SEMINAR 1

- Unificare -

Consideram:

- *variabile*: x, y, z, u, v
- *constante*: a, b, c
- *functii*:
  - aritate **1**: h, g, $()^{-1}$
  - aritate **2**: f, $*$, $+$
  - aritate **3**: p

Aplicati algoritmul de unificare de mai sus pentru a gasi un unificator:

1. $p(a, x, h(g(y)))$ **&** $p(z, h(z), h(u))$
2. $f(h(a), g(x))$ **&** $f(y, y)$
3. $p(a, x, g(x))$ **&** $p(a, y, y)$
4. $p(x, y, z)$ **&** $p(u, f(v, v), u)$
5. $f(x, f(x, x))$ **&** $f(g(y), f(z, g(a)))$
6. $x + (y * y)$ **&** $(y * y) + z$
7. $(x * y) * z$ **&** $u * u^{-1}$
8. $x * y$ **&** $u * u^{-1}$
9. $x * y$ **&** $x * (y * (u * v)^{-1})$
10. $x * y$ **&** $y * (u * v)^{-1}$
11. $f(g(x), x)$ **&** $f(y, y)$
12. $p(x, z, z)$ **&** $p(y, y, b)$
13. $p(a, u, h(x))$ **&** $p(y, f(y, z), z)$

14. $f(x, f(b, x))$ **&** $f(f(y, a), f(b, f(z, z)))$
15. $p(x, b, x)$ **&** $p(y, y, c)$
16. $f(x, y)$ **&** $f(h(x), x)$ **&** $f(x, b)$
17. $f(x, f(x, g(y)))$ **&** $f(u, z)$ **&** $f(g(y), y)$
18. $f(f(x, y), x)$ **&** $f(g(y), z)$ **&** $f(u, h(z))$
19. $f(f(x, y), x)$ **&** $f(v, u)$ **&** $f(u, h(z))$
20. $f(f(x, y), x)$ **&** $f(v, u)$ **&** $f(u, z)$
21. $f(f(g(x), h(y)), h(z))$ **&** $f(f(u, h(h(x))), h(y))$ **&** $f(v, w)$
22. $p(x, x, z)$ **&** $p(f(a, a), y, y)$ **&** $p(f(x, a), b, z)$
23. $p(x, x, z)$ **&** $p(f(a, a), y, y)$ **&** $p(x, b, z)$
24. $p(x, x, z)$ **&** $p(f(a, a), y, y)$ **&** $p(x, f(a, a), z)$
25. $p(f(x, a), g(y), z)$ **&** $p(f(a, a), z, u)$ **&** $p(v, u, z)$

Rezolvari:

1) $p(a, x, h(g(y)))$ **&** $p(z, h(z), h(u))$

S	R	Op
Ø	$p(a, x, h(g(y))) = p(z, h(z), h(u))$	D
Ø	$a = z$ $x = h(z)$ $h(g(y)) = h(u)$	R
$z = a$	$x = h(a)$ $h(g(y)) = h(u)$	R
$z = a$ $x = h(a)$	$h(g(y)) = h(u)$	D
$z = a$ $x = h(a)$	$g(y) = u$	R

S	R	Op
$z = a$ $x = h(a)$ $u = g(y)$	Ø	END

⇒ **cgu** = { $x \leftarrow h(a)$, $z \leftarrow a$, $u \leftarrow g(y)$ }

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2) $f(h(a), g(x))$ & $f(y, y)$

S	R	Op
Ø	$f(h(a), g(x)) = f(y, y)$	D
Ø	$h(a) = y$ $g(x) = y$	R
$y = h(a)$	$g(x) = h(a)$	END

⇒ **cgu** = *INEXISTENT* ($h \neq g$)

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3) $p(a, x, g(x))$ & $p(a, y, y)$

S	R	Op
Ø	$p(a, x, g(x)) = p(a, y, y)$	D
Ø	$a = a$ $x = y$ $g(x) = y$	S

S	R	Op
Ø	$x = y$ $g(x) = y$	R
$y = g(x)$	$x = g(x)$	END

⇒ **cgu** = *INEXISTENT* ($x = g(x)$)

