FLUXONIUM

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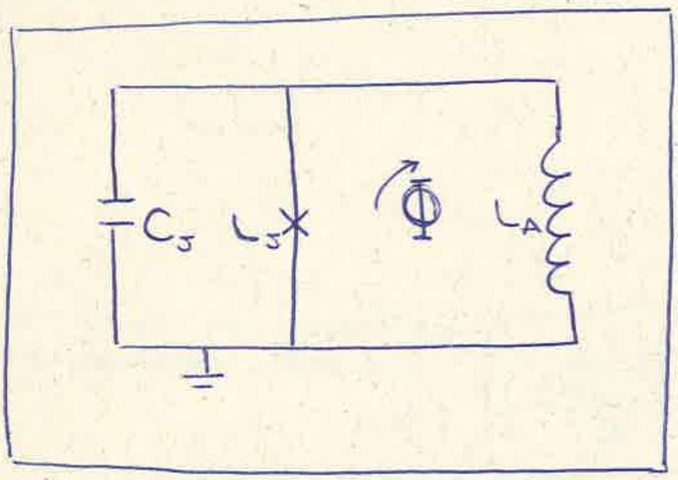
Notas SC - JJ Ripoll

SC23

→ Estudio y cuantización del fluxonium (rf-SQUID)

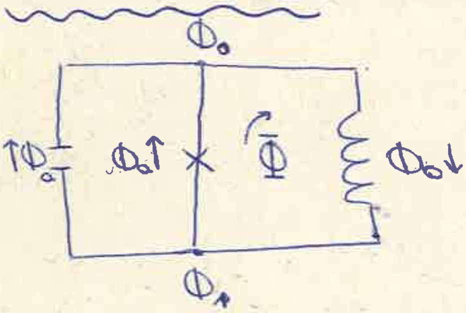


→ Novel single Cooper-pair circuit.
→ Solve inductance and offset charge problems.
→ (inductor) ↔ (chain of junctions) ... in SC23
→ (Conditions to avoid offset charge)
→ Characteristic energies:

$$\begin{cases} E_L = (\Phi_0/2\pi)^2/L_A \\ E_J = (\Phi_0/2\pi)^2/L_J \\ E_C = e^2/2C_J \end{cases}$$

→ Thought to form a BIC when an LC resonator is coupled to measure.

CIRCUITO:



③ $\Phi_b = \Phi_1 - \Phi_0$

④ $\Phi_a + \Phi_b = \bar\Phi \to \Phi_a = \bar\Phi - \Phi_b = \bar\Phi - \Phi_1 + \Phi_0$

⑤ $I_{C,a} = C_J\ddot\Phi_a = C_J(\ddot\Phi_0 - \ddot\Phi_1)$

$I_{J,a} = I_J \sin(\Phi_a/\varphi_0) = I_J \sin\left(\dfrac{\Phi_0 - \Phi_1 + \bar\Phi}{\varphi_0}\right)$

$I_{L,b} = \Phi_b/L_A = \dfrac{\Phi_1 - \Phi_0}{L_A}$

⑥ Nodo 0

$I_{C,a} + I_{J,a} = I_{L,b}$

$$C_J(\ddot\Phi_0 - \ddot\Phi_1) + I_J \sin\left(\frac{\Phi_0 - \Phi_1 + \bar\Phi}{\varphi_0}\right) = \frac{\Phi_1 - \Phi_0}{L_A}$$

↓ trabajo en base variable rama

$$-C_J\ddot\Phi_b + I_J \sin\left(\frac{\bar\Phi - \Phi_b}{\varphi_0}\right) = \frac{\Phi_b}{L_A}$$

⑦ $\Phi_1 = 0 \Rightarrow \Phi_b = -\Phi_0$

↓

$$C_J\ddot\Phi_0 + I_J \sin\left(\frac{\bar\Phi + \Phi_0}{\varphi_0}\right) = -\frac{\Phi_0}{L_A}$$

⑧ $$\frac{d}{dt} C_0\dot\Phi_0 = -\frac{\Phi_0}{L_A} - I_J \sin\left(\frac{\bar\Phi + \Phi_0}{\varphi_0}\right)$$

↓ $$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot\Phi_0} = \frac{\partial\mathcal{L}}{\partial\Phi_0}$$

$$\mathcal{L} = \frac{C_J \dot{\Phi}_0^2}{2} - \frac{\Phi_0^2}{2L_A} + E_J \cos\left(\frac{\bar{\Phi} + \Phi_0}{\varphi_0}\right)$$

⑨ $$Q_0 = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_0} = C_J \dot{\Phi}_0$$

$$H = Q_0 \dot{\Phi}_0 - \mathcal{L}$$

$\downarrow$

$$\boxed{H = \frac{C_J \dot{\Phi}_0^2}{2} + \frac{\Phi_0^2}{2L_A} - E_J \cos\left(\frac{\bar{\Phi} + \Phi_0}{\varphi_0}\right)}$$

$$= \frac{1}{2C_J} Q_0^2 + \frac{\Phi_0^2}{2L_A} - E_J \cos\left(\frac{\bar{\Phi} + \Phi_0}{\varphi_0}\right)$$

→ Tenemos una única variable independiente Φ_0.

REP. NÚMERO (CUANTIZACIÓN):

→ Escribimos H en representación número (forma matricial).

