SEMINAR 1

- Unificare -

Consideram:

• variabile: x, y, z, u, v

• constante: a, b, c

• functii:

• aritate **1**: $h_{i} g_{i} ()^{-1}$

• aritate **2**: f, *, +

• aritate **3**: *p*

Aplicati algoritmul de unificare de mai sus pentru a gasi un unificator:

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1. p(a, x, h(g(y))) & p(z, h(z), h(u))
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2.
$$f(h(a), g(x))$$
 & $f(y, y)$

3.
$$p(a, x, g(x))$$
 & $p(a, y, y)$

4.
$$p(x, y, z)$$
 & $p(u, f(v, v), u)$

5.
$$f(x, f(x, x))$$
 & $f(g(y), f(z, g(a)))$

6.
$$x + (y * y) & (y * y) + z$$

7.
$$(x * y) * z & u * u^{-1}$$

8.
$$x * y & u * u^{-1}$$

9.
$$x * y & x * (y * (u * v)^{-1})$$

10.
$$x * y & y * (u * v)^{-1}$$

11.
$$f(g(x), x) & f(y, y)$$

12.
$$p(x, z, z)$$
 & $p(y, y, b)$

13.
$$p(a, u, h(x))$$
 & $p(y, f(y, z), z)$

14.
$$f(x, f(b, x))$$
 & $f(f(y, a), f(b, f(z, z)))$

15.
$$p(x, b, x)$$
 & $p(y, y, c)$

16.
$$f(x,y)$$
 & $f(h(x),x)$ & $f(x,b)$

17.
$$f(x, f(x, g(y)))$$
 & $f(u, z)$ & $f(g(y), y)$

18.
$$f(f(x,y),x)$$
 & $f(g(y),z)$ & $f(u,h(z))$

19.
$$f(f(x,y),x)$$
 & $f(v,u)$ & $f(u,h(z))$

20.
$$f(f(x,y),x) \& f(v,u) \& f(u,z)$$

21.
$$f(f(g(x), h(y)), h(z))$$
 & $f(f(u, h(h(x))), h(y))$ & $f(v, w)$

22.
$$p(x, x, z)$$
 & $p(f(a, a), y, y)$ & $p(f(x, a), b, z)$

23.
$$p(x, x, z)$$
 & $p(f(a, a), y, y)$ & $p(x, b, z)$

24.
$$p(x, x, z)$$
 & $p(f(a, a), y, y)$ & $p(x, f(a, a), z)$

25.
$$p(f(x, a), g(y), z)$$
 & $p(f(a, a), z, u)$ & $p(v, u, z)$

Rezolvari:

1) p(a, x, h(g(y))) & p(z, h(z), h(u))

S	R	Op
Ø	p(a,x,h(g(y))) = p(z,h(z),h(u))	D
Ø	$egin{aligned} a &= z \ x &= h(z) \ h(g(y)) &= h(u) \end{aligned}$	R
z=a	$x=h(a) \ h(g(y))=h(u)$	R
$egin{aligned} z = a \ x = h(a) \end{aligned}$	h(g(y))=h(u)	D
$egin{aligned} z = a \ x = h(a) \end{aligned}$	g(y)=u	R

S	R	Op
$egin{aligned} z &= a \ x &= h(a) \ u &= g(y) \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ x \leftarrow h(a) \text{, } z \leftarrow a \text{, } u \leftarrow g(y)\ \}$$

2) f(h(a), g(x)) & f(y, y)

S	R	Op
Ø	f(h(a),g(x))=f(y,y)	D
Ø	$egin{aligned} h(a) &= y \ g(x) &= y \end{aligned}$	R
y=h(a)	g(x)=h(a)	END

$$\Rightarrow$$
 cgu = INEXISTENT ($h \neq g$)

3) p(a, x, g(x)) & p(a, y, y)

S	R	Op
Ø	p(a,x,g(x))=p(a,y,y)	D
Ø	$egin{aligned} a &= a \ x &= y \ g(x) &= y \end{aligned}$	S

S	R	Op
Ø	$egin{aligned} x &= y \ g(x) &= y \end{aligned}$	R
y=g(x)	x=g(x)	END

 \Rightarrow cgu = INEXISTENT (x=g(x))

4) p(x, y, z) & p(u, f(v, v), u)