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4) $p(x, y, z)$ **&** $p(u, f(v, v), u)$

S	R	Op
Ø	$p(x, y, z) = p(u, f(v, v), u)$	D
Ø	$x = u$ $y = f(v, v)$ $z = u$	R
$y = f(v, v)$	$x = u$ $z = u$	R
$y = f(v, v)$ $x = u$	$z = u$	R
$y = f(v, v)$ $x = u$ $z = u$	Ø	END

⇒ **cgu** = { $x \leftarrow u$, $y \leftarrow f(v, v)$, $z \leftarrow u$ }

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5) $f(x, f(x, x))$ **&** $f(g(y), f(z, g(a)))$

S	R	Op
Ø	$f(x, f(x, x)) = f(g(y), f(z, g(a)))$	D
Ø	$x = g(y)$ $f(x, x) = f(z, g(a))$	R
$x = g(y)$	$f(g(y), g(y)) = f(z, g(a))$	D
$x = g(y)$	$g(y) = z$ $g(y) = g(a)$	R
$x = g(y)$ $z = g(y)$	$g(y) = g(a)$	D
$x = g(y)$ $z = g(y)$	$y = a$	R
$x = g(a)$ $z = g(a)$ $y = a$	Ø	END

⇒ **cgu** = { $x \leftarrow g(a)$, $y \leftarrow a$, $z \leftarrow g(a)$ }

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6) $x + (y * y)$ **&** $(y * y) + z$

S	R	Op
Ø	$x + (y * y) = (y * y) + z$	D
Ø	$x = y * y$ $y * y = z$	R
$x = y * y$	$y * y = z$	R
$x = y * y$ $z = y * y$	Ø	END

⇒ **cgu** = { $x \leftarrow y * y$, $z \leftarrow y * y$ }

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7) $(x * y) * z$ & $u * u^{-1}$

S	R	Op
Ø	$(x * y) * z = u * u^{-1}$	D
Ø	$x * y = u$ $z = u^{-1}$	R
$z = u^{-1}$	$x * y = u$	R
$u = x * y$ $z = (x * y)^{-1}$	Ø	END

⇒ **cgu** = { $u \leftarrow x * y$, $z \leftarrow (x * y)^{-1}$ }

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8) $x * y$ & $u * u^{-1}$

S	R	Op
Ø	$x * y = u * u^{-1}$	D
Ø	$x = u$ $y = u^{-1}$	R
$x = u$	$y = u^{-1}$	R
$x = u$ $y = u^{-1}$	Ø	R

⇒ **cgu** = { $x \leftarrow u$, $y \leftarrow u^{-1}$ }

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9) $x * y$ & $x * (y * (u * v)^{-1})$

S	R	Op
Ø	$x * y = x * (y * (u * v)^{-1})$	D
Ø	$x = x$ $y = y * (u * v)^{-1}$	S
Ø	$y = y * (u * v)^{-1}$	END

⇒ **cgu** = *INEXISTENT* ($y = y * (u * v)^{-1}$)

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10) $x * y$ $ $y * (u * v)^{-1}$

S	R	Op
Ø	$x * y = y * (u * v)^{-1}$	D
Ø	$x = y$ $y = (u * v)^{-1}$	R
$x = y$	$y = (u * v)^{-1}$	R
$x = (u * v)^{-1}$ $y = (u * v)^{-1}$	Ø	R

⇒ **cgu** = { $x \leftarrow (u * v)^{-1}$, $y \leftarrow (u * v)^{-1}$ }

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11) $f(g(x), x)$ & $f(y, y)$

S	R	Op
Ø	$f(g(x), x) = f(y, y)$	D
Ø	$g(x) = y$ $x = y$	R
$x = y$	$g(y) = y$	END

⇒ **cgu** = *INEXISTENT* ($y = g(y)$)

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12) $p(x, z, z)$ & $p(y, y, b)$

S	R	Op
Ø	$p(x, z, z) = p(y, y, b)$	D
Ø	$x = y$ $z = y$ $z = b$	R
$x = y$	$z = y$ $z = b$	R
$x = y$ $z = y$	$y = b$	R
$x = b$ $z = b$ $y = b$	Ø	END

