Electromagnetics
Part 1

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Course Overview

1. Maxwell's Equations
2. Displacement Current
3. Current Continuity in Optoelectronic Devices
4. Different Forms of Maxwell's Equations
5. Constitutive Equations and Boundary Conditions
6. Energy Conservation
7. Scalar and Vector Potentials
8. Dyadic Green's Functions
9. Near-Field and Far-Field

1. Maxwell's Equations

differential form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (\textit{Faraday's law of induction})$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (\textit{Generalized Ampere's law})$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (\textit{Gauss's law for magnetic field})$$

$$\nabla \cdot \mathbf{D} = \rho \qquad (\textit{Gauss's law for electric field})$$

Why is Maxwell equation important to optoelectronics?

Maxwell equation governs absorption, radiation, propagation, and scattering of light.

Integral form

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\int_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, d\tau$$

Assignment: How about the Faraday's law when a conductor moves in time-varying magnetic field?

$$\xi = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_C (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

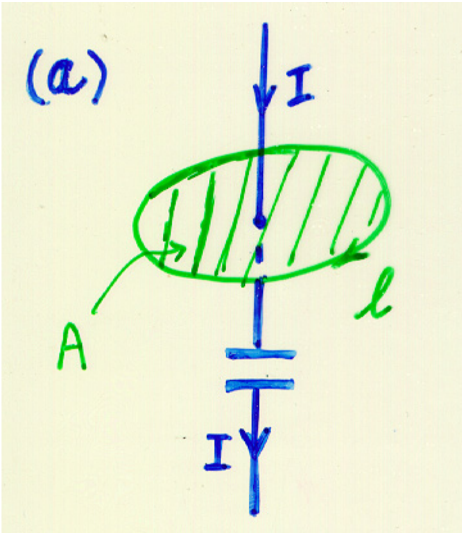
induced + motional

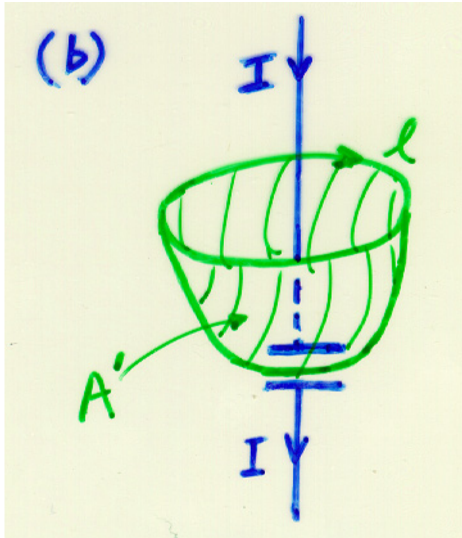
electromotive force

Ref: Xie, Section 2.6

2. Displacement Current (1)

Could time-varying electric field induce magnetic field?
A contradictory thought experiment …





$$\oint_{\ell(A)} \mathbf{H} \cdot d\mathbf{l} = I$$

Ampere's Law

$$\oint_{\ell(A')} \mathbf{H} \cdot d\mathbf{l} = 0$$

Since no current flows across A'

The value of contour integral should be the same for both cases, since the surfaces A & A' are bounded by the same contour.

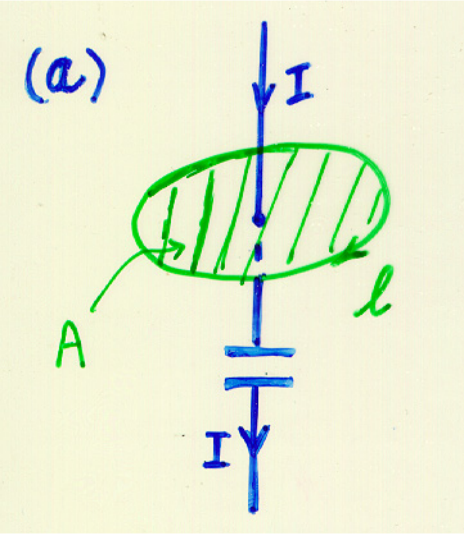
Ref: Xie, Section 2.5.2

2. Displacement Current (2)

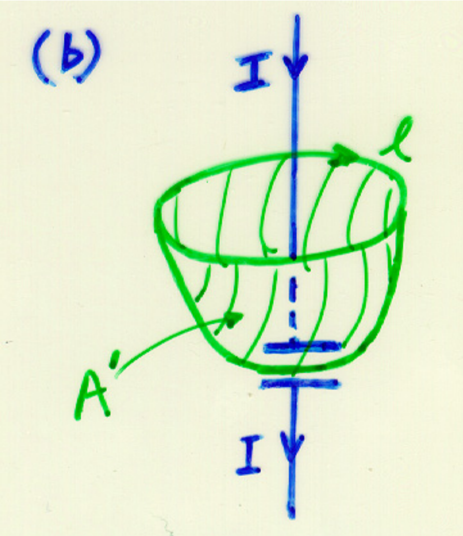
The difficulty can be resolved by assuming the existence of a displacement current through the vacuum between two plates of a capacitor.

