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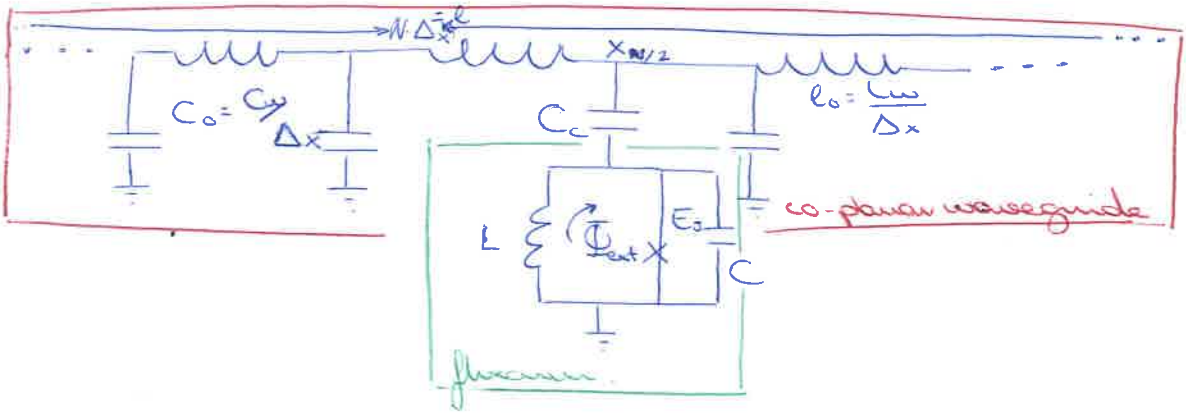
SC23

FLUXONIUM QUTRIT + WAVEGUIDE COUPLING: QUASI-BIC STATE

→ Estudio del acoplo capacitivo entre un fluxonium y una guía de ondas, estudios experimentales han demostrado que aparece un Quasi-BIC ("Bound State In the Continuum") que prohibe la transición directa $|0\rangle \rightarrow |1+2\rangle$ (2^{nd} excited state) cuando el sistema se encuentra en simetría de flujo ($\Phi_{ext}=0$).

CIRCUIT AND HAMILTONIAN:



Cuando hablamos de un circuito de este tipo encontramos que la contribución de la guía de ondas al acoplo viene dado por el potencial creado por la misma en ese punto de modo que el Hamiltoniano completo del sistema viene dado por:

en pto de acoplo.

$$\boxed{\hat{H} = \hat{H}_{waveguide} + \frac{1}{2(C_c+C)}\left(\hat{Q}_J + C_c \hat{V}_{N/2}\right)^2 + \frac{1}{2L}\hat{\Phi}_J^2 - E_J \cos\left(\frac{\hat{\Phi}_J + \Phi_0}{\varphi_0}\right)}$$

Donde $\hat{V}$ es el potencial debido a la guía, que suponiendo que el acoplo es lo suficientemente débil como para no distorsionar los modos de la guía se puede escribir como: (asumiendo ondas planas)

? fallo e signo?

correcto a la unidad?

$$\hat{V} = \sum_{n=0}^{N-1} \sqrt{\frac{\hbar \omega_n}{2 c_0 l}} \left(i b_n - i b_n^{+}\right)(-1)^n$$

Siendo el Hamiltoniano de la guía:

$$\hat{H}_{waveguide} = \sum_n \hbar \omega_n \left(b_n^{+} b_n + \frac{1}{2}\right)$$

De forma que el H completo del sistema puede escribirse como: (autoestados que vamos a usar de la H waveguide: modos desacoplados)

$$\boxed{\begin{aligned}\hat{H} = {} & \overbrace{\frac{1}{2(C_c+C)}\hat{Q}_J^2 + \frac{1}{2L}\hat{\Phi}_J^2 - E_J\cos\left(\frac{\hat{\Phi}_J + \Phi_0}{\varphi_0}\right)}^{H_{fluxonium}} + \overbrace{\sum_{n=0}^{N-1} \hbar\omega_n \left(b_n^{+} b_n + \frac{1}{2}\right)}^{H_{waveguide}} \\ & \underbrace{+ \frac{C_c}{(C_c+C)} Q_J \sum_{n=0}^{N-1} (-1)^n \sqrt{\frac{\hbar\omega_n}{2 c_0 l}} \left(i b_n - i b_n^{+}\right)}_{H_{coupling}}\end{aligned}}$$

