= Temă - LAB 1 =

④ Alegem { limuzinele în C_{11}^{2} moduri = 55
         { furgonetele în C_{6}^{3} moduri = 20
         { cabrioletele în C_{3}^{1} moduri = 3
} ⇒ 3300 c.f.

Alegem un tuplu de 6 autoturisme în C_{20}^{6} moduri =
= 38.760 c.p.

$$\Rightarrow p = \frac{c.f.}{c.p.} = \frac{3300}{38.760} = 0,085$$

⑤ a) Ghicirea a 6 nr. câștigătoare la 6/49 are prob. de

$$\left(C_{49}^{6}\right)^{-1} = \frac{1}{13.983.816}$$

Extragerea unui bob de grâu din 10.000.000 boabe e

$$\frac{1}{10.000.000}$$

b) Ghicirea a 5 sau 6 nr. câștigătoare la 6/49 are prob.

$$\left(C_{49}^{6}\right)^{-1} + \frac{\left(C_{6}^{5} \cdot C_{43}^{1}\right)}{C_{49}^{6}} = \frac{1}{13.983.816} + \frac{1}{54201}$$

Extragerea unui bob de grâu din 150.000 boabe e

$$\frac{1}{150.000}$$

⑥ Presupunem că nu putem analiza aceeași roată de mai multe ori:

Analizăm roțile → roșii (A) în C_6^2 moduri = 15
→ verzi (B) în C_5^1 moduri = 5
→ albastre (C) în C_4^3 moduri = 4
⇒ 300 cf.

Analizăm un grup de 6 roți în C_{15}^6 moduri = 5005 cp.

$\Rightarrow p = \frac{cf}{cp} = \frac{300}{5005} = 0,059$

⑧ Alegerea a celor 2 pagini din $C_{10}^2 = 45$ de moduri =

$\Rightarrow p = \frac{1}{45} = 0,022$

⑨ $C_I: \frac{C_6^6 \cdot C_{43}^0}{C_{49}^6} = \frac{1}{13.983.816}$

$C_{II}: \frac{C_6^5 \cdot C_{43}^1}{C_{49}^6} = \frac{1}{54201}$

$C_{III}: \frac{C_6^4 \cdot C_{43}^2}{C_{49}^6} = \frac{1}{1032}$

$C_{IV}: \frac{C_6^3 \cdot C_{43}^3}{C_{49}^6} = \frac{1763}{99} \quad \frac{8815}{499422}$

(10) a) $A \cap \bar{B} \cap \bar{C}$

b) $A \cap \bar{B} \cap C$

c) $A \cap B \cap C$

d) $A \cup B \cup C$

e) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

f) $(\bar{A} \cap \bar{B} \cap \bar{C}) \cup (A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

g) $\bar{A} \cap \bar{B} \cap \bar{C}$

h) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

(11) a) $(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap A_5) \cup (\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap A_4 \cap \bar{A}_5) \cup$
$(\bar{A}_1 \cap \bar{A}_2 \cap A_3 \cap \bar{A}_4 \cap \bar{A}_5) \cup (\bar{A}_1 \cap A_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap \bar{A}_5) \cup (A_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap \bar{A}_5)$

b) $(A_1 \cap A_2 \cap A_3 \cap A_4 \cap \bar{A}_5) \cup (A_1 \cap A_2 \cap A_3 \cap \bar{A}_4 \cap A_5) \cup$
$(A_1 \cap A_2 \cap \bar{A}_3 \cap A_4 \cap A_5) \cup (A_1 \cap \bar{A}_2 \cap A_3 \cap A_4 \cap A_5) \cup (\bar{A}_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5)$

c) $(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap \bar{A}_5) \cup$ (ce am obținut la a)) $\cup$
$(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap A_4 \cap A_5) \cup (\bar{A}_1 \cap \bar{A}_2 \cap A_3 \cap \bar{A}_4 \cap A_5) \cup \ldots \cup (A_1 \cap A_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap \bar{A}_5)$

