TEMA G.A.L.

- Seminarul 1 -

Apl 1

ii) $AB = BA$; $A, B \in \mathcal{M}_n(\mathbb{C})$

$$(A+B)^k = C_k^0 A^0 B^k + C_k^1 A^1 B^{k-1} + \dots + C_k^{k-1} A^{k-1} B^1 + C_k^k A^k B^0$$

$$= \sum_{j=0}^{k} \left(C_k^j A^j B^{k-j} \right), \quad A^0 = B^0 = I_n$$

Apl 2

$A, B \in \mathcal{M}_n(\mathbb{C})$; $A + B = AB \Rightarrow$

$$\Rightarrow \left.\begin{cases} A + B = AB \\ B + A = BA \\ A + B \overset{asoc}{=\!=} B + A \end{cases}\right| \Rightarrow AB = BA$$

Apl 4

Dacă $A = \begin{pmatrix} \cos e & \sin e \\ -\sin e & \cos e \end{pmatrix} \Rightarrow A^n = \begin{pmatrix} \cos ne & \sin ne \\ -\sin ne & \cos ne \end{pmatrix}, \forall n \in \mathbb{N}^*$

M.i.M:

1) p.p. $p_1 : A^1 = \begin{pmatrix} \cos e & \sin e \\ -\sin e & \cos e \end{pmatrix}$, prop. adevărată, "A"

2) arătăm $p_2 : A^2 = \begin{pmatrix} \cos 2e & \sin 2e \\ -\sin 2e & \cos 2e \end{pmatrix}$, prop. adevărată:

$$A^2 = A \cdot A = \begin{pmatrix} \cos e & \sin e \\ -\sin e & \cos e \end{pmatrix} \begin{pmatrix} \cos e & \sin e \\ -\sin e & \cos e \end{pmatrix} = \begin{pmatrix} \cos^2 e - \sin^2 e & 2 \sin e \cos e \\ -2 \sin e \cos e & \cos^2 e - \sin^2 e \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2e & \sin 2e \\ -\sin 2e & \cos 2e \end{pmatrix}, \quad \text{"A"}$$

3) pp $p_k : A^k = \begin{pmatrix} \cos ke & \sin ke \\ -\sin ke & \cos ke \end{pmatrix}$, prop. adevărată, "A"

4) demonstrăm $p_{k+1} : A^{k+1} = \begin{pmatrix} \cos((k+1)e) & \sin((k+1)e) \\ -\sin((k+1)e) & \cos((k+1)e) \end{pmatrix}$, prop. adevărată:

$$A^{k+1} = A^k \cdot A = \begin{pmatrix} \cos ke & \sin ke \\ -\sin ke & \cos ke \end{pmatrix} \begin{pmatrix} \cos e & \sin e \\ -\sin e & \cos e \end{pmatrix} = \Big(~~\cos ke \cos e + \sin ke \sin e~~$$

$$= \begin{pmatrix} \cos ke \cdot \cos e - \sin ke \cdot \sin e & \cos ke \cdot \sin e + \sin ke \cdot \cos e \\ -\sin ke \cdot \cos e - \cos ke \cdot \sin e & -\sin ke \cdot \sin e \underset{-1-}{+} \cos ke \cos e \end{pmatrix} = \begin{pmatrix} \cos((k+1)e) & \sin((k+1)e) \\ -\sin((k+1)e) & \cos((k+1)e) \end{pmatrix} \text{"A"} \Rightarrow$$

$\Rightarrow$ 5) $A^n = \begin{pmatrix} \cos ne & \sin ne \\ -\sin ne & \cos ne \end{pmatrix}$, $\forall\, n \in \mathbb{N}^*$ (inducție finalizată)

Apl. 9

a) calculați det A, dezvoltând după L_1; $A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 3 & 4 \\ 2 & 5 & 1 & -1 \\ -1 & -2 & -2 & 4 \end{pmatrix}$

$$\det A = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 3 & 4 \\ 2 & 5 & 1 & -1 \\ -1 & -2 & -2 & 4 \end{vmatrix} = (-1)^2 \cdot \begin{vmatrix} 1 & 3 & 4 \\ 5 & 1 & -1 \\ -2 & 2 & 4 \end{vmatrix} + (-1)^3 \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & -1 \\ -1 & 2 & 4 \end{vmatrix} +$$

$$+ (-1)^4 \cdot 2 \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & -1 \\ -1 & -2 & 4 \end{vmatrix} + (-1)^5 \cdot 3 \cdot \begin{vmatrix} 1 & 1 & 3 \\ 2 & 5 & 1 \\ -1 & -2 & 2 \end{vmatrix} =$$

$$= 0 - 5 + \cancel{2 \cdot 15} - \cancel{3 \cdot 10} = -5$$

TEMA G.A.L.