⇒ **cgu** = { $x \leftarrow b$, $z \leftarrow b$, $y \leftarrow b$ }

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13) $p(a, u, h(x))$ & $p(y, f(y, z), z)$

S	R	Op
Ø	$p(a, u, h(x)) = p(y, f(y, z), z)$	D
Ø	$a = y$ $u = f(y, z)$ $h(x) = z$	R
$y = a$	$u = f(a, z)$ $h(x) = z$	R
$y = a$ $z = h(x)$	$u = f(a, h(x))$	R
$y = a$ $z = h(x)$ $u = f(a, h(x))$	Ø	END

⇒ **cgu** = { $y \leftarrow a$, $z \leftarrow h(x)$, $u \leftarrow f(a, h(x))$ }

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14) $f(x, f(b, x))$ & $f(f(y, a), f(b, f(z, z)))$

S	R	Op
Ø	$f(x, f(b, x)) = f(f(y, a), f(b, f(z, z)))$	D

S	R	Op
$\emptyset$	$x = f(y, a)$ $f(b, x) = f(b, f(z, z))$	R
$x = f(y, a)$	$f(b, f(y, a)) = f(b, f(z, z))$	D
$x = f(y, a)$	$b = b$ $f(y, a) = f(z, z)$	S
$x = f(y, a)$	$f(y, a) = f(z, z)$	D
$x = f(y, a)$	$y = z$ $a = z$	R
$x = f(y, a)$ $z = a$	$y = a$	R
$x = f(a, a)$ $z = a$ $y = a$	$\emptyset$	END

⇒ **cgu** = { $x \leftarrow f(a, a)$, $z \leftarrow a$, $y \leftarrow a$ }

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15) $p(x, b, x)$ & $p(y, y, c)$

S	R	Op
$\emptyset$	$p(x, b, x) = p(y, y, c)$	D
$\emptyset$	$x = y$ $b = y$ $x = c$	R
$x = y$	$b = y$ $y = c$	R

S	R	Op
$x = c$ $y = c$	$b = c$	END

⇒ **cgu** = *INEXISTENT* ($b = c$)

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16) $f(x, y)$ **&** $f(h(x), x)$ **&** $f(x, b)$

S	R	Op
Ø	$f(x, y) = f(h(x), x)$ $f(h(x), x) = f(x, b)$	D
Ø	$x = h(x)$ $y = x$ $h(x) = x$ $x = b$	R
Ø	$x = h(x)$ $y = x$ $h(x) = x$ $x = b$	END

⇒ **cgu** = *INEXISTENT* ($x = h(x)$)

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17) $f(x, f(x, g(y)))$ **&** $f(u, z)$ **&** $f(g(y), y)$

S	R	Op

S	R	Op
Ø	$f(x, f(x, g(y))) = f(u, z)$ $f(u, z) = f(g(y), y)$	D
Ø	$x = u$ $f(x, g(y)) = z$ $u = g(y)$ $z = y$	R
$x = u$	$f(u, g(y)) = z$ $u = g(y)$ $z = y$	R
$x = u$ $z = y$	$f(u, g(y)) = y$ $u = g(y)$	R
$x = g(y)$ $z = y$ $u = g(y)$	$f(g(y), g(y)) = y$	END

⇒ **cgu** = *INEXISTENT* ($f(g(y), g(y)) = y$)

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18) $f(f(x, y), x)$ **&** $f(g(y), z)$ **&** $f(u, h(z))$

S	R	Op
Ø	$f(f(x, y), x) = f(g(y), z)$ $f(g(y), z) = f(u, h(z))$	D
Ø	$f(x, y) = g(y)$ $x = z$ $g(y) = u$ $z = h(z)$	R

S	R	Op
Ø	$f(x,y) = g(y)$ $x = z$ $g(y) = u$ $z = h(z)$	END

⇒ **cgu** = *INEXISTENT* ($z = h(z)$)

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19) $f(f(x,y),x)$ & $f(v,u)$ & $f(u,h(z))$

