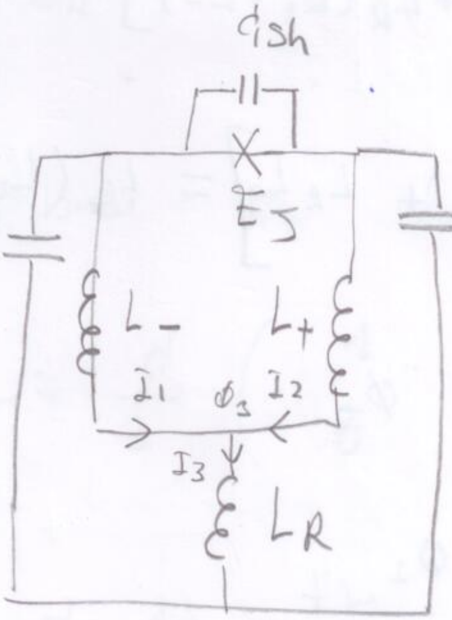
Qubits Iran:

(0) El circuito



$$L_\pm = \frac{L_q}{2} \pm \Delta$$

De las leyes de kirchhoff: $I_3 = I_1 + I_2$.

$$\frac{\Phi_3}{L_R} = \frac{\phi_1 - \phi_3}{L_1} + \frac{\phi_2 - \phi_3}{L_+},$$

$$\phi_3 \left[\frac{1}{L_R} + \frac{1}{L_+} + \frac{1}{L_-} \right] = \frac{\phi_1}{L_-} + \frac{\phi_2}{L_+},$$

$$\frac{\phi_3}{L_+ L_-} \left[\frac{L_+ L_-}{L_R} + L_- + L_+ \right] = \frac{\phi_1}{L_-} + \frac{\phi_2}{L_+} = \frac{1}{L_- L_+} (\phi_1 L_+ + \phi$$

$$\phi_3 = \frac{\cancel{L_R}\ L_R}{L_+ L_- + L_- L_R + L_+ L_R} \left[\phi_1 L_+ + \phi_2 L_- \right].$$

$$\phi_3 = \frac{4L_R}{\ell^2}\left[\phi_1 L_+ + \phi_2 L_-\right]$$

donde : $\ell^2 = 4\left[L_+ L_- + L_R(L_+ + L_-)\right]$

$$\ell^2 = 4\left[\frac{L_q^2}{4} - \Delta^2 + L_R L_q\right] = L_q(L_q + 4L_R) - 4\Delta^2.$$

Pasamos a modos $\phi_\pm$:

$$\phi_\pm = \phi_1 \pm \phi_2$$

$$\phi_1 = \frac{\phi_+ + \phi_-}{2}, \quad \phi_2 = \frac{\phi_+ - \phi_-}{2}$$

$$\boxed{\phi_3 = \frac{4L_R}{\ell^2}\left[\frac{L_q}{2}\phi_+ + \Delta\phi_-\right]}$$

① Escribimos el circuito:

$$\mathcal{L} = \frac{C}{2}(\dot\phi_1^2 + \dot\phi_2^2) + \frac{C_{sh}}{2}(\dot\phi_1 - \dot\phi_2)^2$$

$$+ \frac{(\phi_1 - \phi_3)^2}{L_-} + \frac{(\phi_2 - \phi_3)^2}{L_+} + \frac{\phi_3^2}{L_R}$$

$$- E_J \cos\left[\frac{2\pi}{\ }(\phi_2 - \phi_1 - \phi_{ext})\right]$$

Pasamos a modos $\Phi_\pm = \Phi_1 \pm \Phi_2$

$$\mathcal{L}_K = \frac{C'}{2} \begin{pmatrix} 1+\alpha & -\alpha \\ -\alpha & 1+\alpha \end{pmatrix}$$

en la base $\Phi_\pm$ la energía cinética es:

$$\mathcal{L}_K = \frac{C'}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1+2\alpha \end{pmatrix}$$

