

Nome: Gabriel Gonçalves de Oliveira RA: 2111550021

Professora: Dra. Marisa Atsuko Nitto - 1º-ADS

Lista de Exercícios - Matemática - Aula 14

1) Dada a matriz $A = \begin{bmatrix} 1 & x & 5 \\ 2 & 7 & -4 \\ y & z & -3 \end{bmatrix}_{3 \times 3}$, determinar x, y e z para que $A = A^t$

$$\begin{bmatrix} 1 & x & 5 \\ 2 & 7 & -4 \\ 4 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & y \\ x & 7 & z \\ 5 & -4 & -3 \end{bmatrix} \quad \boxed{x=2} \quad \boxed{z=-4} \quad \boxed{y=5}$$

$$x = 2$$

$$y = 5$$

$$z = -4$$

2) Dadas as matrizes $A = \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}_{2 \times 2}$ e $B = \begin{bmatrix} 0 & 6 \\ 9 & 3 \end{bmatrix}_{2 \times 2}$,
determinar:

$$2.1) \underbrace{A \cdot B}_C = \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 6 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} (1 \cdot 0 + 1 \cdot 9) & (1 \cdot 6 + 1 \cdot 3) \\ (5 \cdot 0 + 7 \cdot 9) & (5 \cdot 6 + 7 \cdot 3) \end{bmatrix}$$

$$= \begin{bmatrix} (0+9) & (6+3) \\ (0+63) & (30+25) \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 63 & 51 \end{bmatrix}_{2 \times 2} \quad C = \begin{bmatrix} 9 & 9 \\ 63 & 51 \end{bmatrix}_{2 \times 2}$$

$$C = \begin{bmatrix} 9 & 9 \\ 63 & 51 \end{bmatrix}_{2 \times 2}$$

2.2) $A^2 + 3(B \cdot A) - 7 \cdot I_2$ (onde I_2 = matriz identidade)

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2 \times 2} = \begin{bmatrix} (a_{11} \cdot b_{11} + a_{12} \cdot b_{21}) & (a_{11} \cdot b_{12} + a_{12} \cdot b_{22}) \\ (a_{21} \cdot b_{11} + a_{22} \cdot b_{21}) & (a_{21} \cdot b_{12} + a_{22} \cdot b_{22}) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} + 3 \left(\begin{bmatrix} 0 & 6 \\ 9 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \right) - 7 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1.1+1.5)(1.1+1.7) \\ (5.1+7.5)(5.1+7.7) \end{bmatrix} + 3 \left(\begin{bmatrix} (0.1+6.5)(0.1+6.7) \\ (9.1+3.5)(9.1+3.7) \end{bmatrix} \right) - 7 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+5) & (1+7) \\ (5+35) & (5+49) \end{bmatrix} + 3 \cdot \begin{bmatrix} (0+30) & (0+42) \\ (9+15) & (9+21) \end{bmatrix} - 7 \begin{bmatrix} 1.7 & 0.7 \\ 0.7 & 1.7 \end{bmatrix}$$

[illegible]

$$= \begin{bmatrix} 6 & 8 \\ 40 & 54 \end{bmatrix} + 3 \cdot \begin{bmatrix} 30 & 42 \\ 24 & 30 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} (6+90-7) & (8+126-0) \\ (40+72-0) & (54+90-7) \end{bmatrix} = \begin{bmatrix} 89 & 134 \\ 112 & 137 \end{bmatrix}$$

$$\det(A) = dp - ds$$

$$\det(A) = a_{11} \cdot a_{22} - \cancel{a_{12} \cdot a_{21}}$$

$$\det(A) = 1 \cdot 7 - (-3/5) \cdot 5$$

$$\det(A) = 7 + 15/5$$

$$\det(A) = 7 + 3 \rightarrow \det(A) = 10$$

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$$\rightarrow \begin{bmatrix} (a - 3/5c = 1) & (b + -3/5d = 0) \\ (5a + 7c = 0) & (5b + 7d = 1) \end{bmatrix}$$

$$\begin{cases} a-3/c=1 & \cdot (7) \\ 5a+7c=0 & \cdot (3/5) \end{cases}$$

$$\begin{cases} a - 3/5c = 1 & \cdot (5) \\ 5a + 7c = 0 & \cdot (-1) \end{cases}$$

$$\begin{cases} 7a - 21/5c = 7 \\ 3a + 21/5c = 0 \end{cases} +$$

$$a = \frac{7}{10}$$

$$\begin{array}{r} 5a - 3c = 5 \\ -5a - 7c = 0 \\ \hline 0 - 10c = 5 \quad (-1) \\ c = \frac{-5}{-10} \Rightarrow c \end{array}$$

