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Nome: Gabriel Gonçalves de Oliveira RA: 2111550021
Professora: Dra. Marisa Atsuko Nitto - 1º ADS

Lista de Exercícios - Matemática - Aula 14

1) Dada a matriz $A = \begin{bmatrix} 1 & x & 5 \\ 2 & 7 & -4 \\ y & z & -3 \end{bmatrix}_{3\times3}$, determinar x, y e z para que $A = A^t$

$A = A^t$

$$\begin{bmatrix} 1 & x & 5 \\ 2 & 7 & -4 \\ y & z & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & y \\ x & 7 & z \\ 5 & -4 & -3 \end{bmatrix} \quad \boxed{x = 2} \quad \boxed{y = 5} \quad \boxed{z = -4}$$

C

2) Dadas as matrizes $A = \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}_{2\times2}$ e $B \begin{bmatrix} 0 & 6 \\ 9 & 3 \end{bmatrix}_{2\times2}$, determinar:

2.1) $\underbrace{A.B}_{C} = \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 6 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} (1.0+1.9) & (1.6+1.3) \\ (5.0+7.9) & (5.6+7.3) \end{bmatrix}$

$= \begin{bmatrix} (0+9) & (6+3) \\ (0+63) & (30+21) \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 63 & 51 \end{bmatrix}_{2\times2} \qquad C = \begin{bmatrix} 9 & 9 \\ 63 & 51 \end{bmatrix}_{2\times2}$

C

2.2) $A^2 + 3(B.A) - 7.I_2$ (onde I_2 = matriz identidade)

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2} \cdot B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2\times2} = \begin{bmatrix} (a_{11}.b_{11} + a_{12}.b_{21}) & (a_{11}.b_{12} + a_{12}.b_{22}) \\ (a_{21}.b_{11} + a_{22}.b_{21}) & (a_{21}.b_{12} + a_{22}.b_{22}) \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} + 3\left(\begin{bmatrix} 0 & 6 \\ 9 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}\right) - 7. \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1.1+1.5) & (1.1+1.7) \\ (5.1+7.5) & (5.1+7.7) \end{bmatrix} + 3\left(\begin{bmatrix} (0.1+6.5) & (0.1+6.7) \\ (9.1+3.5) & (9.1+3.7) \end{bmatrix}\right) - 7. \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+5) & (1+7) \\ (5+35) & (5+49) \end{bmatrix} + 3. \begin{bmatrix} (0+30) & (0+42) \\ (9+15) & (9+21) \end{bmatrix} - \begin{bmatrix} 1.7 & 0.7 \\ 0.7 & 1.7 \end{bmatrix}$$

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$$= \begin{bmatrix} (1+5) & (1+7) \\ (5+35) & (5+49) \end{bmatrix} + 3 \cdot \begin{bmatrix} (0+30) & (0+42) \\ (9+15) & (9+21) \end{bmatrix} - \begin{bmatrix} 1.7 & 0.7 \\ 0.7 & 1.7 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ 40 & 54 \end{bmatrix} + 3 \cdot \begin{bmatrix} 30 & 42 \\ 24 & 30 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ 40 & 54 \end{bmatrix} + \begin{bmatrix} 30.3 & 42.3 \\ 24.3 & 30.3 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 40 & 54 \end{bmatrix} + \begin{bmatrix} 90 & 126 \\ 72 & 90 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} (6+90-7) & (8+126-0) \\ (40+72-0) & (54+90-7) \end{bmatrix} = \begin{bmatrix} 89 & 134 \\ 112 & 137 \end{bmatrix} //$$

3) Determinar a inversa das matrizes dadas:

$\det(A) = dp - ds$

3.1) $A = \begin{bmatrix} 1 & -3/5 \\ 5 & 7 \end{bmatrix}_{2\times 2}$ ds dp

$\det(A) = a_{11} \cdot a_{22} - \cancel{a_{12}} \; a_{12} \cdot a_{21}$

$\det(A) = 1 \cdot 7 - (-3/5) \cdot 5$

$\det(A) = 7 + 15/5$

$\det(A) = 7 + 3 \rightarrow \det(A) = 10$

tem inversa !!

$A \cdot A^{-1} = I_2$

$$\begin{bmatrix} 1 & -3/5 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} (1 \cdot a + (-3/5) \cdot c) & (1 \cdot b + (-3/5) \cdot d) \\ (5 \cdot a + 7 \cdot c) & (5 \cdot b + 7 \cdot d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} (a - 3/5c = 1) & (b + -3/5d = 0) \\ (5a + 7c = 0) & (5b + 7d = 1) \end{bmatrix}$$

$$\begin{cases} a - 3/5c = 1 & \cdot(7) \\ 5a + 7c = 0 & \cdot(3/5) \end{cases}$$