TRANSITION RATES AND DECAY TIME:

The transition probability is given by the Fermi's golden rule:

$$\boxed{\Gamma_{i\to f} = \frac{2\pi}{\hbar}\,|\langle f|\underbrace{\Delta H}_{\text{perturbation}}|i\rangle|^2\,\underbrace{\rho(E_f)}_{\text{density of states at the final state energy}}} \propto \frac{1}{\underbrace{T_{i\to f}}_{\text{decay time (mean lifetime)}}}$$

Density of states:

Wavefunctions → plane waves: $e^{ik_n x_m}\sqrt{\dfrac{\Delta x}{\ell}} = e^{ik_n m\cdot\Delta x}\sqrt{\dfrac{\Delta x}{\ell}}$, $\quad k_n = \dfrac{2\pi}{\ell}\cdot n$

Dispersion relation: $\omega = v|k|$

Density of modes: $\rho(k_n) = \dfrac{\ell}{2\pi}$ (number of modes in a certain distance 1/separation modes)

Tenemos: $\rho(k)\,dk_n = \rho(\omega)\,d\omega$

Luego: $\rho(\omega) = \rho(k)\dfrac{dk}{d\omega} = \dfrac{\ell}{2\pi}\cdot\dfrac{1}{v} = \boxed{\dfrac{\ell}{2\pi v} = \rho(\omega)}$ $\quad (\rho(E) = \hbar^{-1}\rho(\omega))$

Matrix elements: $|\langle f|\Delta H'|i\rangle|$

$$\langle f|\frac{C_c}{(C+C_c)}Q_J\sum_{n=0}^{N-1}(-1)^n\sqrt{\frac{\hbar\omega_n}{2c_0\ell}}\,(+ib_n - ib_n^\dagger)|i\rangle = \left(\frac{C_c}{(C+C_c)}\right)\sqrt{\frac{\hbar\omega_{if}}{2c_0\ell}}\,\langle f|Q_J|i\rangle$$

Entonces:

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\,\frac{\hbar\omega_{if}}{2c_0\ell}\left(\frac{C_c}{C+C_c}\right)^2 4e^2\,\frac{\ell}{\hbar 2\pi v}\,|\langle f|N_J|i\rangle|^2 =$$

$$= \frac{2e^2}{c_0\hbar v}\left(\frac{C_c}{C+C_c}\right)^2\omega_{if}\,|\langle f|N_J|i\rangle|^2 =$$

$$= \frac{2e^2}{\hbar}\left(\frac{C_c}{C+C_c}\right)^2 Z\,\omega_{if}\,|\langle f|N_J|i\rangle|^2 =$$

$Z^{-1} = c_0\cdot v = c_0\cdot\dfrac{1}{\sqrt{c_0 l_0}} = \sqrt{\dfrac{c_0}{l_0}}$ $\qquad G_0 = \dfrac{2e^2}{h} = \dfrac{2e^2}{\hbar\cdot 2\pi}$

$$= \left(\frac{C_c}{C+C_c}\right)^2 2\pi Z G_0\,\omega_{if}\,|\langle f|N_J|i\rangle|^2$$

falla un 2??

$$\rightarrow \boxed{\Gamma_{i\to f} = 2\pi\left(\frac{C_c}{C+C_c}\right)^2 Z G_0\,|\langle f|N_J|i\rangle|^2\,\underbrace{\omega_{if}}_{2\pi\nu_{if}}} \rightarrow$$

→ Decay time: $\boxed{T_{i\to f} = 1/\Gamma_{i\to f}}$

HAMILTONIAN DERIVATION:

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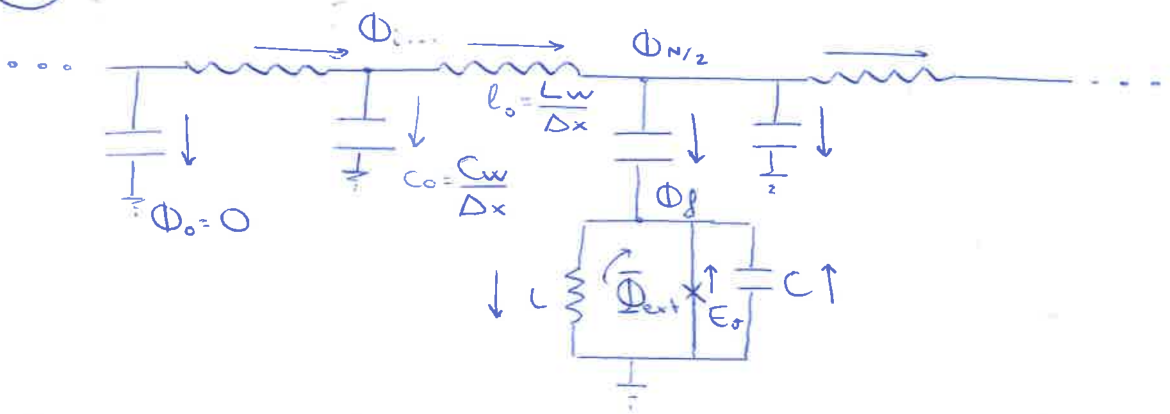
(1-2)



(3) Análisis nodos. (4) Bucles (lo tendremos en cuenta al escribir las intensidades).

(5-6)

Nodos estándar de la guía (i): (menos de N/2)

$$C_i(\ddot{\Phi}_0 - \ddot{\Phi}_i) + \frac{\Phi_{i+1}-\Phi_i}{l_0} = \frac{\Phi_i - \Phi_{i-1}}{l_0} \longrightarrow C_i\ddot{\Phi}_i = \frac{\Phi_{i+1}-\Phi_i}{l_0} - \frac{\Phi_i - \Phi_{i-1}}{l_0}$$

Nodo de acoplo (N/2):

$$\frac{\Phi_{N/2} - \Phi_{-N/2}}{l_0} = C_c(\ddot{\Phi}_g - \ddot{\Phi}_{N/2}) + C_0(\ddot{\Phi}_0 - \ddot{\Phi}_{N/2}) + \frac{\Phi_{3N/2} - \Phi_{N/2}}{l_0} \longrightarrow$$

$$\longrightarrow C_c\ddot{\Phi}_g - (C_c + C_0)\ddot{\Phi}_{N/2} = \frac{\Phi_{N/2} - \Phi_{-N/2}}{l_0} - \frac{\Phi_{3N/2} - \Phi_{N/2}}{l_0}$$

Nodo fluxonium (g):

$$C_c(\ddot{\Phi}_g - \ddot{\Phi}_{N/2}) + I_J \sin\left(\frac{\Phi_g + \Phi_{ext}}{\varphi_0}\right) + C\ddot{\Phi}_g = -\frac{\Phi_g}{L} \longrightarrow$$

$$\longrightarrow (C_c + C)\ddot{\Phi}_g - C_c\ddot{\Phi}_{N/2} = -\frac{\Phi_g}{L} - I_J \sin\left(\frac{\Phi_g + \Phi_{ext}}{\varphi_0}\right)$$

(8)

(i) $$C_i\ddot{\Phi}_i = \frac{\Phi_{i+1}-\Phi_i}{l_0} - \frac{\Phi_i - \Phi_{i-1}}{l}$$

$$\downarrow \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Phi}_j}\right) = \frac{\partial \mathcal{L}}{\partial \Phi_j}$$

$$\mathcal{L}^i = \frac{C_0}{2}\dot{\Phi}_i^2 + \frac{\Phi_{i+1}\Phi_i}{l_0} - \frac{\Phi_i^2}{2l_0} - \frac{\Phi_i^2}{2l_0} + \frac{\Phi_{i-1}\Phi_i}{l_0}$$

menos $i = N/2$

(N/2) $$-C_c\ddot{\Phi}_g + (C_c + C_0)\ddot{\Phi}_{N/2} = \frac{\Phi_{3N/2} - \Phi_{N/2}}{l_0} - \frac{\Phi_{N/2} - \Phi_{-N/2}}{l_0}$$

$$\downarrow$$

$$\mathcal{L}^{N/2} = -C_c\dot{\Phi}_g\dot{\Phi}_{N/2} + \frac{(C_c + C_0)}{2}\dot{\Phi}_{N/2}^2 + \frac{\Phi_{3N/2}\Phi_{N/2}}{l_0} - \frac{\Phi_{N/2}^2}{l_0} + \frac{\Phi_{-N/2}\Phi_{N/2}}{l_0} + \mathcal{L}(\Phi_i, \Phi$$

⑧ $(C_c+C)\,\ddot{\Phi}_J - C_c\ddot{\Phi}_{N/2} = -\dfrac{\Phi_J}{L} - E_J \sin\left(\dfrac{\Phi_J+\bar{\Phi}_{ext}}{\varphi_0}\right)$

