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1) 2.5

2) 2.5

3) 2.5

4) 2.5

Nota = 10

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Segunda Prova

1) (2.5) Determinar a inversa da Matriz A dada.

$$A = \begin{bmatrix} 7/8 & 4 \\ 3 & -10 \end{bmatrix}_{2 \times 2} \rightarrow A \cdot A^{-1} = I_2 \rightarrow \begin{bmatrix} 7/8 & 4 \\ 3 & -10 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \det(A) = d_p - d_s$$

$$\det(A) = (7/8) \cdot (-10) - 4 \cdot 3$$

$$\det(A) = -70/8 - 12$$

$$\Rightarrow \begin{bmatrix} (7/8 \cdot a + 4 \cdot c) & (7/8 \cdot b + 4 \cdot d) \\ (3 \cdot a + (-10) \cdot c) & (3 \cdot b + (-10) \cdot d) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \det(A) = -70/8 - 96/8$$

$$\det(A) = -166/8 : 2$$

$$\det(A) = -83/4$$

tem inversa (11)

$$\Rightarrow \begin{bmatrix} 7/8a + 4c = 1 & 7/8b + 4d = 0 \\ 3a - 10c = 0 & 3b - 10d = 1 \end{bmatrix}$$

$$\begin{cases} 7/8a + 4c = 1 & \cdot (3) \\ 3a - 10c = 0 & \cdot (-7/8) \end{cases}$$

$$\begin{cases} 7/8a + 4c = 1 & \cdot (10) \\ 3a - 10c = 0 & \cdot (4) \end{cases}$$

$$\begin{cases} 21/8a + 12c = 3 \\ -21/8a + 70/8c = 0 \end{cases} +$$

$$\begin{cases} 70/8a + 40c = 10 \\ 12a - 40c = 0 \end{cases}$$

$$\begin{array}{r} 0 + \frac{12c}{8} + \frac{70c}{8} = \frac{3}{8} \\ \hline 96c + 70c = 3 \\ \hline 166c = 3 \end{array}$$

$$\begin{array}{r} \frac{70a}{8} + \frac{12a}{8} + 0 = \frac{10}{8} \\ \hline 70a + 12a = 10 \\ \hline 82a = 10 \end{array}$$

$$\frac{166c}{8} = \frac{3}{8} \Rightarrow c = \frac{3 \cdot 8}{166}$$

$$\frac{82a}{8} = \frac{10}{8} \Rightarrow a = \frac{10 \cdot 8}{82}$$

$$c = \frac{24}{166} \Rightarrow c = \frac{12}{83}$$

$$a = \frac{80}{82} \Rightarrow a = \frac{40}{41}$$



$$\begin{cases} 7/8b + 4d = 0 & \cdot (3) \\ 3b - 10d = 1 & \cdot (-7/8) \end{cases}$$

$$\begin{cases} 7/8b + 4d = 0 & \cdot (10) \\ 3b - 10d = 1 & \cdot (4) \end{cases}$$

$$\begin{cases} 21/8b + 12d = 0 \\ -21/8b + 70/8d = -7/8 \end{cases} +$$

$$\begin{cases} 70/8b + 40d = 0 \\ 12b - 40d = 4 \end{cases} +$$

$$0 + \frac{12d}{1} + \frac{70d}{8} = \frac{-7}{8}$$

$$0 + \frac{70b}{8} + \frac{12b}{1} = \frac{4}{1}$$

$$\frac{96d + 70d}{8} = \frac{-7}{8}$$

$$\frac{70b + 96b}{8} = 4$$

$$\frac{166d}{8} = \frac{-7}{8}$$

$$\frac{166b}{8} = 4$$

$$\boxed{d = \frac{-7}{166}}$$

$$b = \frac{4 \cdot 8}{166} \Rightarrow b = \frac{32}{166} \Rightarrow b = \frac{16}{83}$$

$$\boxed{b = \frac{16}{83}}$$

$$\text{Portanto, } A^{-1} = \begin{bmatrix} 40/83 & 12/83 \\ 16/83 & -7/166 \end{bmatrix}_{2 \times 2}$$