* Rep. número:

$$\hat{Q} = -2e\hat{n} = -2e\sum_n n|n\rangle\langle n|$$

$$e^{-i\hat{\varphi}} = e^{-i\hat{\Phi}/\varphi_0} = \sum_n |n+1\rangle\langle n|$$

$$H = \frac{1}{2C_J} 4e^2 \sum_n n^2 |n\rangle\langle n| + \frac{\hat{\Phi}^2 \cdot \varphi_0^2}{2L_A \varphi_0^2} - \frac{E_J}{2}\left[\sum_n e^{-i\bar{\Phi}'} |n+1\rangle\langle n| + e^{+i\bar{\Phi}'} |n\rangle\langle n+1|\right] =$$

$$= 4E_c \sum_n n^2 |n\rangle\langle n| + \frac{\hat{\varphi}^2 \Phi_0^2}{2L_A (2\pi)^2} - \frac{E_J}{2}\left[\sum_n e^{-i\bar{\Phi}'} |n+1\rangle\langle n| + e^{+i\bar{\Phi}'} |n\rangle\langle n+1|\right] \Rightarrow$$

?

$$H = 4E_c \sum_n n^2 |n\rangle\langle n| + \frac{E_L}{2}\hat{\varphi}^2 - \frac{E_J}{2}\left[\sum_n e^{-i\bar{\Phi}'} |n+1\rangle\langle n| + e^{i\bar{\Phi}'} |n\rangle\langle n+1|\right], \quad \bar{\Phi}' = \frac{\bar{\Phi}}{\varphi_0}$$

? No podemos escribir el operador fase de forma directa en rep. número. Una buena <u>solución</u> es: expandir el potencial $\hat{\varphi}^2$ en base de Fourier. Para ello habría que encontrar el cutoff en φ y el orden de la expansión que no modificaran la física del sistema.

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REP. FASE (CUANTIZACIÓN):

* Rep. fase:
$$\hat{Q} = -i2e\partial_\varphi \qquad \hat{\phi} = \frac{\Phi_0}{2\pi}\varphi$$

$$\boxed{H = -4E_c\partial_\varphi^2 + \frac{E_L}{2}\varphi^2 - E_J\cos(\varphi + \Phi')} \quad \Phi' = \frac{\Phi}{\varphi_0}$$

→ Dada esta ecuación podemos trabajar en la representación fase.

→ Forma matricial: el flujo se escribe como un "grid" de valores $[-\Phi_{max}, \Phi_{max}+\delta, \ldots, \Phi_{max}]$ de forma que se construye una matriz diagonal con la que trabajar → se discretiza el flujo → el operador de carga también queda discretizado.

$$q \to \frac{i\hbar}{2\delta}\begin{pmatrix} 0 & 1 & & \\ -1 & 0 & 1 & \\ & & -1 & 0 \end{pmatrix} \qquad q^2 \to \frac{-\hbar^2}{\delta^2}\begin{pmatrix} -2 & 1 & \\ 1 & -2 & 1 \\ & 1 & -2 \end{pmatrix}$$

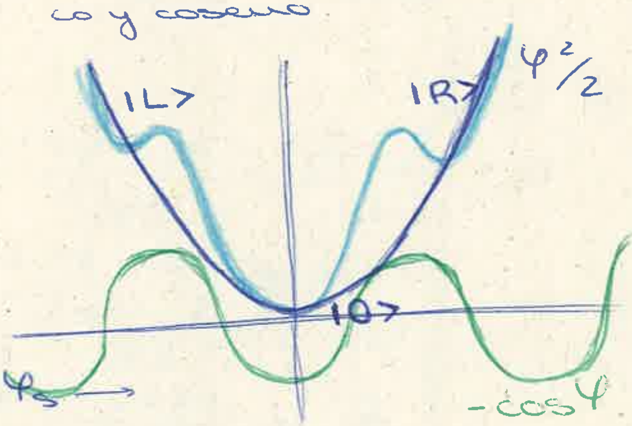
→ Es lo que hace circuitQ.
→ Es útil cuando hay inductores pero no deja de ser una aproximación a lo "bruto".

ESTUDIO DEL POTENCIAL:

→ Estudio del potencial del sistema para encontrar los mínimos

$$V(\varphi) = \frac{E_L}{2}\varphi^2 - E_J\cos(\varphi + \Phi')$$

Superposición del pot. armónico y coseno



* Para $\Phi' = 0$ (fluxonium):

$$\frac{dV}{d\varphi} = 0 \Rightarrow E_L\varphi^* = E_J\sin(\varphi^*)$$

→ Sol trivial: $\varphi^* = 0 \to |0\rangle$ state

→ Dos soluciones simétricas: $\varphi^* = \pm\varphi_s$ → $|L\rangle, |R\rangle$ se encuentran numéricamente

$$\frac{d^2V}{d\varphi^2} = E_L + E_J\cos(\varphi)$$

$$\left.\frac{d^2V}{d\varphi^2}\right|_0 = E_L + E_J > 0 \to \text{mínimo global}$$

$$\left.\frac{d^2V}{d\varphi^2}\right|_{\pm\varphi_s} = E_L + E_J\cos(\varphi_s)$$

→ $E_L > E_J \Rightarrow \left.\frac{d^2V}{d\varphi^2}\right|_{\pm s} > 0 \to$ mínimo → $L_A < L_J$ → no cumple las condiciones de fluxonium.

→ $E_J < E_L \Rightarrow$ no asegura nada pero podemos asumir que los mínimos surgen en la parte negativa de $(-\cos\varphi)$ por lo que $\cos\varphi_s > 0 \Rightarrow \left.\frac{d^2V}{d\varphi^2}\right|_{\pm\varphi_s} > 0$ → mínimo local.

* Para $\bar{\Phi}' \neq 0$ los mínimos se desplazan y cambia la forma del potencial:

$\Phi_{ext} = \pi$:

$\varphi^* = 0 \Rightarrow \frac{d^2 V}{d\varphi^2} < 0 \longrightarrow$ máximo global → qubit

$(\bar{\Phi}' = \Phi_{ext})$

FLUXONIUM AS RESONATOR:

→ Cuando los niveles de baja energía están profundos en el potencial coseno, podemos aproximar los autoestados con las autofunciones de un oscilador ligeramente armónico.

