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2) Dada a Matriz $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3\times3}$, determinar:

2.1) A^2

↳ Solução:

$$A^2 = A \cdot A$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} (0\wedge0\vee0\wedge1\vee1\wedge1)(0\wedge0\vee0\wedge0\vee1\wedge0)\ldots \\ (2^{a} L \text{ e } 1^{a} C)(2^{a} L \text{ e } 2^{a} C)(2^{a} L \text{ e } 3^{a} C) \\ (3^{a} L \text{ e } 1^{a} C)(3^{a} L \text{ e } 2^{a} C)(3^{a} L \text{ e } 3^{a} C) \end{bmatrix}$$

$$= \begin{bmatrix} (0\vee0\vee1) & (0\vee0\vee0) & (0\vee0\vee0) \\ (0\vee0\vee1) & (0\vee0\vee0) & (1\vee0\vee0) \\ (0\vee0\vee0) & (0\vee0\vee0) & (1\vee0\vee0) \end{bmatrix}, \text{ Portanto: } A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

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Lista de Exercícios - Matemática - Aula 15

1) Dadas as matrizes booleanas $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3\times3}$ e $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$ determinar:
(Deixar na resolução o resultado da operação $\vee$ (ou))

1.1) $A \cdot B$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} (1\wedge1\vee1\wedge0\vee0\wedge1) & (1\wedge0\vee1\wedge0\vee0\wedge1) & (1\wedge1\vee1\wedge1\vee0\wedge1) \\ (0\wedge1\vee1\wedge0\vee1\wedge1) & (0\wedge0\vee1\wedge0\vee1\wedge1) & (0\wedge1\vee1\wedge1\vee1\wedge1) \\ (1\wedge1\vee0\wedge0\vee0\wedge1) & (1\wedge0\vee0\wedge0\vee0\wedge1) & (1\wedge1\vee0\wedge1\vee0\wedge1) \end{bmatrix}$$

$$= \begin{bmatrix} (1\vee0\vee0) & (0\vee0\vee0) & (1\vee1\vee0) \\ (0\vee0\vee1) & (0\vee0\vee1) & (0\vee1\vee1) \\ (1\vee0\vee0) & (0\vee0\vee0) & (1\vee0\vee0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{3\times3}$$

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Portanto, $A \cdot B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{3\times3}$

1.2) $B \cdot A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} =$

$$= \begin{bmatrix} (1\wedge1\vee0\wedge0\vee1\wedge1) & (1\wedge1\vee0\wedge1\vee1\wedge0) & (1\wedge0\vee0\wedge1\vee1\wedge0) \\ (0\wedge1\vee0\wedge0\vee1\wedge1) & (0\wedge1\vee0\wedge1\vee1\wedge0) & (0\wedge0\vee0\wedge1\vee1\wedge0) \\ (1\wedge1\vee1\wedge0\vee1\wedge1) & (1\wedge1\vee1\wedge1\vee1\wedge0) & (1\wedge0\vee1\wedge1\vee1\wedge0) \end{bmatrix}$$

$$= \begin{bmatrix} (1\vee0\vee1) & (1\vee0\vee0) & (0\vee0\vee0) \\ (0\vee0\vee1) & (0\vee0\vee0) & (0\vee0\vee0) \\ (1\vee0\vee1) & (1\vee1\vee0) & (0\vee1\vee0) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$$

Portanto, $B \cdot A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$

2) Determinar a inversa das matrizes dadas:

2.1) $A = \begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix}_{2\times2} \rightarrow A \cdot A^{-1} = I_2 \rightarrow \begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} (-1 \cdot a + 5 \cdot c) & (-1 \cdot b + 5 \cdot d) \\ (2 \cdot a + 3 \cdot c) & (2 \cdot b + 3 \cdot d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(A) = (-1) \cdot 3 - 5 \cdot 2$

$\det(A) = -3 - 10 = -13$

$\det(A) = -13 \neq 0.$

$\Rightarrow \begin{bmatrix} -1a + 5c = 1 & -1b + 5d = 0 \\ 2a + 3c = 0 & 2b + 3d = 1 \end{bmatrix}$

