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Abstract

We study the spreading of a droplet of surfactant solution on a thin suspended soap film as a function of dynamic surface tension and volume of the droplet. Radial growth of the leading edge (R) shows a power-law dependence on time with exponents ranging roughly from 0.1 to 1 for different surface tension differences ($\Delta\sigma$) between the film and the droplet. When the surface tension of the droplet is lower than the surface tension of the film ($\Delta\sigma>0$), we observe rapid spreading of the droplet with $R \sim t^{\alpha}$ where α ($0.4 < \alpha < 1$) is highly dependent on $\Delta\sigma$. Balance arguments assuming the spreading process is driven by Marangoni stresses versus inertial stresses yield $\alpha=2/3$. When the surface tension difference does not favour spreading ($\Delta\sigma<0$), spreading still occurs but is slow with $0.1 < \alpha < 0.2$. This phenomenon could be used for stretching droplets in 2D and modifying thin suspended films.

1. Introduction

The dynamics of the spreading of a droplet over a solid or liquid surface is a phenomenon that has attracted much attention in the past decades [1-2]. Beside its relevance to many fields of technology [3-5], studying the dynamics of spreading can provide understanding of the various forces acting on an interface.

It has been shown that depositing a droplet on a solid surface will initially result in the formation of a precursor film [6-7] with a molecular thickness [8], which decreases the spreading rate of the main droplet. However, Cazabat *et al.* [9] showed that spreading a droplet on a liquid substrate exhibits faster dynamics, and an even faster spreading has been observed in a drop-on-drop geometry [10]. In these previous studies, the radius of the spreading front increases as a function of time according to a power law. The exponent of this power-law growth depends on physicochemical properties of both the spreading liquid and the substrate; e.g. surface energy, roughness, fluidity[11], miscibility [12], immiscibility [13] and the depth of the substrate for liquid substrates [9, 14-15]. When capillary forces are driving the spreading of a drop on a solid substrate and inertial effects are negligible, a balance between viscous forces and capillary forces predicts a slow time evolution of the spreading front, $R \sim t^{1/10}$, known as Tanner's law [16-18]. When two different liquids are brought into contact with each other, it is often the difference in the surface tensions (Marangoni stress) that drives the spreading [19]. For spreading on a thin liquid substrate where the thickness of the film is much smaller than the radius of the spreading front, the

1   lubrication approximation applies. Jensen *et al.* [20] have shown that the when viscous effects dominate

2   inertial effects the width of a planar monolayer strip containing a fixed mass of surfactant spreads in

3   time as $t^{1/3}$, and the radius of an axisymmetric monolayer droplet evolves as $t^{1/4}$. Numerical studies

4   [21] also confirmed the exponent of 1/4 for spreading of a droplet on a rigid substrate.

5   Although spreading has been studied under various conditions, spreading a droplet on a soap film of a

6   few micrometres thickness, suspended in the air, hasn't been studied yet to the best of our knowledge.

7   Here, we study a specific case of the droplet spreading on a liquid substrate: we deposit droplets of

8   surfactant solution on a suspended thin soap film and document the spreading dynamics as a function

9   of surface tension and droplet size using high-speed imaging. In contrast to our experimental system,

10 thin liquid substrates studied in previous works [15] were at least a few millimetres thick, and in contact

11 with a solid boundary, which causes significant viscous dissipation due to the no-slip boundary condi-

12 tion.

13

2.   Experimental section

15 To make a horizontal soap film, we used a concentric cylinder made of stainless steel with inner and

16 outer diameters of 39.5 and 40.0 mm respectively. The concentric cylinder was dipped into sodium

17 dodecyl sulphate (SDS) (Merck) solution to produce a horizontal soap film. Then a droplet of ammo-

18 nium lauryl sulphate (ALS) (Fluka) solution or SDS solution was deposited on the soap film using a

19 capillary tube. The average volume of deposited droplets was 6±0.5 μL, although in one series of

20 experiments, we varied the deposited volume to see how spreading depends on the droplet size. To

21 vary the droplet size capillary tubes with different diameters were used; for very small droplet sizes,

22 the liquid was sprayed very close to the soap film. SDS solutions with molar concentrations of 0.005

23 M were used to make the initial soap films, and SDS solutions with concentrations of 0.005, 0.008,

24 0.01 and 0.02 M and ALS solutions with concentrations of 0.001, 0.002, 0.003, 0.005, 0.01, 0.04, 0.1