$\downarrow$

$$\mathcal{L}^J = \frac{(C_c+C)}{2}\dot{\Phi}_J^2 - C_c\dot{\Phi}_J\dot{\Phi}_{N/2} - \frac{\Phi_J^2}{2L} + E_J\cos\left(\frac{\Phi_J+\bar{\Phi}_{ext}}{\varphi_0}\right) + \mathcal{L}(\Phi_i)$$

("$\mathcal{L} = \sum\limits_{\substack{i \\ (-N/2)}} \mathcal{L}^i + \mathcal{L}^{N/2} + \mathcal{L}^J$" (quitando los términos repetidos) (es una simplificación a la hora de escribirlo))

$$\mathcal{L} = \sum_{i=0}^{N-1}\frac{C_0}{2}\dot{\Phi}_i^2 - \sum_{i=0}^{N-1}\frac{(\Phi_{i+1}-\Phi_i)^2}{2\ell_0} + \underbrace{\frac{C_c}{2}\dot{\Phi}_{N/2}^2}_{\text{renormaliza }\dot{\Phi}^2_{N/2}\text{ solo tenemos en cuenta.}} - C_c\dot{\Phi}_J\dot{\Phi}_{N/2} +$$

(ya incluye el $N/2$)

$$+ \frac{(C_c+C)}{2}\dot{\Phi}_J^2 - \frac{\Phi_J^2}{2L} + E_J\cos\left(\frac{\Phi_J+\bar{\Phi}_{ext}}{\varphi_0}\right)$$

⑨ $q_i = \dfrac{\partial\mathcal{L}}{\partial\dot{\Phi}_i} = C_0\dot{\Phi}_i \longrightarrow \dot{\Phi}_i = \dfrac{q_i}{C_0}$ (incluye a $N/2$ al no contar la renormalización)

$Q_J = \dfrac{\partial\mathcal{L}}{\partial\dot{\Phi}_J} = (C_c+C)\dot{\Phi}_J - C_c\dot{\Phi}_{N/2} \longrightarrow \dot{\Phi}_J = \dfrac{Q_J + (C_c/C_0)q_{N/2}}{(C_c+C)}$

$$H = \sum q_i\dot{\Phi}_i + Q_J\dot{\Phi}_J - \mathcal{L} = \sum_{i=1}^{N}\frac{q_i^2}{2C_0} + \sum_{i=1}^{N-1}\frac{(\Phi_{i+1}-\Phi_i)^2}{2\ell_0} +$$

$$+ \frac{Q_J^2}{(C_c+C)} + \frac{(C_c/C_0)q_{N/2}Q_J}{(C_c+C)} + C_c\frac{Q_J+(C_c/C_0)q_{N/2}}{(C_c+C)}\cdot\frac{q_{N/2}}{C_0}$$

$$-\frac{(C_c+C)}{2}\left(\frac{Q_J+(C_c/C_0)q_{N/2}}{(C_c+C)}\right)^2 + \frac{\Phi_J^2}{2L} - E_J\cos\left(\frac{\Phi_J+\bar{\Phi}_{ext}}{\varphi_0}\right)$$

$\downarrow$ $V_{N/2} = \dfrac{d}{dt}\Phi_{N/2} = \dfrac{q_{N/2}}{C_0}$

$$H = \sum_{i=1}^{N}\frac{q_i^2}{2C_0} + \sum_{i=1}^{N-1}\frac{(\Phi_{i+1}-\Phi_i)^2}{2\ell_0} + \frac{1}{2(C_c+C)}Q_J^2 + \frac{C_c}{(C_c+C)}V_{N/2}Q_J +$$

$$+\frac{C_c^2}{2(C_c+C)}V_{N/2}^2 + \ldots = \sum_{i=1}^{N}\frac{q_i^2}{2C_0} + \sum_{i=1}^{N-1}\frac{(\Phi_{i+1}-\Phi_i)^2}{2\ell_0} + \frac{1}{2(C_c+C)}(Q_J + C_cV_{N/2}Q_J)^2$$

$$+\frac{\Phi_J^2}{2L} - E_J\cos\left(\frac{\Phi_J+\bar{\Phi}_{ext}}{\varphi_0}\right) = H_{waveguide} + H_{fluxonium}\,(V = V_{N/2})$$