$\boxed{\bar{\Phi}' \simeq 0}$

$$H = \frac{1}{2C_J} Q^2 + \frac{\Phi^2}{2L_A} - E_J \cos\left(\frac{\Phi}{\varphi_0} + \cancel{\bar{\Phi}'}^{\,0}\right)$$

cos Taylor expansion:

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

$$H \simeq \frac{1}{2C_J} \hat{Q}^2 + \frac{\hat{\Phi}^2}{2L_A} - E_J + \frac{E_J}{2\varphi_0^2} \hat{\Phi}^2 - \frac{E_J}{24\varphi_0^4} \hat{\Phi}^4 + O(\Phi^6)$$

$$H \simeq \frac{1}{2C_J} \hat{Q}^2 + \frac{(E_L + E_J)}{2\varphi_0^2} \hat{\Phi}^2 - E_J - \frac{E_J}{24\varphi_0^4} \hat{\Phi}^4 + \dots$$

Fock operators for diagonalization:

$$\hat{Q} = 2e \left(\frac{E_L + E_J}{8E_C}\right)^{1/4} \frac{i}{\sqrt{2}} (\hat{a}^+ - \hat{a})$$

$$\hat{\Phi} = \varphi_0 \left(\frac{8E_C}{E_L + E_J}\right)^{1/4} \frac{1}{\sqrt{2}} (\hat{a}^+ + \hat{a})$$

$$H = \frac{(2e)^2}{2C_J} \left(\frac{E_L + E_J}{8E_C}\right)^{1/2} \frac{1}{2} (\hat{a}^+ - \hat{a})(\hat{a} - \hat{a}^+) + \frac{(E_L + E_J)}{2\varphi_0^2} \varphi_0^2 \cdot$$

$$\left(\frac{8E_C}{E_L + E_J}\right)^{1/2} \frac{1}{2} (\hat{a}^+ + \hat{a})(\hat{a}^+ + \hat{a}) - \frac{E_J}{24\varphi_0^4} \varphi_0^4 \left(\frac{8E_C}{E_L + E_J}\right) \frac{1}{4}$$

$$(\hat{a}^+ + \hat{a})^4 + \dots \simeq$$

$$\simeq 2E_C \sqrt{\frac{E_L + E_J}{8E_C}} \left[2\hat{a}^+\hat{a} - \hat{a}^+\hat{a}^+ - \hat{a}\hat{a}\right] + \frac{1}{4} \sqrt{8E_C(E_L + E_J)} \cdot$$

$$\left[2\hat{a}^+\hat{a} + \hat{a}^+\hat{a}^+ + \hat{a}\hat{a}\right] - \frac{E_J \cdot E_C}{12(E_J + E_L)} \hat{a}^+\hat{a}^+\hat{a}\hat{a} + \dots =$$

$$= \sqrt{8E_C(E_L + E_J)} \left[\hat{a}^+\hat{a}\right] - \frac{E_J E_C}{(E_J + E_L)} \hat{a}^+\hat{a}^+\hat{a}\hat{a} + \dots =$$

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$= \hbar\omega_{01}\hat{a}^{+}\hat{a} - \hbar\alpha\hat{a}^{+}\hat{a}^{+}\hat{a}\hat{a} + \dots$

Osc. frec: $\boxed{\hbar\omega_{01} = \sqrt{8E_C(E_L+E_J)}}$ equal to the equivalent resonator frequency

$$\omega_{12} = \omega_{01} - \alpha \longrightarrow \hbar\alpha = \hbar(\omega_{01}-\omega_{12}) = \frac{E_J E_C}{(E_J+E_L)}$$

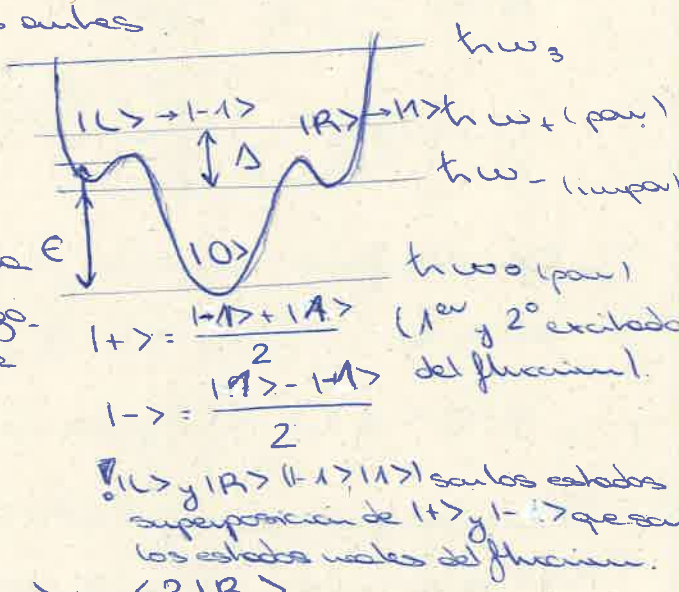
Relative anharmonicity:

$$\alpha_r = \frac{\alpha}{\omega_{01}} = \frac{E_J E_C}{(E_J+E_L)\sqrt{8E_C(E_L+E_J)}} = \boxed{\frac{E_J}{(E_J+E_L)}\sqrt{\frac{E_C}{8(E_J+E_L)}} = \alpha_r}$$

$\alpha_r <<< 1 \longrightarrow$ small anharmonicity to fulfil the approximation

LOW-ENERGY HAMILTONIAN (Qutrit Hamiltonian)

→ Trabajando cerca de $\boxed{\Phi' \simeq 0}$ $\left(\Phi' = \frac{\Phi_{ext}}{\varphi_0}\right)$ y considerando una inductancia lineal lo suficientemente grande, obtenemos un potencial con dos mínimos locales y uno global, como el que veíamos antes.



→ Intuitivamente uno pensaría que $|L\rangle$, $|R\rangle$, $|0\rangle$ son entonces la base del qutrit sobre la que expandir el Hamiltoniano efectivo. Sin embargo, uno se encuentra con que estos estados no son del todo ortogonales a los estados superiores por lo que se hace necesario ortogonalizarlos usando Gram-Schmidt.

$$|+\rangle = \frac{|-1\rangle + |1\rangle}{2} \qquad |-\rangle = \frac{|1\rangle - |-1\rangle}{2}$$

(1^{er} y 2^{o} excitado del fluxonium).