$\begin{cases} -1a + 5c = 1 \\ 2a + 3c = 0 \end{cases}$

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$$\begin{cases} -1a+5c=1 & \cdot(2) \\ 2a+3c=0 & \cdot(1) \end{cases}$$

$$\begin{cases} -2a+10c=2 \\ 2a+3c=0 \end{cases} +$$

$$0 \quad 13c=2$$

$$\boxed{c=\frac{2}{13}}$$

$$\begin{cases} -1a+5c=1 & \cdot(-3) \\ 2a+3c=0 & \cdot(5) \end{cases}$$

$$\begin{cases} 3a-15c=-3 \\ 10a+15c=0 \end{cases} +$$

$$13a+0=-3$$

$$\boxed{a=\frac{-3}{13}}$$

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$$\begin{cases} -1b+5d=0 & \cdot(2) \\ 2b+3d=1 & \cdot(1) \end{cases}$$

$$\begin{cases} -2b+10d=0 \\ 2b+3d=1 \end{cases} +$$

$$0+13d=1$$

$$\boxed{d=\frac{1}{13}}$$

$$\begin{cases} -1b+5d=0 & \cdot(-3) \\ 2b+3d=1 & \cdot(5) \end{cases}$$

$$\begin{cases} 3b-15d=0 \\ 10b+15d=5 \end{cases} +$$

$$13b+0=5$$

$$\boxed{b=\frac{5}{13}}$$

$$A^{-1}=\begin{bmatrix} -3/13 & 5/13 \\ 2/13 & 1/13 \end{bmatrix}_{2\times2}$$

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~~2.1)~~

2.2) $B=\begin{bmatrix} 3 & -1 \\ -5 & 4 \end{bmatrix}_{2\times2} \rightarrow B \cdot B^{-1} = I_2 \rightarrow \begin{bmatrix} 3 & -1 \\ -5 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\rightarrow \begin{bmatrix} (3.a+(-1).c) & (3.b+(-1).d) \\ (-5.a+4.c) & (-5.b+4.d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\det(B) = 3.4 - (-5).-1$

$\det(B) = 12-5 = 7.$

$\det(B) = 7 \neq 0.$

$$\rightarrow \begin{bmatrix} 3a-1c=1 & 3b-1d=0 \\ -5a+4c=0 & -5b+4d=1 \end{bmatrix}$$

$$\begin{cases} 3a-1c=1 \\ -5a+4c=0 \end{cases}$$

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$$\begin{cases} 3a - 1c = 1 & .(5) \\ -5a + 4c = 0 & .(3) \end{cases}$$

$$\begin{cases} 15a - 5c = 5 \\ -15a + 12c = 0 \end{cases} +$$

$$0 + 7c = 5$$

$$\boxed{c = \frac{5}{7}}$$

$$\begin{cases} 3a - 1c = 1 & .(4) \\ -5a + 4c = 0 & .(1) \end{cases}$$

$$\begin{cases} 12a - 4c = 4 \\ -5a + 4c = 0 \end{cases} +$$

$$7a + 0 = 4$$

$$\boxed{a = \frac{4}{7}}$$

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$$\begin{cases} 3b - 1d = 0 & .(5) \\ -5b + 4d = 1 & .(3) \end{cases}$$

$$\begin{cases} 15b - 5d = 0 \\ -15b + 12d = 3 \end{cases} +$$

$$0 + 7d = 3$$

$$\boxed{d = \frac{3}{7}}$$

$$\begin{cases} 3b - 1d = 0 & .(4) \\ -5b + 4d = 1 & .(1) \end{cases}$$

$$\begin{cases} 12b - 4d = 0 \\ -5b + 4d = 1 \end{cases} +$$

$$7b + 0 = 1$$

$$\boxed{b = \frac{1}{7}}$$

$$B^{-1} = \begin{bmatrix} 4/7 & 1/7 \\ 5/7 & 3/7 \end{bmatrix}_{2\times2}$$

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"Bem-aventurado o povo que conhece os gritos de alegria, que anda, ó Senhor, na luz da tua presença. Em teu nome se alegra o dia todo e na tua justiça se exalta, porque tu és a glória de sua força; no teu favor é exaltado o nosso poder. Pois ao Senhor pertence o nosso escudo, e ao Santo de Israel, o nosso ~~Rei~~. rei."

Salmos 89:15-18