S	R	Op
Ø	$f(f(x,y),x) = f(v,u)$ $f(v,u) = f(u,h(z))$	D
Ø	$f(x,y) = v$ $x = u$ $v = u$ $u = h(z)$	R
$u = h(z)$	$f(x,y) = v$ $x = h(z)$ $v = h(z)$	R
$u = h(z)$ $v = h(z)$	$f(x,y) = h(z)$ $x = h(z)$	END

⇒ **cgu** = *INEXISTENT* ($f(x,y) = h(z)$)

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20) $f(f(x,y),x)$ **&** $f(v,u)$ **&** $f(u,z)$

S	R	Op
Ø	$f(f(x,y),x) = f(v,u)$ $f(v,u) = f(u,z)$	D
Ø	$f(x,y) = v$ $x = y$ $v = u$ $u = z$	R
$u = z$	$f(x,y) = v$ $x = y$ $v = z$	R
$u = z$ $v = z$	$f(x,y) = z$ $x = y$	R
$u = z$ $v = z$ $x = y$	$f(y,y) = z$	R
$u = f(y,y)$ $v = f(y,y)$ $x = y$ $z = f(y,y)$	Ø	END

⇒ **cgu** = { $u \leftarrow f(y,y)$, $v \leftarrow f(y,y)$, $x \leftarrow y$, $z \leftarrow f(y,y)$ }

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21) $f(f(g(x),h(y)),h(z))$ **&** $f(f(u,h(h(x))),h(y))$ **&** $f(v,w)$

S	R	Op
Ø	$f(f(g(x),h(y)),h(z)) = f(f(u,h(h(x))),h(y))$ $f(f(u,h(h(x))),h(y)) = f(v,w)$	D

S	R	Op
Ø	$f(g(x), h(y)) = f(u, h(h(x)))$ $h(z) = h(y)$ $f(u, h(h(x))) = v$ $h(y) = w$	R
$w = h(y)$	$f(g(x), h(y)) = f(u, h(h(x)))$ $h(z) = h(y)$ $f(u, h(h(x))) = v$	D
$w = h(y)$	$g(x) = u$ $h(y) = h(h(x))$ $z = y$ $f(u, h(h(x))) = v$	R
$w = h(y)$ $u = g(x)$	$h(y) = h(h(x))$ $z = y$ $f(g(x), h(h(x))) = v$	R
$w = h(y)$ $u = g(x)$ $z = y$	$h(y) = h(h(x))$ $f(g(x), h(h(x))) = v$	D
$w = h(y)$ $u = g(x)$ $z = y$	$y = h(x)$ $f(g(x), h(h(x))) = v$	R
$w = h(h(x))$ $u = g(x)$ $z = h(x)$ $y = h(x)$	$f(g(x), h(h(x))) = v$	R
$w = h(h(x))$ $u = g(x)$ $z = h(x)$ $y = h(x)$ $v = f(g(x), h(h(x)))$	Ø	END

⇒ **cgu** = { $w \leftarrow h(h(x))$, $u \leftarrow g(x)$, $z \leftarrow h(x)$, $y \leftarrow h(x)$, $v \leftarrow f(g(x), h(h(x)))$ }

22) $p(x,x,z)$ **&** $p(f(a,a),y,y)$ **&** $p(f(x,a),b,z)$

S	R	Op
Ø	$p(x,x,z) = p(f(a,a),y,y)$ $p(f(a,a),y,y) = p(f(x,a),b,z)$	D
Ø	$x = f(a,a)$ $x = y$ $z = y$ $f(a,a) = f(x,a)$ $y = b$ $y = z$	R
$x = f(a,a)$	$f(a,a) = y$ $z = y$ $f(a,a) = f(f(a,a),a)$ $y = b$ $y = z$	R
$x = f(a,a)$ $y = f(a,a)$	$z = f(a,a)$ $f(a,a) = f(f(a,a),a)$ $f(a,a) = b$ $f(a,a) = z$	R
$x = f(a,a)$ $y = f(a,a)$ $z = f(a,a)$	$f(a,a) = f(f(a,a),a)$ $f(a,a) = b$ $f(a,a) = f(a,a)$	S
$x = f(a,a)$ $y = f(a,a)$ $z = f(a,a)$	$f(a,a) = f(f(a,a),a)$ $f(a,a) = b$	END