The displacement current $\boldsymbol{J}_{\mathbf{D}}$ in the vacuum must be the same as the summation of the conduction current $\boldsymbol{J}$ and displacement current $\boldsymbol{J}_{\mathbf{Dw}}$ in the wire, although $\boldsymbol{J}_{\mathbf{Dw}} << \boldsymbol{J}$. For perfect electric conductor, $\sigma \to \infty$, $\mathbf{E} \to 0$, $\mathbf{J}$ is finite, $\mathbf{J}_{Dw} \to 0$.

$$\oint_{\ell(A)} \mathbf{H} \cdot d\mathbf{l} = \iint_{A(\ell)} \mathbf{J} \cdot d\mathbf{S} + \iint_{A(\ell)} \mathbf{J}_{\mathbf{D}} \cdot d\mathbf{S}$$



$$I_D = I$$



$$\oint_{\ell(A)} \mathbf{H} \cdot d\mathbf{l} = I$$

$$\oint_{\ell(A')} \mathbf{H} \cdot d\mathbf{l} = I_D$$

2. Displacement Current (3)

Displacement current is defined in terms of the rate of change of electric displacement field. Displacement current has the units of electric current density, and it has an associated magnetic field just as actual currents do. However it is not an electric current of moving charges (conduction current), but a time-varying electric field. The idea was conceived by James Clerk Maxwell in his 1861.

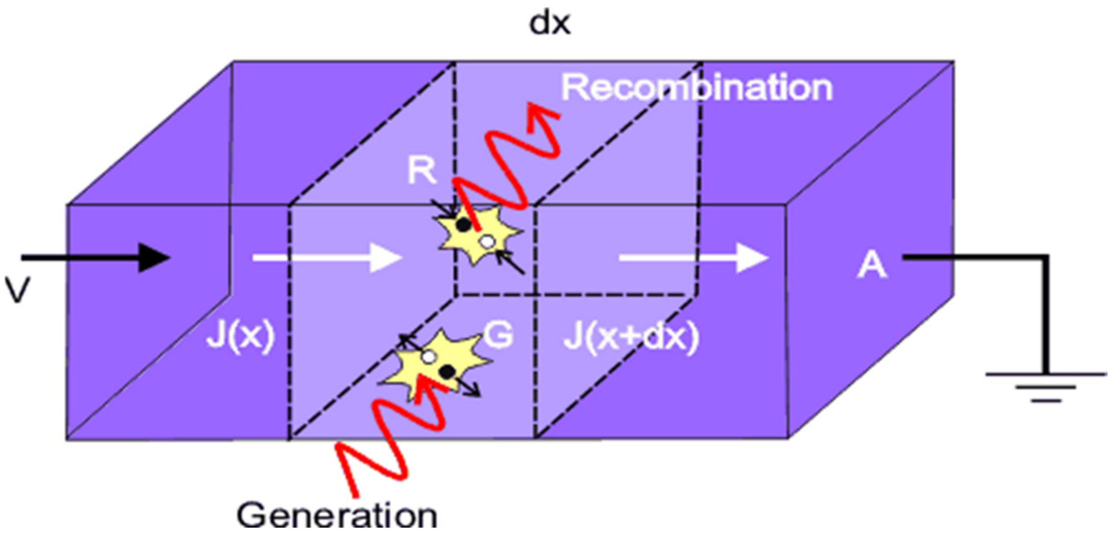
Current continuity $$I = \frac{dQ}{dt} \rightarrow \oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{d}{dt}\int_v \rho dv \rightarrow \nabla \cdot \mathbf{J} = -\frac{d\rho}{dt}$$

Gauss law $$\nabla \cdot \mathbf{D} = \rho$$

Ampère-Maxwell Equation $$\boxed{\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}}$$

Total current continuity $$\nabla \cdot \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) = 0$$

3. Current Continuity in Optoelectronic Devices



$$\nabla \cdot \mathbf{J} = -\frac{d\rho}{dt} + q(G-R)$$

$$\frac{d\rho}{dt} = -\nabla \cdot \mathbf{J} + q(G-R)$$

G: generation rate; R: recombination rate

$$\frac{dp}{dt} = -\frac{\nabla \cdot \mathbf{J}_p}{q} + G - R \quad (\text{Hole})$$

$$\frac{dn}{dt} = \frac{\nabla \cdot \mathbf{J}_n}{q} + G - R \quad (\text{Electron})$$

$$\xrightarrow[\text{DC current}]{\text{steady case}} \quad \nabla \cdot (\mathbf{J}_n + \mathbf{J}_p) = 0$$

$$\int (\mathbf{J}_n + \mathbf{J}_p) \cdot d\mathbf{S} = 0$$

4. Different Forms of Maxwell's Equations (1)

<u>Maxwell's equations (time-harmonic form by Fourier transform)</u>

$$\boxed{\begin{aligned}&\nabla\times\mathbf{E}=-j\omega\mathbf{B}\\&\nabla\times\mathbf{H}=\mathbf{J}+j\omega\mathbf{D}\\&\nabla\cdot\mathbf{B}=0\\&\nabla\cdot\mathbf{D}=\rho\end{aligned}}$$

$$\boxed{\begin{aligned}&\nabla\times\mathbf{E}=i\omega\mathbf{B}\\&\nabla\times\mathbf{H}=\mathbf{J}-i\omega\mathbf{D}\\&\nabla\cdot\mathbf{B}=0\\&\nabla\cdot\mathbf{D}=\rho\end{aligned}}$$

$$\boxed{\exp(j\omega t)}$$

$$\boxed{\exp(-i\omega t)}$$

$$\boxed{A(r,t)=\mathrm{Re}\left[A(r)\exp(j\omega t)\right]}$$

$$\boxed{A(r,t)=\mathrm{Re}\left[A(r)\exp(-i\omega t)\right]}$$

Maxwell's equations is linear !