S	R	Ор
Ø	p(x,y,z)=p(u,f(v,v),u)	D
Ø	$egin{aligned} x &= u \ y &= f(v,v) \ z &= u \end{aligned}$	R
y=f(v,v)	$egin{aligned} x &= u \ z &= u \end{aligned}$	R
$y=f(v,v) \ x=u$	z=u	R
$y=f(v,v) \ x=u \ z=u$	Ø	END

 $\Rightarrow \mathbf{cgu} = \{\ x \leftarrow u \text{, } y \leftarrow f(v,v) \text{, } z \leftarrow u\ \}$

5) f(x, f(x, x)) & f(g(y), f(z, g(a)))

S	R	Op
Ø	f(x,f(x,x))=f(g(y),f(z,g(a)))	D
Ø	$x=g(y) \ f(x,x)=f(z,g(a))$	R
x=g(y)	f(g(y),g(y))=f(z,g(a))	D
x=g(y)	$egin{aligned} g(y) &= z \ g(y) &= g(a) \end{aligned}$	R
$egin{aligned} x &= g(y) \ z &= g(y) \end{aligned}$	g(y)=g(a)	D
$egin{aligned} x &= g(y) \ z &= g(y) \end{aligned}$	y=a	R
$egin{aligned} x &= g(a) \ z &= g(a) \ y &= a \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ x \leftarrow g(a) \text{, } y \leftarrow a \text{, } z \leftarrow g(a)\ \}$$

6) x + (y * y) & (y * y) + z

S	R	Op
Ø	x+(y*y)=(y*y)+z	D
Ø	$egin{aligned} x &= y * y \ y * y &= z \end{aligned}$	R
x = y * y	y*y=z	R
$egin{aligned} x &= y * y \ z &= y * y \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ x \leftarrow y * y \text{, } z \leftarrow y * y\ \}$$

7)
$$(x * y) * z & u * u^{-1}$$

S	R	Op
Ø	$(xst y)st z=ust u^{-1}$	D
Ø	$egin{aligned} x*y &= u \ z &= u^{-1} \end{aligned}$	R
$z=u^{-1}$	x*y=u	R
$u = x * y$ $z = (x * y)^{-1}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ u \leftarrow x * y \text{, } z \leftarrow (x * y)^{-1}\ \}$$

8)
$$x * y & u * u^{-1}$$

S	R	Op
Ø	$x*y=u*u^{-1}$	D
Ø	$egin{aligned} x &= u \ y &= u^{-1} \end{aligned}$	R
x=u	$y=u^{-1}$	R
$egin{aligned} x &= u \ y &= u^{-1} \end{aligned}$	Ø	R

$$\Rightarrow \mathbf{cgu} = \{\ x \leftarrow u \text{, } y \leftarrow u^{-1}\ \}$$

9)
$$x * y & x * (y * (u * v)^{-1})$$

S	R	Op
Ø	$x*y = x*(y*(u*v)^{-1})$	D
Ø	$egin{aligned} x &= x \ y &= y*(u*v)^{-1} \end{aligned}$	S
Ø	$y = y \ast (u \ast v)^{-1}$	END

$$\Rightarrow$$
 cgu = INEXISTENT $(y = y * (u * v)^{-1})$

10)
$$x * y$$
\$ $y * (u * v)^{-1}$

S	R	Ор
Ø	$x*y=y*(u*v)^{-1}$	D
Ø	$egin{aligned} x &= y \ y &= (u * v)^{-1} \end{aligned}$	R
x = y	$y=(ust v)^{-1}$	R
$x=(ust v)^{-1} \ y=(ust v)^{-1}$	Ø	R

$$\Rightarrow \mathbf{cgu} = \{\ x \leftarrow (u*v)^{-1} \text{, } y \leftarrow (u*v)^{-1}\ \}$$

11) f(g(x), x) & f(y, y)

S	R	Op
Ø	f(g(x),x)=f(y,y)	D
Ø	$g(x)=y \ x=y$	R
x = y	g(y)=y	END

 \Rightarrow cgu = INEXISTENT (y = g(y))

12) p(x, z, z) & p(y, y, b)

S	R	Op
Ø	p(x,z,z)=p(y,y,b)	D
Ø	$egin{aligned} x &= y \ z &= y \ z &= b \end{aligned}$	R
x = y	$egin{aligned} z &= y \ z &= b \end{aligned}$	R
$egin{array}{c} x=y \ z=y \end{array}$	y = b	R
$egin{aligned} x &= b \ z &= b \ y &= b \end{aligned}$	Ø	END

13) p(a, u, h(x)) & p(y, f(y, z), z)