! $|L\rangle$ y $|R\rangle$ ($|-1\rangle$, $|1\rangle$) son los estados superposición de $|+\rangle$ y $|-\rangle$ que son los estados reales del fluxonium.

Los nuevos estados son entonces:

$$\boxed{\begin{aligned} |-1\rangle &= \frac{|L\rangle - a|3\rangle}{\sqrt{1-a^2}} \\ |0\rangle &= |0\rangle \\ |+1\rangle &= \frac{|R\rangle + a|3\rangle}{\sqrt{1-a^2}} \end{aligned}}$$

con $a = \langle 3|L\rangle = -\langle 3|R\rangle$

! We do not need to orthogonalize $|0\rangle$ due to its different parity respect to $|3\rangle$.

→ Otros términos podrían tenerse en cuenta pero disminuyen rápido con la energía creciente debido a las rápidas oscilaciones.

Proyectando el Hamiltoniano en este subespacio se tiene:

(El desarrollo completo está al final)

$$\boxed{H \approx \underbrace{\epsilon S_z^2}_{} + \underbrace{\frac{\Delta}{2}(S_+^2 + S_-^2)}_{} + \overset{\text{aproximación } \approx 0 \; (1^{st} \text{ order})}{\Phi_{ext}}\, I_0 \sin\left(\tilde{\Phi}_* S_z + b\,(S_z S_z S_x)\right)}$$

da las energías del sistema

conecta los estados $|-1\rangle, |+1\rangle$

$P_0 H(\Phi_{ext}=0) P_0$

operador de flujo (lo desarrollamos a continuación)

(normalmente $\sin(\Phi) \to \Phi$) (1^{st} order)

LOW-ENERGY OPERATORS:

→ Para poder hacer un estudio del sistema y escribir su Hamiltoniano efectivo es interesante e imprescindible conocer la forma de sus operadores de flujo y carga en la base del qutrit.

Operador de flujo ($\hat{\varphi}$):

Necesitamos saber cómo actúa $\hat{\varphi}$ sobre $|L\rangle, |0\rangle, |R\rangle$. Para ello asumimos que son gaussianas centradas en $\mu = -\Phi_*, 0, \Phi_*$ con varianza $\sigma = \sqrt{\frac{\hbar}{m \cdot \omega}}$

$$|\psi\rangle = \frac{1}{\sqrt{A}} e^{\frac{-(\varphi-\mu)^2}{2\sigma^2}} \qquad \langle\psi|\psi\rangle = 1$$

$$\langle\psi|\psi\rangle = \frac{1}{A}\int_{-\infty}^{\infty} e^{\frac{-(\varphi-\mu)}{\sigma^2}} d\varphi = \frac{1}{A}\cdot\sqrt{\pi}\cdot\sigma = 1 \to A = \sqrt{\pi}\,\sigma$$

De esta forma cuando $\mu >>> \sigma$ se cumple que $\varphi|\psi\rangle = \mu|\psi\rangle$, por lo que: $\varphi|L\rangle = -\Phi_*|L\rangle$ y $\varphi|R\rangle = \Phi_*|R\rangle$ pero $\varphi|0\rangle \neq 0\,|0\rangle$ pues no cumple la aproximación. ~~(pues Φ ...)~~

Ahora suponiendo que $\langle R|L\rangle = \langle R|0\rangle = \langle L|0\rangle \approx 0$ (son ortogonales) ya podemos obtener el operador de flujo:

$$\hat{\varphi} = \sum_{i,j} \langle i|\hat{\varphi}|j\rangle |i\rangle\langle j| \quad \text{con} \quad |i\rangle, |j\rangle = |-1\rangle, |0\rangle, |+1\rangle$$

$$\langle -1|\hat{\varphi}|0\rangle = \left(\frac{\langle L| - a\langle 3|}{\sqrt{1-a^2}}\right)\hat{\varphi}|0\rangle = \frac{1}{\sqrt{1-a^2}}\left[\langle L|\hat{\varphi}|0\rangle - a\langle 3|\hat{\varphi}|0\rangle\right] = \frac{1}{\sqrt{1-a^2}}\left[-\Phi_* \cancelto{0}{\langle L|0\rangle} - a\langle 3|\hat{\varphi}|0\rangle\right] =$$

$$= -\frac{a}{\sqrt{1-a^2}}\langle 3|\hat{\varphi}|0\rangle = \alpha$$

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$$\langle -1|\hat{\varphi}|-1\rangle = \left(\frac{\langle L| - a\langle 3|}{\sqrt{1-a^2}}\right)\hat{\varphi}\left(\frac{|L\rangle - a|3\rangle}{\sqrt{1-a^2}}\right) =$$

$$= \frac{1}{1-a^2}\Big[\langle L|\hat{\varphi}|L\rangle - a\langle 3|\hat{\varphi}|L\rangle - a\langle L|\hat{\varphi}|3\rangle +$$

$$+ a^2 \overset{0\ (\text{paridad})}{\cancel{\langle 3|\hat{\varphi}|3\rangle}}\Big] = \frac{\Phi_*}{1-a^2}\Big[-\langle L|L\rangle + a\langle 3|L\rangle -$$

$$+ a\langle L|3\rangle = \frac{2a^2-1}{1-a^2}\,\Phi_* = \beta$$

$$\langle -1|\hat{\varphi}|+1\rangle = \left(\frac{\langle L| - a\langle 3|}{\sqrt{1-a^2}}\right)\hat{\varphi}\left(\frac{|R\rangle + a|3\rangle}{\sqrt{1-a^2}}\right) =$$

$$= \frac{1}{1-a^2}\Big[\langle L|\hat{\varphi}|R\rangle + a\langle L|\hat{\varphi}|3\rangle - a\langle 3|\hat{\varphi}|R\rangle -$$

$$- a^2 \overset{0}{\cancel{\langle 3|\hat{\varphi}|3\rangle}}\Big] = \frac{\Phi_*}{1-a^2}\Big[-\overset{0}{\cancel{\langle L|R\rangle}} - a\overbrace{\langle L|3\rangle}^{a} -$$