Ref: Xie, Section 4.5.1-4.5.3

4. Different Forms of Maxwell's Equations (2)

Maxwell's equations (microscopic form or general form)

$$\boxed{\begin{aligned}
&\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
&\nabla \times \mathbf{B} = \mu_0 \underline{(\mathbf{J} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M})} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\
&\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \underline{(\rho - \nabla \cdot \mathbf{P})} \\
&\nabla \cdot \mathbf{B} = 0
\end{aligned}}$$

$$\boxed{\mathbf{J}_{tot} = \mathbf{J} + \mathbf{J}_{pol} + \mathbf{J}_{mag} = \mathbf{J} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}}$$

$$\boxed{\rho_{total} = \rho + \rho_{pol} = \rho - \nabla \cdot \mathbf{P}}$$

What is the fundamental difference between acoustic wave and electromagnetic wave?

Electromagnetic fields are matters themselves which are independent of any media.

Sound waves are not independent matter but the disturbance of material media.

5. Constitutive Equations and Boundary Conditions (1)

Linear, non-dispersive, isotropic

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P} \qquad \mathbf{B} = \mu\mathbf{H} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$$

low frequency

Linear, dispersive, isotropic

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega) \qquad \mathbf{B}(\omega) = \mu(\omega)\mathbf{H}(\omega)$$

Complex permittivity

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D} = \sigma(\omega)\mathbf{E} + j\omega\varepsilon_0\varepsilon_r'(\omega)\mathbf{E}$$

$$= j\omega\varepsilon_0\left(\varepsilon_r'(\omega) - j\frac{\sigma(\omega)}{\omega\varepsilon_0}\right)\mathbf{E} = j\omega\varepsilon_0\varepsilon_r(\omega)\mathbf{E}$$

complex

Complex refractive index

Refractive index n

Extinction coefficient k

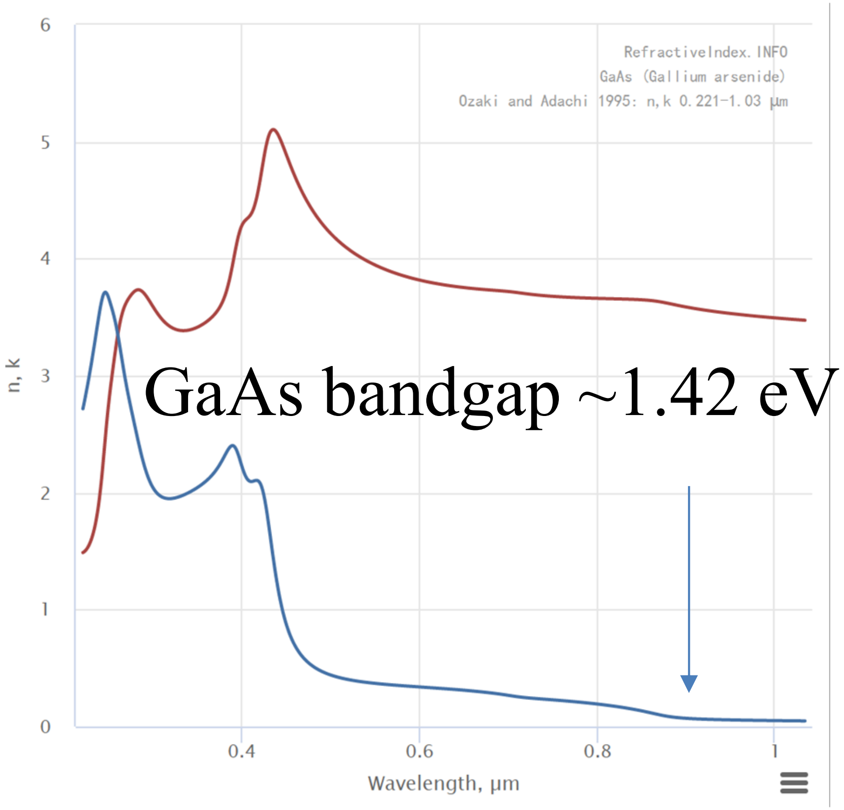
$$n_c(\omega) = \sqrt{\varepsilon_r(\omega)\mu_r(\omega)} = n(\omega) - jk(\omega)$$

Ref: Xie, Section 2.5.2

5. Constitutive Equations and Boundary Conditions (2)

Active materials in optoelectronic devices are highly dispersive at optical frequencies !

Silicon, GaAs, GaP, AlGaAs, polymer, perovskite, etc…



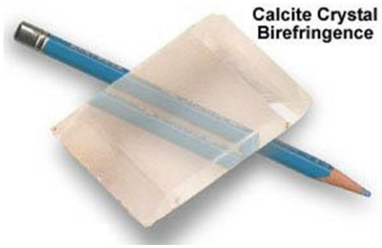
Why do Maxwell's equations have strong prediction capability?

complex refractive index is the only parameter !

5. Constitutive Equations and Boundary Conditions (3)

Material is rich! [An advanced nanocoating technique]

Linear, non-dispersive, anisotropic



$$\mathbf{D} = \overleftrightarrow{\boldsymbol{\varepsilon}}\mathbf{E} \qquad \mathbf{B} = \overleftrightarrow{\boldsymbol{\mu}}\mathbf{H}$$

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}$$

Linear, non-dispersive, bi-anisotropic

$$\mathbf{D} = \overleftrightarrow{\boldsymbol{\varepsilon}}\mathbf{E} + \overleftrightarrow{\boldsymbol{\xi}}\mathbf{H} \qquad \mathbf{B} = \overleftrightarrow{\boldsymbol{\mu}}\mathbf{H} + \overleftrightarrow{\boldsymbol{\varsigma}}\mathbf{E}$$