S	R	Op
Ø	p(a,u,h(x))=p(y,f(y,z),z)	D
Ø	$egin{aligned} a &= y \ u &= f(y,z) \ h(x) &= z \end{aligned}$	R
y=a	$u=f(a,z) \ h(x)=z$	R
$egin{aligned} y &= a \ z &= h(x) \end{aligned}$	u=f(a,h(x))	R
$egin{aligned} y &= a \ z &= h(x) \ u &= f(a,h(x)) \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ y \leftarrow a \text{, } z \leftarrow h(x) \text{, } u \leftarrow f(a,h(x))\ \}$$

14) f(x, f(b, x)) & f(f(y, a), f(b, f(z, z)))

S	R	Ор
Ø	f(x,f(b,x))=f(f(y,a),f(b,f(z,z)))	D

S	R	Op
Ø	$egin{aligned} x &= f(y,a) \ f(b,x) &= f(b,f(z,z)) \end{aligned}$	R
x=f(y,a)	f(b,f(y,a))=f(b,f(z,z))	D
x=f(y,a)	$b=b \ f(y,a)=f(z,z)$	S
x=f(y,a)	f(y,a)=f(z,z)	D
x=f(y,a)	$egin{aligned} y &= z \ a &= z \end{aligned}$	R
$egin{aligned} x &= f(y,a) \ z &= a \end{aligned}$	y=a	R
$egin{aligned} x &= f(a,a) \ z &= a \ y &= a \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ x \leftarrow f(a,a) \text{, } z \leftarrow a \text{, } y \leftarrow a\ \}$$

15) p(x, b, x) & p(y, y, c)

S	R	Op
Ø	p(x,b,x)=p(y,y,c)	D
Ø	$egin{aligned} x &= y \ b &= y \ x &= c \end{aligned}$	R
x = y	$egin{aligned} b &= y \ y &= c \end{aligned}$	R

S	R	Op
x = c	b=c	END
y = c	0 — 0	LIVE

 \Rightarrow cgu = INEXISTENT (b = c)

16) f(x,y) & f(h(x),x) & f(x,b)

S	R	Op
Ø	$f(x,y) = f(h(x),x) \ f(h(x),x) = f(x,b)$	D
Ø	$egin{aligned} x &= h(x) \ y &= x \ h(x) &= x \ x &= b \end{aligned}$	R
Ø	$egin{aligned} x &= h(x) \ y &= x \ h(x) &= x \ x &= b \end{aligned}$	END

 \Rightarrow cgu = INEXISTENT (x = h(x))

17) f(x, f(x, g(y))) & f(u, z) & f(g(y), y)

S	R	Op
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S	R	Op
Ø	$f(x,f(x,g(y)))=f(u,z) \ f(u,z)=f(g(y),y)$	D
Ø	$egin{aligned} x &= u \ f(x,g(y)) &= z \ u &= g(y) \ z &= y \end{aligned}$	R
x = u	$egin{aligned} f(u,g(y)) &= z \ u &= g(y) \ z &= y \end{aligned}$	R
$egin{array}{c} x = u \ z = y \end{array}$	$f(u,g(y))=y \ u=g(y)$	R
$egin{aligned} x &= g(y) \ z &= y \ u &= g(y) \end{aligned}$	f(g(y),g(y))=y	END

 $\Rightarrow \mathbf{cgu} = \mathit{INEXISTENT} \; (f(g(y), g(y)) = y)$

18) f(f(x,y),x) & f(g(y),z) & f(u,h(z))

S	R	Op
Ø	$f(f(x,y),x)=f(g(y),z) \ f(g(y),z)=f(u,h(z))$	D
Ø	$egin{aligned} f(x,y) &= g(y) \ x &= z \ g(y) &= u \ z &= h(z) \end{aligned}$	R

S	R	Op
Ø	$egin{aligned} f(x,y) &= g(y) \ x &= z \ g(y) &= u \ z &= h(z) \end{aligned}$	END

 \Rightarrow cgu = INEXISTENT (z=h(z))

19) f(f(x,y),x) & f(v,u) & f(u,h(z))

S	R	Ор
Ø	$f(f(x,y),x)=f(v,u) \ f(v,u)=f(u,h(z))$	D
Ø	$egin{aligned} f(x,y) &= v \ x &= u \ v &= u \ u &= h(z) \end{aligned}$	R
u=h(z)	$egin{aligned} f(x,y) &= v \ x &= h(z) \ v &= h(z) \end{aligned}$	R
$u=h(z) \ v=h(z)$	$f(x,y) = h(z) \ x = h(z)$	END