$$- a\overbrace{\langle 3|R\rangle}^{-a}\Big] = 0$$

$$\langle 0|\hat{\varphi}|0\rangle = 0 \overset{(\text{por paridad})}{=} \int_{-\infty}^{\infty} \varphi\, e^{-\frac{(\varphi)^2}{2\sigma^2}} e^{-\frac{(\varphi^2)}{2\sigma^2}} \frac{1}{\sqrt{\pi}\,\sigma} = 0$$

$$\langle 0|\hat{\varphi}|-1\rangle = \langle 0|\hat{\varphi}\left(\frac{|L\rangle - a|3\rangle}{\sqrt{1-a^2}}\right) = \frac{-a}{\sqrt{1-a^2}}\langle 0|\hat{\varphi}|3\rangle = \alpha$$

$$\langle 0|\hat{\varphi}|+1\rangle = \langle 0|\hat{\varphi}\left(\frac{|R\rangle + a|3\rangle}{\sqrt{1-a^2}}\right) = \frac{1}{\sqrt{1-a^2}}\Big[\Phi_* \overset{0}{\cancel{\langle 0|R\rangle}} + a$$

$$\langle 0|\hat{\varphi}|3\rangle\Big] = \frac{a}{\sqrt{1-a^2}}\langle 0|\hat{\varphi}|3\rangle = -\alpha$$

$$\langle +1|\hat{\varphi}|0\rangle = \left(\frac{\langle R| + a\langle 3|}{\sqrt{1-a^2}}\right)\hat{\varphi}\,|0\rangle = \frac{a}{\sqrt{1-a^2}}\langle 3|\hat{\varphi}|0\rangle = -\alpha$$

$$\langle +1|\hat{\varphi}|-1\rangle = \left(\frac{\langle R| + a\langle 3|}{\sqrt{1-a^2}}\right)\hat{\varphi}\left(\frac{|L\rangle - a|3\rangle}{\sqrt{1-a^2}}\right) = \frac{a\,\Phi_*}{\sqrt{1-a^2}}$$

$$\Big[-a\overbrace{\langle 3|L\rangle}^{a} - a\overbrace{\langle R|3\rangle}^{-a}\Big] = 0$$

$$\langle +1|\hat{\varphi}|+1\rangle = \left(\frac{\langle R| + a\langle 3|}{\sqrt{1-a^2}}\right)\hat{\varphi}\left(\frac{|R\rangle + a|3\rangle}{\sqrt{1-a^2}}\right) =$$

$$= \frac{1}{1-a^2}\left[\langle R|\hat{\varphi}|R\rangle + a\langle 3|\hat{\varphi}|R\rangle + a\langle R|\hat{\varphi}|3\rangle + a^2\cancel{\langle 3|\hat{\varphi}|3\rangle}\right.$$

$$= \frac{\bar{\Phi}_*}{1-a^2}\left[1 - a^2 - a^2\right] = \frac{1-2a^2}{1-a^2}\,\bar{\Phi}_* = -\beta$$

$$\hat{\varphi} = \begin{pmatrix} -\beta & -\alpha & 0 \\ -\alpha & 0 & \alpha \\ 0 & \alpha & \beta \end{pmatrix} = -\beta S_z - \alpha\left[S_x S_z + S_z S_x\right]$$

$$-\beta = \tilde{\bar{\Phi}}_* = \frac{1-2a^2}{1-a^2}\bar{\Phi}_*$$

$$-\alpha = \frac{b}{\sqrt{2}} = \frac{a}{\sqrt{1-a^2}}\langle 3|\varphi|0\rangle \longrightarrow b = \frac{a\sqrt{2}}{\sqrt{1-a^2}}\langle 3|\varphi|0\rangle$$

$$\boxed{\hat{\varphi} = \tilde{\bar{\Phi}}_* S_z + b\left(S_x S_z + S_z S_x\right)}$$

Operador de carga $(\hat{Q})$:

→ Una vez conocemos el operador de flujo, el operador de carga es fácil de obtener utilizando la ecuación de Heisenberg:

$$\hat{Q} = \frac{iC}{\hbar}\left[H, \hat{\varphi}\right]$$

Calculamos primero los conmutadores:

$$[\;\ldots\;\tilde{\bar{\Phi}}_* S_z + b(S_x S_z + S_z S_x),\ \tilde{\bar{\Phi}}_* S_z + b(S_x S_z + S_z S_x)] = 0$$

$$[\epsilon S_z^2, \tilde{\bar{\Phi}}_* S_z] = 0$$

$$[\epsilon S_z^2, b S_x S_z] = \epsilon b[S_z^2, S_x S_z] = \epsilon b[S_z^2, S_x]S_z =$$

$$= \epsilon b\left\{S_z[S_z, S_x] + [S_z, S_x]S_z\right\}S_z =$$

$$= \epsilon b\left\{i S_z S_y S_z + i S_y S_z^2\right\} = \left(i\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} + i\begin{pmatrix}0&0&0\\i&0&-i\\0&0&0\end{pmatrix}\right)\epsilon b =$$

$$= \begin{pmatrix}0&0&0\\-1&0&1\\0&0&0\end{pmatrix}\epsilon b\cdot\frac{1}{\sqrt{2}}$$