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$$\begin{cases} b - 3/5d = 0 & \cdot (5) \\ 5b + 7d = 1 & \cdot (-1) \end{cases}$$

$$\begin{cases} b - 3/5d = 0 & \cdot (7) \\ 5b + 7d = 1 & \cdot (3/5) \end{cases}$$

$$\begin{cases} 5b - 3d = 0 \\ -5b - 7d = -1 \end{cases} +$$

$$\begin{cases} 7b - 21/5 = 0 \\ 3b + 21/5 = 3/5 \end{cases} +$$

$$0 - 10d = -1 \cdot (-1)$$

$$10b + 0 = 3/5$$

$$10d = 1$$

$$b = \frac{3}{5 \cdot 10} \Rightarrow b = \frac{3}{50}$$

$$\boxed{d = \frac{1}{10}}$$

$$A^{-1} = \begin{bmatrix} 7/10 & 3/50 \\ -1/2 & 1/10 \end{bmatrix}_{2 \times 2}$$

$$3.27 \quad B = \begin{bmatrix} 1 & -2 \\ 3/4 & 7 \end{bmatrix}_{2 \times 2}$$

$$\det(B) = d_p - d_s$$

$$\det(B) = b_{11} \cdot b_{22} - b_{12} \cdot b_{21}$$

$$\det(B) = 1 \cdot 7 - (-2) \cdot 3/4$$

$$B \cdot B^{-1} = I_2$$

$$\det(B) = 7 + 6/4$$

$$\det(B) = 7 + 3/2$$

$$\det(B) = 14/2 + 3/2$$

$$\boxed{\det(B) = \frac{17}{2}}$$

$$\begin{bmatrix} 1 & -2 \\ 3/4 & 7 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} (1 \cdot a + (-2) \cdot c) & (1 \cdot b + (-2) \cdot d) \\ (3/4 \cdot a + 7 \cdot c) & (3/4 \cdot b + 7 \cdot d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{cases} (1a - 2c = 1) & (1b - 2d = 0) \\ (3/4a + 7c = 0) & (3/4b + 7d = 1) \end{cases}$$

$$\begin{cases} a - 2c = 1 & \cdot (3/4) \\ 3/4a + 7c = 0 & \cdot (-1) \end{cases}$$

$$\frac{-3c - 14c}{2} = \frac{3}{4}$$

$$\frac{-17c}{2} = \frac{3}{4} \cdot (-1)$$

$$\begin{cases} 3/4a - 3/2c = 3/4 \\ -3/4a - 7c = 0 \end{cases} +$$

$$\frac{17c}{2} = -\frac{3}{4}$$

$$c = \frac{-3 \cdot 2}{17 \cdot 4} = \frac{-3}{34}$$

$$\boxed{c = -\frac{3}{34}}$$

$$\frac{0 - 3c - 7c}{2} = \frac{3}{4}$$

$$\begin{cases} a - 2c = 1 & \cdot (7) \\ 3/4 a + 7c = 0 & \cdot (2) \end{cases}$$

$$\begin{cases} 7a - 14c = 7 \\ 3/2 a + 14c = 0 \end{cases} + \rightarrow \frac{14a + 3a = 7}{2}$$

$$\frac{7a}{1} + \frac{3a}{2} = 7$$

$$17a = 2 \cdot 7$$

$$\boxed{a = \frac{7 \cdot 2}{17}}$$

$$\rightarrow \boxed{a = \frac{14}{17}}$$

$$\begin{cases} b - 2d = 0 & \cdot (-3/4) \\ 3/4 b + 7d = 1 & \cdot (1) \end{cases}$$

$$\begin{cases} b - 2d = 0 & \cdot (+7) \\ 3/4 b + 7d = 1 & \cdot (2) \end{cases}$$

$$\begin{cases} -3/4 b + 3/2 d = 0 \\ 3/4 b + 7d = 1 \end{cases} +$$

$$\begin{cases} -7b + 14d = 0 \\ 3/4 b \end{cases}$$

$$0 + \frac{3d}{2} + \frac{7d}{1} = 1$$

$$\begin{cases} 7b - 14d = 0 \\ 3/2 b + 14d = 2 \end{cases} +$$

$$\frac{3d + 14d}{2} = 1$$

$$\frac{7b}{1} + \frac{3b}{2} = 2$$

$$\frac{17d}{2} = 1$$

$$\frac{14b + 3b}{2} = 2$$

$$\boxed{d = \frac{2}{17}}$$

$$\frac{17b}{2} = 2$$

$$\boxed{b = \frac{2 \cdot 2}{17}}$$

$$\boxed{b = \frac{4}{17}}$$

$$B^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$$

$$B^{-1} = \begin{bmatrix} 14/17 & 4/17 \\ -3/34 & 2/17 \end{bmatrix}_{2 \times 2}$$