25 and 0.3 M were used for the droplets deposited on the soap film. The critical micelle concentrations

26 (CMC) of ALS and SDS solutions at 25ºC are 0.0065 M [22] and 0.0082 M [23], respectively. For the

27 deposited droplets, the concentrations of solutions corresponded to a range from 0.15 CMC to 46 CMC

28 and 0.6 CMC to 2.4 CMC for ALS and SDS, respectively. Dynamic surface tensions of solutions were

29 measured via the bubble pressure method using a Krüss BP50 bubble surface tensiometer. The thick-

30 ness of the initial soap film was estimated indirectly from the speed of a traveling mechanical wave on

31 the soap film using the equation for wave speed on an elastic sheet under tension (see supporting

32 information). After depositing an ALS droplet, the evolution of the deposited drop on the soap film

33 was recorded using a high-speed camera (Phantom) at a rate of 4000 frames per second. To illuminate

34 the surface of the soap film, we used a white light source with the incident and reflected angles of

35 ~45º. The radius of the leading edge was measured by pixel counting on the frames.

36

3.   Results and discussion

38 We focus on the spreading of a droplet on a suspended liquid film of a few microns thickness. This is

39 different from other work on Marangoni spreading, where the minimum thickness of liquid substrates

[15] was typically about a few millimetres, and where the substrates are in contact with a solid boundary. Therefore, we expect the viscous dissipation in our system to be much smaller, which can lead to faster spreading. Furthermore, having a suspended soap film allows us to confine the liquid-liquid interface solely to a circular rim.

To investigate the position of the droplet and the soap film during the course of spreading, we first made qualitative observations. Fig. 1 shows an example of spreading a droplet of 0.4 M ALS on a 0.005 M SDS soap film. In Fig. 2, spreading of a droplet of ALS 0.4 M on an SDS 0.005 M soap film is shown in which a fluorescent dye was added to the SDS solution. The sequence of images reveals that, after deposition of the droplet, a non-fluorescent circular film is formed in the centre, gradually growing and pushing the fluorescent SDS soap film towards the edge of the container. The absence of fluorescence in the spreading film indicates that the ALS droplet is hardly contaminated by the initial soap film.

This qualitative observation is confirmed by calculating the ratio of advection rate to diffusion rate of our system. This ratio, the Peclet number, is given by $Pe=\frac{lu}{D}$ where l is a characteristic length of the system, u is the average flow velocity and D is the diffusion coefficient. By considering the size of the droplet (~1 mm) as the characteristic length, the lowest spreading speed (~0.1 m/s) and diffusion co-efficient of $D \sim 10^{-9}$ m^{2}s^{-1} (for a typical surfactant in water [24]), we arrive at a Peclet number significantly greater than 1, confirming that diffusion at the boundaries is negligible during the course of fast spreading.

3.1. Effect of surface tension

Fig. 3 shows the dynamic surface tension for different concentrations of ALS and SDS as a function of the surface age measured using a bubble pressure surface tensiometer. To determine the effective surface tension difference ($\Delta\sigma$) between the droplets and the soap film in our experiments, we use the time-dependent data of Fig. 3. For the ALS droplet, we consider a characteristic time scale given by $\tau = {L}/{u}$ where u is the average speed of the spreading film and L is the maximum radius of the spreading film (~1 cm). For the SDS substrate, in each experiment there was at least a few seconds between making the soap film and depositing the ALS droplet, therefore the surface tension of the SDS at 5 s (Fig. 3) is used to calculate $\Delta\sigma$. Taking this into account, the surface tension difference between droplets (ALS) and soap film (SDS) cover a wide range, from -19.4 mN/m for ALS 0.001 M to 6.3 mN/m for ALS 0.3 M.

In order to identify how surface tension regulates spreading, droplets with different concentrations of ALS and SDS (different surface tensions) were deposited on the soap film containing SDS 0.005 M. After the moment of deposition, the radius (R) of the leading front was measured as a function of time using high-speed imaging. The results are shown in Fig. 4a and b for ALS and SDS droplets spreading on an SDS film, respectively. For the experiments where the radius of the leading front went beyond 1 cm, a cross-over regime is observed (open symbols in Fig. 4), probably due to boundary effects (the radius of the cylinder is 2 cm). After discarding the data points at and beyond the cross-over region, a clear power-law behaviour for R versus time is observed ($R \propto t^{\alpha}$). By fitting power-law functions to the closed-symbol data sets, we find exponents α ranging from 0.11 to 0.9 for different surface tension

3

1 differences (Fig. 5a). Power-law growth of spreading fronts has been observed in many other spreading
2 phenomena [9, 12, 15, 25].