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$$[\epsilon S_z^2, bS_zS_x] = \epsilon b S_z [S_z^2, S_x] = \epsilon b S_z \{ iS_zS_y + i\cancel{S_yS_z}^{0} \} =$$

$$= i\epsilon b S_z^2 S_y = i\epsilon b \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ 0 & 0 & 0 \\ 0 & i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \epsilon b \cdot \frac{1}{\sqrt{2}}$$

$$[\tfrac{\Delta}{2} S_+^2, \tilde{\bar{\Phi}}_a S_z] = \frac{\Delta}{2} \tilde{\bar{\Phi}}_a \{ S_+[S_+,S_z] + [S_+,S_z]S_+ \} =$$

$$= -\cancel{2}\Delta \tilde{\bar{\Phi}}_a S_+^2$$

$$\downarrow$$

$$[S_+,S_z] = [S_x,S_z] + i[S_y,S_z] = -iS_y - S_x = -S_+$$

$$[\tfrac{\Delta}{2} S_-^2, \tilde{\bar{\Phi}}_a S_z] = \frac{\Delta}{\cancel{2}} \tilde{\bar{\Phi}}_a \{ S_-[S_-,S_z] + [S_-,S_z]S_- \} =$$

$$= 2\cancel{\Delta}\bar{\Phi}_a S_-^2$$

$$\downarrow$$

$$[S_-,S_z] = [S_x,S_z] - i[S_y,S_z] = -iS_y + S_x = S_-$$

$$[\tfrac{\Delta}{2} S_+^2, bS_xS_z] = \frac{\Delta}{2} b \{ S_x \underbrace{[S_+^2,S_z]}_{-2S_+^2} + S_+[S_+,S_x]S_z + [S_+,S_x]S_+S_z \}$$

$$= \frac{\Delta}{2} b \{ -2S_xS_+^2 + S_+S_z^2 + \cancel{S_zS_+S_z}^{0} \} =$$

$$\downarrow$$

$$[S_+,S_x] = [iS_y,S_x] = S_z$$

$$= \frac{\Delta}{2} b \left\{ -2\sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \right\} = -\frac{\sqrt{2}}{2} \Delta b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\tfrac{\Delta}{2} S_+^2, bS_zS_x] = \frac{\Delta}{2} b \{ -2S_+^2S_x + \cancel{S_zS_+S_z}^{0} + S_z^2 S_+ \} =$$

$$= \frac{\Delta}{2} b \left\{ -2\sqrt{2} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \sqrt{2} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} = -\frac{\sqrt{2}}{2} \Delta b \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\tfrac{\Delta}{2} S_-^2, bS_xS_z] = \frac{\Delta}{2} b \{ S_x \underbrace{[S_-^2,S_z]}_{2S_-^2} + S_-[S_-,S_x]S_z + [S_-,S_x]S_-S_z \}$$

$$= \frac{\Delta}{2} b \{ 2S_xS_-^2 - S_-S_z^2 - \cancel{S_zS_-S_z}^{0} \} =$$

$$\downarrow$$

$$[S_-,S_x] = [-iS_y,S_x] = -S_z$$

$$= \frac{\Delta}{2} b \left\{ 2\sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} = \frac{\sqrt{2}}{2} \Delta b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\tfrac{\Delta}{2} S_-^2, bS_zS_x] = \frac{\Delta}{2} b \{ 2S_-^2S_x - S_z^2S_- - \cancel{S_zS_-S_z}^{0} \} =$$

$$= \frac{\Delta}{2} b \left\{ 2\sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} - \sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \right\} = \sqrt{2} \frac{\Delta}{2} b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Then:

$$\hat{Q} = \begin{pmatrix} 0 & \frac{\epsilon b}{\sqrt{2}} - \frac{\sqrt{2}}{2}\Delta b & -2\Delta\tilde{\bar{\Phi}}_* \\ -\frac{\epsilon b}{\sqrt{2}} + \sqrt{2}\Delta b & 0 & \frac{\epsilon b}{\sqrt{2}} - \sqrt{2}\Delta b \\ 2\Delta\tilde{\bar{\Phi}}_* & -\frac{\epsilon b}{\sqrt{2}} + \frac{\sqrt{2}}{2}\Delta b & 0 \end{pmatrix} = \frac{iC}{\hbar} =$$

$$\boxed{\hat{Q} = \frac{iC}{\hbar}\left\{ i(\epsilon b - \Delta b) S_y + \Delta\tilde{\bar{\Phi}}_* (S_-^2 - S_+^2) \right\}}$$

O de igual forma:

$$\hat{H} = \begin{pmatrix} \epsilon & 0 & -\Delta \\ 0 & 0 & 0 \\ \Delta & 0 & \epsilon \end{pmatrix} \qquad \hat{\varphi} = \begin{pmatrix} \tilde{\bar{\Phi}}_* & \frac{b}{\sqrt{2}} & 0 \\ b/\sqrt{2} & 0 & -b/\sqrt{2} \\ 0 & -b/\sqrt{2} & -\tilde{\bar{\Phi}}_* \end{pmatrix}$$

$$[\hat{H}, \hat{\varphi}] = \hat{H}\hat{\varphi} - \hat{\varphi}\hat{H} =$$

$$= \begin{pmatrix} \epsilon\tilde{\bar{\Phi}}_* & \frac{\epsilon b}{\sqrt{2}} - \frac{\Delta b}{\sqrt{2}} & -\Delta\tilde{\bar{\Phi}}_* \\ 0 & 0 & 0 \\ \Delta\tilde{\bar{\Phi}}_* & \frac{\Delta b}{\sqrt{2}} - \frac{\epsilon b}{\sqrt{2}} & -\epsilon\tilde{\bar{\Phi}}_* \end{pmatrix} - \begin{pmatrix} \epsilon\tilde{\bar{\Phi}}_* & 0 & \Delta\tilde{\bar{\Phi}}_* \\ \frac{\epsilon b}{\sqrt{2}} - \frac{2\Delta b}{\sqrt{2}} & 0 & \frac{\Delta b}{\sqrt{2}} - \frac{\epsilon b}{\sqrt{2}} \\ -\Delta\tilde{\bar{\Phi}}_* & 0 & -\epsilon\tilde{\bar{\Phi}}_* \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & \frac{\epsilon b}{\sqrt{2}} - \frac{\Delta b}{\sqrt{2}} & -2\Delta b\tilde{\bar{\Phi}}_* \\ -\frac{\epsilon b}{\sqrt{2}} + \frac{\Delta b}{\sqrt{2}} & 0 & \frac{\epsilon b}{\sqrt{2}} - \frac{\Delta b}{\sqrt{2}} \\ +2\Delta b\tilde{\bar{\Phi}}_* & \frac{\Delta b}{\sqrt{2}} - \frac{\epsilon b}{\sqrt{2}} & 0 \end{pmatrix}$$

$$\hat{Q} = \frac{iC}{\hbar} \cdot \left\{ i(\epsilon b - 2\Delta b) S_y + \Delta\tilde{\bar{\Phi}}_* (S_-^2 - S_+^2) \right\}$$

Maria Milla Pérez

DIPOLAR INTERACTION WITH A MAGNETIC FLUX:

→ We study the effect of adding an external flux to the fluxonium.

