= Tema LAB 3 =

1) $X \sim \begin{pmatrix} 1 & \boxed{2} & \dots & \boxed{5} & 6 & \boxed{7} & \dots & 12 \\ \frac{0}{36} & \frac{1}{36} & \dots & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \dots & \frac{1}{36} \end{pmatrix}$, variabilă aleatoare

a) $P(5) = \frac{4}{36} \Leftarrow \{(1,4),(4,1),(2,3),(3,2)\}$

$P(7) = \frac{6}{36} \Leftarrow \{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

$P(2) = \frac{1}{36} \Leftarrow \{(1,1)\}$

$cp = 6 \cdot 6 = 36$ cazuri posibile

→ probabilitatea ca în primele $n-1$ aruncări să nu pice suma 5 sau 7 : $P(\cancel{X}, X \notin \{5,7\}) = \left(\frac{26}{36}\right)^{n-1}$;

iar în a n-a aruncare să pice 5: $P(A) = \left(\frac{26}{36}\right)^{n-1} \cdot \frac{4}{36}$

b) → probabilitatea ca în ——〃—— suma 2 sau 7 : $P(X, X \notin \{2,7\}) = \left(\frac{29}{36}\right)^{n-1}$

iar în a n-a aruncare să pice 2 : $P(B) = \left(\frac{29}{36}\right)^{n-1} \cdot \frac{1}{36}$

2) $X \sim \begin{pmatrix} -1 & 0 & 1 \\ 0,3 & 0,2 & 0,5 \end{pmatrix}$; $P(A|B) = \frac{P(A \cap B)}{P(B)}$

a) $P\left(X > -\frac{1}{3}\right) = \sum_{n > -1}^{+\infty} P(n) = P(0) + P(1) = 0,7$

b) $P\left(X < \frac{1}{4} \mid X \geq -\frac{1}{2}\right) = \frac{P\left(-\frac{1}{2} \leq X \leq \frac{1}{4}\right)}{P\left(X \geq -\frac{1}{2}\right)} = \frac{\sum_{n > -1}^{0} P(n)}{\sum_{n > -1}^{+\infty} P(n)} = \frac{P(0)}{P(0)+P(1)} =$

$= \frac{0,2}{0,7} = 0,28$

3) a) din (i) $\Rightarrow p_n = \frac{\lambda^n e^{-\lambda}}{n!} \Rightarrow \frac{p_n}{p_{n-1}} = \frac{\lambda^n e^{-\lambda}}{n!} \cdot \frac{(n-1)!}{\lambda^{n-1} e^{-\lambda}} = \frac{\lambda}{n}$ (ii)

din (ii) $\Rightarrow \frac{p_n}{p_{n-1}} = \frac{\lambda}{n} \Leftrightarrow (p_n)$ progr. geom cu $r = \frac{\lambda}{n} \Rightarrow$

$\Rightarrow$ fie $p_0 = X \Rightarrow p_1 = \frac{X \cdot \lambda}{1!}$

$p_2 = \frac{\lambda^2 X}{2!}$

$p_3 = \frac{\lambda^3 X}{3!}$ și $\sum_{k=0}^{+\infty} p_k = 1 \Rightarrow$

$\vdots$

$p_n = \frac{\lambda^n X}{n!}$

$\Rightarrow X = e^{-\lambda} = p_0 \Rightarrow p_n = \frac{\lambda^n e^{-\lambda}}{n!}$ (i)

4) a) $\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \cancel{\int_0^{+\infty} P(k \geq x)\,dx} = \int_0^{+\infty} \alpha x^2 e^{-kx}\,dx \underset{t=kx}{\overset{\text{subst.}}{=}}$

$= \frac{\alpha}{k^3} \int_0^{+\infty} t^2 e^{-t}\,dt = \frac{2\alpha}{k^3} = 1 - \int_{-\infty}^{0} 0\,dx = 1 \Rightarrow \boxed{\alpha = \frac{k^3}{2}}$

b) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

$$F(x) = \begin{cases} \int_{-\infty}^{0} f(t)\,dt = \int_{-\infty}^{0} 0\,dt = [0, x \leq 0] \\ \int_0^1 \frac{k^3}{2} t^2 e^{-kt}\,dt = \left[1 - \frac{(kx(kx+2)+2)e^{-kx}}{2}, 0 < x \leq 1\right] \\ \int_{-\infty}^{+\infty} f(t)\,dt = [1, x > 1] \end{cases}$$

c) $P(0 < X < \frac{1}{k}) = F(\frac{1}{k}) - F(0) = F(\frac{1}{k}) = 1 - \frac{\frac{k}{k} \cdot (\frac{k}{k}+2)+2}{2e}$

$0 < \frac{1}{k} \leq 1$

$= \boxed{1 - \frac{5}{2e}}$

5) a) $E[X] = \sum_{k=0}^{+\infty} k(P(X=k)) = \sum_{k=1}^{+\infty} k \frac{(1-p)^k}{-k\log(p)} =$

$= -\frac{1}{\log(p)} \sum_{k=1}^{+\infty} (1-p)^k = -\frac{1}{\log(p)} \left[\frac{1}{1-(1-p)} - 1\right] = -\frac{1-p}{p\log(p)}$

b) $E[X^2] = \sum_{k=0}^{+\infty} k^2 P(X=k) = \sum_{k=1}^{+\infty} k^2 \frac{(1-p)^k}{-k\log(p)} = -\frac{1}{\log(p)} \sum_{k=1}^{+\infty} k(1-p)^k =$

$= -\frac{1}{\log(p)} (1-p) \sum_{k=1}^{+\infty} ((1-p)^k)' = -\frac{1-p}{p^2\log(p)}$

c) $Var[X] = E[X^2] - E[X]^2 = -\frac{1-p}{p^2\log(p)} - \left(\frac{1-p}{p\log(p)}\right)^2 =$

$= \frac{(1-p)(1-p+\log(p))}{-p^2\log^2(p)}$

6) a) $P(X>n) = P(X=n+1) + P(X=n+2) + \ldots = \sum_{k=n+1}^{+\infty} P(X=k)$

$\sum_{n=0}^{+\infty} \left(\sum_{k=n+1}^{+\infty} P(X=k)\right) = (P(X=1) + P(X=2) + \ldots) + (P(X=2) + P(X=3)) +$

$+ \ldots = P(X=1) + 2P(X=2) + \ldots =$

$= \sum_{k=1}^{+\infty} kP(X=k) = \sum_{k=0}^{+\infty} kP(X=k) = E[X]$

b) $E[X] = \int_0^{+\infty} P(X \geq x)\,dx = \int_0^{+\infty} E[\,?\,]\,dx$ | nu știu să rezolv!