d) $(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) \cup$ (ce am obținut la b)) $\cup$
$(\bar{A}_1 \cap \bar{A}_2 \cap A_3 \cap A_4 \cap A_5) \cup (\bar{A}_1 \cap A_2 \cap \bar{A}_3 \cap A_4 \cap A_5) \cup \ldots \cup (A_1 \cap A_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap \bar{A}_5)$

e) $(A_1 \cap A_2 \cap A_3 \cap \bar{A}_4 \cap \bar{A}_5) \cup (A_1 \cap A_2 \cap \bar{A}_3 \cap A_4 \cap \bar{A}_5) \cup$
$(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4 \cap A_5) \cup (A_1 \cap \bar{A}_2 \cap A_3 \cap A_4 \cap \bar{A}_5) \cup (\bar{A}_1 \cap A_2 \cap A_3 \cap A_4 \cap \bar{A}_5) \cup$
$\ldots \cup (\bar{A}_1 \cap \bar{A}_2 \cap A_3 \cap A_4 \cap A_5)$

(12) a) Notăm nr. șosete = n și nr. șosete roșii = X

$$\Rightarrow \left.\begin{array}{l} f = C_X^2 = \frac{X(X-1)}{2} \\ \Rightarrow C_p = C_n^2 = \frac{n(n-1)}{2} \end{array}\right\} \Rightarrow p = \frac{cf}{cp} = \frac{X(X-1)}{n(n-1)} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow 2X(X-1) = n(n-1)$$

C I: verif. pt $n=3$, cu $X=2 \Rightarrow p = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3} \neq \frac{1}{2}$

C II: Verif. pt $n=4$, cu $X=3 \Rightarrow p = \frac{3}{4} \cdot \frac{2}{3} = \boxed{\frac{1}{2}} \Rightarrow$

$\Rightarrow$ minim 4 șosete, din care 3 roșii

b) nr. șosete negre: 2

$$\left.\begin{array}{l} \text{șos. roșii} = x \\ \text{șos. negre} = n \end{array}\right\} \Rightarrow 2x(x-1) = (x+n)(x+n-1) \Leftrightarrow$$

$$\Leftrightarrow 2x^2 - 2x = x^2 + n^2 + 2nx - x - n \Leftrightarrow$$

$$\Leftrightarrow \frac{x^2}{2n} + x\left(\frac{1}{2n} + 1\right) - \frac{n-1}{2} = 0 \text{ ; rezolvăm pt } x:$$

fie $n=2 \Rightarrow \Delta = \frac{1}{4} + 1 - \sqrt{\Delta}$ / $\frac{1}{2}$; $\frac{1}{4} + 1 + \sqrt{\Delta}$ / $\frac{1}{4}$

fie $n=4 \Rightarrow \ldots$

(Desmos)

fie $n=6 \Rightarrow x=15$; $t=21 \Rightarrow n=6$ minim

(13) Nr. format cu cele 5 cartonase este $\overline{abcde}$:

$X = 10.000a + 1.000b + 100c + 10d + e$

$\Rightarrow$ ~~cazuri favorabile~~: $A_{10}^{5} = 30.240$

cazuri posibile:

$X : 495 \Leftrightarrow \begin{cases} X : 5 \\ X : 9 \\ X : 11 \end{cases}$

$X : 5 \Rightarrow e \in \{0, 5\}$

$X : 9$

$X = (a+b+c+d+e) + (9999a + 999b + 99c + 9d) \Big\} \Rightarrow (a+b+c+d+e) : 9$

$X : 11$

$X = (a-b+c-d+e) + (9999a + 1001b + 99c + 11d) \Big\} \Rightarrow (a-b+c-d+e) : 11$