General Hamiltonian:

$$H(\Phi_{ext}) = \frac{1}{2C_J} Q^2 + \frac{\Phi^2}{2L_A} - E_J \cos\left(\varphi + \frac{\Phi_{ext}}{\varphi_0}\right)$$

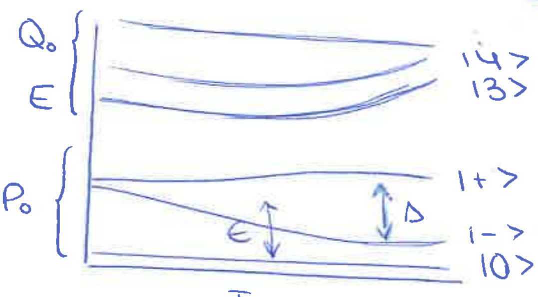
For $\Phi_{ext} \approx 0$ (Taylor expansion):

$$H(\Phi_{ext}\approx 0) = \underbrace{\frac{1}{2C_J} Q^2 + \frac{\Phi^2}{2L_A} - E_J \cos(\varphi)}_{H_0(\Phi_{ext}=0)} + \underbrace{\overbrace{\frac{E_J}{\varphi_0}}^{I_0} \Phi_{ext} \sin(\varphi)}_{\Delta H = \underbrace{\Phi_{ext}}_{\lambda} V}$$

Perturbation theory to find the effective Hamiltonian: $H = H_0 + \lambda V$.

$$H_{eff} \approx \underbrace{P_0 H_0 P_0}_{H_{0\,eff}} + \underbrace{\lambda\, P_0 V P_0}_{1^{st}\text{ order}} + \underbrace{\frac{\lambda^2}{2} \sum_{i,j,\alpha} \left(\frac{1}{E_i^0 - E_\alpha^0} + \frac{1}{E_j^0 - E_\alpha^0}\right)}_{2^{nd}\text{ order}}$$

$$\underbrace{|i^0\rangle\langle i^0|}_{P_0(\Sigma)} V \underbrace{|\alpha^0\rangle\langle\alpha^0|}_{Q_0(\Sigma)} V \underbrace{|j^0\rangle\langle j^0|}_{P_0(\Sigma)} + \dots$$



Q → high-energy subspace.

P → qubit subspaces ($|0\rangle, |-1\rangle, |1\rangle$)

$\Phi_{ext} \sim \lambda$

We have:

$$H_{0eff}: \sum_{i,j=0,1,-1} \langle i | H(\Phi_{ext}=0) | j\rangle |i\rangle\langle j| = \epsilon S_z^2 + \frac{\Delta}{2}(S_+^2 + S_-^2)$$

$\sin\varphi \sim \varphi$

$$1^{st}\text{ order}: \Phi_{ext} \sum_{i,j=0,+1,-1} \langle i | \sin(\varphi) I_0 | j\rangle |i\rangle\langle j| =$$

$$= \Phi_{ext}\left[\tilde{\bar{\Phi}}_A S_z + b(S_x S_z + S_z S_x)\right]$$

2^{nd} order: (numérical) $I_0^2 \frac{\bar{\Phi}_{ext}^2}{2} \underbrace{\left[\eta S_x + \xi(S_+^2 + S_-^2) + \chi S_z^2\right]}_{\sum_{i,j,\alpha}\left(\frac{1}{E_i^0 - E_\alpha^0} + \frac{1}{E_j^0 - E_\alpha^0}\right) |i^0\rangle\langle i^0| \sin(\tilde{\varphi}) |\alpha^0\rangle\langle\alpha^0| \sin(\varphi)|j^0\rangle\langle j^0|}$

(el cálculo analítico se complica).

Hence:

$$\boxed{\Delta H_{eff} = \bar{\Phi}_{ext}\left[\tilde{\bar{\Phi}}_x S_z + b(S_x S_z + S_z S_x)\right] + \frac{\bar{\Phi}_{ext}^2}{2} I_0^2 \cdot \left[\eta S_x + \xi(S_+^2 + S_-^2) + \chi S_z^2\right] + \dots}$$

DIPOLAR INTERACTION WITH ELECTRIC FIELD

→ We study the effect of adding an external voltage to the Hamiltonian.

General Hamiltonian:

$$H(\Delta V, \Phi_{ext} = 0) = \frac{1}{2(C_J + C_V)}(Q + C_V \Delta V)^2 + \frac{\Phi^2}{2L_A} - E_J \cos(\varphi) =$$

$$= \underbrace{\frac{1}{2(C_J+C_V)} Q^2 + \frac{\Phi^2}{2L_A} - E_J\cos(\varphi)}_{H_0(\Delta V = 0,\ \Phi_{ext} = 0)\ \text{(renormalizado)}} + \left(\underbrace{C_V \Delta V Q}_{\Delta H = \underbrace{\lambda}_{C_V \Delta V} V} + \underbrace{C_V^2 \Delta V^2}_{\text{cte (sólo desplaza los E)}}\right)\frac{1}{2(C_J+C_V)}$$

Perturbation theory:

H_0 eff: $\epsilon S_z^2 + \Delta(S_+^2 + S_-^2)$

1^{st} order: $\frac{C_V \Delta V}{2(C_J + C_V)} \underbrace{P_0 \hat{Q} P_0}_{\text{lo obtenemos por la ec. Heisenberg.}} = \frac{C_V \Delta V}{2(C_J + C_V)} \frac{i(C_J + C_V)}{\hbar}\left\{2\Delta\tilde{\bar{\Phi}}_x(S_-^2 - S_+^2) + i\left(\epsilon b - \frac{3}{2}\Delta b\right)S_y\right\}$

2^{nd} order: (numerical) $\frac{C_V^2 \Delta V^2}{2^2(C_J + C_V)} \underbrace{\left[\tau S_x + \mu(S_+^2 + S_-^2) + \nu S_z^2\right]}_{\sum_{i,j,\alpha}\left(\frac{1}{E_i^0 - E_\alpha^0} + \frac{1}{E_j^0 - E_\alpha^0}\right)|i^0\rangle\langle i^0| Q |\alpha^0\rangle\langle\alpha^0| Q|j^0\rangle\langle j^0|}$

(para hacer el analítico necesitaríamos también de $|3\rangle$, $|4\rangle$ ...)

