/1 = -12/5 \end{aligned}$$

$$\begin{aligned} a_{23} &= (-2^2) - (-3^2) - 3/5 (2 \cdot 3) = -4 + 9 - 3/5 \cdot 6 = +5 - 3/5 \cdot 6/1 = +5/1 - 18/5 = +25/5 - 18/5 = +7/5 \\ a_{24} &= (-2^2) - (-4^2) - 3/5 (2 \cdot 4) = -4 + 16 - 3/5 \cdot 8 = +12 - 3/5 \cdot 8/1 = +12/1 - 24/5 = +60/5 - 24/5 = +36/5 \end{aligned}$$

$$\begin{aligned} a_{31} &= (-3^2) - (-1^2) - 3/5(3 \cdot 1) = -9 + 1 - 3/5 \cdot 3 = -8 - 3/5 \cdot 3/1 = -8/1 - 9/5 = -40/5 - 9/5 = -49/5 \\ a_{32} &= (-3^2) - (-2^2) - 3/5(3 \cdot 2) = -9 + 4 - 3/5 \cdot 6 = -5 - 3/5 \cdot 6/1 = -5/1 - 18/5 = -25/5 - 18/5 = -43/5 \end{aligned}$$

$$\begin{aligned} a_{33} &= (-3^2) - (-3^2) - 3/5(3 \cdot 3) = -9 + 9 - 3/5 \cdot 9 = 0 - 3/5 \cdot 9 = -3/5 \cdot 9/1 = -27/5 \\ a_{34} &= (-3^2) - (-4^2) - 3/5(3 \cdot 4) = -9 + 16 - 3/5 \cdot 12 = +7 - 3/5 \cdot 12/1 = +7/1 - 36/5 = +35/5 - 36/5 = -1/5 \end{aligned}$$

1.2) $A = (a_{ij})_{3 \times 3}$ tal que $a_{ij} = 2 \cdot f(i) + 3 \cdot f(j)$, para $f(x) = 2 \cdot (-x^2) + 5$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3 \times 3}; A = \begin{bmatrix} 15 & -3 & -33 \\ 3 & -15 & -45 \\ -17 & -35 & -65 \end{bmatrix}_{3 \times 3}$$

$$\begin{aligned} f(1) &= 2 \cdot (-1^2) + 5 = 2 \cdot (-1) + 5 = -2 + 5 = +3 \\ f(2) &= 2 \cdot (-2^2) + 5 = 2 \cdot (-4) + 5 = -8 + 5 = -3 \\ f(3) &= 2 \cdot (-3^2) + 5 = 2 \cdot (-9) + 5 = -18 + 5 = -13 \end{aligned}$$

$$\begin{aligned} a_{11} &= 2 \cdot f(1) + 3 \cdot f(1) = 2 \cdot 3 + 3 \cdot 3 = 6 + 9 = +15 \\ a_{12} &= 2 \cdot f(1) + 3 \cdot f(2) = 2 \cdot 3 + 3 \cdot (-3) = 6 - 9 = -3 \end{aligned}$$

$$\begin{aligned} a_{13} &= 2 \cdot f(1) + 3 \cdot f(3) = 2 \cdot 3 + 3 \cdot (-13) = 6 - 39 = -33 \\ a_{21} &= 2 \cdot f(2) + 3 \cdot f(1) = 2 \cdot (-3) + 3 \cdot 3 = -6 + 9 = +3 \end{aligned}$$

$$\begin{aligned} a_{22} &= 2 \cdot f(2) + 3 \cdot f(2) = 2 \cdot (-3) + 3 \cdot (-3) = -6 + (-9) = -6 - 9 = -15 \\ a_{23} &= 2 \cdot f(2) + 3 \cdot f(3) = 2 \cdot (-3) + 3 \cdot (-13) = -6 + (-39) = -6 - 39 = -45 \end{aligned}$$