 \Rightarrow cgu = INEXISTENT (f(x,y) = h(z))

20) f(f(x,y),x) & f(v,u) & f(u,z)

S	R	Op
Ø	$f(f(x,y),x)=f(v,u) \ f(v,u)=f(u,z)$	D
Ø	$egin{aligned} f(x,y) &= v \ x &= y \ v &= u \ u &= z \end{aligned}$	R
u=z	$egin{aligned} f(x,y) &= v \ x &= y \ v &= z \end{aligned}$	R
$egin{aligned} u &= z \ v &= z \end{aligned}$	$f(x,y)=z \ x=y$	R
$egin{aligned} u &= z \ v &= z \ x &= y \end{aligned}$	f(y,y)=z	R
$egin{aligned} u &= f(y,y) \ v &= f(y,y) \ x &= y \ z &= f(y,y) \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ u \leftarrow f(y,y) \text{, } v \leftarrow f(y,y) \text{, } x \leftarrow y \text{, } z \leftarrow f(y,y)\ \}$$

21) f(f(g(x), h(y)), h(z)) & f(f(u, h(h(x))), h(y)) & f(v, w)

S	R	Op
Ø	$f(f(g(x),h(y)),h(z)) = f(f(u,h(h(x))),h(y)) \ f(f(u,h(h(x))),h(y)) = f(v,w)$	D

S	R	Ор
Ø	$egin{split} f(g(x),h(y))&=f(u,h(h(x)))\ h(z)&=h(y)\ f(u,h(h(x)))&=v\ h(y)&=w \end{split}$	R
w=h(y)	$egin{split} f(g(x),h(y))&=f(u,h(h(x)))\ h(z)&=h(y)\ f(u,h(h(x)))&=v \end{split}$	D
w=h(y)	$egin{aligned} g(x) &= u \ h(y) &= h(h(x)) \ z &= y \ f(u,h(h(x))) &= v \end{aligned}$	R
$egin{aligned} w &= h(y) \ u &= g(x) \end{aligned}$	$egin{aligned} h(y) &= h(h(x)) \ z &= y \ f(g(x), h(h(x))) &= v \end{aligned}$	R
$egin{aligned} w &= h(y) \ u &= g(x) \ z &= y \end{aligned}$	$h(y) = h(h(x)) \ f(g(x), h(h(x))) = v$	D
$egin{aligned} w &= h(y) \ u &= g(x) \ z &= y \end{aligned}$	$y=h(x) \ f(g(x),h(h(x)))=v$	R
$egin{aligned} w &= h(h(x)) \ u &= g(x) \ z &= h(x) \ y &= h(x) \end{aligned}$	f(g(x),h(h(x)))=v	R
$egin{aligned} w &= h(h(x)) \ u &= g(x) \ z &= h(x) \ y &= h(x) \ v &= f(g(x), h(h(x))) \end{aligned}$	Ø	END

 $\Rightarrow \mathbf{cgu} = \{ \ w \leftarrow h(h(x)) \text{, } u \leftarrow g(x) \text{, } z \leftarrow h(x) \text{, } y \leftarrow h(x) \text{, } v \leftarrow f(g(x), h(h(x))) \ \}$

22) p(x, x, z) & p(f(a, a), y, y) & p(f(x, a), b, z)

S	R	Ор
Ø	$p(x,x,z)=p(f(a,a),y,y) \ p(f(a,a),y,y)=p(f(x,a),b,z)$	D
Ø	$egin{aligned} x &= f(a,a) \ x &= y \ z &= y \ f(a,a) &= f(x,a) \ y &= b \ y &= z \end{aligned}$	R
x=f(a,a)	$f(a,a)=y \ z=y \ f(a,a)=f(f(a,a),a) \ y=b \ y=z$	R
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \end{aligned}$	$egin{aligned} z &= f(a,a) \ f(a,a) &= f(f(a,a),a) \ f(a,a) &= b \ f(a,a) &= z \end{aligned}$	R
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \ z &= f(a,a) \end{aligned}$	$f(a,a) = f(f(a,a),a) \ f(a,a) = b \ f(a,a) = f(a,a)$	S
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \ z &= f(a,a) \end{aligned}$	$f(a,a) = f(f(a,a),a) \ f(a,a) = b$	END

 \Rightarrow cgu = INEXISTENT (b=f(a,a))