- Seminarul 2 -

Apl 1

a) $$\det A = \begin{vmatrix} 1 & 3 & 0 & -2 \\ 2 & 1 & -1 & 3 \\ 1 & 2 & 3 & -1 \\ -1 & 4 & 0 & 1 \end{vmatrix} \begin{matrix} L_1+2L_4 \\ \underline{\underline{L_2+L_1+L_3}} \\ L_3+L_4 \end{matrix} \begin{vmatrix} -1 & 11 & 0 & 0 \\ 4 & 6 & -2 & 0 \\ 0 & 6 & 3 & 0 \\ -1 & 4 & 0 & 1 \end{vmatrix} = (-1)^8 \begin{vmatrix} -1 & 11 & 0 \\ 4 & 6 & -2 \\ 0 & 6 & 3 \end{vmatrix} =$$

$$= -18 + 0 + 0 - 0 - 12 - 132 = -162$$

Apl 2

c) $$A_3(1,2) = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \det A_3(1,2) = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4 \neq 0 \Rightarrow$$

$$\Rightarrow \exists A_3(1,2)^{-1} \in \mathcal{M}_3(\mathbb{C}) \text{ inversa lui } A_3(1,2)$$

$$\overset{t}{A}_3(1,2) = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} = A_3(1,2)$$

$$\overset{*}{A}_3(1,2) = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

$a = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$ $\quad d = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1$ $\quad g = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$

$b = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1$ $\quad e = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$ $\quad h = -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -1$

$c = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - 1$ $\quad f = -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -1$ $\quad i = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$

$$A^{-1}_3(1,2) = \frac{1}{\det A_3(1,2)} \cdot \overset{*}{A}_3(1,2) = \frac{1}{4} \cdot \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

Apl 3

a) $$\Delta_1 = \begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} \overset{?}{=} 0$$

$$\underset{L_1+L_2+L_3+L_4}{\Delta_1} = \begin{vmatrix} x+3a & x+3a & x+3a & x+3a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x+3a)\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} \begin{matrix} C_2-C_1 \\ C_3-C_1 \\ \hline \\ C_4-C_1 \end{matrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ a & x-a & 0 & 0 \\ a & 0 & x-a & 0 \\ a & 0 & 0 & x-a \end{vmatrix} \xrightarrow{\text{dezv. } L_1} (x+3a)\cdot(-1)^{2}\cdot 1\cdot \begin{vmatrix} x-a & 0 & 0 \\ 0 & x-a & 0 \\ 0 & 0 & x-a \end{vmatrix} = (x+3a)(x-a)^3$$

$$\Delta_1 = 0 \Rightarrow (x+3a)(x-a)^3 = 0 \Rightarrow \begin{cases} x_1 = -3a \\ x_2 = a \\ x_3 = a \\ x_4 = a \end{cases}$$

Apl 4

$$\Delta_n = \begin{vmatrix} -1 & a & a & \cdots & a \\ a & -1 & a & \cdots & a \\ a & a & -1 & \cdots & a \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a & a & a & & -1 \end{vmatrix} \begin{matrix} L_2-L_1 \\ L_3-L_1 \\ \vdots \\ \hline \\ \vdots \\ L_n-L_1 \end{matrix} \begin{vmatrix} -1 & a & a & \cdots & a \\ a+1 & -a-1 & 0 & \cdots & 0 \\ a+1 & \cdots 0 \cdots & -a-1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a+1 & \cdots 0 \cdots & 0 & \cdots & -a-1 \end{vmatrix} \begin{matrix} C_1+C_2 \\ C_1+C_3 \\ \vdots \\ \hline \\ \vdots \\ C_1+C_n \end{matrix}$$

$$= \begin{vmatrix} (n-1)(a-1) & a & a & \cdots & a \\ 0 & -a-1 & 0 & \cdots & 0 \\ 0 & 0 & -a-1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & -a-1 \end{vmatrix} = (-a-1)^{n-1}\cdot\left[(n-1)a-1\right]$$

$$\Rightarrow \Delta_n = (-1)^{n-1}\cdot(a+1)^{n-1}\left[(n-1)a-1\right]$$

Apl 5

~~$b)\ \Delta_2 = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix} = (x_1 x_2 x_3)^3 - (x_1^3 + x_2^3 + x_3^3)$~~

~~$x_1, x_2, x_3 =$ rad. ec. $x^3 - 2x^2 + 2x + 14 = 0 \Rightarrow$~~ $\left[\begin{matrix} S_1 = -\frac{b}{a} = 2 \\ S_2 = \frac{c}{a} = 2 \end{matrix}\right]$

~~$\Rightarrow$ Viete: $x_1^3 + x_2^3 + x_3^3 =$~~

$x_1 x_2 x_3 =$

$$b)\ \Delta_2 = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix} = (x_1+x_2+x_3)\begin{vmatrix} 1 & x_2 & x_3 \\ 1 & x_3 & x_1 \\ 1 & x_1 & x_2 \end{vmatrix} =$$

$$= (x_1+x_2+x_3)\left((x_1x_2 + x_2x_3 + x_3x_1) - (x_1^2 + x_2^2 + x_3^2)\right) =$$

$$= S_1\left(S_2 - (S_1^2 - 2S_2)\right) = 3S_1S_2 - S_1^3 = 3\cdot 2\cdot 2 - 2^3 = 12 - 8 = 4$$

