data / fecha 17.06.21

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Segunda Prova

1) (2.5) Determinar a inversa da Matriz A dada.

$A = \begin{bmatrix} 7/8 & 4 \\ 3 & -10 \end{bmatrix}_{2\times2} \rightarrow A \cdot A^{-1} = I_2 \rightarrow \begin{bmatrix} 7/8 & 4 \\ 3 & -10 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} (a_{11}.b_{11} + a_{12}.b_{21}) & (a_{11}.b_{12} + a_{12}.b_{22}) \\ (a_{21}.b_{12} + a_{22}.b_{21}) & (a_{21}.b_{12} + a_{22}.b_{22}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} (7/8.a + 4.c) & (7/8.b + 4.d) \\ (3.a + (-10).c) & (3.b + (-10.d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7/8a + 4c = 1 & 7/8b + 4d = 0 \\ 3a - 10c = 0 & 3b - 10d = 1 \end{bmatrix}$

$det(A) = d_p - d_s$
$det(A) = (7/8).(-10) - 4.3$
$det(A) = -70/8 - 12$
$det(A) = -70/8 - 96/8$
$det(A) = -166/8 \ :2$
$det(A) = -83/4$
tem inversa

$\begin{cases} 7/8a + 4c = 1 & .(3) \\ 3a - 10c = 0 & .(-7/8) \end{cases}$

$\begin{cases} 21/8a + 12c = 3 \\ -21/8a + 70/8c = 0 \end{cases} +$

$0 + \frac{12c}{1} + \frac{70c}{8} = \frac{3}{1}$

$\frac{96c + 70c}{8} = 3$

$\frac{166c}{8} = 3$

$c = \frac{3.8}{166}$

$c = \frac{24}{166} \frac{:2}{:2} \Rightarrow \boxed{c = \frac{12}{83}}$

$\begin{cases} 7/8a + 4c = 1 & .(10) \\ 3a - 10c = 0 & .(4) \end{cases}$

$\begin{cases} 70/8a + 40c = 10 \\ 12a - 40c = 0 \end{cases} +$

$\frac{70a}{8} + \frac{12a}{1} + 0 = \frac{10}{1}$

$\frac{70a + 96a}{8} = 10$

$\frac{166a}{8} = 10$

$a = \frac{10.8}{166}$

$a = \frac{80}{166} \frac{:2}{:2} \Rightarrow \boxed{a = \frac{40}{83}}$

data fecha 17.06.21

$$\begin{cases} \frac{7}{8}b + 4d = 0 & \cdot(3) \\ 3b - 10d = 1 & \cdot(-7/8) \end{cases}$$

$$\begin{cases} \frac{21}{8}b + 12d = 0 \\ -\frac{21}{8}b + \frac{70}{8}d = -\frac{7}{8} \end{cases} +$$

$$0 + \frac{12d}{1} + \frac{70d}{8} = \frac{-7}{8}$$

$$\frac{96d + 70d}{8} = \frac{-7}{8}$$

$$\frac{166d}{8} = \frac{-7}{8}$$

$$\boxed{d = \frac{-7}{166}}$$

$$\begin{cases} \frac{7}{8}b + 4d = 0 & \cdot(10) \\ 3b - 10d = 1 & \cdot(4) \end{cases}$$

$$\begin{cases} \frac{70}{8}b + 40d = 0 \\ 12b - 40d = 4 \end{cases} +$$

$$0 + \frac{70b}{8} + \frac{12b}{1} = \frac{4}{1}$$

$$\frac{70b + 96b}{8} = 4$$

$$\frac{166b}{8} = 4$$

$$b = \frac{4.8}{166} \Rightarrow b = \frac{32 : 2}{166 : 2} \Rightarrow \boxed{b = \frac{16}{83}}$$

Portanto, $A^{-1} = \begin{bmatrix} 40/83 & 12/83 \\ 16/83 & -7/166 \end{bmatrix}_{2\times2}$

2) (2.5) Escreva em forma de tabela as matrizes A e B dadas por:

$A = (a_{ij})_{3\times3}$ tal que $a_{ij} = (-i^2) - (-j^2)$

$B = (b_{ij})_{3\times3}$ tal que $b_{ij} = f(i) + f(j)$, para $f(x) = x + 2$

Determinar $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} = A = \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}_{3\times3}$