2) (2.5) Escreva em forma de tabela as matrizes A e B dadas por:

$$A = (a_{ij})_{3 \times 3} \text{ tal que } a_{ij} = (-i^2) - (-j^2)$$

$$B = (b_{ij})_{3 \times 3} \text{ tal que } b_{ij} = f(i) + f(j), \text{ para } f(x) = x + 2$$

$$\text{Determinar } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3 \times 3} = A = \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}_{3 \times 3}$$

$$a_{11} = (-1^2) - (-1^2) = -1 - (-1) = -1 + 1 = 0$$

$$a_{12} = (-1^2) - (-2^2) = -1 - (-4) = -1 + 4 = 3$$

$$a_{13} = (-1^2) - (-3^2) = -1 - (-9) = -1 + 9 = 8$$

$$a_{21} = (-2^2) - (-1^2) = -4 - (-1) = -4 + 1 = -3$$

$$a_{22} = (-2^2) - (-2^2) = -4 - (-4) = -4 + 4 = 0$$

$$a_{23} = (-2^2) - (-3^2) = -4 - (-9) = -4 + 9 = 5$$

$$a_{31} = (-3^2) - (-1^2) = -9 - (-1) = -9 + 1 = -8$$

$$a_{32} = (-3^2) - (-2^2) = -9 - (-4) = -9 + 4 = -5$$

$$a_{33} = (-3^2) - (-3^2) = -9 - (-9) = -9 + 9 = 0$$



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$$a_{31} = (-3^2) - (-1^2) = -9 - (-1) = -9 + 1 = -8$$

$$a_{32} = (-3^2) - (-2^2) = -9 - (-4) = -9 + 4 = -5$$

$$a_{33} = (-3^2) - (-3^2) = -9 - (-9) = -9 + 9 = 0$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3 \times 3} \Rightarrow B = \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix}_{3 \times 3}$$

$$f(x) = x + 2; \quad f(1) = 1 + 2 = 3$$

$$f(2) = 2 + 2 = 4$$

$$f(3) = 3 + 2 = 5$$

$$b_{11} = f(1) + f(1) = 3 + 3 = 6$$

$$b_{12} = f(1) + f(2) = 3 + 4 = 7$$

$$b_{13} = f(1) + f(3) = 3 + 5 = 8$$

$$b_{21} = f(2) + f(1) = 4 + 3 = 7$$

$$b_{22} = f(2) + f(2) = 4 + 4 = 8$$

$$b_{23} = f(2) + f(3) = 4 + 5 = 9$$

$$b_{31} = f(3) + f(1) = 5 + 3 = 8$$

$$b_{32} = f(3) + f(2) = 5 + 4 = 9$$

$$b_{33} = f(3) + f(3) = 5 + 5 = 10$$

→ Determinar:

$$2.1 \rightarrow A + B \Rightarrow \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} (0+6) & (3+7) & (8+8) \\ (-3+7) & (0+8) & (5+9) \\ (-8+8) & (-5+9) & (0+10) \end{bmatrix} = \begin{bmatrix} 6 & 10 & 16 \\ 4 & 8 & 14 \\ 0 & 4 & 10 \end{bmatrix}_{3 \times 3}$$

Por tanto,  $A + B = \begin{bmatrix} 6 & 10 & 16 \\ 4 & 8 & 14 \\ 0 & 4 & 10 \end{bmatrix}_{3 \times 3}$



2.2)  $2B - A^t$   $2 \cdot \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 8 \\ -3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$  para transpor

$$= 2 \cdot \begin{bmatrix} 6 & 7 & 8 \\ 7 & 8 & 9 \\ 8 & 9 & 10 \end{bmatrix} - \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \cdot 2 & 7 \cdot 2 & 8 \cdot 2 \\ 7 \cdot 2 & 8 \cdot 2 & 9 \cdot 2 \\ 8 \cdot 2 & 9 \cdot 2 & 10 \cdot 2 \end{bmatrix} - \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 14 & 16 \\ 14 & 16 & 18 \\ 16 & 18 & 20 \end{bmatrix} - \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 12-0 & 14-(-3) & 16-(-8) \\ 14-3 & 16-0 & 18-(-5) \\ 16-8 & 18-5 & 20-0 \end{bmatrix} = \begin{bmatrix} 12 & 17 & 24 \\ 11 & 16 & 23 \\ 8 & 13 & 20 \end{bmatrix}$$

Portanto,  $2B - A^t = \begin{bmatrix} 12 & 17 & 24 \\ 11 & 16 & 23 \\ 8 & 13 & 20 \end{bmatrix}_{3 \times 3}$

3) (2.5) Dadas as matrizes booleanas, determinar:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3 \times 3} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3 \times 3}$$