$$\boxed{\begin{aligned} C_+ &= \frac{C'}{2} \\ C_- &= \frac{C'}{2}\left[1+\frac{2C_s'}{C'}\right] \end{aligned}}$$

donde α is the ratio of capacitances:
changing to Hamiltonian via $q_\pm = C_\pm \dot{\Phi}_\pm$:

$$H = \frac{1}{2C_+} q_+^2 + \frac{1}{2C_-} q_-^2 - E_J \cos\left[\frac{2\pi}{\Phi_0}(\Phi_- - \Phi_{ext})\right]$$

$$+ \underbrace{\frac{1}{2L_-}(\Phi_1 - \Phi_3)^2 + \frac{1}{2L_+}(\Phi_2 - \Phi_3)^2 + \frac{1}{2L_R}\Phi_3^2}_{U}.$$

Simplificamos ahora la energía potencial cuadrática

$$U(\Phi_i R) = \frac{1}{2L_+L_-}\left(L_+\Phi_1^2 + L_-\Phi_2^2\right) + \frac{1}{2L_+L_-}\left(L_+ + L_- + \frac{L_+L_-}{L_R}\right)\Phi_3^2$$

$$- \frac{\Phi_3}{L_+L_-}\left(L_+\Phi_1 + L_-\Phi_2\right)$$

* Nota, las capacitancias cambian factor $\frac{1}{2}$ debido a

$$u = \frac{1}{2L_+L_-}\left(L_+\phi_1^2 + L_-\phi_2^2\right) + \frac{1}{2L_+L_-}\left(L_+ + L_- + \frac{L_+L_-}{L_R}\right)\phi_3^2$$

$$- \frac{\phi_3}{L_+L_-}\left(L_+\phi_1 + L_-\phi_2\right) =$$

Simplificam:

$$L_+\phi_1^2 + L_-\phi_2^2 = \left(\frac{L_q}{2} + \Delta\right)\frac{\phi_+^2 + \phi_-^2 + 2\phi_+\phi_-}{4} + \frac{\phi_+^2 + \phi_-^2 - 2\phi_+\phi_-}{4}\cdot\left(\frac{L_q}{2} - \Delta\right)$$

$$= \frac{L_q}{4}\left(\phi_+^2 + \phi_-^2\right) + \Delta\,\phi_+\phi_-$$

$$L_+\phi_1 + L_-\phi_2 = \left(\frac{L_q}{2} + \Delta\right)\frac{\phi_+ + \phi_-}{2} + \left(\frac{L_q}{2} - \Delta\right)\frac{\phi_+ - \phi_-}{2}$$

$$= \frac{L_q}{2}\phi_+ + \Delta\,\phi_- .$$

$$\frac{1}{2}\left(L_+ + L_- + \frac{L_+L_-}{L_R}\right) = \frac{1}{2L_R}\left[L_R\left(L_+ + L_-\right) + L_+L_-\right]$$

$$= \frac{e^2}{8L_R}$$

$$u = \frac{1}{2L_+L_-}\left[\frac{L_q}{4}\left(\phi_+^2+\phi_-^2\right) + \Delta\phi_+\phi_-\right] +$$

$$+ \frac{\phi_3}{2L_+L_-}\left[\left(\frac{L_q}{2}\phi_+ + \Delta\phi_-\right) - \frac{L_q}{2}\phi_+ - 2\Delta\phi_-\right]$$

$$= \frac{1}{2L_+L_-}\left[\frac{L_q}{4}\left(\phi_+^2+\phi_-^2\right) + \Delta\phi_+\phi_-\right]$$

$$- \frac{\phi_3}{2L_+L_-}\left[\frac{L_q}{2}\phi_+ + \Delta\phi_-\right]$$

$$= \frac{1}{2L_+L_-}\left[\frac{L_q}{4}\left(\phi_+^2+\phi_-^2\right) + \Delta\phi_+\phi_- - \frac{4L_R}{e^2}\left(\frac{L_q}{2}\phi_+ + \Delta\phi_-\right.\right.$$