3 For droplets with low concentrations of ALS, where the surface tension of the droplets is larger than
4 or equal to the surface tension of the soap film ($\Delta\sigma \leq 0$), a slow spreading regime was observed. Here
5 the surface tension difference is not favourable for the spreading as the new stretched liquid film has
6 a higher surface tension than the original film. For deposited droplets of ALS 0.001, 0.002, and 0.003
7 M, for which $\Delta\sigma$ = -19.4, -10.7, -3 mN/m, respectively, we obtain power-law exponents of 0.12±0.04,
8 0.14±0.04 and 0.19± 0.04, respectively (Fig. 5a).

9 Fig. 6 shows the situation when the droplet and soap film are of the same composition and concentra-
10 tion: SDS 0.005 M. After deposition of the droplet, it forms a lens and spreads slowly (black squares
11 in Fig. 4b) with a power-law time dependence and an exponent of 0.15±0.04. This is in contrast with
12 an earlier experimental work by Aarts *et al.* [26] for a three-dimensional configuration where coalescence
13 of two droplets of the same liquid was studied and a linear dependence of radius on time (for early
14 times) was measured, although the current work considers a system with surfactant molecules at the
15 surface and a different surface geometry.

16 When a drop of SDS spreads slowly on a film of the same liquid ($\Delta\sigma = 0$), there is a clear energetic
17 reason due to lowering the total surface area. This slow spreading with small exponents may be ex-
18 plained by the regime of Tanner's law. In Tanner's regime, the driving force of spreading is due to
19 capillary pressure induced by the curvature at the rim. We note that Tanner's law applies for spreading
20 on a rigid boundary. If the surface exhibits slip then one might expect a slightly different result [27]. We
21 have an almost similar situation for spreading a droplet on the same liquid film, where large curvatures
22 are observed at the rim of the liquid lens. The exponent predicted by Tanner's law is shown by a
23 horizontal dashed line on Fig. 5a; it is in agreement with the spreading exponents observed for ($\Delta\sigma$=0).
24 For droplets with larger surface tension with respect to the film ($\Delta\sigma$<0) we still see slow spreading
25 dynamics with similar exponents ($\alpha < 0.2$). At first glance, it seems that the spreading should not occur
26 due to higher surface tension of the final film. However, if we consider the very slow dynamics we
27 can expect that the surface tension difference between the droplet and film rapidly decreases and
28 reaches a uniform surface tension, which is lower than the original surface tension of the film surface,
29 shortly after the coalescence. In particular, in the slow spreading regime the spreading speed is about
30 1 mm/s, so that it takes several seconds to spread about 1 cm. The data in Fig. 3 shows that the surface
31 tension decreases considerably over this time scale so we anticipate that the surface tension everywhere
32 becomes uniform in the slow spreading regime. Therefore, even in case of $\Delta\sigma$<0 the slow spreading
33 continues with the Tanner-like dynamics discussed above.

34 For the droplets with lower surface tension than the soap film ($\Delta\sigma > 0$), the exponent α strongly in-
35 creases with increasing $\Delta\sigma$ (Fig. 5a). Here the positive surface tension difference favours drop-
36 let spreading. Due to very fast stretching of the droplet, we expect a gradient of surface tension from
37 the outside film toward the centre of the spreading droplet.

38 We estimate the average spreading speed by fitting a linear function to the data in Fig. 4a and b. The
39 results are shown as a function of $\Delta\sigma$ in Fig. 5b, ranging from 0.003 m/s for negative $\Delta\sigma$ to about 0.7

4

1   m/s for $\Delta\sigma$=6.3 mN/m. From the average spreading speed we determine an experimental time scale

2   for the spreading. Using this time scale we obtain the dynamical surface tensions in Fig. 3.

3   In our system, we observe a range of exponents from $\alpha \sim 0.1$ to linear growth of the front radius (α

4    $\sim$1). A similar range of spreading exponents was reported previously by Rafai *et al.* [28] in a completely

5   different system consisting of a surfactant solution (Trisiloxan) spreading on a solid substrate where

6   by increasing the concentration of surfactant α increased from 0.2 to 1. This wide range of exponents

7   was associated with Marangoni effects.