$a_{11} = (-1^2) - (-1^2) = -1 - (-1) = -1 + 1 = 0.$

$a_{12} = (-1^2) - (-2^2) = -1 - (-4) = -1 + 4 = 3.$

$a_{13} = (-1^2) - (-3^2) = -1 - (-9) = -1 + 9 = 8.$

~~$= (-1^2) - (-1^2) = -1 - ($~~

$a_{21} = (-2^2) - (-1^2) = -4 - (-1) = -4 + 1 = -3.$

$a_{22} = (-2^2) - (-2^2) = -4 - (-4) = -4 + 4 = 0.$

$a_{23} = (-2^2) - (-3^2) = -4 - (-9) = -4 + 9 = 5.$

data
fecha 17.06.21

pg3

$a_{31} = (-3^2) - (-1^2) = -9 - (-1) = -9 + 1 = -8$
$a_{32} = (-3^2) - (-2^2) = -9 - (-4) = -9 + 4 = -5$
$a_{33} = (-3^2) - (-3^2) = -9 - (-9) = -9 + 9 = 0$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times3} \Rightarrow B = \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix}_{3\times3}$$

$f(x) = x + 2;$ $f(1) = 1 + 2 = 3$
$f(2) = 2 + 2 = 4$
$f(3) = 3 + 2 = 5$

$b_{11} = f(1) + f(1) = 3 + 3 = 6$
$b_{12} = f(1) + f(2) = 3 + 4 = 7$
$b_{13} = f(1) + f(3) = 3 + 5 = 8$
$b_{21} = f(2) + f(1) = 4 + 3 = 7$
$b_{22} = f(2) + f(2) = 4 + 4 = 8$
$b_{23} = f(2) + f(3) = 4 + 5 = 9$
$b_{31} = f(3) + f(1) = 5 + 3 = 8$
$b_{32} = f(3) + f(2) = 5 + 4 = 9$
$b_{33} = f(3) + f(3) = 5 + 5 = 10$

→ Determinar:

2.1) $A + B \Rightarrow \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix}$

$$= \begin{bmatrix} (0+6) & (3+7) & (8+8) \\ (-3+7) & (0+8) & (5+9) \\ (-8+8) & (-5+9) & (0+10) \end{bmatrix} = \begin{bmatrix} 6 & 10 & 16 \\ 4 & 8 & 14 \\ 0 & 4 & 10 \end{bmatrix}_{3\times3}$$

Portanto, $A + B = \begin{bmatrix} 6 & 10 & 16 \\ 4 & 8 & 14 \\ 0 & 4 & 10 \end{bmatrix}_{3\times3}$

17 06 21

2.2) $2B - A^t$ $\quad 2 \cdot \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$ para transpor

$$= 2 \cdot \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix} - \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6.2 & 7.2 & 8.2 \\ 7.2 & 8.2 & 9.2 \\ 8.2 & 9.2 & 10.2 \end{bmatrix} - \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 14 & 16 \\ 14 & 16 & 18 \\ 16 & 18 & 20 \end{bmatrix} - \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (12-0) & (14 \overset{+}{-} (-3)) & (16 \overset{+}{-} (-8)) \\ (14-3) & (16-0) & (18-(-5)) \\ (16-8) & (18-5) & (20-0) \end{bmatrix} = \begin{bmatrix} 12 & 14+3 & 16+8 \\ 11 & 16 & 18+5 \\ 8 & 13 & 20 \end{bmatrix}$$

Portanto, $2B - A^t = \begin{bmatrix} 12 & 17 & 24 \\ 11 & 16 & 23 \\ 8 & 13 & 20 \end{bmatrix}_{3 \times 3}$

3) (2.5) Dadas as matrizes booleanas, determinar:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3 \times 3} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3 \times 3}$$

3.1) $A \times B$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} (0\wedge1 \vee 0\wedge0 \vee 1\wedge1) & (0\wedge0 \vee 0\wedge1 \vee 1\wedge0) & (0\wedge1 \vee 0\wedge1 \vee 1\wedge0) \\ (1\wedge1 \vee 1\wedge0 \vee 1\wedge1) & (1\wedge0 \vee 1\wedge1 \vee 1\wedge0) & (1\wedge1 \vee 1\wedge1 \vee 1\wedge0) \\ (1\wedge1 \vee 1\wedge0 \vee 0\wedge1) & (1\wedge0 \vee 1\wedge1 \vee 0\wedge0) & (1\wedge1 \vee 1\wedge1 \vee 0\wedge0) \end{bmatrix}$$