Nonlinear, dispersive

$$\mathbf{D}(\omega) = \varepsilon_0\mathbf{E}(\omega) + \varepsilon_0\chi^{(1)}(\omega)\mathbf{E}(\omega) + \varepsilon_0\overleftrightarrow{\boldsymbol{\chi}}^{(2)}(\omega_i,\omega_j;\omega=\omega_i+\omega_j)\mathbf{E}(\omega_i)\mathbf{E}(\omega_j) + \cdots$$

Nonlinear effect is weak $\overleftrightarrow{\boldsymbol{\chi}}^{(2)} \ll \chi^{(1)}$

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Boundary Conditions (General)

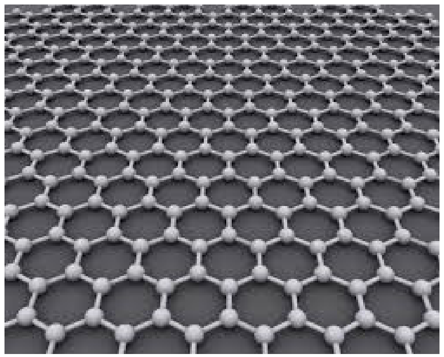
$$
\boxed{
\begin{aligned}
&\mathbf{n}\times(\mathbf{E}_1-\mathbf{E}_2)=0\\
&\mathbf{n}\times(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_S\\
&\mathbf{n}\cdot(\mathbf{D}_1-\mathbf{D}_2)=\rho_S\\
&\mathbf{n}\cdot(\mathbf{B}_1-\mathbf{B}_2)=0
\end{aligned}
}
$$

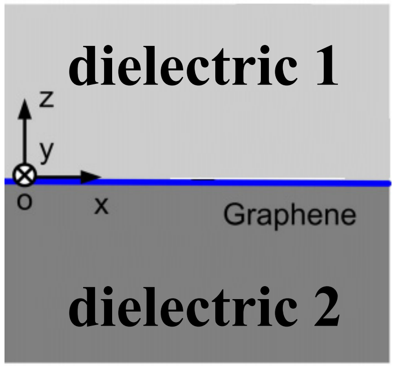
Boundary Conditions (**Dielectric-Dielectric**)

$$\mathbf{J}_S=0 \qquad \rho_S=0$$

$\mathbf{J}_s$: surface conduction current $\qquad$ ρ_s: surface free charge

An interesting case: how to write the boundary condition for a dielectric-graphene -dielectric structure? Graphene is a very thin sheet compared to dielectric layer; and can be seen as an infinitely thin layer.





$$\mathbf{n}\times(\mathbf{H}_1-\mathbf{H}_2)=\sigma_S\mathbf{E}_t$$

$$\mathbf{E}_t=\text{-}\mathbf{n}\times\mathbf{n}\times\mathbf{E}$$

Ref: Xie, Section 2.7

6. Energy Conservation (1)

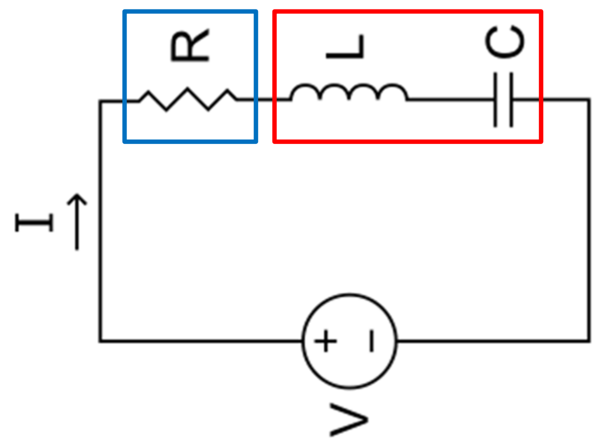
Energy Conservation (Harmonic Field)

$$\langle \mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t)\rangle = \frac{1}{T}\int_0^T \mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t)dt = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}(\mathbf{r})\times\mathbf{H}^*(\mathbf{r})\right]$$

$$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^*$$ <u>Complex Poynting vector</u>

$$\nabla\cdot\left(\mathbf{E}\times\mathbf{H}^*\right) = \mathbf{H}^*\cdot(\nabla\times\mathbf{E}) - \mathbf{E}\cdot\left(\nabla\times\mathbf{H}^*\right)$$

$$\boxed{-\nabla\cdot\mathbf{S} = \frac{1}{2}\mathbf{E}\cdot\mathbf{J}^* + 2j\omega(W_m - W_e)}$$



$$\boxed{W_m = \frac{\mathbf{H}^*\cdot\mathbf{B}}{4}} \qquad \boxed{W_e = \frac{\mathbf{E}\cdot\mathbf{D}^*}{4}}$$