8

9   *3.2. Effect of droplet volume*

10   In order to study the effect of the volume of the droplets on the spreading process, droplets with dif-

11   ferent diameters (*D*) were deposited on an SDS 0.005M film. We used ALS 0.04 M droplets for fast

12   spreading and SDS 0.005 M droplets for slow spreading. Fig. 7 shows the radius of spreading as a

13   function of time for droplets with different volumes deposited on the soap film. We fit the time-de-

14   pendencies with a power law and find that the power-law exponent is roughly the same for all droplet

15   volumes in each regime ($\alpha \sim 0.9$ for fast spreading and $\alpha \sim 0.11$. for the slow regime) as shown in

16   Fig. 8a. Only for the smallest droplet of ALS, created by spraying, an exponent $\alpha = 0.8 \pm 0.05$ was

17   obtained, which is a little smaller than the other experiments in the fast regime. For the fast spreading

18   experiments since the exponent α is close to unity, we can estimate the spreading speed from the slope

19   of linear fits to the data points. In Fig. 8b, the spreading speed is reported as a function of droplet

20   diameter. By increasing the size of the droplets, the average speed of spreading increases slowly before

21   reaching a plateau. The capillary length of the liquid ($L_c = \sqrt{\dfrac{\sigma}{\rho g}} \sim 2$ mm, where σ=41.2 mN/m is the

22   dynamic surface tension of the ALS 0.04 M), determines where the plateau starts. For the droplets with

23   diameters smaller than the capillary length of the system, the average velocity grows linearly with

24   increasing radius of the droplet (slope 1 in the log-log plot of Fig. 8b). After that, the speed remains

25   constant at 0.5±0.15 m/s. L_c is indicated in Fig. 8b as a vertical dashed line. According to our results

26   for the fast spreading, the volume of the spreading droplet does not affect the spreading exponent but

27   the spreading speed can be influenced by the droplet volume when the diameter of the droplet is smaller

28   than the capillary length of the liquid.

29   For the slow spreading although the power law exponents are significantly smaller than unity ($\alpha \sim$

30   0.11) we still are able to linearly fit the time evolution data of Fig. 7 and estimate a spreading speed.

31   The results are shown by square symbols in Fig. 8b and indicate that the average spreading speed

32   remains approximately constant with an increase of the volume of the droplet.

33   *3.3. Scaling arguments*

34   Bringing two miscible liquids into contact with each other will generate a gradient of surface tension

35   and induces a flow known as a Marangoni-driven flow. The driving force in such flows is the tangential

36   stress associated with gradients of surface tension at the interface which should be balanced by viscous

37   stresses. Based on scaling the Marangoni stresses for viscosity-limited spreading on a deep bath (dis-

38   tance as a function of time), a spreading exponent of ¾ can be derived [29-31].

5

1 The Reynolds number ($Re = \frac{\rho u h}{\eta}$, with h the film thickness) of the system in the rapid spreading regime

2 ($\Delta\sigma > 0$) is always larger than one, indicating that the inertial terms should be considered in the balance

3 of stresses. In a first approximation, we ignore the viscous dissipation and balance the surface tension

4 gradient by the inertial term: $|\nabla\sigma| \sim \rho u^2$ where $u \sim \dot{R}$ with a dot denoting the time derivative. The surface

5 tension gradient term scales with the dynamic surface tension difference divided by the radius of the

6 spreading front ($|\nabla\sigma| \sim \frac{\Delta\sigma}{R}$), so we arrive at $\frac{\Delta\sigma}{R} \sim \rho\dot{R}^2$ and hence $R \sim \left(\frac{\Delta\sigma}{\rho}\right)^{1/3} t^{2/3}$. The exponent 2/3 is

7 within the range of exponents we observed for the fast spreading experiments as shown in Fig. 5a by

8 a dashed line marked by "inertial".

4. Conclusions

11 Droplets will spread when deposited on a soap film. Spreading evolves as a function of time in a

12 power-law manner with exponents ranging from about 0.1 to about 1 depending on the dynamic sur-

13 face tension differences between the film and the droplet. Small spreading exponents with slow spread-

14 ing dynamics occur when the surface tension of the droplet is greater than or equal to the surface

15 tension of the film so that the tension difference does not favour spreading. This slow spreading regime

16 resembles the Tanner's spreading on a solid substrate with exponent 0.1, as driven by capillary pres-

17 sure at the rim. By increasing the surface tension difference, the spreading exponent increases towards

18 1. In this regime, the spreading speed increases significantly with surface tension difference. Here, the

19 driving mechanism is the Marangoni stress. By scaling the Marangoni stress with inertial terms, we

20 can derive spreading exponents of $\alpha = 2/3$. Although this exponent is in the range of experimentally

21 observed exponents for the fast spreading regime, however it does not explain the very fast spreading

22 dynamics with exponents about one. This might be due to that fact that in our scaling argument we

23 have simplified all the kinetics effects and surfactant absorption dynamics in a surface tension gradient.