Apl 6

a) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} \cdot (yz+zx+xy) = (-1)^2(xy+yz+zx)\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} =$

$\overset{\text{det. VM.}}{=} (xy+yz+zx)\cdot(-1)^2((y-x)(z-x)(z-y)) =$

$= (xy+yz+zx)(x-y)(y-z)(z-x)$

b) $\begin{vmatrix} a+b & b+c & c+a \\ a^2+b^2 & b^2+c^2 & c^2+a^2 \\ a^3+b^3 & b^3+c^3 & c^3+a^3 \end{vmatrix} = 2(a^3+b^3+c^3)\begin{vmatrix} 1 & 1 & 1 \\ a+b & b+c & c+a \\ a^2+b^2 & b^2+c^2 & c^2+a^2 \end{vmatrix} =$

$\overset{\text{det VM.}}{=} 2(a^3+b^3+c^3)((b-a)(c-a)(c-b))$

$\begin{vmatrix} a+b & b+c & c+a \\ a^2+b^2 & b^2+c^2 & c^2+a^2 \\ a^3+b^3 & b^3+c^3 & c^3+a^3 \end{vmatrix} \underset{\substack{C_2-C_3 \\ C_2-C_1 \\ C_3-C_2 \\ C_3-C_1}}{\overset{\substack{C_1-C_2 \\ C_1-C_3}}{=}} 2\begin{vmatrix} -c & b & a \\ -c^2 & b^2 & a^2 \\ -c^3 & b^3 & a^3 \end{vmatrix} =$

$= 2(abc(-a^2b-b^2c-c^2a+c^2b+b^2a+c^2a)) =$

$= 2abc(a-b)(b-c)(c-a)$

Apl 7

a) $\Delta_3 = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 3 & 0 & 1 \end{vmatrix} = 1+27 = 28$

$\Delta_4 = \begin{vmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \\ 3 & 0 & 0 & 1 \end{vmatrix} \overset{L_3-3L_4}{=} \begin{vmatrix} 1 & 3 & 0 & \cancel{0} \\ 0 & 1 & 3 & \cancel{0} \\ -9 & 0 & 1 & \cancel{0} \\ \cancel{3} & \cancel{0} & \cancel{0} & \cancel{1} \end{vmatrix} = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ -9 & 0 & 1 \end{vmatrix} = 1-81 = -80$

b) $\Delta_n = \begin{vmatrix} 1 & 3 & 0 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots 1 & 3 \\ 3 & 0 & 0 & \dots 0 & 1 \end{vmatrix} \overset{\text{dezv } C_1}{=} (-1)^2 \cdot 1 \cdot \underbrace{\begin{vmatrix} 1 & 3 & 0 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{vmatrix}}_{\Delta'_{n-1}} + (-1)^{n+1} \cdot 3 \underbrace{\begin{vmatrix} 3 & 0 & 0 & \dots & 0 \\ 1 & 3 & 0 & \dots & 0 \\ 0 & 1 & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \dots 1 & 3 \end{vmatrix}}_{\Delta''_{n-1}}$

$= \Delta'_{n-1} + (-1)^{n-1} \cdot 3\Delta''_{n-1}$

$\Delta'_{n-1} = 1^{n-1} = 1$

$\Delta''_{n-1} = 3^{n-1}$

$\Rightarrow \boxed{\Delta_n = 1+(-1)^{n+1} \cdot 3^n}$

Apl. 8

a) $\Delta_n = \begin{vmatrix} 4 & 3 & & & & 0 \\ 1 & 4 & 3 & & & \\ & 1 & 4 & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & 4 & 3 \\ 0 & & & & 1 & 4 \end{vmatrix} \overset{\text{dezv. } L_1}{=} 4\cdot(-1)^2 \begin{vmatrix} 4 & 3 & & 0 \\ 1 & 4 & \ddots & \\ & \ddots & 4 & 3 \\ 0 & & 1 & 4 \end{vmatrix} + 3\cdot(-1)^3 \begin{vmatrix} 1 & 3 & 0 & \dots & 0 & 0 \\ 0 & 4 & 3 & \dots & 0 & 0 \\ 0 & 1 & 4 & \dots & 0 & 0 \\ 0 & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 4 & 3 \\ 0 & 0 & 0 & \dots & 1 & 4 \end{vmatrix}$

$= 4\Delta_{n-1} - 3\cdot 1\cdot(-1)^2\Delta_{n-2} = 4\Delta_{n-1} - 3\Delta_{n-2} \Rightarrow$