(No tengo claro que esto esté del todo bien lo correcto sería:
$C_0 + C_g \longrightarrow C_0$ (pues $C_g \lll$) y $C = (A+\underbrace{B}_{\text{términos con } C_g})^{-1} = A^{-1} + \frac{1}{1+g}A^{-1}BA^{-1}$ con $g = \mathrm{tr}(BA^{-1})$)

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$$C = \begin{pmatrix} C+C_c & 0 & \cdots & -C_c & \cdots & 0 \\ 0 & C_0 & & 0 & & \vdots \\ \vdots & & \ddots & & & \vdots \\ -C_c & 0 & & C_0+C_c & & 0 \\ \vdots & & & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \cdots & C_0 \end{pmatrix} \simeq \underbrace{\begin{pmatrix} C & & & 0 \\ & C_0 & & \\ & & C_0 & \\ 0 & & & C_0 \end{pmatrix}}_{A} +$$

$C_c \lll C_0$

$$+ \underbrace{\begin{pmatrix} C_c & 0 & \cdots & -C_c & \cdots & 0 \\ 0 & & & & & \vdots \\ \vdots & & & 0 & & \vdots \\ -C_c & & & & & \\ \vdots & & & & \ddots & \\ 0 & \cdots & \cdots & 0 & \cdots & 0 \end{pmatrix}}_{B}$$

con

$$\mathcal{L} = \frac{1}{2}\dot{\vec{\Phi}}^T C \vec{\Phi} + L(\Phi)$$

y $\vec{\Phi} = \begin{pmatrix} \Phi_g \\ \Phi_1 \\ \Phi_2 \\ \Phi_{N/2} \\ \vdots \\ \Phi_N \end{pmatrix}$

$$A^{-1} = \begin{pmatrix} 1/C & & 0 \\ & 1/C_0 & \\ 0 & & 1/C_0 \end{pmatrix} \qquad \mathrm{tr}(BA^{-1}) = \frac{C_c}{C} = g$$

$$A^{-1}BA^{-1} = \begin{pmatrix} C_c/C^2 & 0 & \cdots & -\frac{C_c}{C_0 C} & \cdots & 0 \\ 0 & 0 & & & & \vdots \\ \vdots & & & & & \\ -\frac{C_c}{C_0 C} & & & 0 & & \vdots \\ \vdots & & & & \ddots & \\ 0 & \cdots & \cdots & 0 & \cdots & 0 \end{pmatrix}$$

$$C^{-1} = A^{-1} - \underbrace{\frac{1}{1+g}}_{\frac{C}{C_c+C}} A^{-1}BA^{-1} = \begin{pmatrix} \overbrace{\frac{1}{C} - \frac{C_c}{C(C_c+C)}}^{\frac{1}{C_c+C}} & 0 & \cdots & +\frac{C_c}{C_0(C_c+C)} & \cdots & 0 \\ 0 & 1/C_0 & & 0 & & \vdots \\ \vdots & & \ddots & & & \\ +\frac{C_c}{C_0(C_c+C)} & & & & & \vdots \\ \vdots & & & & \ddots & \\ 0 & & & & & 1/C_0 \end{pmatrix}$$

$$\mathcal{H} = \frac{1}{2}\vec{Q}^T C^{-1}\vec{Q} - L(\Phi) = \overbrace{\sum_{i=1}^{N} \frac{q_i^2}{2C_0} + \sum_{i=1}^{N} \frac{(\Phi_{i+1}-\Phi_i)^2}{2l_0}}^{H_{waveguide}} + \frac{1}{2(C_c+C)} Q_g^2$$

$$\underbrace{+ \frac{C_c}{(C_c+C)C_0} q_{N/2} Q_g}_{H_{coupling}} + \frac{\Phi_g^2}{2L} - E_J \cos\left(\frac{\Phi_g + \Phi_{ext}}{\Phi_0}\right) =$$

$$\mathcal{H} = H_{\text{waveguide}} + \tilde{H}_{\text{fluxonium}}(C \to C_c + C = C_\Sigma) + \frac{C_c}{(C_c + C)} \frac{q_{N/2}}{c_0} Q_f =$$

$$= \boxed{H_{\text{waveguide}} + \tilde{H}_{\text{fluxonium}}(C \to C_\Sigma) + \frac{C_c}{(C_c + C)} V_{N/2} Q_f = \mathcal{H}}$$