$$= \frac{1}{2L_+L_-}\left[\left(\frac{L_q}{4} - \frac{L_RL_q^2}{e^2}\right)\phi_+^2 + \left(\frac{L_q}{4} - \frac{4L_R}{e^2}\Delta^2\right)\phi_-\right.$$

$$\left. + \left(\Delta - \frac{4L_RL_q\Delta}{e^2}\right)\phi_+\phi_-\right]$$

Coefficient ϕ_+^2:

$$\frac{1}{2L_+L_-}\left(\frac{L_q}{4} - \frac{L_RL_q^2}{e^2}\right) = \frac{L_q}{2e^2L_+L_-}\left(\frac{e^2}{4} - L_RL_q\right) =$$

Coeficiente ϕ_-:

$$\frac{1}{2L_+L_-}\left(\frac{L_q}{4}-\frac{4L_R\Delta^2}{\ell^2}\right)=\frac{1}{\ell^2 2L_+L_-}\left[\frac{L_q\ell^2}{4}-4L_R\Delta^2\right]$$

$$=\frac{1}{2L_+L_-\ell^2}\left[L_q\left(L_+L_-+L_R(L_++L_-)\right)-4L_R\Delta^2\right]$$

$$=\frac{1}{2L_+L_-\ell^2}\left[L_q(L_+\cdot L_-)+L_R\left(L_q^2-4\Delta^2\right)\right]$$

$$=\frac{1}{2L_+L_-\ell^2}\left[L_q(L_+L_-)+L_R 4L_+L_-\right]$$

$$=\frac{1}{2\ell^2}\left(L_q+4L_R\right),$$

Coeficiente de $\phi_+\phi_-$

$$\frac{\Delta}{2L_+L_-}\left(1-\frac{4L_RL_q}{\ell^2}\right)=\frac{\Delta}{2L_+L_-\ell^2}\left(\ell^2-4L_RL_q\right)$$

$$=\frac{\Delta}{2L_+L_-\ell^2}\left(4L_+L_-\right)=\frac{2\Delta}{\ell^2}$$

||| Si

El Hamiltoniano del circuito es:

$$\mathcal{H} = \frac{1}{2C_+} q_+^2 + \frac{1}{2L_+} \phi_+^2 +$$

$$\frac{1}{2C_-} q_-^2 + \frac{1}{2L_-} \phi_-^2 - E_J \cos\left[\frac{2\pi}{\Phi_0}(\phi_- - \Phi_{ext})\right]$$

$$+ \frac{1}{M_c} \phi_+ \phi_-$$

donde $C_+ = \frac{C}{2}$, $C_- = \frac{C}{2}\left(1 + 2\frac{C_{sh}}{C}\right)$

$$L_+ = \frac{\ell^2}{L_q}, \quad L_- = \frac{\ell^2}{(L_q + 4L_R)}$$

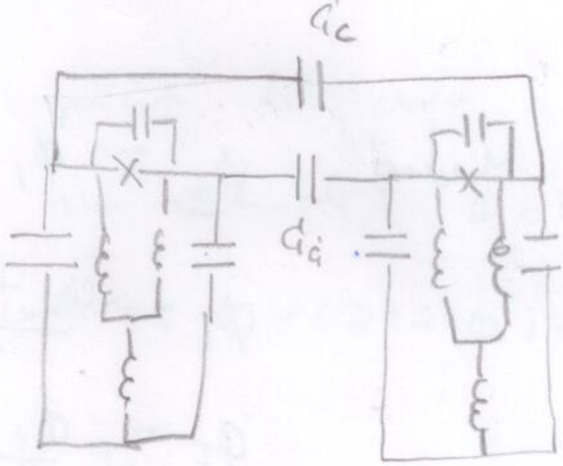
$$M_c = \frac{\cancel{2}\ell^2}{2\Delta}$$ (¡Comprobar factor 2!

Notas: * No hay factor 2 en las notas de Gabriel.

* Comprobar las capacitancias de los modos ✓

* El acoplo es pequeño por lo que podemos tratarlo perturbativamente.