$\Rightarrow \Delta_n = 4\Delta_{n-1} - 3\Delta_{n-2}, \forall n \geq 3 \Rightarrow$ rel. de recurență

b) $\Delta_n - 4\Delta_{n-1} + 3\Delta_{n-2} = 0$

$x^2 - 4x + 3 = 0 \Rightarrow x_1 = 1$; $\Delta_n = A x_1^n + B x_2^n = A\cdot 1^n + B\cdot 3^n$

$\Rightarrow x_2 = 3$ $\quad = B\cdot 3^n + A$; $A, B \in \mathbb{R}$

$$\left.\begin{aligned} \Delta_3 = 40 &= 27B + A \\ \Delta_4 = 121 &= 81B + A \end{aligned}\right\} \Rightarrow 54B = 81 \Rightarrow \boxed{B = \frac{3}{2}}$$

$$A = 40 - \frac{81}{2} \Rightarrow \boxed{-\frac{1}{2} = A}$$

$$\Rightarrow \boxed{\Delta_n = \frac{3^{n+1}}{2} - \frac{1}{2}, \forall n \geq 3}$$

TEMA G.A.L.

- Seminarul 3 -

①

a) $\begin{cases} x+y-z=2 \\ 2x+y-3z=2 \\ x-y-z=0 \end{cases}$

$$\tilde{A}=\left(\begin{array}{ccc|c} \boxed{1} & 1 & -1 & 2 \\ 2 & 1 & -3 & 2 \\ 1 & -1 & -1 & 0 \end{array}\right) \underset{L_3-L_1}{\overset{L_2-2L_1}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & \boxed{-1} & -1 & -2 \\ 0 & -2 & 0 & -2 \end{array}\right) \underset{L_3-2L_2}{\overset{L_1+L_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 2 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \underset{L_2-L_3}{\overset{L_1+2L_3}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{cases} x=2 \\ y=1 \\ z=1 \end{cases}$$

b) $\begin{cases} x+y+2z=1 \\ x+y+3z=1 \\ x+y-2z=1 \end{cases}$

$$\tilde{A}=\left(\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & -2 & 1 \end{array}\right) \underset{L_3-L_1}{\overset{L_2-L_1}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & -4 & 0 \end{array}\right) \underset{L_3+4L_2}{\overset{L_1-2L_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$$

$\Rightarrow \begin{cases} x+y=1 \\ z=0 \end{cases}$

notam $y=\alpha$

$\Rightarrow S=(1-\alpha, \alpha, 0)$

c) $\begin{cases} x+y+z+t=1 \\ 2x-y+z-t=2 \\ x-2y-2t=-1 \end{cases}$

$$\tilde{A}=\left(\begin{array}{cccc|c} \boxed{1} & 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & -1 & 2 \\ 1 & -2 & 0 & -2 & -1 \end{array}\right) \underset{L_3-L_1}{\overset{L_2-2L_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & \boxed{-3} & -1 & -3 & 0 \\ 0 & -3 & -1 & -3 & -2 \end{array}\right) \underset{L_3-L_2}{\overset{3L_1+L_2}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & 2/3 & 0 & 1 \\ 0 & 1 & 1/3 & 1 & 0 \\ 0 & 0 & 0 & 0 & \boxed{-2} \end{array}\right)$$

$\Rightarrow \exists$ pivot pe coloana 5 $\Rightarrow$ S incompatibil

③ d) $\begin{cases} x+2y+3z-2t=6 \\ 2x-y-2z-3t=8 \\ 3x+2y-z+2t=4 \\ 2x-3y+2z+t=-8 \end{cases}$

$$\tilde{A}=\left(\begin{array}{cccc|c} \boxed{1} & 2 & 3 & -2 & 6 \\ 2 & -1 & -2 & -3 & 8 \\ 3 & 2 & -1 & 2 & 4 \\ 2 & -3 & 2 & 1 & -8 \end{array}\right) \underset{\substack{L_3-3L_1 \\ L_4-2L_1}}{\overset{L_2-2L_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 2 & 3 & -2 & 6 \\ 0 & \boxed{-7} & -11 & 3 & -10 \\ 0 & -4 & -10 & 8 & -14 \\ 0 & -7 & -4 & 5 & -20 \end{array}\right) \underset{\substack{7L_3-4L_2 \\ L_4-L_2}}{\overset{7L_1+2L_2}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & -\frac{1}{7} & -\frac{8}{7} & \frac{22}{7} \\ 0 & -7 & -11 & 3 & -10 \\ 0 & 0 & -\frac{26}{7} & \frac{44}{7} & -\frac{58}{7} \\ 0 & 0 & 7 & 2 & -10 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -\frac{1}{7} & -\frac{8}{7} & \frac{22}{7} \\ 0 & 1 & \frac{11}{7} & -\frac{3}{7} & \frac{10}{7} \\ 0 & 0 & \boxed{1} & -\frac{22}{13} & \frac{29}{13} \\ 0 & 0 & 7 & 2 & -10 \end{array}\right) \underset{L_4-7L_3}{\overset{\substack{7L_1+L_3 \\ 7L_2-11L_3}}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{9}{10} \\ 0 & 1 & 0 & 0 & \frac{41}{20} \\ 0 & 0 & 1 & 0 & -\frac{9}{10} \\ 0 & 0 & 0 & 1 & -\frac{37}{20} \end{array}\right) \Rightarrow \begin{cases} x=\frac{9}{10} \\ y=\frac{41}{20} \\ z=-\frac{9}{10} \\ t=-\frac{37}{20} \end{cases}$$