$a \neq b \neq c \neq d \neq e$

$\Rightarrow$ $10 \leq a+b+c+d+e \leq 35 \Rightarrow a+b+c+d+e \in \{18, 27\}$

CI: $a+b+c+d+e = 18$

$a-b+c-d+e = 18 - 2(b+d) \Rightarrow (a-b+c-d+e) : 2 \Big\} \Rightarrow$

$\Rightarrow |a-b+c-d+e| < 18 \Rightarrow a-b+c-d+e = 0 \Rightarrow a+c+e = b+d = 9$

CI.1: $e = 0 \Rightarrow a+c = b+d = 9 \Rightarrow (a,c) ; (b,d) \in \{(1,8), (2,7), \ldots, (8,1)\}$

$\Rightarrow 8 \cdot 6 = 48$ nr. (*)

CI.2: $e = 5 \Rightarrow a+c = 4 \text{ si } b+d = 9 \Rightarrow (a,c) ; (b,d) \in$

$\in \{(0,4)/(4,0); (1,8)/(2,7)/(3,6)/(6,3)/(7,2)/(8,1); (1,3)/(3,1), (0,9)/(2,7)/(7,2)/(9,0)\}$

$\Rightarrow 2 \cdot 6 + 2 \cdot 4 = 20$ nr. (**)

(*), (**) $\Rightarrow 68$ cf.

CII: $a+b+c+d+e = 27$

$a-b+c-d+e = 27-2(b+d) \Rightarrow a-b+c-d+e \,\vdots\, 2$

$\Rightarrow 27 \geq a-b+c-d+e \geq 27-2(9+8) = -7$

$\Rightarrow a-b+c-d+e \in \{0, 11, 22\} \Rightarrow$

$\Rightarrow a-b+c-d+e = 27-2(b+d) = 11 \Rightarrow \begin{cases} b+d=8 \\ a+c+e=19 \end{cases}$

CII.1: $e=0 \Rightarrow a+c=19$ ✂

$a+c \leq 17$

CII.2: $e=5 \Rightarrow a+c=14 \Rightarrow (a,c), (b,d) \in \{(8,6)/(6,8), (1,7)/(7,1)\}$

$\Rightarrow 2 \cdot 2 = 4$ poss. (***)

(*), (**), (***) $\Rightarrow 68+4 = 72$ cf

$\Rightarrow p = \frac{cf}{cp} = \frac{72}{30.240} = 0,23\%$

(15) $P(A \cup B) = 0,5 \Rightarrow P(A)+P(B)-P(A)P(B|A) = 0,5$

$P(A \cup B) = 0,5 \Rightarrow P(A)+P(B)-P(A \cap B) = 0,5$

$P(A \cup \bar{B}) = 0,1 \Rightarrow P(A)+1-P(B)-P(A \cap \bar{B}) = 0,1$

$P(A)+1-P(B)-P(A)P(\bar{B}|A) = 0,1$

$\Rightarrow 1+2P(A)-P(A)\underbrace{(P(B|A)+P(\bar{B}|A))}_{=1} = 0,6 \Rightarrow 2P(A) = 0,6$

$\Rightarrow P(A) = 0,3$

(14) a) $A = \frac{3}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{5}{7} \cdot C_4^1 = 0,5 = 50\%$

b) $B = \frac{3}{10} \cdot \frac{7}{9} \cdot \frac{6}{8} \cdot \frac{5}{7} \cdot C_4^1 + \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{7}{8} \cdot \frac{6}{7} \cdot C_4^2 + \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} \cdot \frac{7}{7} \cdot C_4^3 = 83\%$

c) $C = \frac{3}{10} = 30\%$

d) $D = \frac{3}{10} \cdot \frac{2}{9} + \frac{7}{10} \cdot \frac{3}{9} = 30\%$

e) $E = \frac{3}{10} \cdot \frac{2}{9} = 6,66\%$

f) $F = \frac{7}{10} \cdot \frac{3}{9} + \frac{3}{10} \cdot \frac{7}{9} + \frac{3}{10} \cdot \frac{2}{9} = 53,33\%$

g) $G = \frac{C_{10}^4}{A_{10}^4} = 4,16\%$