24 We also find that the spreading speed depends on the size of the droplet in the fast spreading regime.

25 The spreading speed depends linearly on the droplet size for drops smaller than the capillary length of

26 the liquid, and remains constant for larger droplet sizes, however, the spreading exponent is not influ-

27 enced by the droplet size. Our experimental results may help in understanding of the various forces

28 acting on a droplet deposited on a liquid interface. Using a more detailed theoretical model in which

29 the surfactant absorption kinetics is considered may elucidate all our experimental observations. In

30 addition, our methodology can be used for stretching complex liquid droplets in 2D and therefore may

31 have applications in modifying thin suspended films. For example, using this approach the effect of

32 elongational viscosity on stretching a complex liquid droplet in 2D can be studied.

Authors' contributions:

R.S. observed this phenomenon for the first time. M.E., M.S. and R.S. performed preliminary experi-
ments that are not reported here. R.S., E.L., D.B., H.S. M.M. and M.H. contributed in designing the
research and the interpretation of the data. M.M. performed all the experiments and data analysis of
this paper. M.M., M.H. and H.S. developed the theoretical models. M.M. and M.H. wrote the manu-
script. All the authors reviewed and commented on the manuscript and discussed the results and mod-
els.

6

Supporting Information: S1. Dynamic surface tension measurements, S2. Determination of soap film thickness.

Figures:

a)   b)   c)
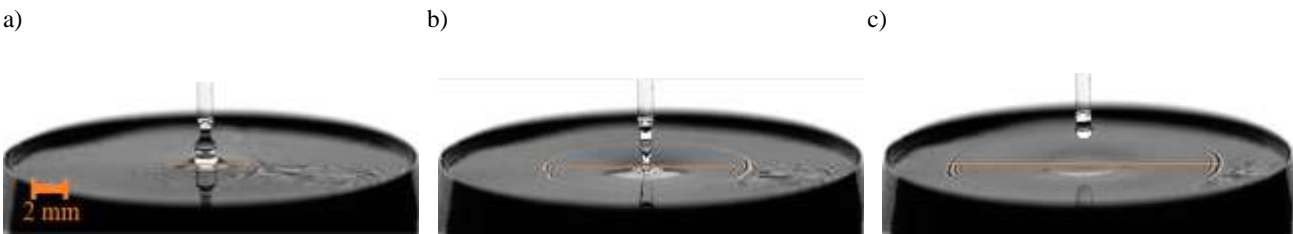

Fig. 1. Spreading a droplet of 0.3 M ALS on a soap film of 0.005 M SDS. Images from left to right show the instant of deposition and subsequent stretching of the droplet. From left to right, the images are taken at (a) 5, (b) 12 and (c) 24 ms after deposition.