②

a) $\begin{cases} x+y-z=0 \\ 2x+y-3z=0 \\ x-y-z=0 \end{cases}$

$$\tilde{A}=\left(\begin{array}{ccc|c} \boxed{1} & 1 & -1 & 0 \\ 2 & 1 & -3 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right) \underset{L_3-L_1}{\overset{L_2-2L_1}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & \boxed{-1} & -1 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right) \underset{L_3-2L_2}{\overset{L_1+L_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & \boxed{2} & 0 \end{array}\right)$$

$$\underset{2L_2+L_3}{\overset{L_1+L_3}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right) \Rightarrow \begin{cases} x=0 \\ -y=0 \\ 2z=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y=0 \\ z=0 \end{cases}$$

b) $\begin{cases} x+y-z+t=0 \\ x-y+z+t=0 \\ 2x+y+2z-t=0 \end{cases}$

$$\tilde{A}=\left(\begin{array}{cccc|c} \boxed{1} & 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 2 & 1 & 2 & -1 & 0 \end{array}\right) \underset{L_3-2L_1}{\overset{L_2-L_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & -1 & 4 & -3 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & \boxed{1} & -1 & 0 & 0 \\ 0 & -1 & 4 & -3 & 0 \end{array}\right)$$

$$\underset{L_3+L_2}{\overset{L_1-L_2}{\sim}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & \boxed{1} & -1 & 0 \end{array}\right) \overset{L_2+L_3}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} x=-\alpha \\ y=0 \\ z=\alpha \\ t=\alpha \end{cases} \Rightarrow S=(-\alpha, 0, \alpha, \alpha)$$

③

b) $B = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \in \mathcal{M}_3(\mathbb{R})$; $B^{-1} = ?$

$$(B|I_3) = \left(\begin{array}{ccc|ccc} \boxed{1} & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \underset{L_3-2L_1}{\overset{L_2-L_1}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 2 & -1 & 1 & 0 \\ 0 & \boxed{-1} & 4 & -2 & 0 & 1 \end{array}\right) \underset{\substack{L_1+L_2 \\ L_3-2L_2}}{\overset{L_3 \leftrightarrow L_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & -1 & 0 & 1 \\ 0 & -1 & 4 & -2 & 0 & 1 \\ 0 & 0 & \boxed{-6} & 3 & 1 & -2 \end{array}\right)$$

$$\underset{3L_2+2L_3}{\overset{2L_1+L_3}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & -6 & 3 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{6} & \frac{1}{3} \end{array}\right) \Rightarrow B^{-1} = -\frac{1}{6}\begin{pmatrix} -3 & -3 & 0 \\ 0 & 4 & -2 \\ 3 & 1 & -2 \end{pmatrix}$$

$$\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3} \quad \underbrace{\begin{matrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{1}{3} \end{matrix}}_{B^{-1}}$$

④

a) $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$; $A^{-1} = ?$; $\det A = \frac{1}{5} \neq 0 \Rightarrow \exists A^{-1} \in \mathcal{M}_2(\mathbb{R})$ aî $A \cdot A^{-1} = I_2$

$$\widetilde{(A|I_2)} = \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} \textcircled{1} & 3 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}\right) \overset{L_2-2L_1}{\sim} \left(\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & \boxed{-5} & 1 & -2 \end{array}\right) \underset{\frac{L_2}{-5}}{\overset{L_1+\frac{3}{5}L_2}{\sim}} \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & -\frac{1}{5} & \frac{2}{5} \end{array}\right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$$

b) $B = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$, $B^{-1} = ?$; $\det B = -3 \neq 0 \Rightarrow \exists B^{-1} \in \mathcal{M}_3(\mathbb{R})$ aî $B \cdot B^{-1} = I_3$

$$\widetilde{(B|I_3)} = \left(\begin{array}{ccc|ccc} 2 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \overset{L_1 \leftrightarrow L_2}{\sim} \left(\begin{array}{ccc|ccc} \textcircled{1} & 1 & 2 & 0 & 1 & 0 \\ 2 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \underset{L_3-L_1}{\overset{L_2-2L_1}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & \textcircled{-3} & -3 & 1 & -2 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right)$$