(puede ser un artefacto del modelo → en realidad sólo debe haber acoplo a primer orden)

Hence:

$$\boxed{\Delta H_{eff} = C_V \Delta V \frac{i\cancel{(C_J + C_V)}}{2\hbar}\left\{\Delta\tilde{\bar{\Phi}}_x(S_-^2 - S_+^2) + i(\epsilon b - \ \Delta b)S_y\right\} + \left(+\frac{C_V^2 \Delta V^2}{2^2(C_J+C_V)}\left[\tau S_x + \mu(S_+^2 + S_-^2) + \nu S_z^2\right]\right) ?}$$

Maria Hita Pérez

CHARGE AND FLUX OPERATORS:

→ In order to find the matrix form of the charge and flux operators, one can study the variation of the Hamiltonian when adding a perturbation in charge or flux.

Operador de carga ($\hat{Q}$):

→ We add a perturbation in Q, by adding an external voltage (V) connected through a capacitance (C_V).

$$H(V, \Phi_{ext}=0) = \frac{1}{2(C_J+C_V)}(Q+C_V V)^2 + \frac{\Phi^2}{2L_A} - E_J\cos(\varphi) =$$

$$= \frac{1}{2(C_J+C_V)}Q^2 + \frac{\Phi^2}{2L_A} - E_J\cos(\varphi) + (C_V V Q + C_V^2 V^2)\frac{1}{2(C_V+C_J)}$$

$$\Rightarrow \boxed{Q = \frac{2(C_V+C_J)}{C_V}\left.\frac{dH}{dV}\right|_{V=0}}$$

Operador de flujo ($\hat{\varphi}$):

$$H(V=0, \Phi_{ext}) = \frac{1}{2C_J}Q^2 + \frac{\Phi^2}{2L_A} - E_J\cos\left(\varphi + \frac{\Phi_{ext}}{\varphi_0}\right)$$

→ we add a perturbation in $\hat{\varphi}$, by threading the loop with an external flux (Φ_{ext})

For ($\Phi_{ext} \simeq 0$), Taylor expansion: $f(x \simeq a) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$

$$H(\Phi_{ext} \simeq 0) = \frac{1}{2C_J}Q^2 + \frac{\Phi^2}{2L_A} - E_J\cos(\varphi) + \Phi_{ext} I_0 \sin(\varphi) + \frac{\Phi_{ext}^2}{2}\frac{I_0}{\varphi_0}\cos(\varphi)$$

$$- \frac{\Phi_{ext}^3}{6}\frac{I_0}{\varphi_0^2}\sin(\varphi) - \frac{\Phi_{ext}^4}{6}\frac{I_0}{\varphi_0^2}\cos(\varphi) + \dots$$

$$\Rightarrow \boxed{\sin(\varphi) = \frac{1}{I_0}\left.\frac{dH}{d\Phi_{ext}}\right|_{\Phi_{ext}=0}}, \quad \cos(\varphi) = \frac{\varphi_0}{I_0}\left.\frac{d^2H}{d\Phi_{ext}^2}\right|_{\Phi_{ext}=0} \dots$$

EFFECTIVE HAMILTONIAN DERIVATION:

Hamiltonian: $H = H(\Phi_{ext}=0) + \underbrace{\Phi_{ext} I_0 \sin(\tilde{\Phi}_a S_z + b(S_x S_z + S_z S_x))}_{\text{we already know the effective form of this operator.}}$

Projection:

$$\langle -1|H(\Phi_{ext}=0)|-1\rangle = U(\bar{\Phi}_a) = \epsilon = \langle 1|H(0)|1\rangle$$

$$\langle -1|H(\Phi_{ext}=0)|0\rangle = 0 = \langle 1|H(\Phi_{ext}=0)|0\rangle = \langle 0|H(0)|-1\rangle = \langle 0|H(0)|1\rangle$$

$$\langle -1|H(\Phi_{ext}=0)|1\rangle = \left(\frac{\langle L| - a\langle 3|}{\sqrt{1-a^2}}\right) H(\Phi_{ext}=0) \left(\frac{|R\rangle + a|3\rangle}{\sqrt{1-a^2}}\right)$$

↳ no lapen with la barrera = de potencial

$$= \frac{1}{1-a^2}\left\{U(\Phi_a)\cancelto{0}{\langle L|R\rangle} - a\cancel{\langle 3|H(\Phi_{ext}=0)|R\rangle} + a\cancel{\langle L|H(\Phi_{ext}=0)|3\rangle}\right.$$

$$\left. + a^2\langle 3|H(\Phi_{ext}=0)|3\rangle\right\} = \frac{E_3 a^2}{1-a^2} = \Delta = \langle 1|H(\Phi_{ext}=0)|-1\rangle$$

$$\langle 0 | H(\Phi_{ext}=0) | 0 \rangle = E_0 = 0.$$

Then:

$$H_{eff}(\Phi_{ext}=0) = \begin{pmatrix} \epsilon & 0 & \Delta \\ 0 & 0 & 0 \\ \Delta & & \epsilon \end{pmatrix} = \epsilon S_z^2 + \frac{\Delta}{2}(S_+^2 + S_-^2)$$