a)   b)   c)
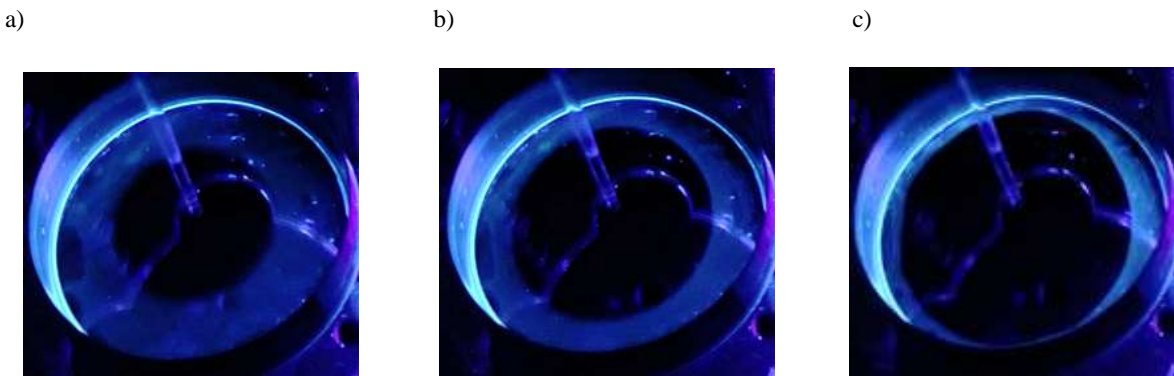

Fig. 2. Spreading of a droplet containing ALS 0.4 M on a soap film of SDS 0.005 M. A fluorescent dye (blue) is added to the initial SDS soap film. From left to right: the droplet (black circle) is stretched towards the edges of the container. The two liquids do not mix during spreading. From left to right images are taken at (a) 5, (b) 15 and (c) 30 ms after deposition of the droplet.
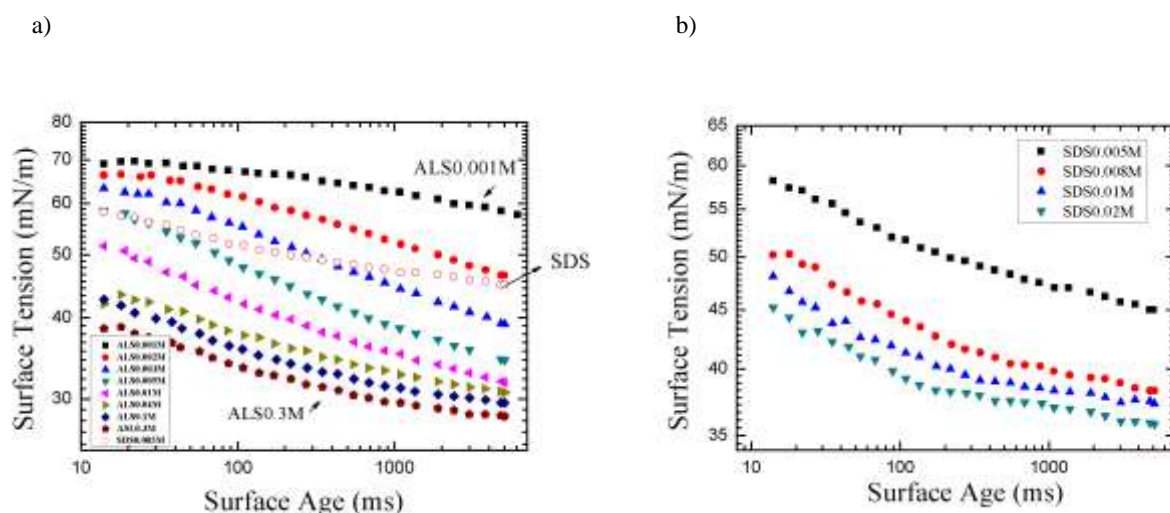
7

a)

b)

Fig. 3. Dynamic surface tension as a function of surface age for a) different concentration of ALS solutions and 0.005 M SDS (open symbols) and b) different concentrations of SDS solutions. Details of the dynamic surface tension measurements are explained in supporting information.
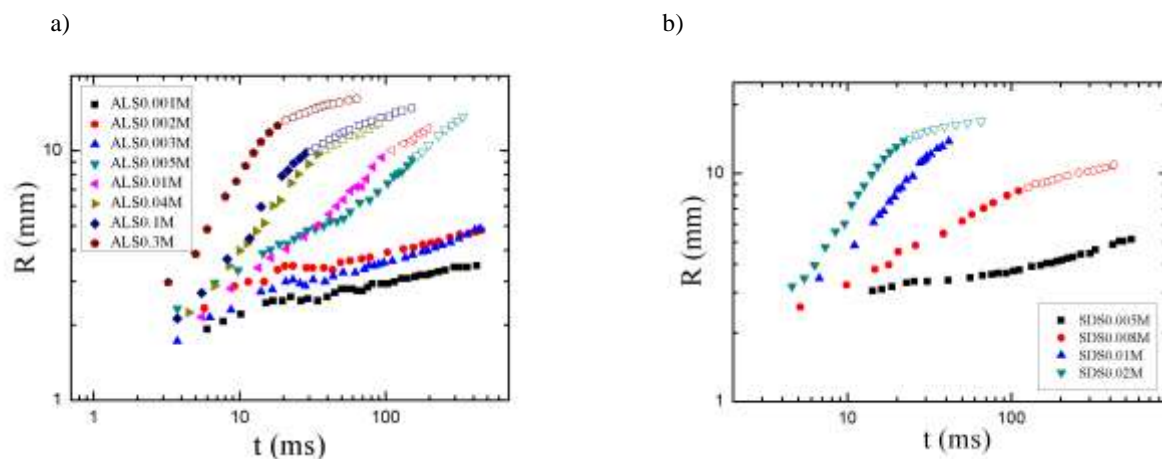
a)

b)