$$\begin{cases} a - 3/5c = 1 & \cdot(5) \\ 5a + 7c = 0 & \cdot(-1) \end{cases}$$

$$\begin{cases} 5a - 3c = 5 \\ -5a - 7c = 0 \end{cases} +$$

$0 - 10c = 5 \quad \cdot(-1)$

$c = \frac{-5}{10} \Rightarrow \boxed{c = \frac{-1}{2}} //$

$$\begin{cases} 7a - 21/5c = 7 \\ 3a + 21/5c = 0 \end{cases} +$$

$10a + 0 = 7$

$$\boxed{a = \frac{7}{10}} //$$

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$$\begin{cases} b - 3/5d = 0 & .(5) \\ 5b + 7d = 1 & .(-1) \end{cases}$$

$$\begin{cases} 5b - 3d = 0 \\ -5b - 7d = -1 \end{cases} +$$

$$0 - 10d = -1 \quad .(-1)$$

$$10d = 1$$

$$\boxed{d = \frac{1}{10}}$$

$$\begin{cases} b - 3/5\, d = 0 & .(7) \\ 5b + 7d = 1 & .(3/5) \end{cases}$$

$$\begin{cases} 7b - 21/5 = 0 \\ 3b + 21/5 = 3/5 \end{cases} +$$

$$10b + 0 = 3/5$$

$$b = \frac{3}{5.10} \Rightarrow \boxed{b = \frac{3}{50}}$$

$$A^{-1} = \begin{bmatrix} 7/10 & 3/50 \\ -1/2 & 1/10 \end{bmatrix}_{2\times2}$$

3.2) $B = \begin{bmatrix} 1 & -2 \\ 3/4 & 7 \end{bmatrix}_{2\times2}$

$$\det(B) = dp - ds$$

$$\det(B) = b_{11}.b_{22} - b_{12}.b_{21}$$

$$\det(B) = 1.7 - (-2).3/4$$

$$\det(B) = 7 + 6/4$$

$$\det(B) = 7 + 3/2 \quad {:}2 \; {:}2$$

$$\det(B) = 14/2 + 3/2$$

$$\boxed{\det(B) = \frac{17}{2}}$$

$B.B^{-1} = I_2$

$$\begin{bmatrix} 1 & -2 \\ 3/4 & 7 \end{bmatrix} . \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} (1.a + (-2).c) & (1.b + (-2).d) \\ (3/4.a + 7.c) & (3/4.b + 7.d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} (1a - 2c = 1) & (1b - 2d = 0) \\ (3/4a + 7c = 0) & (3/4b + 7d = 1) \end{bmatrix}$$

$$\begin{cases} a - 2c = 1 & .(3/4) \\ 3/4a + 7c = 0 & .(-1) \end{cases}$$

$$\begin{cases} 3/4a - 3/2c = 3/4 \\ -3/4a - 7c = 0 \end{cases} +$$

$$0 - \frac{3c}{2} - \frac{7c}{1} = \frac{3}{4}$$

$$\frac{-3c - 14c}{2} = \frac{3}{4}$$

$$-\frac{17c}{2} = \frac{3}{4} \quad .(-1)$$

$$\frac{17c}{2} = -\frac{3}{4}$$

$$c = -\frac{3.2}{17.4} \quad {:}2$$

$$\boxed{c = -\frac{3}{34}}$$

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$$\begin{cases} a-2c=1 & .(7) \\ 3/4a+7c=0 & .(2) \end{cases}$$

$$\begin{cases} 7a-14c=7 \\ 3/2a+14c=0 \end{cases} +$$

$$\frac{7a}{1}+\frac{3a}{2}=7$$

$$\frac{14a+3a}{2}=7$$

$$17a=2.7$$

$$\boxed{a=\frac{7.2}{17}} \rightarrow \boxed{a=\frac{14}{17}}$$

$$\begin{cases} b-2d=0 & .(-3/4) \\ 3/4b+7d=1 & .(1) \end{cases}$$

$$\begin{cases} -3/4b+3/2d=0 \\ 3/4b+7d=1 \end{cases} +$$

$$0+\frac{3d}{2}+\frac{7d}{1}=1$$

$$\frac{3d+14d}{2}=1$$

$$\frac{17d}{2}=1$$

$$\boxed{d=\frac{2}{17}}$$

$$\begin{cases} b-2d=0 & .(+7) \\ 3/4b+7d=1 & .(2) \end{cases}$$

$$\cancel{\begin{cases} -7b+14d=0 \\ 3/4b \end{cases}}$$

$$\begin{cases} 7b-14d=0 \\ 3/2b+14d=2 \end{cases} +$$

$$\frac{7b}{1}+\frac{3b}{2}=2$$

$$\frac{14b+3b}{2}=2$$

$$\frac{17b}{2}=2$$

$$b=\frac{2.2}{17}$$

$$\boxed{b=\frac{4}{17}}$$

$$B^{-1}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} \qquad B^{-1}=\begin{bmatrix} 14/17 & 4/17 \\ -3/34 & 2/17 \end{bmatrix}_{2\times2}$$

Gabriel Gonçalves de Oliveira 2111550021 1ºADS