time averaged stored energy density

<u>reactive energy density</u>

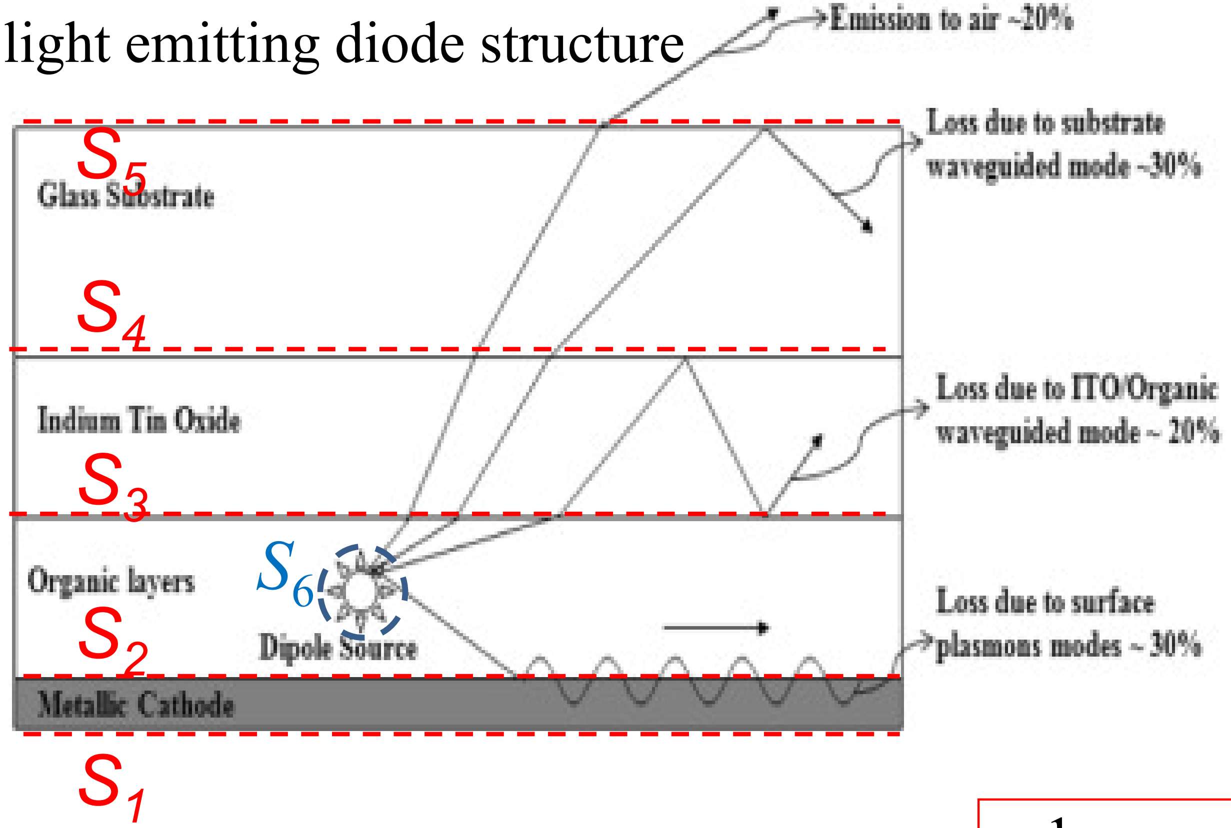
$$\boxed{\frac{1}{2}\mathbf{E}\cdot\mathbf{J}^*}$$

<u>active/real power density</u>

Ref: Xie, Section 4.3 and 4.5.6

6. Energy Conservation (2)

1D light emitting diode structure



EP of a dipole source = EP in air (20%) + DP in device (80%)

EP: emission power

DP: dissipation power

total emitted power from a dipole	$\int_{S_6} \frac{1}{2} \mathrm{Re}\left[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})\right] \cdot d\mathbf{S}$
dissipation power in metal cathode	$-\int_{S_1+S_2} \frac{1}{2} \mathrm{Re}\left[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})\right] \cdot d\mathbf{S}$
emission power in air	$\int_{S_{upper}^{\infty}} \frac{1}{2} \mathrm{Re}\left[\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})\right] \cdot d\mathbf{S}$

7. Scalar and Vector Potentials (1)

Vector potential **A**

$$\nabla \cdot \mathbf{B} = 0 \qquad\qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Scalar potential $\boldsymbol{\phi}$

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B} \qquad \nabla \times (\mathbf{E} + j\omega\mathbf{A}) = 0 \qquad \mathbf{E} = -j\omega\mathbf{A} - \nabla\phi$$

Generalized Ampere's law by **A** and $\boldsymbol{\phi}$ (homogeneous space)

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D} \qquad\qquad \frac{1}{\mu}\nabla \times \nabla \times \mathbf{A} = \mathbf{J} + j\omega\varepsilon(-j\omega\mathbf{A} - \nabla\phi)$$

$$\nabla \times \nabla \times \mathbf{A} = \mu\mathbf{J} + k^2\mathbf{A} - j\omega\mu\varepsilon\nabla\phi \qquad\qquad \nabla\nabla \cdot \mathbf{A} - \nabla^2\mathbf{A} = \mu\mathbf{J} + k^2\mathbf{A} - j\omega\mu\varepsilon\nabla\phi$$

Ref: Xie, Section 4.2 and 4.5.5; Chew Section 23.1.3

7. Scalar and Vector Potentials (2)

Lorenz Gauge

$$\boxed{\nabla \cdot \mathbf{A} = -j\omega\mu\varepsilon\phi}$$

Governing equation of vector potential

$$\boxed{\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J}}$$

Governing equation of scalar potential

$$\nabla \cdot \mathbf{D} = \rho \quad \rightarrow \quad \nabla \cdot (-j\omega\mathbf{A} - \nabla\phi) = \frac{\rho}{\varepsilon} \quad \rightarrow \quad \boxed{\nabla^2 \phi + k^2 \phi = -\frac{\rho}{\varepsilon}}$$

Gauge Transformation

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{E} = -j\omega\mathbf{A} - \nabla\phi$$

$$\boxed{\begin{array}{l} A \rightarrow A + \nabla\Lambda \\ \phi \rightarrow \phi - j\omega\Lambda \end{array}} \qquad \boxed{\nabla^2 \Lambda + k^2 \Lambda = 0}$$

Gauge is not unique but **E** and **B** are unique!

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7. Scalar and Vector Potentials (3)

Gauge has deeper meaning …. (Optional)

Symmetry (momentum, energy, angular momentum conservation, ..)