(ii) Dos qubits:



$$\mathcal{L} = \sum_{i=1}^{2} \frac{C^i}{2} \left[(\dot{\phi}_1^i)^2 + (\dot{\phi}_2^i)^2 \right] + \frac{C_{sh}}{2} \left[\dot{\phi}_1^i - \dot{\phi}_1^i \right]^2$$

$$+ \frac{C_c}{2} \left[(\dot{\phi}_1^2 - \dot{\phi}_1^1)^2 + (\dot{\phi}_1^1 - \dot{\phi}_2^2)^2 \right] + U(\vec{\phi})$$

Pasamos a modos $\pm$:

$$\mathcal{L} = \sum_{i=1}^{2} \mathcal{L}_i + \frac{C_c}{8} \left[(\dot{\phi}_+^2 + \dot{\phi}_-^2 - \dot{\phi}_+^1 + \dot{\phi}_-^1)^2 + (\dot{\phi}_+^2 - \dot{\phi}_-^2 - \dot{\phi}_+^1 - \dot{\phi}_-^1)^2 \right]$$

$$= \sum_{i=1}^{2} \frac{1}{2} \left[(C_+ + \frac{C_c}{2}) (\dot{\phi}_+^i)^2 + (C_- + \frac{C_c}{2}) (\dot{\phi}_-^i)^2 + U(\vec{\phi}) \right.$$

$$- \frac{C_c}{2} \left(\dot{\phi}_+^2 \dot{\phi}_+^1 - \dot{\phi}_-^2 \dot{\phi}_-^1 \right) + \frac{1}{M_c} \sum_{i=1}^{2} \phi_+^i \phi_-^i$$

Obtenemos el Lagrangiano:

$$\mathcal{L} = \sum_{i=1}^{2} \tilde{\mathcal{L}}_i - \frac{C_c}{2} \left[\dot{\phi}_+^2 \dot{\phi}_+^1 - \dot{\phi}_-^2 \dot{\phi}_-^1 \right]$$

La matriz de capacitancias es:

$$
\mathcal{C} = \begin{pmatrix} \tilde{C}_+ & 0 & -\frac{C_c}{2} & 0 \\ 0 & \tilde{C}_- & 0 & +\frac{C_c}{2} \\ -\frac{C_c}{2} & 0 & \tilde{C}_+ & 0 \\ 0 & +\frac{C_c}{2} & 0 & \tilde{C}_- \end{pmatrix}
$$

$$\tilde{C}_+ = \frac{C}{2} + \frac{C_c}{2} = \frac{C}{2}(1+\epsilon) \, , \qquad \epsilon = \frac{C_c}{C} .$$

$$C_- = C\left(\frac{1}{2}+\alpha\right) + \frac{C_c}{2} = \frac{C}{2}(1+2\alpha+\epsilon) \, , \qquad \alpha = \frac{C_{sh}}{C} .$$

El Lagrangiano:

$$\mathcal{L} = \frac{1}{2} \vec{\dot\phi}\, \mathcal{C}\, \vec{\dot\phi} + U(\vec{\phi}) + \frac{1}{M_c}\left[\vec{\phi}_+^1 \phi_-^1 + \phi_+^2 \phi_-^2\right]$$

Invertimos $\mathcal{C}$: Expresamos $\mathcal{C} = C(D+\epsilon\tilde{R})$ donde D es diagonal

$$\frac{1}{\mathcal{C}} = \frac{1}{C}\frac{1}{(D+\epsilon R)} = \frac{1}{C}\left(\frac{1}{D} - \frac{1}{D^2}\epsilon\tilde{R}\right) + \mathcal{O}^2(\epsilon)$$

Al orden menor no nulo en ϵ:

$$\frac{1}{\mathcal{C}} = \frac{1}{C}\frac{1}{D}\left(1 - \frac{\epsilon\tilde{R}}{D}\right)^{-1} =$$

$$= \frac{\epsilon}{C}\begin{pmatrix} \frac{1}{\tilde{C}_+} & & 0 & \\ & \frac{1}{\tilde{C}_-} & & \\ 0 & & \frac{1}{\tilde{C}_+} & \\ & & & \ddots \end{pmatrix} \qquad \frac{1}{C} R \frac{1}{D^2}$$