17 06 21

$$= \begin{bmatrix} (0 \vee 0 \vee 1) & (0 \vee 0 \vee 0) & (0 \vee 0 \vee 0) \\ (1 \vee 0 \vee 1) & (0 \vee 1 \vee 0) & (1 \vee 1 \vee 0) \\ (1 \vee 0 \vee 0) & (0 \vee 1 \vee 0) & (1 \vee 1 \vee 0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$, Portanto, $A \times B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$

3.2) $B \times A$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} (0 \vee 0 \vee 1) & (0 \vee 0 \vee 1) & (1 \vee 0 \vee 0) \\ (0 \vee 1 \vee 1) & (0 \vee 1 \vee 1) & (0 \vee 1 \vee 0) \\ (0 \vee 0 \vee 0) & (0 \vee 0 \vee 0) & (1 \vee 0 \vee 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$, Portanto, $B \times A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

4-) (2,5) Determinar x e y para que as matrizes sejam iguais:

$$\begin{bmatrix} 1/2x + 7y \\ -7x + 3/5y + 3 \end{bmatrix}_{2\times1} = \begin{bmatrix} 2y + 7 \\ 1/3x - 5 \end{bmatrix}_{2\times1} \rightarrow \begin{bmatrix} 1/2x + 7y = 2y + 7 \\ -7x + 3/5y + 3 = 1/3x - 5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1/2x + 7y - 2y = 7 \\ -7x - 1/3x + 3/5y = -5 - 3 \end{bmatrix} = \begin{bmatrix} 1/2x + 5y = 7 \\ -21/3x - 1/3x + 3/5y = -8 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1/2x + 5y = 7 \\ -20/3x + 3/5y = -8 \end{bmatrix}_{2\times1}$$

*Descobri que errei uma subtração... Vou refazer a partir disso.

$$\begin{cases} 1/2x + 5y = 7 & .(20/3) \\ -20/3x + 3/5y = -8 & .(1/2) \end{cases}$$

$$\begin{cases} 20/6x + 100/3y = 140/3 \\ -20/6x + 3/10y = \underbrace{-8/2}_{4} \end{cases}$$

$$\rightarrow \begin{cases} 20/6x + 100/3y = 140/3 \\ -20/6x + 3/10y = 4 \end{cases} +$$

$$0 + \frac{100y}{3} + \frac{3y}{10} = \frac{4}{1}$$

17 06 21

$$\begin{bmatrix} 1/2x + 5y = 7 \\ -7x - 1/3x + 3/5y = -5-3 \end{bmatrix} = \begin{bmatrix} 1/2x + 5y = 7 \\ -21/3x - 1/3x + 3/5y = -8 \end{bmatrix}$$

$$\begin{cases} 1/2x + 5y = 7 & .(22/3) \\ -22/3x + 3/5y = -8 & .(1/2) \end{cases}$$

$$\begin{cases} 22/6x + 110/3y = 154/3 \\ -22/6x + 3/10y = -4 \end{cases} +$$

$$0 + \frac{110y}{3} + \frac{3y}{10} = \frac{-4}{1} + \frac{154}{3}$$

$$\frac{1100y + 9y}{30} = \frac{-12+154}{3}$$

$$\frac{1109y}{30:3} = \frac{142}{3:3}$$

$$\frac{1109y}{10} = \frac{142}{1}$$

$$y = \frac{142 \cdot 10}{1109}$$

$$\boxed{y = \frac{1420}{1109}}$$

$$\begin{cases} 1/2x + 5y = 7 & .(3/5) \\ -22/3x + 3/5y = -8 & .(-5) \end{cases}$$

$$\begin{cases} 3/10x + 15/5y = 21/5 \\ 110/3x - 15/5y = 40 \end{cases} +$$

$$\frac{3x}{10} + \frac{110x}{3} + 0 = \frac{21}{5} + \frac{40}{1}$$

$$\frac{9x + 1100x}{30} = \frac{21+200}{5}$$

$$\frac{1109x}{30:5} = \frac{221}{5:5}$$

$$\frac{1109x}{6} = 221$$

$$x = \frac{221 \cdot 6}{1109}$$

$$\boxed{x = \frac{1326}{1109}}$$

Portanto, para que as matrizes sejam iguais, $x = \frac{1326}{1109}$ e $y = \frac{1420}{1109}$.

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