Electromagnetism (A), Strong interaction (B), Weak interaction (C)

A: Quantum electrodynamics (1940) — Symmetry group U(1)
B-: Yang-Mills gauge theory (1954) — Symmetry group SU(2)
C: Electroweak theory (1971) — Symmetry group SU(2)×U(1)
B+: Quantum chromodynamics (1973) — Symmetry group SU(3)
A & B & C: Standard model (1975) — Symmetry group SU(3)×SU(2)× U(1)

Final target (Optional)

Unify Gravitation (D) with Electromagnetism, Strong interaction, and Weak interaction.

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8. Dyadic Green's Functions (1)

$$\nabla\times\mathbf{E} = -j\omega\mathbf{B}$$
$$\nabla\times\mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$$

$$\longrightarrow \quad \nabla\times\nabla\times\mathbf{E} = -j\omega\nabla\times\mathbf{B} = -j\omega\mu_0\mathbf{J} + \omega^2\mu_0\varepsilon_0\mathbf{E}$$

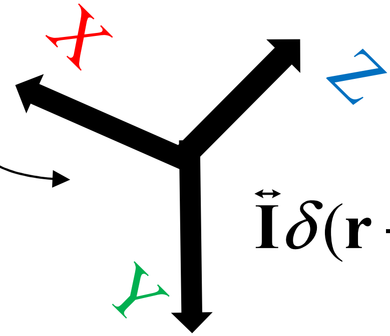
$$\boxed{\nabla\times\nabla\times\mathbf{E} - k_0^2\mathbf{E} = -j\omega\mu_0\mathbf{J}}$$

Vector wave equation (free space)

$$\boxed{\nabla\times\nabla\times\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') - k_0^2\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \overleftrightarrow{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}')}$$

Dyadic Green's function (free space)

Hertzian dipole source



$$\overleftrightarrow{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}')$$

Radiation field calculation

$$\boxed{\mathbf{E} = -j\omega\mu_0\int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')d\tau'}$$

$$\boxed{\mathbf{H} = \nabla\times\int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')d\tau'}$$

E-field produced by y polarized source

$$\overleftrightarrow{G} = \begin{pmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{pmatrix}$$

Ref: Xie, Section 8.1 and 8.2

8. Dyadic Green's Functions (2)

Scalar Green's function

$$\boxed{\nabla^2 g(\mathbf{r},\mathbf{r}') + k_0^2 g(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')}$$

$$\boxed{g(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}}$$

Solutions to scalar and vector potentials

$$\nabla^2\phi + k_0^2\phi = -\frac{\rho}{\varepsilon_0} \longrightarrow \phi(\mathbf{r}) = \frac{1}{\varepsilon_0}\int_V g(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')d\tau'$$

$$\nabla^2\mathbf{A} + k_0^2\mathbf{A} = -\mu_0\mathbf{J} \qquad \mathbf{A}(\mathbf{r}) = \mu_0\int_V g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')d\tau'$$

Solution to E-field

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\phi = -j\omega\mu_0\int_V g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')d\tau' + \frac{\nabla}{j\omega\varepsilon_0}\int_V g(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{J}(\mathbf{r}')d\tau'$$

$$= -j\omega\mu_0\int_V g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')d\tau' + \frac{1}{j\omega\varepsilon_0}\int_V \nabla\nabla g(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')d\tau'$$

$$= -j\omega\mu_0\int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')d\tau'$$

Dyadic Green's function

$$\boxed{\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left(\overleftrightarrow{\mathbf{I}} + \frac{\nabla\nabla}{k_0^2}\right)g(\mathbf{r},\mathbf{r}')}$$

8. Dyadic Green's Functions (3)

$$\mathbf{E}(\mathbf{r}) = -j\omega\mu_0 \int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\tau'$$

$$\mathbf{J} = Il\delta(\mathbf{r}')\mathbf{a}_z$$

Near-field	$\mathbf{E}(\mathbf{r}) \sim \dfrac{1}{R^3}$
Mid-field	$\mathbf{E}(\mathbf{r}) \sim \dfrac{1}{R^2}$
Far-field	$\mathbf{E}(\mathbf{r}) \sim \dfrac{1}{R}$

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left(\overleftrightarrow{\mathbf{I}} + \frac{\nabla\nabla}{k_0^2} \right) g(\mathbf{r},\mathbf{r}')$$

$$\overleftrightarrow{\mathbf{G}}(R) = G_1(R)\overleftrightarrow{\mathbf{I}} + G_2(R)\mathbf{a}_R\mathbf{a}_R$$

$$R = |\mathbf{r} - \mathbf{r}'|$$

$$G_1(R) = (-1 - jkR + k^2R^2)\frac{e^{-jkR}}{4\pi k^2 R^3}$$

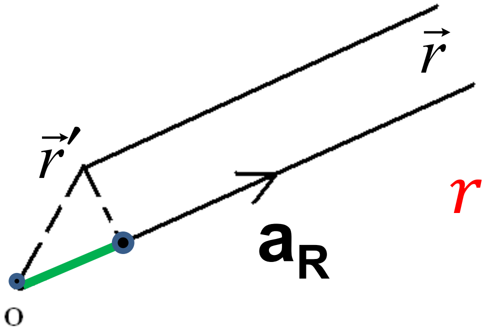
$$G_2(R) = (3 + 3jkR - k^2R^2)\frac{e^{-jkR}}{4\pi k^2 R^3}$$

Why does E-field decay 1/R at far field and $1/R^3$ at near field?

The former is due to energy conservation. The latter is due to electrostatic physics by the Hertzian dipole source.