El Hamiltoniano así:

$$\frac{1}{G} = \frac{1}{G}\,\frac{1}{\left(D+\frac{\epsilon}{2}R\right)} \cong \frac{1}{C}\left(\frac{1}{D} - \frac{\epsilon}{2D^2}R\right) + \Theta(\epsilon^2)$$

$$= \frac{1}{GD}\left(\mathbb{1} - \frac{\epsilon}{2D}R\right)$$

$$R = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} \frac{\tilde{C}_+}{G} & 0 & 0 & 0 \\ 0 & \frac{\tilde{C}_-}{G} & 0 & 0 \\ 0 & 0 & \frac{\tilde{C}_+}{G} & 0 \\ 0 & 0 & 0 & \frac{\tilde{C}_-}{G} \end{pmatrix}$$

$$\frac{R}{D} = \begin{pmatrix} \frac{G}{\tilde{C}_+} & 0 & 0 & 0 \\ 0 & \frac{G}{\tilde{C}_-} & 0 & 0 \\ 0 & 0 & \frac{G}{\tilde{C}_+} & 0 \\ 0 & 0 & 0 & \frac{G}{\tilde{C}_-} \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 0 & 0 & \frac{G}{\tilde{C}_+} \\ 0 & 0 & -\frac{G}{\tilde{C}_-} & 0 \\ \frac{G}{\tilde{C}_+} & 0 & 0 & 0 \\ 0 & -\frac{1}{\tilde{C}_-} & 0 & 0 \end{pmatrix}$$

$$\frac{1}{D}\left(\mathbb{1} - \frac{\epsilon R}{D}\right) = \begin{pmatrix} \frac{G}{\tilde{C}_+} & 0 & 0 & 0 \\ 0 & \frac{G}{\tilde{C}_+} & 0 & 0 \\ 0 & 0 & \frac{G}{\tilde{C}} & 0 \\ 0 & 0 & 0 & \frac{G}{\tilde{C}_-} \end{pmatrix} \begin{pmatrix} 1 & 0 & \frac{G'\epsilon}{\tilde{G}_+} & 0 \\ 0 & 1 & 0 & -\frac{G'\epsilon}{\tilde{G}_-} \\ \frac{G'\epsilon}{\tilde{G}_+} & 0 & 1 & 0 \\ 0 & -\frac{G'\epsilon}{\tilde{G}_-} & 0 & 1 \end{pmatrix}$$

$=$

$$G^{-1} = \frac{1}{G}\begin{pmatrix} \frac{1}{\tilde{C}_+} & 0 & \frac{G'\epsilon}{\tilde{G}_+^2} & 0 \\ 0 & \frac{1}{\tilde{C}_-} & 0 & -\frac{G'\epsilon}{\tilde{G}_-^2} \\ \frac{G'\epsilon}{\tilde{G}_+^2} & 0 & \frac{1}{\tilde{C}_+} & 0 \\ 0 & -\frac{G'\epsilon}{\tilde{G}_-^2} & 0 & \frac{1}{\tilde{C}_-} \end{pmatrix}$$

El Hamiltoniano:

$$H = \sum_{i=1}^{2} \tilde{H}_i - \frac{\epsilon G'}{\tilde{G}_+^2} q_+^1 q_+^2 + \frac{\epsilon G'}{\tilde{C}_-^2} q_-^1 q_-^2$$