Fig. 4. Spreading radius versus time for droplets of ALS (a) and SDS (b) with different concentrations on a soap film of 0.005 M SDS. The open symbols show the cross-over regime due to the boundary effects. Error bars are smaller than the size of the symbols.
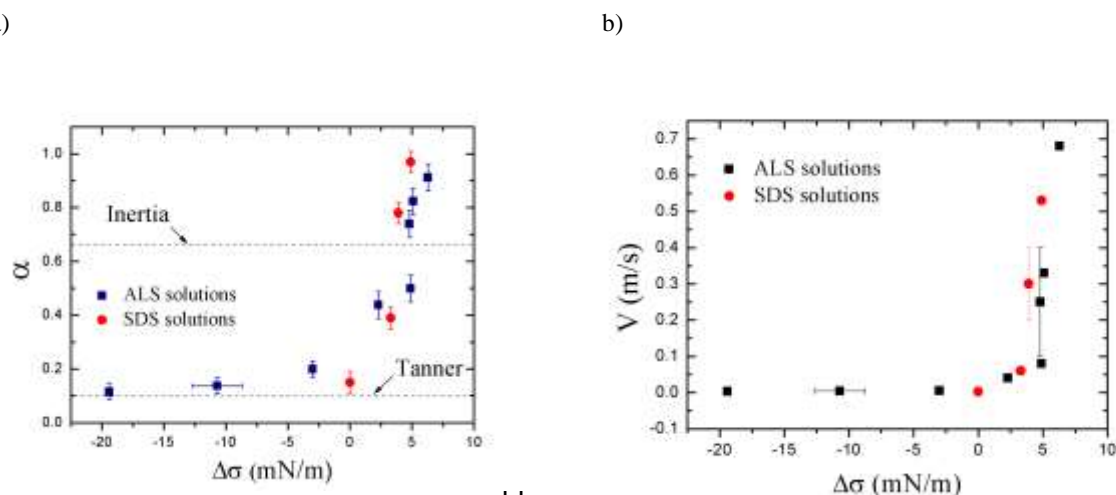
a)

b)

1 Fig. 5. a) Exponent α as a function of surface tension difference for deposited droplets with different concentrations of ALS and
2 SDS on 0.005 M SDS soap film. α is calculated by a power-law fit to data in Fig. 4. b) Average spreading speed calculated by
3 linear fits to the solid data points in Fig. 4. For the sake of clarity, only several typical error bars for the velocity and $\Delta\sigma$ are
4 shown.

5

6 Fig. 6. A droplet of 0.005 M SDS is deposited on the same soap film. After deposition, a liquid lens is formed that spreads slowly
7 in time. R as a function of time for this experiment is shown by black squares in Fig. 4b.
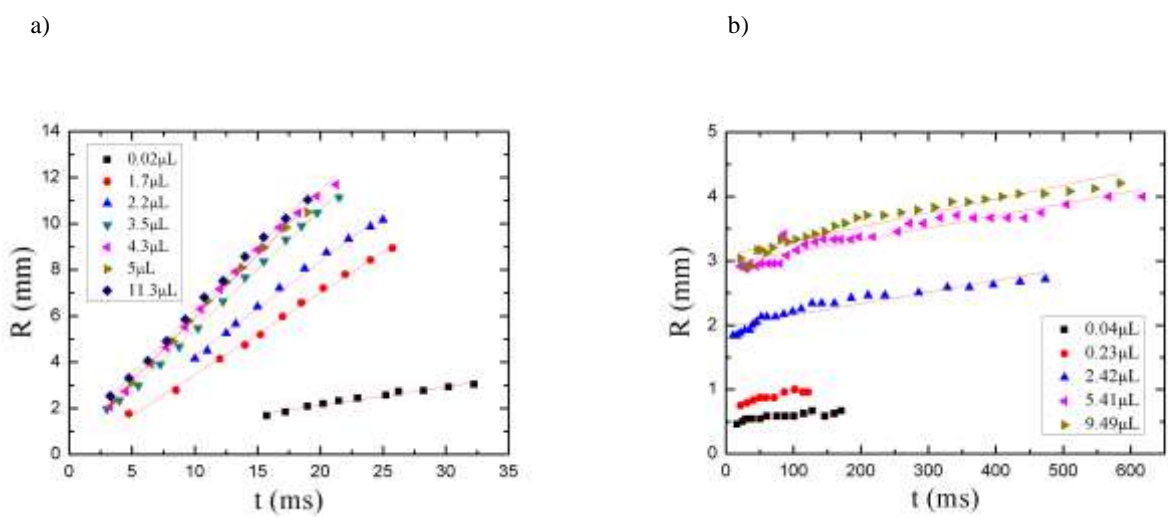
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9 a)   b)