23) p(x, x, z) & p(f(a, a), y, y) & p(x, b, z)

S	R	Op
Ø	$p(x,x,z) = p(f(a,a),y,y) \ p(f(a,a),y,y) = p(x,b,z)$	D
Ø	$egin{aligned} x &= f(a,a) \ x &= y \ z &= y \ f(a,a) &= x \ y &= b \ y &= z \end{aligned}$	R
x=f(a,a)	$egin{aligned} f(a,a) &= y \ z &= y \ f(a,a) &= f(a,a) \ y &= b \ y &= z \end{aligned}$	S
x=f(a,a)	$egin{aligned} f(a,a) &= y \ z &= y \ y &= b \ y &= z \end{aligned}$	R
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \end{aligned}$	$egin{aligned} z &= f(a,a) \ f(a,a) &= b \ f(a,a) &= z \end{aligned}$	R
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \ z &= f(a,a) \end{aligned}$	$f(a,a) = b \ f(a,a) = f(a,a)$	S
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \ z &= f(a,a) \end{aligned}$	f(a,a)=b	END

24) p(x, x, z) & p(f(a, a), y, y) & p(x, f(a, a), z)

S	R	Ор
Ø	$p(x,x,z)=p(f(a,a),y,y) \ p(f(a,a),y,y)=p(x,f(a,a),z)$	D
Ø	$egin{aligned} x &= f(a,a) \ x &= y \ z &= y \ f(a,a) &= x \ y &= f(a,a) \ y &= z \end{aligned}$	R
x=f(a,a)	$egin{aligned} f(a,a) &= y \ z &= y \ f(a,a) &= f(a,a) \ y &= f(a,a) \ y &= z \end{aligned}$	S
x=f(a,a)	$egin{aligned} f(a,a) &= y \ z &= y \ y &= f(a,a) \ y &= z \end{aligned}$	R
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \end{aligned}$	$egin{aligned} f(a,a) &= f(a,a) \ z &= f(a,a) \ f(a,a) &= z \end{aligned}$	S
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \end{aligned}$	$egin{aligned} z &= f(a,a) \ f(a,a) &= z \end{aligned}$	R

S	R	Op
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \ z &= f(a,a) \end{aligned}$	f(a,a)=f(a,a)	S
$egin{aligned} x &= f(a,a) \ y &= f(a,a) \ z &= f(a,a) \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{ \ x \leftarrow f(a,a) \text{, } y \leftarrow f(a,a) \text{, } z \leftarrow f(a,a) \ \}$$

25) p(f(x,a),g(y),z) & p(f(a,a),z,u) & p(v,u,z)

S	R	Ор
Ø	$p(f(x,a),g(y),z)=p(f(a,a),z,u) \ p(f(a,a),z,u)=p(v,u,z)$	D
Ø	$egin{aligned} f(x,a) &= f(a,a) \ g(y) &= z \ z &= u \ f(a,a) &= v \ z &= u \ u &= z \end{aligned}$	R
v=f(a,a)	$egin{aligned} f(x,a) &= f(a,a) \ g(y) &= z \ z &= u \ z &= u \ u &= z \end{aligned}$	R

S	R	Op
$egin{aligned} v &= f(a,a) \ z &= g(y) \end{aligned}$	$egin{aligned} f(x,a) &= f(a,a) \ g(y) &= u \ g(y) &= u \ u &= g(y) \end{aligned}$	R
$egin{aligned} v &= f(a,a) \ z &= g(y) \ u &= g(y) \end{aligned}$	$egin{aligned} f(x,a) &= f(a,a) \ g(y) &= g(y) \ g(y) &= g(y) \end{aligned}$	S
$egin{aligned} v &= f(a,a) \ z &= g(y) \ u &= g(y) \end{aligned}$	f(x,a)=f(a,a)	D
$egin{aligned} v &= f(a,a) \ z &= g(y) \ u &= g(y) \end{aligned}$	$egin{aligned} x &= a \ a &= a \end{aligned}$	S
$egin{aligned} v &= f(a,a) \ z &= g(y) \ u &= g(y) \end{aligned}$	x=a	R
$egin{aligned} v &= f(a,a) \ z &= g(y) \ u &= g(y) \ x &= a \end{aligned}$	Ø	END

$$\Rightarrow \mathbf{cgu} = \{\ v \leftarrow f(a,a) \text{, } z \leftarrow g(y) \text{, } u \leftarrow g(y) \text{, } x \leftarrow a\ \}$$