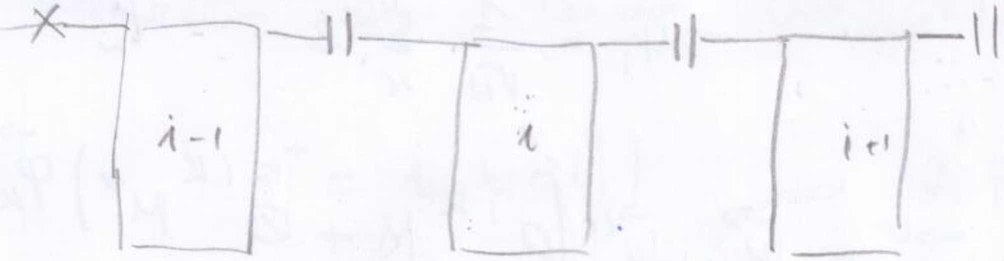
$$+ \quad + \frac{1}{\mu}\left(\phi_+^1 \phi_-^1 + \phi_+^2 \phi_-^2\right).$$

$$H = \frac{1}{2\tilde{G}}\left[(q_+^1)^2 + (q_+^2)^2\right] + U(\vec{\phi}_+) + \frac{1}{\mu}\sum_{i=1}^{2} \phi_+^i \phi_-^i$$

$$+ \frac{G'_q}{\tilde{C}_+^2} q_+^1 q_+^2 + \frac{G_q}{\tilde{C}_-^2} q_-^1 q_-^2$$

Chain of qubits

$$\phi_1 = \frac{1}{2}(\phi_+ + \phi_-),$$
$$\phi_\pm = \phi_1 \pm \phi_2$$
$$\phi_2 = \frac{1}{2}(\phi_+ - \phi_-)$$



$$\mathcal{L} = \sum_{i=1}^{N} \frac{1}{2}\left[C_+(\dot\phi_+^i)^2 + C_-(\dot\phi_-^i)^2\right] + U(\vec\phi_i) + \frac{1}{M}\phi_+^i\phi_-^i$$
$$+ \frac{1}{4} C_c \underbrace{\left[(\dot\phi_1^{i+1} - \dot\phi_2^i)^2 + (\dot\phi_2^i - \dot\phi_1^{i-1})^2\right]}_{\circledast}$$

El término inductivo da renormalización de las masas y acoplo:

$$\circledast = \frac{C_c}{4}\left(\frac{(\dot\phi_+^{i+1} + \dot\phi_-^{i+1} - \dot\phi_+^i + \dot\phi_-^i)^2}{4} + \frac{(\dot\phi_+^i + \dot\phi_-^i - \dot\phi_+^{i-1} + \dot\phi_-^{i-1})^2}{4}\right)$$

$$= \frac{C_c}{8}\sum_{k=i-1}^{i+1}(\dot\phi_+^k)^2 + (\dot\phi_-^k)^2 + \frac{C_c}{4}\left[(\dot\phi_+^{i+1} + \dot\phi_-^{i+1})(-\dot\phi_+^i + \dot\phi_-^i) + (\dot\phi_+^i + \dot\phi_-^i)(-\dot\phi_+^{i-1} + \dot\phi_-^{i-1})\right]$$

Reescribimos el Hamiltoniano:

$$\mathcal{L} = \sum_{i=1}$$

$$\mathcal{L} = \sum_{i=1}^{N} \frac{1}{2} \left[\left(C_+ + \frac{C_q}{2} \right) \dot{\phi}_+^2 + \left(C_- + \frac{C_q}{2} \right) \dot{\phi}_-^2 \right] + U(\varphi_i) + \frac{1}{\mu} \varphi_+^i \varphi$$

$$+ \frac{C_c}{2} \left[-\dot{\phi}_+^{i+1} \dot{\phi}_+^i + \dot{\phi}_-^{i+1} \dot{\phi}_-^i - \dot{\phi}_-^{i+1} \dot{\phi}_+^i + \dot{\phi}_-^{i+1} \dot{\phi}_-^{i+1} \right]$$

~~Hac~~ capacitancias renormalizadas $\tilde{C}_\pm = C_\pm + \frac{C_q}{2}$.

Para simplificar notación hacemos $\Gamma_\pm = \frac{C'}{\tilde{C}_\pm}$.