$$\overset{L_1-\frac{L_2}{3}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/3 & 1/3 & 0 \\ 0 & 1 & 1 & -\frac{1}{3} & 2/3 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/3 & 1/3 & 0 \\ 0 & 1 & 1 & -1/3 & 2/3 & 0 \\ 0 & 0 & \boxed{1} & 0 & 1 & -1 \end{array}\right) \underset{L_2-L_3}{\overset{L_1-L_3}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1 \\ 0 & 1 & 0 & -1/3 & -1/3 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right) \Rightarrow$$

$$\Rightarrow B^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & 1 \\ -\frac{1}{3} & -\frac{1}{3} & 1 \\ 0 & 1 & -1 \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} -1 & 2 & -3 \\ 1 & 1 & -3 \\ 0 & -3 & 3 \end{pmatrix}$$

c) $C = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & -2 \\ 1 & -1 & 4 \end{pmatrix}$; $C^{-1} = ?$; $\det C = -17 \neq 0 \Rightarrow \exists\, C^{-1} \in \mathcal{M}_3(\mathbb{R})$ aî $C \cdot C^{-1} = I_3$

$$(\widetilde{C} \mid I_3) = \left(\begin{array}{ccc|ccc} \boxed{1} & 1 & 3 & 1 & 0 & 0 \\ 2 & 1 & -2 & 0 & 1 & 0 \\ 1 & -1 & 4 & 0 & 0 & 1 \end{array}\right) \underset{L_3-L_1}{\overset{L_2-2L_1}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & \boxed{-1} & -8 & -2 & 1 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right) \underset{\substack{L_3+2L_2 \\ L_2\cdot(-1)}}{\overset{L_1-L_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & -5 & -1 & 1 & 0 \\ 0 & 1 & 8 & 2 & -1 & 0 \\ 0 & 0 & \boxed{17} & -5 & 2 & 1 \end{array}\right)$$

$$\underset{-17L_1+8L_3}{\overset{17L_1+5L_3}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/17 & 7/17 & 5/17 \\ 0 & 1 & 0 & 10/17 & -1/17 & -8/17 \\ 0 & 0 & 1 & 3/17 & -2/17 & 1/17 \end{array}\right)$$

$$\underbrace{}_{I_3} \qquad \underbrace{}_{C^{-1}}$$

$$C^{-1} = \begin{pmatrix} -2/17 & 7/17 & 5/17 \\ 10/17 & -1/17 & -8/17 \\ 3/17 & -2/17 & 1/17 \end{pmatrix} = -\frac{1}{17} \begin{pmatrix} 2 & -7 & -5 \\ -10 & 1 & 8 \\ -3 & 2 & -1 \end{pmatrix}$$

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— Seminarul 4 —

① a) $S_3 = \{v_1 = (1,1,0),\ v_2 = (1,0,1),\ v_3 = (0,1,1)\} \in \mathbb{R}^3/\mathbb{R}$

S_3 sist. de generatori $(\Rightarrow)$ $\forall\, v \in \mathbb{R}^3$, $\exists\, \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ a.î.
$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$

fie $v = (x, y, z) \in \mathbb{R}^3$ vector arbitrar

$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 \ (\Rightarrow)\ (x, y, z) = \alpha_1(1,1,0) + \alpha_2(1,0,1) + \alpha_3(0,1,1)$

$$(\Rightarrow) \begin{cases} \alpha_1 + \alpha_2 = x \\ \alpha_1 + \alpha_3 = y \\ \alpha_2 + \alpha_3 = z \end{cases} \Rightarrow \begin{cases} \alpha_1 = \frac{x+y-z}{2} \\ \alpha_2 = \frac{x-y+z}{2} \\ \alpha_3 = \frac{-x+y+z}{2} \end{cases};$$

Deci $(\exists) \begin{cases} \alpha_1 = \frac{x+y-z}{2} \\ \alpha_2 = \frac{x-y+z}{2} \\ \alpha_3 = \frac{-x+y+z}{2} \end{cases}$ a.î. $\forall\, v = (x, y, z) \in \mathbb{R}$
$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$ $\Rightarrow$

$\Rightarrow S_3 = \{v_1, v_2, v_3\} \subset \mathbb{R}^3/\mathbb{R}$ sist. de generatori pt. $\mathbb{R}^3/\mathbb{R}$

b) $S_4 = \{v_1 = 1,\ v_2 = X-1,\ v_3 = (X-1)^2\} \subset \mathbb{R}_2[X]$

S_4 sist de generatori $(\Rightarrow)$ $\forall\, v \in \mathbb{R}_2[X]$, $\exists\, \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ a.î.
$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$

fie $v = aX^2 + bX + c \in \mathbb{R}_2[X]$ polinom arbitrar

$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 \ (\Rightarrow)\ aX^2 + bX + c = \alpha_1 + \alpha_2(X-1) + \alpha_3(X-1)^2 =$
$= \alpha_3 X^2 + (\alpha_2 - 2\alpha_3)X + x_1 - x_2 + x_3$

$$(\Rightarrow) \begin{cases} a = \alpha_3 \\ b = \alpha_2 - 2\alpha_3 \\ c = \alpha_1 - \alpha_2 + \alpha_3 \end{cases} \Rightarrow \begin{cases} \alpha_1 = a+b+c \\ \alpha_2 = b+2a \\ \alpha_3 = a \end{cases};$$