⇒ **cgu** = *INEXISTENT* ($b = f(a,a)$)

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23) $p(x,x,z)$ **&** $p(f(a,a),y,y)$ **&** $p(x,b,z)$

S	R	Op
Ø	$p(x,x,z) = p(f(a,a),y,y)$ $p(f(a,a),y,y) = p(x,b,z)$	D
Ø	$x = f(a,a)$ $x = y$ $z = y$ $f(a,a) = x$ $y = b$ $y = z$	R
$x = f(a,a)$	$f(a,a) = y$ $z = y$ $f(a,a) = f(a,a)$ $y = b$ $y = z$	S
$x = f(a,a)$	$f(a,a) = y$ $z = y$ $y = b$ $y = z$	R
$x = f(a,a)$ $y = f(a,a)$	$z = f(a,a)$ $f(a,a) = b$ $f(a,a) = z$	R
$x = f(a,a)$ $y = f(a,a)$ $z = f(a,a)$	$f(a,a) = b$ $f(a,a) = f(a,a)$	S
$x = f(a,a)$ $y = f(a,a)$ $z = f(a,a)$	$f(a,a) = b$	END

⇒ **cgu** = *INEXISTENT* ($b = f(a, a)$)

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24) $p(x, x, z)$ **&** $p(f(a, a), y, y)$ **&** $p(x, f(a, a), z)$

S	R	Op
Ø	$p(x, x, z) = p(f(a, a), y, y)$ $p(f(a, a), y, y) = p(x, f(a, a), z)$	D
Ø	$x = f(a, a)$ $x = y$ $z = y$ $f(a, a) = x$ $y = f(a, a)$ $y = z$	R
$x = f(a, a)$	$f(a, a) = y$ $z = y$ $f(a, a) = f(a, a)$ $y = f(a, a)$ $y = z$	S
$x = f(a, a)$	$f(a, a) = y$ $z = y$ $y = f(a, a)$ $y = z$	R
$x = f(a, a)$ $y = f(a, a)$	$f(a, a) = f(a, a)$ $z = f(a, a)$ $f(a, a) = z$	S
$x = f(a, a)$ $y = f(a, a)$	$z = f(a, a)$ $f(a, a) = z$	R

S	R	Op
$x = f(a,a)$ $y = f(a,a)$ $z = f(a,a)$	$f(a,a) = f(a,a)$	S
$x = f(a,a)$ $y = f(a,a)$ $z = f(a,a)$	Ø	END

⇒ **cgu** = { $x \leftarrow f(a,a)$, $y \leftarrow f(a,a)$, $z \leftarrow f(a,a)$ }

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25) $p(f(x,a), g(y), z)$ & $p(f(a,a), z, u)$ & $p(v,u,z)$

S	R	Op
Ø	$p(f(x,a), g(y), z) = p(f(a,a), z, u)$ $p(f(a,a), z, u) = p(v,u,z)$	D
Ø	$f(x,a) = f(a,a)$ $g(y) = z$ $z = u$ $f(a,a) = v$ $z = u$ $u = z$	R
$v = f(a,a)$	$f(x,a) = f(a,a)$ $g(y) = z$ $z = u$ $z = u$ $u = z$	R

S	R	Op
$v = f(a, a)$ $z = g(y)$	$f(x, a) = f(a, a)$ $g(y) = u$ $g(y) = u$ $u = g(y)$	R
$v = f(a, a)$ $z = g(y)$ $u = g(y)$	$f(x, a) = f(a, a)$ $g(y) = g(y)$ $g(y) = g(y)$	S
$v = f(a, a)$ $z = g(y)$ $u = g(y)$	$f(x, a) = f(a, a)$	D
$v = f(a, a)$ $z = g(y)$ $u = g(y)$	$x = a$ $a = a$	S
$v = f(a, a)$ $z = g(y)$ $u = g(y)$	$x = a$	R
$v = f(a, a)$ $z = g(y)$ $u = g(y)$ $x = a$	Ø	END

⇒ **cgu** = { $v \leftarrow f(a, a)$, $z \leftarrow g(y)$, $u \leftarrow g(y)$, $x \leftarrow a$ }
