1. NURBS in 1D

For convenience, we let,

$$N_A^{(k)} = w_A \, \frac{d^k N_A}{d\xi^k}, \quad \Sigma^{(k)} = \sum_A N_A^{(k)}$$

0th derivative.

$$R_A^{(0)} = \frac{1}{\Sigma^{(0)}} \left[N_A^{(0)} \right]$$

1st derivative.

$$R_A^{(1)} = \frac{1}{\Sigma^{(0)}} \left[N_A^{(1)} - \Sigma^{(1)} R_A^{(0)} \right]$$

2nd derivative.

$$R_A^{(2)} = \frac{1}{\Sigma^{(0)}} \left[N_A^{(2)} - \Sigma^{(2)} R_A^{(0)} - 2\,\Sigma^{(1)} R_A^{(1)} \right]$$

3rd derivative.

$$R_A^{(3)} = \frac{1}{\Sigma^{(0)}} \left[N_A^{(3)} - \Sigma^{(3)} R_A^{(0)} - 3\,\Sigma^{(2)} R_A^{(1)} - 3\,\Sigma^{(1)} R_A^{(2)} \right]$$

k-th derivative.

$$R_A^{(k)} = \frac{1}{\Sigma^{(0)}} \left[N_A^{(k)} - \sum_{j=0}^{k-1} \binom{k}{j} \Sigma^{(k-j)} R_A^{(j)} \right]$$

2. NURBS in 2D

For convenience, we let,

$$N_A^{(k_1 k_2)} = w_A \, \frac{\partial^k N_A}{\partial \xi^{k_1} \partial \eta^{k_2}}, \quad \Sigma^{(k_1 k_2)} = \sum_A N_A^{(k_1 k_2)}$$

0th derivative.

$$R_A^{(00)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(00)} \right]$$

1st derivatives.

$$R_A^{(10)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(10)} - \Sigma^{(10)} R_A^{(00)} \right]$$

$$R_A^{(01)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(01)} - \Sigma^{(01)} R_A^{(00)} \right]$$

2nd derivatives.

$$R_A^{(20)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(20)} - \Sigma^{(20)} R_A^{(00)} - 2\,\Sigma^{(10)} R_A^{(10)} \right]$$

$$R_A^{(11)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(11)} - \Sigma^{(11)} R_A^{(00)} - \Sigma^{(01)} R_A^{(10)} - \Sigma^{(10)} R_A^{(01)} \right]$$

$$R_A^{(02)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(02)} - \Sigma^{(02)} R_A^{(00)} - 2\,\Sigma^{(01)} R_A^{(01)} \right]$$

3rd derivatives.

$$R_A^{(30)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(30)} - \Sigma^{(30)} R_A^{(00)} - 3\,\Sigma^{(20)} R_A^{(10)} - 3\,\Sigma^{(10)} R_A^{(20)} \right]$$

$$R_A^{(21)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(21)} - \Sigma^{(21)} R_A^{(00)} - 2\,\Sigma^{(11)} R_A^{(10)} - \Sigma^{(01)} R_A^{(20)} - \Sigma^{(20)} R_A^{(01)} - 2\,\Sigma^{(10)} R_A^{(11)} \right]$$

$$R_A^{(12)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(12)} - \Sigma^{(12)} R_A^{(00)} - \Sigma^{(02)} R_A^{(10)} - 2\,\Sigma^{(11)} R_A^{(01)} - 2\,\Sigma^{(01)} R_A^{(11)} - \Sigma^{(10)} R_A^{(02)} \right]$$

$$R_A^{(03)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(03)} - \Sigma^{(03)} R_A^{(00)} - 3\,\Sigma^{(02)} R_A^{(01)} - 3\,\Sigma^{(01)} R_A^{(02)} \right]$$

k-th derivatives.

$$R_A^{(k_1 k_2)} = \frac{1}{\Sigma^{(00)}} \left[N_A^{(k_1 k_2)} - \sum_{j_2=0}^{J_2} \sum_{j_1=0}^{J_1} \binom{k_1}{j_1} \binom{k_2}{j_2} \Sigma^{(k_1-j_1)(k_2-j_2)} R_A^{(j_1 j_2)} \right]$$

$$J_2 = \min(k-1,\, k_2)$$

$$J_1 = \min(k-1-j_2,\, k_1)$$

3. NURBS in 3D

For convenience, we let,

$$N_A^{(k_1 k_2 k_3)} = w_A \, \frac{\partial^k N_A}{\partial \xi^{k_1} \partial \eta^{k_2} \partial \zeta^{k_3}}, \quad \Sigma^{(k_1 k_2 k_3)} = \sum_A N_A^{(k_1 k_2 k_3)}$$

0th derivative.

$$R_A^{(000)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(000)} \right]$$

1st derivatives.

$$R_A^{(100)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(100)} - \Sigma^{(100)} R_A^{(000)} \right]$$

$$R_A^{(010)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(010)} - \Sigma^{(010)} R_A^{(000)} \right]$$

$$R_A^{(001)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(001)} - \Sigma^{(001)} R_A^{(000)} \right]$$

2nd derivatives.

$$R_A^{(200)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(200)} - \Sigma^{(200)} R_A^{(000)} - 2\,\Sigma^{(100)} R_A^{(100)} \right]$$

$$R_A^{(110)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(110)} - \Sigma^{(110)} R_A^{(000)} - \Sigma^{(010)} R_A^{(100)} - \Sigma^{(100)} R_A^{(010)} \right]$$

$$R_A^{(101)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(101)} - \Sigma^{(101)} R_A^{(000)} - \Sigma^{(001)} R_A^{(100)} - \Sigma^{(100)} R_A^{(001)} \right]$$

$$R_A^{(020)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(020)} - \Sigma^{(020)} R_A^{(000)} - 2\,\Sigma^{(010)} R_A^{(010)} \right]$$

$$R_A^{(011)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(011)} - \Sigma^{(011)} R_A^{(000)} - \Sigma^{(001)} R_A^{(010)} - \Sigma^{(010)} R_A^{(001)} \right]$$

$$R_A^{(002)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(002)} - \Sigma^{(002)} R_A^{(000)} - 2\,\Sigma^{(001)} R_A^{(001)} \right]$$

k-th derivatives.

$$R_A^{(k_1 k_2 k_3)} = \frac{1}{\Sigma^{(000)}} \left[N_A^{(k_1 k_2 k_3)} - \sum_{j_3=0}^{J_3} \sum_{j_2=0}^{J_2} \sum_{j_1=0}^{J_1} \binom{k_1}{j_1} \binom{k_2}{j_2} \binom{k_3}{j_3} \Sigma^{(k_1-j_1)(k_2-j_2)(k_3-j_3)} R_A^{(j_1 j_2 j_3)} \right]$$

$$J_3 = \min(k-1,\, k_3)$$

$$J_2 = \min(k-1-j_3,\, k_2)$$

$$J_1 = \min(k-1-j_3-j_2,\, k_1)$$