Deci $(\exists) \begin{cases} \alpha_1 = a+b+c \\ \alpha_2 = b+2a \\ \alpha_3 = a \end{cases}$ a.î. $\forall\, v = aX^2 + bX + c \in \mathbb{R}_2[X]$
$v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$ $\Rightarrow$

$\Rightarrow S_4 = \{v_1, v_2, v_3\} \subset \mathbb{R}_2[X]$ sist. de generatori pt. $\mathbb{R}_2[X]$

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- Seminarul 5 -

① a) $U=\{(x,y,z)\in\mathbb{R}^3 \mid -x+3y+z=0\}\subset \mathbb{R}^3/\mathbb{R}$

fie $v_1, v_2 \in U \Rightarrow \begin{cases} v_1=(x_1,y_1,z_1), & -x_1+3y_1+z_1=0 \\ v_2=(x_2,y_2,z_2), & -x_2+3y_2+z_2=0\end{cases}$

$\alpha_1,\alpha_2\in\mathbb{R}$

$\alpha_1v_1+\alpha_2v_2 \overset{?}{\in} U$:

$$\alpha_1v_1+\alpha_2v_2=(\underbrace{\alpha_1x_1+\alpha_2x_2}_{x}, \underbrace{\alpha_1y_1+\alpha_2y_2}_{y}, \underbrace{\alpha_1z_1+\alpha_2z_2}_{z})$$

$$-x+3y+z=-(\alpha_1x_1+\alpha_2x_2)+3(\alpha_1y_1+\alpha_2y_2)+(\alpha_1z_1+\alpha_2z_2)=$$

$$=\alpha_1(\underbrace{-x_1+3y_1+z_1}_{0})+\alpha_2(\underbrace{-x_2+3y_2+z_2}_{0})=0 \Rightarrow$$

$\Rightarrow \alpha_1v_1+\alpha_2v_2\in U$; deci $U\subset\mathbb{R}^3$ subsp. vectorial

b) fie $v\in U$ vector arbitrar; $v=(x,y,z)$; $-x+3y+z=0 \Rightarrow x=3y+z$

$v=(3y+z,y,z)=y(\underbrace{3,1,0}_{v_1})+z(\underbrace{1,0,1}_{v_2})=yv_1+zv_2\Rightarrow$

$\Rightarrow S=\{v_1,v_2\}\subset U$ sist. de generatoare (*)

fie $\alpha_1,\alpha_2\in\mathbb{R}$ a.î. $\alpha_1v_1+\alpha_2v_2=0\Rightarrow$

$\Rightarrow \alpha_1(3,1,0)+\alpha_2(1,0 1)=(0,0,0)\Rightarrow \begin{cases}3\alpha_1+\alpha_2=0\\ \alpha_2=0\\ \alpha_1=0\end{cases} \Rightarrow \alpha_1=\alpha_2=0$

$\Rightarrow S=\{v_1,v_2\}\subset\mathbb{R}^3$ spațiu vectorial liniar independent (**)

$\xrightarrow{(*),(**)} S\subset U$ (bază) $\Rightarrow \dim_{\mathbb{R}} U=2$

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- Seminarul 6 -

① $V_1 = \{(x,y,0) \mid x,y \in \mathbb{R}\}$
$V_2 = \{(u,0,v) \mid u,v \in \mathbb{R}\}$

a) V_2 subsp. vect. $\Leftrightarrow \forall\, \alpha,\beta \in \mathbb{R}$ avem că $\alpha(u_1, 0 v_1) + \beta(u_2, 0, v_2) \in V_2$

$(\alpha u_1 + \beta u_2, 0, \alpha v_1 + \beta v_2) \in V_2 \Rightarrow V_2$ subsp. vectorial

$$\left.\begin{array}{l} \{(u,0,v)\} = \{u\cdot(1,0,0) + v\cdot(0,0,1) \mid u,v \in \mathbb{R}\} = \langle (1,0,0),(0,0,1)\rangle \\ \alpha(1,0,0) + \beta(0,0,1) = 0 \Rightarrow \alpha = \beta = 0 \end{array}\right\} \Rightarrow$$

$\Rightarrow \dim_{\mathbb{R}} V_2 = 2$

b.i) $V_1 + V_2 = \langle V_1 \cup V_2 \rangle = \langle (x,y,0),(u,0,v)\rangle$

~~$\forall(\alpha,\beta,\gamma) = a(x,y,0) + b(u,0,v) \Rightarrow \begin{cases} \alpha = ax + bu \\ \beta = ay \\ \gamma = bv \end{cases}$~~

$$\left.\begin{array}{l} V_1 = \langle (1,0,0),(0,1,0)\rangle \\ V_2 = \langle (1,0,0),(0,0,1)\rangle \end{array}\right| \Rightarrow V_1 \cup V_2 = \langle \underbrace{(1,0,0)}_{e_1}, \underbrace{(0,1,0)}_{e_2}, \underbrace{(0,0,1)}_{e_3} \rangle \Rightarrow$$