10

11 Fig. 7. Time evolution of spreading droplets with different volume on SDS soap films. a) Spreading of 0.04 M ALS droplets with
12 different volumes (0.02, 1.7, 2.2, 3.5, 4.3, 5, 11.3 µL) on a 0.005 M SDS soap film (fast spreading). b) Spreading of 0.005 M SDS
13 droplets with different volumes (0.04,0.23, 2.42, 5.41 and 9.49 µL) on a 0.005 M SDS soap film (Slow spreading). Error bars
14 are smaller than the size of the symbols.

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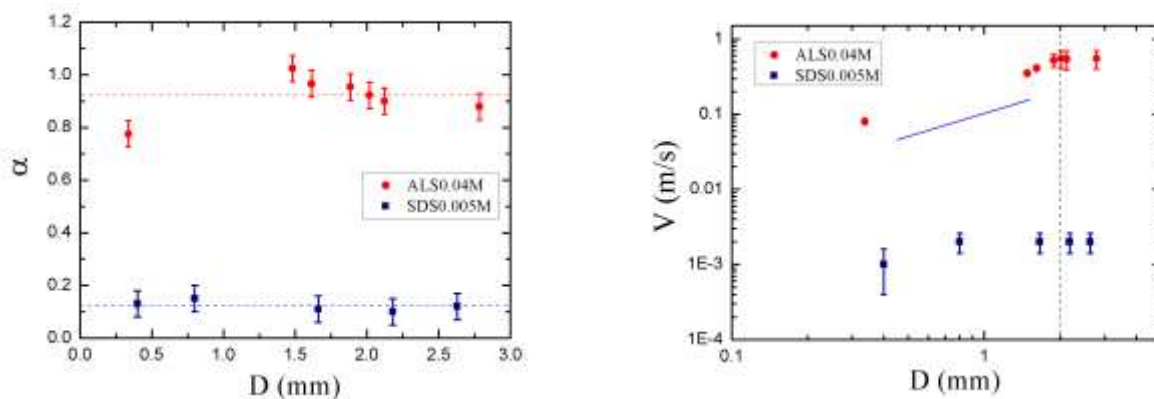16   a)   b)

Fig. 8. a) Exponent α calculated by a power-law fit to the data in Fig. 7 as a function of the diameter of the droplets (*D*). The upper and lower horizontal dashed lines represent the average value of α for rapid and slow spreading, respectively. b) Average spreading speed as a function of droplet size with logarithmic scales for fast (0.04 M ALS droplet on a soap film of 0.005 M SDS) and slow spreading (0.005 M SDS droplet on 0.005 M SDS soap film). The vertical dashed line illustrates the capillary length of the system. A blue line with slope one is shown as a guide to the eye.

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3 1 Graphical abstract:
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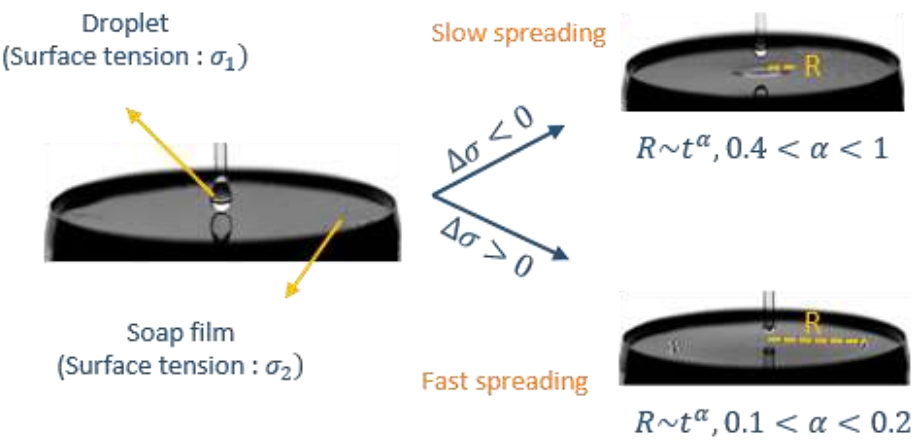
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