$$a_{31} = 2 \cdot f(3) + 3 \cdot f(1) =$$

$$2 \cdot (-13) + 3 \cdot 3 =$$

$$-26 + 9 = -17$$

$$a_{32} = 2 \cdot f(3) + 3 \cdot f(2) =$$

$$2 \cdot (-13) + 3 \cdot (-3) =$$

$$-26 - 9 = -35$$

$$a_{33} = 2 \cdot f(3) + 3 \cdot f(3) =$$

$$2 \cdot (-13) + 3 \cdot (-13) =$$

$$-26 - 39 = -65$$

2) (2.5) Determinar x, y, a e b tal que:

$$\begin{bmatrix} 1/2x + 5y & 3a - 5b \\ -7x + 3/4y + 3 & 2a + 7b \end{bmatrix}_{2 \times 2} = \begin{bmatrix} 2y + 2 & 0 \\ 1/3x - 5 & 1 \end{bmatrix}_{2 \times 2}$$

$$\begin{cases} 1/2x + 5y = 2y + 2 \\ -7x + 3/4y + 3 = 1/3x - 5 \end{cases}$$

$$\begin{cases} 22/6x + 66/3y = 44/3 \\ -22/6x + 3/8y = -8/2 \end{cases}$$

$$\begin{cases} 1/2x + 5y - 2y = +2 \\ -7x + 3/4y - 1/3x = -5 - 3 \end{cases}$$

$$\begin{cases} 22/6x + 22y = 44/3 \\ -22/6x + 3/8y = -4 \end{cases}$$

$$\begin{cases} 1/2x + 3y = +2 \\ -7/1x - 1/3x + 3/4y = -8 \end{cases}$$

$$0 + 22y + 3y = \frac{44}{3} - 4 =$$

$$\frac{176y + 3y}{8 \quad 3} = \frac{44 - 12}{3} =$$

$$\frac{179y}{8 \quad 3} = \frac{32}{3} \Rightarrow y = \frac{32 \cdot 8}{3 \cdot 179} =$$

$$\begin{cases} 1/2x + 3y = +2 \\ -22/3x + 3/4y = -8 \end{cases}$$

$$y = \frac{256}{537}$$

eliminando o x :

$$\begin{cases} 1/2x + 3y = +2 \quad \cdot (22/3) \\ -22/3x + 3/4y = -8 \quad \cdot (1/2) \end{cases}$$

$$\begin{cases} 1/2x(22/3) + 3y(22/3) = 2 \cdot (22/3) \\ -22/3x(1/2) + 3/4y(1/2) = -8 \cdot (1/2) \end{cases}$$

eliminando o y :

$$\begin{cases} 1/2x + 3y = +2 \quad \cdot 3/4 \\ -22/3x + 3/4y = -8 \quad \cdot (-3) \end{cases}$$

$$\begin{cases} 1/2x(3/4) + 3y(3/4) = 2 \cdot (3/4) \\ -22/3x(-3) + 3/4y(-3) = -8(-3) \end{cases}$$

$$\begin{cases} 3/8x + 9/4y = 6/4 \\ +66/3x + (-9/4y) = +24 \end{cases} \rightarrow \begin{matrix} 176x + 3x = 3 + 48 = \\ 8 \qquad \qquad 2 \end{matrix}$$

$$\begin{cases} 3/8x + 9/4y = 3/2 + \\ 22x - 9/4y = 24 \end{cases} \rightarrow \begin{matrix} 179x = 51 \Rightarrow x = \frac{51.8}{179.2} \\ 8 \times 2 \end{matrix}$$

$$\begin{matrix} 22x + 3x + 0 = \frac{3}{2} + \frac{24}{1} = \\ 1 \qquad 8 \qquad 2 \end{matrix} \rightarrow x = \frac{408:2}{358:2} \Rightarrow \boxed{x = \frac{204}{179}}$$

$$\begin{cases} 3a - 5b = 0 \\ 2a + 7b = 1 \end{cases} \quad \text{eliminando o } b:$$