$\Rightarrow \langle V_1 \cup V_2 \rangle = \mathbb{R}^3 \Rightarrow V_1 + V_2 = \mathbb{R}^3$

b.ii) $V_1 \oplus V_2 = \mathbb{R}^3$ nu este "A" pt că $V_1 \cap V_2 = \langle e_1 \rangle \neq \{0_{\mathbb{R}^3}\}$

$\Rightarrow$ Afirmație FALSĂ

② $V_1=\{A\in \mathcal{M}_n(\mathbb{R}) \mid Tr\,A=0\}$

$V_2=\{A\in \mathcal{M}_n(\mathbb{R}) \mid A=\lambda I_n,\ \lambda\in\mathbb{R}\}$

a) V_1 subsp. vect $\Leftrightarrow \forall \alpha,\beta\in\mathbb{R}$ și $A,B\in V_1$, avem că:

$Tr(\alpha A+\beta B)=Tr(\alpha A)+Tr(\beta B)=\alpha\,Tr\,A+\beta\,Tr\,B=0+0=0$

$\Rightarrow V_1$ subsp. vect din $\mathcal{M}_n(\mathbb{R})$

V_2 subsp. vect $\Leftrightarrow \forall \lambda_{1,2}\in\mathbb{R}$ și $A,B\in V_2$, avem că:
$A=\lambda_1 I_n$ și $B=\lambda_2 I_n$ $\Leftrightarrow$ ~~$\alpha\,Tr(A)$~~ $\alpha A+\beta B=\alpha\lambda_1 I_n+\beta\lambda_2 I_n=$
$=(\alpha\lambda_1+\beta\lambda_2)I_n \Rightarrow V_2$ subsp. vect. din $\mathcal{M}_n(\mathbb{R})$

b) $V_1\oplus V_2 \overset{?}{=} \mathcal{M}_n(\mathbb{R})$; $V_1\oplus V_2=\mathcal{M}_n(\mathbb{R}) \Leftrightarrow \begin{cases} V_1\oplus V_2\supset \mathcal{M}_n(\mathbb{R})\\ V_1\oplus V_2\subset \mathcal{M}_n(\mathbb{R})\end{cases}$

1) fie $A\in V_1$ și $B\in V_2 \Rightarrow A+B\in\mathcal{M}_n(\mathbb{R})$, $\forall A,B\in V_1$, resp. V_2
$\Rightarrow V_1\oplus V_2\subset\mathcal{M}_n(\mathbb{R})$

2) fie $C\in\mathcal{M}_n(\mathbb{R})$ a.î. $C=A+B$, cu $A\in V_1$ și $B=V_2$

~~$A=\{a_{ij}=\begin{cases} c_{ij}, & i\neq j\\ c_{ij}-\frac{Tr}{n}, & i=j\end{cases}\}$~~ $A=\left\{a_{ij} \,\middle|\, \begin{matrix} a_{ij}=c_{ij}, & i\neq j\\ a_{ij}=c_{ij}-\frac{Tr\,C}{n}, & i=j\end{matrix}\right\}$

~~$B=\{b_{ij}=\begin{cases}\frac{Tr\,C}{n}, & i=j\\ 0, & i\neq j\end{cases}\}$~~ $B=\left\{b_{ij} \,\middle|\, \begin{matrix} b_{ij}=\frac{Tr\,C}{n}, & i=j\\ 0, & i\neq j\end{matrix}\right\}$ $\Bigg| \Rightarrow$

$\Rightarrow \forall C\in\mathcal{M}_n(\mathbb{R})$, $\exists A\in V_1$ și $B\in V_2$ a.î. $C=A+B \Rightarrow \mathcal{M}_n(\mathbb{R})\subset V_1\oplus V_2$

1), 2) $\Rightarrow V_1\oplus V_2=\mathcal{M}_n(\mathbb{R})$

c) $\dim_{\mathbb{R}}\mathcal{M}_n(\mathbb{R})=\dim_{\mathbb{R}}A+\dim_{\mathbb{R}}B$

$\dim_{\mathbb{R}}A=n^2-1$, deoarece un singur element depinde de celelalte element din $Tr\,A$
$\dim_{\mathbb{R}}B=1$, deoarece e generat de e_1 $\Bigg\}\Rightarrow$

$\Rightarrow \dim_{\mathbb{R}}A+\dim_{\mathbb{R}}B=n^2=\dim_{\mathbb{R}}\mathcal{M}_n(\mathbb{R})$ "A"