3.1)  $A \times B$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} (0 \wedge 1 \vee 0 \wedge 0 \vee 1 \wedge 1) & (0 \wedge 0 \vee 0 \wedge 1 \vee 1 \wedge 0) & (0 \wedge 1 \vee 0 \wedge 1 \vee 1 \wedge 0) \\ (1 \wedge 1 \vee 1 \wedge 0 \vee 1 \wedge 1) & (1 \wedge 0 \vee 1 \wedge 1 \vee 1 \wedge 0) & (1 \wedge 1 \vee 1 \wedge 1 \vee 1 \wedge 0) \\ (1 \wedge 1 \vee 1 \wedge 0 \vee 0 \wedge 1) & (1 \wedge 0 \vee 1 \wedge 1 \vee 0 \wedge 0) & (1 \wedge 1 \vee 1 \wedge 1 \vee 0 \wedge 0) \end{bmatrix}$$



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$$= \begin{bmatrix} (0V0V1) & (0V0V0) & (0V0V0) \\ (1V0V1) & (0V1V0) & (1V1V0) \\ (1V0V0) & (0V1V0) & (1V1V0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \text{Portanto, } A \times B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3 \times 3}$$

3.2) B x A

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} (0V0V1) & (0V0V1) & (1V0V0) \\ (0V1V1) & (0V1V1) & (0V1V0) \\ (0V0V0) & (0V0V0) & (1V0V0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}, \text{Portanto, } B \times A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

4-) (2.5) Determinar x e y para que as matrizes sejam iguais:

$$\begin{bmatrix} 1/2x + 7y \\ -7x + 3/5y + 3 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 2y + 7 \\ 1/3x - 5 \end{bmatrix}_{2 \times 1} \rightarrow \begin{bmatrix} 1/2x + 7y = 2y + 7 \\ -7x + 3/5y + 3 = 1/3x - 5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1/2x + 7y - 2y = 7 \\ -7x - 1/3x + 3/5y = -5 - 3 \end{bmatrix} = \begin{bmatrix} 1/2x + 5y = 7 \\ -21/3x - 1/3x + 3/5y = -8 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1/2x + 5y = 7 \\ -20/3x + 3/5y = -8 \end{bmatrix}_{2 \times 1}$$

+Descobri que errei uma subtração... Vou refazer a partir disso.

$$\begin{cases} 1/2x + 5y = 7 & \cdot (20/3) \\ -20/3x + 3/5y = -8 & \cdot (1/2) \end{cases}$$

$$\begin{cases} 20/6x + 100/3y = 140/3 \\ -20/6x + 3/10y = -8/2 \end{cases}$$

$$\begin{cases} 20/6x + 100/3y = 140/3 \\ -20/6x + 3/10y = 4 \end{cases} +$$

$$0 + \frac{100x}{3} + \frac{3y}{10} = \frac{4}{1}$$



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$$\begin{bmatrix} 1/2x + 5y = 7 \\ -7x - 1/3x + 3/5y = -5-3 \end{bmatrix} = \begin{bmatrix} 1/2x + 5y = 7 \\ -22/3x - 1/3x + 3/5y = -8 \end{bmatrix}$$

$$\begin{cases} 1/2x + 5y = 7 & \cdot (22/3) \\ -22/3x + 3/5y = -8 & \cdot (1/2) \end{cases} \quad \begin{cases} 1/2x + 5y = 7 & \cdot (3/5) \\ -22/3x + 3/5y = -8 & \cdot (-5) \end{cases}$$

$$\begin{cases} 22/6x + 110/3y = 154/3 + \\ -22/6x + 3/10y = -4 \end{cases}$$

$$\begin{cases} 3/10x + 15/5y = 21/5 + \\ 110/3x - 15/5y = 40 \end{cases}$$

$$0 + \frac{110y}{3} + \frac{3y}{10} = \frac{-4}{1} + \frac{154}{3}$$

$$\frac{3x}{10} + \frac{110x}{3} + 0 = \frac{21}{5} + \frac{40}{1}$$

$$\frac{1100y + 9y}{30} = \frac{-12 + 154}{3}$$

$$\frac{9x + 1100x}{30} = \frac{21 + 200}{5}$$

$$\frac{1109y}{30:3} = \frac{142}{3:3}$$

$$\frac{1109x}{30:5} = \frac{221}{5:5}$$

$$\frac{1109y}{10 \times 3} = \frac{142}{1}$$

$$\frac{1109x}{6 \times 5} = \frac{221}{1}$$

$$y = \frac{142 \cdot 10}{1109}$$

$$x = \frac{221 \cdot 6}{1109}$$

$$y = \frac{1420}{1109}$$

$$x = \frac{1326}{1109}$$

Portanto, para que as matrizes sejam iguais,  
 $x = \frac{1326}{1109}$  e  $y = \frac{1420}{1109}$ .

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