$$\begin{cases} 3a - 5b = 0 \quad \cdot 7 \\ 2a + 7b = 1 \quad \cdot 5 \end{cases}$$

$$\text{eliminando o } a:$$

$$\begin{cases} 3a - 5b = 0 \quad \cdot (-2) \\ 2a + 7b = 1 \quad \cdot 3 \end{cases} \rightarrow \begin{cases} 21a - 35b = 0 + \\ 10a + 35b = 3 \end{cases}$$

$$\begin{cases} -6a + 10b = 0 + \\ 6a + 21b = 3 \end{cases} \rightarrow \begin{matrix} 31a + 0 = 3 \\ a = \frac{3}{31} \end{matrix}$$

$$\begin{matrix} 0 + 31b = 3 \\ b = \frac{3}{31} \end{matrix}$$

$$\begin{matrix} x = \frac{204}{179} & y = \frac{256}{537} \\ a = \frac{3}{31} & b = \frac{3}{31} \end{matrix}$$

3) (2.5) Determinar x, y, z e t tal que se tenha

$$\begin{bmatrix} x^2 & 2x & y \\ 4 & 5 & t^2 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} x & x & 3 \\ z & 5t & t \end{bmatrix}_{2 \times 3}$$

$x^2 = x$	$2x = x$	$y = 3$	$4 = z$	$5 = 5t$	$t^2 = t$
$x = \pm \sqrt{x}$	$x = \frac{x}{2}$		$z = 4$	$5 = t$	$t = \pm \sqrt{t}$
$a_{11} \uparrow$	$a_{12} \uparrow$	$a_{13} \uparrow$	$a_{21} \uparrow$	$t = 1$	$t = \pm 1$
			$a_{22} \uparrow$		$a_{23} \uparrow$

4.1) (2.5) Dada a matriz $A_{3 \times 3}$, determinar:

$$\begin{bmatrix} 3/5 & 2 & 4/5 \\ 5 & 1/2 & -7 \\ 2/3 & 9 & 0 \end{bmatrix}_{3 \times 3}$$

4.1.1) O valor de $a_{11} - \frac{[a_{11} \cdot a_{13} - (a_{13})^2]}{(a_{13} + a_{11})} =$

$$\frac{3}{5} - \frac{\left[\frac{3}{5} \cdot \frac{4}{5} - \left(\frac{4}{5} \right)^2 \right]}{\frac{4}{5} + \frac{3}{5}} = \frac{3}{5} - \frac{\left[\frac{12}{25} - \frac{16}{25} \right]}{\frac{7}{5}} = \begin{array}{l|l} 5,175 & 5 \\ 1,35 & 5 \\ 7 & 7 \\ 1 & 175 \end{array}$$

$$\frac{3}{5} - \frac{\left[\frac{-4}{25} \right]}{\frac{7}{5}} = \frac{3 \cdot 1}{5 \cdot 1} - \frac{(-4) \cdot 5}{25 \cdot 7} = \frac{3}{5} - \frac{(-20)}{175} = \rightarrow$$

$$\frac{105 + 20}{175} = \frac{125}{175} : 25 = \frac{5}{7}$$

4.2) O valor de $a_{13} + a_{22} \cdot a_{23} - (a_{31})^2 =$

$$\frac{4}{5} + \frac{1}{2} \cdot \frac{(-7)}{1} - \left(\frac{2}{3} \right)^2 = \frac{4}{5} + \frac{(-7)}{2} - \frac{4}{9} = \frac{4}{5} - \frac{7}{2} - \frac{4}{9} =$$

$$\frac{+72 - 315 - 40}{90} = \frac{+72 - 355}{90} = \frac{-283}{90}$$

$$\begin{array}{l|l} 2,5,9 & 2 \\ 1,5,9 & 3 \\ 1,5,3 & 3 \\ 1,5,1 & 5 \\ 1,1,1 & 90 \end{array}$$

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