9. Near-Field and Far-Field (1)

Far-field condition



$r \to \infty$

$$R \approx r - \boxed{\mathbf{r}' \cdot \mathbf{a_R}}$$

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \mathbf{H} \qquad \nabla \times \mathbf{H} = j\omega\varepsilon_0 \mathbf{E}$$

$$\xrightarrow{r \to \infty}$$

$$j\mathbf{k}_0 \times \mathbf{E} = -j\omega\mu_0 \mathbf{H} \qquad j\mathbf{k}_0 \times \mathbf{H} = j\omega\varepsilon_0 \mathbf{E}$$

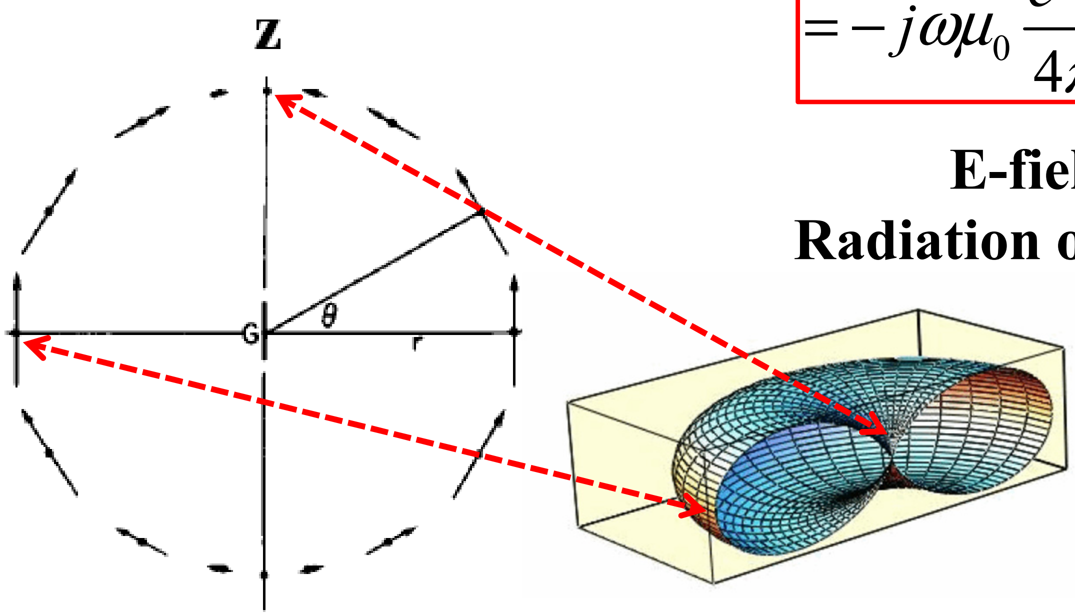
angular spectrum

$$\begin{aligned}
\mathbf{E}(\mathbf{k}) &= -j\omega\mu_0 \int_v \left(\overleftrightarrow{\mathbf{I}} - \mathbf{a}_R\mathbf{a}_R\right) g(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\tau' \\
&= -j\omega\mu_0 \int_v \left(\mathbf{a}_\theta\mathbf{a}_\theta + \mathbf{a}_\varphi\mathbf{a}_\varphi\right) g(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\tau' \\
&= -j\omega\mu_0 \frac{e^{-jkr}}{4\pi r} \int_v \left(\mathbf{a}_\theta\mathbf{a}_\theta + \mathbf{a}_\varphi\mathbf{a}_\varphi\right) \cdot \mathbf{J}(\mathbf{r}') \exp(jk\mathbf{r}' \cdot \mathbf{a}_R) d\tau' \\
&= -j\omega\mu_0 \frac{e^{-jkr}}{4\pi r} \int_v \left(\mathbf{a}_\theta J_\theta(\mathbf{r}') + \mathbf{a}_\varphi J_\varphi\right) \exp(jk\mathbf{r}' \cdot \mathbf{a}_R) d\tau'
\end{aligned}$$

Radiative E-field of dipole source

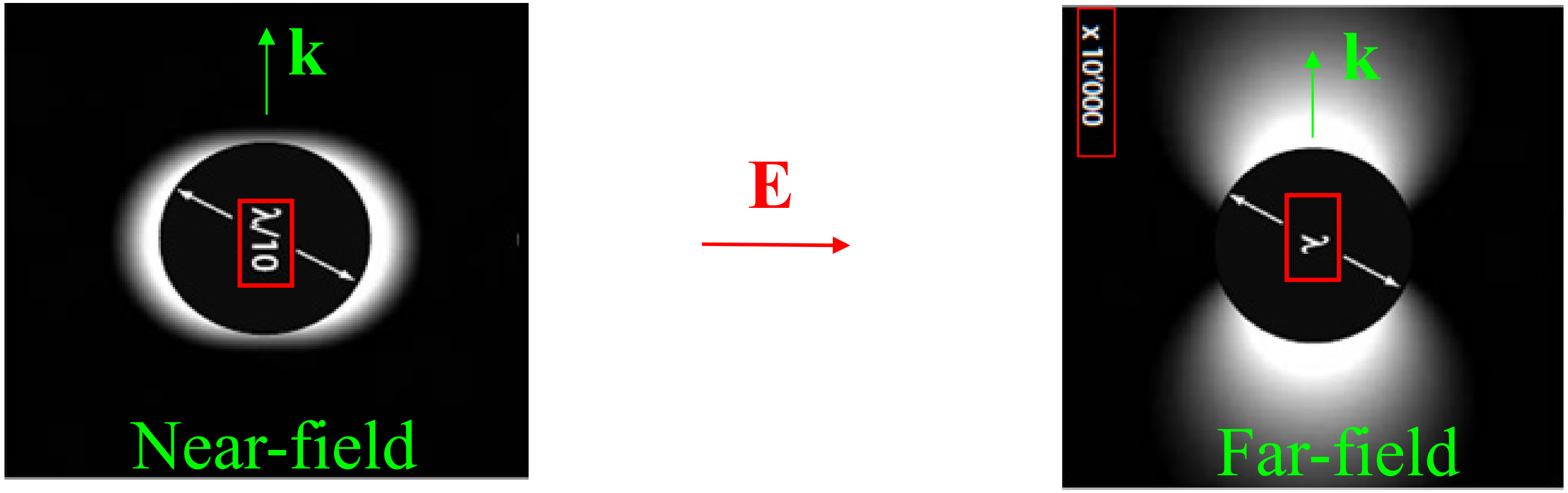
E-field is transverse at far-field region!