La matriz de capacitancias: por sitios $i, i+1$:

$$\mathcal{C} = \begin{pmatrix} \tilde{C}_+ & 0 & -\frac{C_c}{2} & \frac{C_c}{2} & & \\ 0 & \tilde{C}_- & -\frac{C_c}{2} & \frac{C_c}{2} & 0 & \\ -\frac{C_c}{2} & \frac{C_c}{2} & \tilde{C}_+ & 0 & -\frac{C_c}{2} & \frac{C_c}{2} \\ -\frac{C_c}{2} & \frac{C_c}{2} & 0 & \tilde{C}_- & -\frac{C_c}{2} & \frac{C_c}{2} \\ & & -\frac{C_c}{2} & \frac{C_c}{2} & \tilde{C}_+ & 0 \\ & 0 & -\frac{C_c}{2} & \frac{C_c}{2} & 0 & \tilde{C}_- \end{pmatrix}$$

Invertimos $\mathcal{C} = C'(D + \epsilon R)$, $\epsilon = \frac{C_c}{2C'}$

$D = \text{diagonal} \left\{ \frac{\tilde{C}_+}{C'}, \frac{\tilde{C}_-}{C'} \right\}$, $R = \begin{pmatrix} \boxed{0} & \begin{matrix} -1 & 1 \\ -1 & 1 \end{matrix} & 0 \\ \begin{matrix} -1 & 1 \end{matrix} & \boxed{0} & \begin{matrix} -1 & 1 \\ -1 & 1 \end{matrix} \end{pmatrix}$

$$\frac{1}{\mathcal{C}} = \frac{1}{G}\left[\frac{1}{D} - \frac{\epsilon R}{D^2}\right] \qquad \Gamma_{\pm} = \frac{G}{\tilde{C}_{\pm}}$$

$$\frac{R}{D^2} = \begin{pmatrix} \Gamma_+^2 & 0 & 0 & 0 \\ 0 & \Gamma_-^2 & 0 & 0 \\ 0 & 0 & \Gamma_+^2 & 0 \\ 0 & 0 & 0 & \Gamma_-^2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 0 & -\Gamma_+^2 & \Gamma_+^2 \\ 0 & 0 & -\Gamma_-^2 & \Gamma_-^2 \\ -\Gamma_+^2 & \Gamma_+^2 & 0 & 0 \\ -\Gamma_-^2 & \Gamma_-^2 & 0 & 0 \end{pmatrix}$$

$$\mathcal{C}^{-1} = \frac{1}{G}\begin{pmatrix} \Gamma_+ & 0 & \epsilon\Gamma_+^2 & -\epsilon\Gamma_+^2 \\ 0 & \Gamma_- & \epsilon\Gamma_-^2 & -\epsilon\Gamma_-^2 \\ \epsilon\Gamma_+^2 & -\epsilon\Gamma_+^2 & \Gamma_+ & 0 \\ \epsilon\Gamma_-^2 & -\epsilon\Gamma_-^2 & 0 & \Gamma_- \end{pmatrix} + O^2(\epsilon)$$

El Hamiltoniano es:

$$\mathcal{H} = \sum_{1=1}^{N} \tilde{\mathcal{H}}_i + \frac{1}{M} \phi_+^i \phi_-^i +$$

$$+ \frac{1}{G} \sum_{1=1}^{N} \Big[\epsilon r_+^2 \left(q_+^{i+1} q_+^i + q_-^{i+1} q_+^i \right)$$

$$- \epsilon r_-^2 \left(q_+^{i+1} q_-^i + q_-^{i+1} q_-^i \right) \Big]$$

Simplifica $\frac{\epsilon}{G} = G_q$

$$\mathcal{H} = \sum \tilde{\mathcal{H}}_i + \frac{1}{M} \phi_+^i \phi_-^i + \frac{G_q}{2} \sum_{1=1}^{N} \Big[r_+^2 (q_+^{i+1} q_+^i + q_-^{i+1} q_+^i$$

$$- r_-^2 (q_+^{i+1} q_-^i + q_-^{i+1} q_-^i$$

Pasamos a osciladores armónicos con RWA:

$$q_+^i = i(b_i - b_i^+) \qquad q_-^i = i(a_i - a_i^+)$$

$$\phi_+^i = (b_i + b_i^+), \quad \phi_+^i = (a_i + a_i^+).$$

$$H \cong \sum_{1=1}^{N} \omega_+ b_i^+ b_i + \omega_- a_i^+ a_i + \frac{1}{M} (b_i a_i^+ + b_i^+ a_i)$$

$$- \frac{G_q}{2} \sum_{1=1}^{N} r_+^2 (b_{i+1} b_i^+ + b_{i+1}^+ b_i + a_{i+1} b_i^+ + a_{i+1}^+ b$$

$$- r_-^2 (b_{i+1} a_i^+ + b_{i+1}^+ a_i + a_{i+1}^+ a_i + a_{i+1} a_i^+$$

Expresamos en la base de k:

$$\vec{\psi}_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}, \quad \vec{\psi}_i = \frac{1}{\sqrt{N}} \sum e^{ikr_i} \vec{\psi}_k .$$

tenemos que: $H = \vec{\psi}_i^{+} P \psi_i + \psi_{i+1}^{+} M \psi_i + \psi_i^{+} M_2^{*} \psi_{i+1}$

$$H = \sum_{i=1}^{N-} \vec{\psi}_i^{+} P \vec{\psi}_i + \psi_{i+1}^{+} M \psi_i + \psi_i^{+} \overset{M_2}{\overset{\shortparallel}{M^{tr}}} \psi_{i+1}$$

$$P = \begin{pmatrix} \omega_- & \frac{1}{\mu} \\ \frac{1}{\mu} & \omega_+ \end{pmatrix} = \begin{pmatrix} \frac{\omega_- - \omega_+}{2} & \frac{1}{\mu} \\ \frac{1}{\mu} & -\frac{\omega_- - \omega_+}{2} \end{pmatrix} =$$

$$P = \frac{\omega_- - \omega_+}{2} \sigma^z + \frac{1}{\mu} \sigma^x$$

$$M_1 = \begin{pmatrix} \Gamma_-^2 & -\Gamma_+^2 \\ +\Gamma_-^2 & -\Gamma_+^2 \end{pmatrix}, \quad M_2 = \begin{pmatrix} \Gamma_-^2 & +\Gamma_-^2 \\ -\Gamma_+^2 & -\Gamma_+^2 \end{pmatrix}$$

Transformamos a k:

$$H = \vec{\Psi}_k^{\dagger} \, P \, \vec{\Psi}_k + e^{ik} \, \vec{\Psi}_k^{\dagger} M \vec{\Psi}_k + e^{-ik} \, \Psi_k^{\dagger} M^{tr} \Psi_k$$

$$= \vec{\Psi}_k^{\dagger} \left[P + e^{ik} M + e^{-ik} M^{tr} \right] \vec{\Psi}_k$$

Descompuesta en matrices de Pauli:

$$P = \Delta \sigma^z + \frac{1}{\mu} \sigma^x$$

$$e^{ik} M + e^{-ik} M^{tr} = \begin{pmatrix} 2r_-^2 \cos(k) & -r_+^2 e^{ik} + r_-^2 e^{-ik} \\ r_-^2 e^{ik} - r_+^2 e^{-ik} & -2 r_+^2 \cos(k) \end{pmatrix}$$

$$= +(r_-^2 - r_+^2) \cos(k) \mathbb{1} + (r_-^2 + r_+^2) \cos(k) \, \sigma^z$$

$$+ \, (r_-^2 - r_+^2) \cos(k) \, \sigma^x + (r_+^2 + r_-^2) \sin k \, \sigma^y$$

$$\Rightarrow H = \vec{\Psi}_k M_k \vec{\Psi}_k :$$

$$M_k = \left[\Delta + (r_-^2 + r_+^2) \cos(k) \right] \sigma^z + (r_-^2 - r_+^2) \cos(k)$$

$$\left[1 + (r_-^2 - r_+^2) \cos(k) \right] \sigma^x + (r_+^2 + r_-^2) \sin k \, \sigma^y .$$