Radiation of E-field is Fourier transform of source!

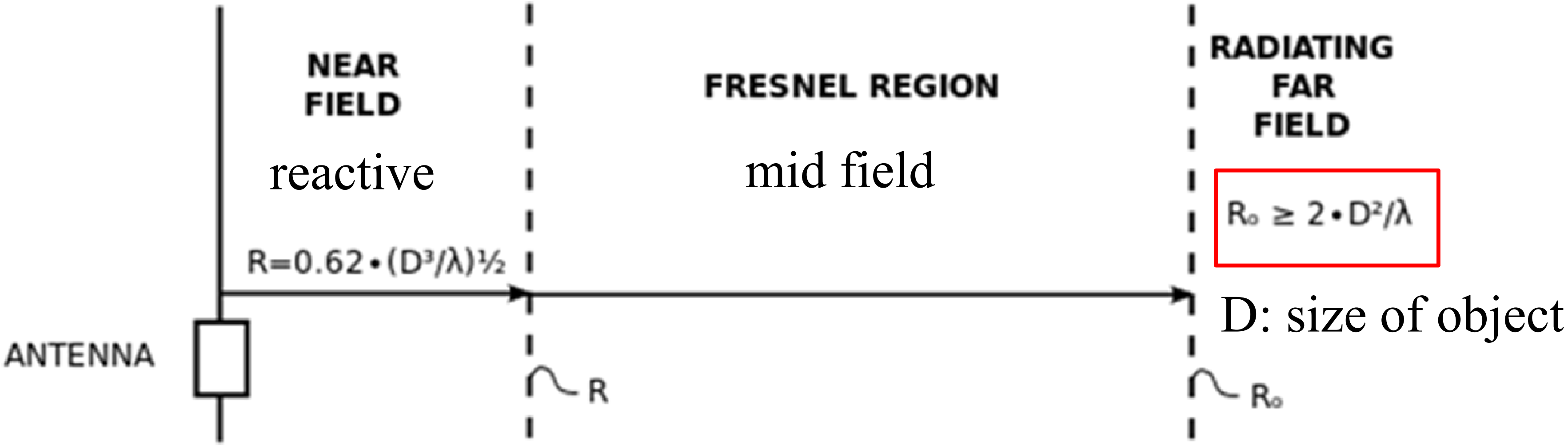


far-field radiation pattern

9. Near-Field and Far-Field (2)



1. E-field is concentrated along the polarization (horizontal) direction at near-field and along the propagation (vertical) direction at far-field;
2. E-field intensity at near-field is much stronger than that at far-field (x10,000 times in figure).



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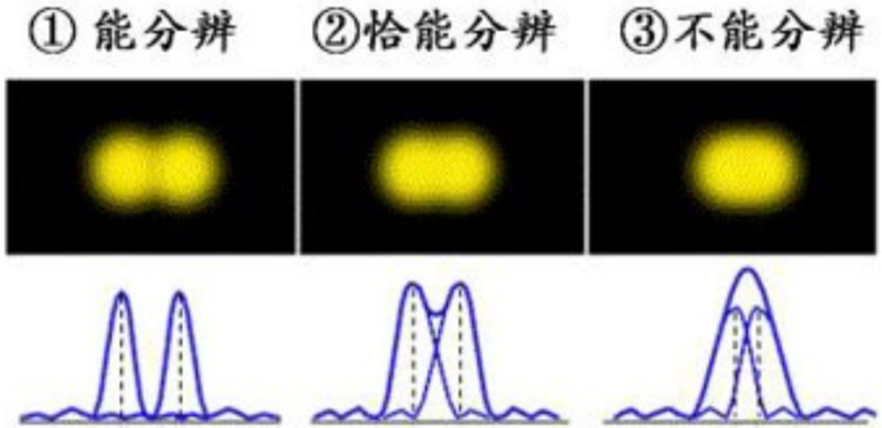
9. Near-Field and Far-Field (3) —— Applications

1. Radar and Laser

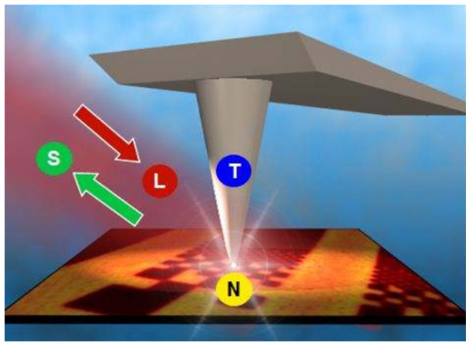
2. Imaging and Scanning Near-Field Optical Microscope (SNOM)

3. Organic Solar Cells (OSCs)

imaging



SNOM



OSCs